

# ***Information Provision in Road Transport with Elastic Demand***

## ***A Welfare Economic Approach***

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### **1. Introduction**

Congestion severely affects most metropolitan areas around the world. Numerous instruments to tackle congestion have been studied in the past: electronic road pricing, fuel taxation, regulatory parking policies, improving public transport, and so on. Another instrument, widely viewed to be able to relieve part of the congestion problem, is the use of new information technologies in transport networks (the DRIVE I and II programmes of the European Community and the Intelligent Vehicle Highway Systems (IVHS) efforts in the United States are examples). Information provision to drivers is believed to improve their knowledge of the traffic situation on the roads and thus to improve drivers' decision-making (Ben-Akiva *et al.*, 1991; Bonsall, 1992). Unfortunately, the real picture of the effects of information provision to drivers is less clear than such intuitive reasoning suggests. At an aggregate level, the improved decision-making might imply that information will direct traffic flows to the user equilibrium (Emmerink *et al.*, 1995a; Wardrop, 1952), in other words, a situation characterised by driver optimal decisions. However, the inequality between Wardrop's first (user equilibrium) and second (system optimum) principles in congested situations (Sheffi, 1985) indicates that information provided to drivers need not direct the traffic flows towards the system optimum, that is, the most effective use of the transport network. This potential discrepancy arises because of the existence of the congestion externality. This observation leaves the interesting (and still open) question: to what extent is information able to improve network efficiency, and hence to diminish the external costs caused by traffic congestion?

In the literature, sparse attention has been paid to this question. Most papers use either a simulation approach to infer conclusions on network efficiency (El Sanhoury, 1994; Emmerink *et al.*, 1995b; Mahmassani and Jayakrishnan, 1991) or an empirical analysis

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in which the impacts of information on drivers' individual behaviour is studied (Caplice and Mahmassani, 1992; Conquest *et al.*, 1993; Emmerink, Nijkamp, Rietveld and Van Ommeren, 1994; Mannering *et al.*, 1994; Spyridakis *et al.*, 1991; Van Berkum and Van Der Mede, 1993).

The objective of this paper is to enhance insight into the welfare economic effects of information provision to a group of drivers, and to gain insight into the mechanisms affecting the impact of providing information. We will do so by theoretically analysing the impact of information provision on network efficiency. In addition, we will consider the equity aspects of information provision by answering questions such as "who benefits (or disbenefits) most from information?" and "do uninformed drivers also benefit?" To study these questions, we will confine ourselves to the economic fundamentals of information provision and therefore limit the analysis to using simple economic equilibrium models rather than complex equilibrium assignment models. Although the latter can be extremely useful, particularly in the well-known four-stage transport model (Ortúzar and Willumsen, 1994), they are less appropriate for our purpose, since the complexity of these models may prevent us from obtaining clear insight into the key fundamentals of information provision.

The model presented here is a static economic equilibrium model, allowing for elastic demand. An increasing cost curve represents the costs for travel (including congestion costs). In particular, the elasticity of demand distinguishes our approach from the existing literature on information provision. The equilibrium models reported on in the literature generally assume that demand is fixed or inelastic (Al-Deek and Kanafani, 1993; Arnott *et al.*, 1990, 1991, 1992, 1994; Tsuji *et al.*, 1985). As will be shown in this paper, the assumption of inelastic demand is a severe limitation of the analysis of information provision, because changes in use due to changes in costs are then ignored. In the past, Ben-Akiva *et al.* (1986) studied an elastic demand version of Vickrey's (1969) bottleneck model, as did Arnott *et al.* (1993). The current paper is complementary to their work since it adds (a) information provision to drivers, and (b) stochasticity in terms of link travel costs. The latter point has not been given much attention in the literature, but proves to be important in a driver information systems context.

The paper is organised as follows. To provide the necessary background, Section 2 compares the traditional equilibrium concepts with the methodology proposed in this paper. Next, in Section 3, the general model is presented. In Section 4, the impact of information provision on equity and efficiency is studied. Finally, Section 5 contains some concluding comments and future research directions.

## **2. Traditional Equilibrium Concepts and the Proposed Methodology**

In the 1960s, the classical four-stage (trip generation, distribution, modal split and assignment) transport model was developed. Despite major improvements in transport modelling techniques during the 1970s and 1980s, the basic structure of the model has remained unaltered (Ortúzar and Willumsen, 1994). The fourth stage of the model — assigning the origin-destination (OD) matrix to the transport network — has usually been

based on Wardrop's (1952) first principle: the user equilibrium. According to this principle, each user of the network attempts to maximise utility. In the literature, the standard user equilibrium principle is often referred to as the *deterministic user equilibrium*. A widely recognised restriction of Wardrop's concept is that it assumes that all users of the transport network have full information on the traffic situation; something that is unlikely to happen in practice due to all types of stochastic incidents.

Daganzo and Sheffi (1977) recognised this shortcoming in the deterministic user equilibrium concept, and introduced the concept of *stochastic user equilibrium*. This is defined as a situation in which "no traveller believes that his travel time can be improved by unilaterally changing routes" (Sheffi, 1985, p.20). Sheffi and Powell (1981) were the first to apply the stochastic user equilibrium to a relatively simple test network. The essence of the difference between the two equilibrium concepts concerns the difference between *beliefs* (user *perceptions of reality*) and *actual values*. The discrepancy between the drivers' perceptions and the actual traffic situation would however, disappear by the provision of perfect information. Therefore, it might be argued that the stochastic user equilibrium concept distinguishes itself from its deterministic counterpart by allowing for limited (incomplete) information from the traveller's point of view.

The above mentioned equilibrium concepts have been applied in analysing the impacts of driver information systems on network efficiency. First, the deterministic user equilibrium may be viewed as the *ideal* situation in which all users have perfect information on the traffic situation. Then, the stochastic user equilibrium characterises a situation in which drivers are uncertain about route travel costs. The difference between the two concepts is a measure for the (potential) impacts of driver information systems. In other words, driver information systems reduce driver uncertainty, and bring the stochastic user equilibrium closer to the deterministic user equilibrium. In the past, different researchers have been using stochastic network assignment models to assess the impact of driver information systems on network performance (Hicks *et al.*, 1992; Ran and Boyce, 1994; Tsuji *et al.*, 1985; Van Vuren and Van Vliet, 1992).

It is worthwhile to analyse the source of the uncertainty in the models that are based on the stochastic user equilibrium concept. In these models, the uncertainty stems from modelling the behavioural mechanism underlying the traffic assignment process as a discrete choice situation. The deterministic user equilibrium then follows under the assumption that travellers choose the least cost route and, in addition, have full information regarding the whole traffic situation. Conversely, stochastic models arise when it is assumed that, "due to variations in perception and exogenous factors (such as weather, lighting, and so on) the path times are perceived differently by each driver" (Sheffi, 1985, p.272). Put differently, the intrinsic path travel times may well be constant in these models, but it is the case that drivers have different perceptions of them.

These two concepts are given in the first row of Table 1. The traditional deterministic and stochastic user equilibrium concepts are characterised as models in which the network is assumed to be deterministic, while the beliefs of the drivers (the assumed knowledge of the travellers) may either be according to reality (full information and therefore deterministic) or subjective perceptions (limited information and therefore stochastic).

**Table 1**  
*Combination of Features of Network and Individual Characteristics*

		<i>Assumed Knowledge of Traveller</i>	
		<i>Full (deterministic)</i>	<i>Limited (stochastic)</i>
Network Characteristics	Deterministic	DUE	SUE
	Stochastic	SNDUE	SNSUE

Hence, in the traditional DUE (deterministic user equilibrium) and SUE (stochastic user equilibrium) concepts, the stochasticity stems from the travellers themselves (see the first row of Table 1).

However, as indicated by the second row of Table 1, one might adopt an alternative perspective. Given the large variation in daily travel times witnessed on real road networks, it is reasonable to assume that the route travel costs are stochastic rather than deterministic (see also Mirchandani and Soroush, 1987). In fact this is, for three reasons, a natural approach for analysing the impact of driver information systems. First, this approach acknowledges that the daily fluctuations in travel costs for different road users are highly *correlated*, something that is not considered by the traditional class of models. Second, in this approach non-recurrent congestion is modelled explicitly by using stochastic route travel costs. Since driver information systems are viewed as beneficial particularly in non-recurrent congested situations, this is an attractive feature. Third, this approach allows us to provide informed travellers with *realisations* of the stochastic travel costs, which is a natural way of modelling information. Consequently, the impact of information on network performance is modelled as an *endogenous* result of the drivers' responses to the information provided. Conversely, when the discrepancy between the DUE and SUE concepts is used as a measure of the impact of information, then it is assumed that information provision *exogenously* reduces the uncertainty on the network situation.

The models in the second row of Table 1 are referred to as the stochastic network deterministic user equilibrium (SNDUE) and stochastic network stochastic user equilibrium (SNSUE). In this article, a simple model based on the SNDUE is discussed.

### 3. The One-link Equilibrium Model with Stochastic Congestion

In this section, the model will be presented in its simplest form. In this form, the network is limited to one link only. Clearly, in such a simple network the potentially beneficial route-split effects of information provision cannot be analysed, that is, beneficial effects of information provision owing to the fact that informed drivers change route in

circumstances of incidents (see Verhoef *et al.*, 1996, for a simultaneous consideration of route split and modal split in relation to information). Mode-split effects however, can be addressed in the current context and these are precisely the ones that have previously been ignored in the literature, because of the underlying assumption in most models that demand is inelastic. Furthermore, in Verhoef *et al.*, (1996) it is shown that the welfare implications of information provision on a two-link network are often practically the same as those on a one-link network, with significant differences only occurring at rather inelastic demand. Hence, when studying elastic demand a one-link network is sufficient to provide important insights.

The rest of this section is structured as follows. In Section 3.1, the assumptions in the model are presented. In sub-sections 3.2 and 3.3 model *P* and model *N* are studied, respectively. In model *P* it is assumed that a certain group of travellers is provided with information, while in model *N* no information is available. The properties of these two models are explored in sub-section 3.4.

### 3.1 Assumptions in the model

In the model, the demand side is modelled using two groups of drivers, labelled group *x* and group *y*. The inverse demand function for *x*-travellers is denoted by  $D_x$ , and for *y*-travellers by  $D_y$ . These functions relate the number of users in the network to total travel costs. Hence, for all levels of travel costs  $\kappa$  the total demand function is by definition given by  $D^{-1}(\kappa) = D_x^{-1}(\kappa) + D_y^{-1}(\kappa)$ .

The supply side of the system is modelled using link travel cost functions. It is assumed that the link travel cost function has either the functional form  $C^0$  or  $C^1$ , depending on the state of the system. State 1 reflects *low capacity* congestion occurring with a probability  $p$ , while state 0 denotes *high capacity* congestion which occurs with a probability  $1-p$ . The distinction between these states lies in the higher travel costs in state 1, that is,  $C^1(N) \geq C^0(N)$  for all  $N$ , where  $N$  denotes the number of drivers using the one-link network. In addition, it is assumed that  $dC^1(N)/dN \geq dC^0(N)/dN$ . Consequently, both the travel cost itself and the rate at which travel cost increases are higher under state 1. This increase in travel costs is caused by random (unpredictable) incidents such as traffic accidents, sudden lane closures, and so on.

This random cost component in combination with the elastic demand functions render an analysis of the impacts of information provision relevant. To visualise the kind of information that is provided, the best example is *pre-trip* information, as the model in the present paper is static. Then, without the provision of *pre-trip* information, it is assumed that uninformed drivers use the probability  $p$  to determine the *expected* cost function in the transport system, while informed drivers base their behaviour on the *actual* cost function as they are provided with *pre-trip* information. An informed road-user will use the network if private benefits are at least equal to actual private costs for the prevailing state of the transport system. An uninformed driver uses the network if private benefits are at least equal to *expected* private costs. The equilibrium that is reached in this way conforms to Wardrop's first principle, the user equilibrium (Wardrop, 1952), as both may be characterised by individual maximising behaviour.

To study the impact of information provision two models (model *P* and model *N*) are compared. In model *P* information is provided to the group of *x*-travellers, while no information is provided to *y*-travellers. In model *N* neither *x*- nor *y*-travellers are supplied with information. Then, a comparison of these models allows us to isolate the impact of information provision on both the informed (*x*) and uninformed (*y*) travellers. Furthermore, this gives us the opportunity to study the effects of information on network performance.

### 3.2 Model P

In model *P*, *x*-travellers are provided with information, whereas *y*-travellers are not. Therefore, *x*-travellers base their behaviour on *actual* travel costs, whereas *y*-travellers use *expected* travel costs instead.

Transferring the verbally explained equilibrium conditions into mathematical expressions yields model *P* in expressions (1) to (3), where  $N_{p,x}^0$  and  $N_{p,x}^1$  denote the number of informed (*x*) drivers using the one-link network in state 0 and state 1, respectively; and  $N_{p,y}$  the number of uninformed (*y*) drivers. Subscript *p* (referring to model *P*) denotes the equilibrium road-use values of model *P*. Equations (1) and (2) ensure that the marginal informed driver, that is, the informed driver who is indifferent between using the one-link network and an alternative (implying zero marginal net private benefits), equates marginal private costs and marginal private benefits for both state 0 and state 1. In a similar fashion expression (3) guarantees that the marginal uninformed driver experiences zero expected marginal net private benefits. For the non-marginal drivers (expected) net private benefits are larger than zero, because of the downward-sloping demand function. Finally, the additional condition that road use is non-negative has to be imposed, that is,  $N_{p,x}^0, N_{p,x}^1$  and  $N_{p,y}$  have to be greater than or equal to zero.

$$D_x(N_{p,x}^0) \leq C^0(N_{p,x}^0 + N_{p,y}), N_{p,x}^0 \geq 0 \text{ and } N_{p,x}^0 \cdot [D_x(N_{p,x}^0) - C^0(N_{p,x}^0 + N_{p,y})] = 0 \quad (1)$$

$$D_x(N_{p,x}^1) \leq C^1(N_{p,x}^1 + N_{p,y}), N_{p,x}^1 \geq 0 \text{ and } N_{p,x}^1 \cdot [D_x(N_{p,x}^1) - C^1(N_{p,x}^1 + N_{p,y})] = 0 \quad (2)$$

$$D_y(N_{p,y}) \leq (1-p) \cdot C^0(N_{p,x}^0 + N_{p,y}) + p \cdot C^1(N_{p,x}^1 + N_{p,y}), N_{p,y} \geq 0 \text{ and } N_{p,y} \cdot \{D_y(N_{p,y}) - [(1-p) \cdot C^0(N_{p,x}^0 + N_{p,y}) + p \cdot C^1(N_{p,x}^1 + N_{p,y})]\} = 0 \quad (3)$$

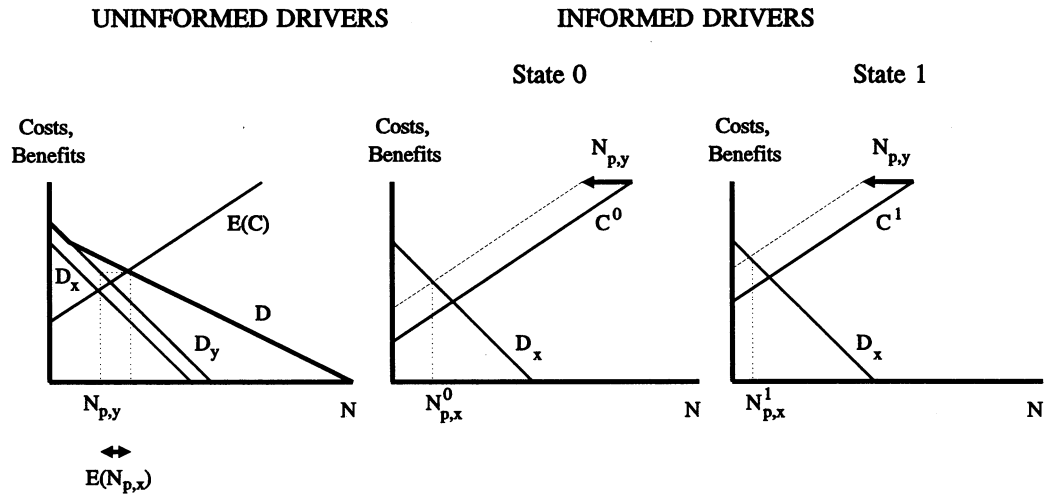
In the analysis below, it is assumed that the *group-regularity* condition applies for each group, that is, for each state and each group of drivers the network will at least be marginally used. For a one-link network this is a plausible assumption, since it seems likely that for each state at least some uninformed and informed drivers will use the network. Imposing this restriction implies that model equations (1) to (3) can be rewritten to equations (4) to (6).

$$D_x(N_{p,x}^0) = C^0(N_{p,x}^0 + N_{p,y}) \quad (4)$$

$$D_x(N_{p,x}^1) = C^1(N_{p,x}^1 + N_{p,y}) \quad (5)$$

$$D_y(N_{p,y}) = (1-p) \cdot C^0(N_{p,x}^0 + N_{p,y}) + p \cdot C^1(N_{p,x}^1 + N_{p,y}) \quad (6)$$

Figure 1 gives a diagrammatic representation for the situation where  $C^j(N)$  ( $j=0,1$ ) is linear and, in addition, the slopes of the cost functions are identical.



**Figure 1**  
*Equilibrium Model with Informed and Uninformed Individuals*

In the left-hand panel of Figure 1, uninformed drivers ( $y$ -travellers) equate expected marginal link travel costs to their marginal benefits. In doing so, they take account of the effect that the expected number of informed drivers ( $x$ -travellers) will have on their costs. Given the assumptions on the cost functions, the expected number of uninformed drivers can be found by equating their demand to expected user cost.<sup>1</sup> This leads to a total number of  $N_{p,y}$  uninformed individuals using the network. Next, informed drivers shift the prevailing cost curve  $C_j$  ( $j=0,1$ ) with an amount  $N_{p,y}$  to the left (see the dashed cost curves in Figure 1) to account for travel demand of uninformed road users. Then for each state, the number of informed road users is found by equating demand with prevailing costs (as given by the dashed cost curve), leading to  $N_{p,x}^0$  informed drivers using the network in state 0, and  $N_{p,x}^1$  informed drivers in state 1. Under the assumptions on the cost functions  $C^0$  and  $C^1$ , it follows that  $N_{p,x}^1$  is smaller than  $N_{p,x}^0$ : when low capacity prevails, some informed drivers will not use the car but the other transport mode (or remain at home).

It can easily be demonstrated that  $N_{p,y}$ ,  $N_{p,x}^0$  and  $N_{p,x}^1$  are the only equilibrium values. In general, in user equilibrium models it can be shown that the equilibrium link travel flows are unique (Sheffi, 1985). Conversely, path travel flows are not unique. In our one-link network however, a path is equivalent to a link, and hence we witness the similarity between our result and the literature.

<sup>1</sup> A meaningful graphical illustration is impossible when these conditions (linear  $C$  [ $j=0,1$ ] and the same slope) are not imposed on the cost functions. In such a situation the expected cost function (as shown in the left-hand panel of Figure 1) does not solely depend on  $E(N_{p,x})$  but also on the exact values of  $N_{p,x}^0$  and  $N_{p,x}^1$ .

### 3.3 Model N

Next, the model in which no information is available to both  $x$ - and  $y$ -travellers is investigated. This model is referred to as model  $N$  (no information). The equilibrium values of road use of model  $N$  are studied in order to assess the welfare economic impacts of information provision. In model  $N$ , both  $x$ - and  $y$ -travellers base their behaviour on *expected* travel costs. The equilibrium conditions of model  $N$  are given in (7) and (8).

$$\begin{aligned} D_x(N_{n,x}) &\leq (1-p) \cdot C^0(N_{n,x} + N_{n,y}) + p \cdot C^1(N_{n,x} + N_{n,y}), N_{n,x} \geq 0 \text{ and} \\ N_{n,x} \cdot \{D_x(N_{n,x}) - [(1-p) \cdot C^0(N_{n,x} + N_{n,y}) + p \cdot C^1(N_{n,x} + N_{n,y})]\} &= 0 \end{aligned} \quad (7)$$

$$\begin{aligned} D_y(N_{n,y}) &\leq (1-p) \cdot C^0(N_{n,x} + N_{n,y}) + p \cdot C^1(N_{n,x} + N_{n,y}), N_{n,y} \geq 0 \text{ and} \\ N_{n,y} \cdot \{D_y(N_{n,y}) - [(1-p) \cdot C^0(N_{n,x} + N_{n,y}) + p \cdot C^1(N_{n,x} + N_{n,y})]\} &= 0 \end{aligned} \quad (8)$$

The subscript  $n$  (referring to model  $N$ ) is used to distinguish the equilibrium values of this model with the equilibrium values of the model described in subsection 3.1. Expression (7) shows that the  $x$ -travellers now also base their behaviour on the *expected* link travel costs, rather than on the *actual* link travel costs. By imposing the group-regularity condition, as defined in the previous section, these expressions can be simplified to two equalities.

### 3.4 Properties of models P and N

In the current section, the properties of the models  $P$  and  $N$  specified in subsections 3.2 and 3.3 are explored. In order to keep the analysis manageable and the outcomes tractable we will assume linear demand and cost functions over the relevant ranges considered (that is, the ranges containing the levels of use in each of the possible states and in each of the possible regulatory regimes). Although the use of linear functions may be criticised, they are in any case sufficient to serve the general goal of the current paper, which is to enhance our insight into the welfare economic effects of information provision to a group of drivers. Furthermore, it might be interesting to note that for inelastic demand Arnott *et al.* (1992) have proved that the equilibrium travel cost functions in Vickrey's dynamic congestion model (Vickrey, 1969) of the morning rush hour with two groups and two parallel routes are special cases of our linear cost functions.

An analytical comparison of the model  $P$  (in which information is available to  $x$ -travellers, and no information is available to  $y$ -travellers) and the model  $N$  (in which no information is available to both  $x$  and  $y$ -travellers) leads us to the following proposition for a system with linear demand and cost functions.

*Proposition:*<sup>2</sup>

Assuming linear demand ( $D_x$ ,  $D_y$ ) and cost ( $C^0$ ,  $C^1$ ) functions and  $C^1(N) \geq C^0(N)$  and  $dC^1(N)/dN \geq dC^0(N)/dN$ , then the following relationships hold:

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<sup>2</sup> Proof of this proposition is available from the authors on request.



- *expected road use increases due to information:*  

$$N_{n,x} + N_{n,y} \leq N_{p,y} + (1-p)N_{p,x}^0 + pN_{p,x}^1;$$
- *road use in state 0 increases due to information:*  

$$N_{n,x} + N_{n,y} \leq N_{p,y} + N_{p,x}^0, \text{ hence } C^0(N_{n,x} + N_{n,y}) \leq C^0(N_{p,y} + N_{p,x}^0);$$
- *road use in state 1 decreases due to information:*  

$$N_{n,x} + N_{n,y} \geq N_{p,y} + N_{p,x}^1, \text{ hence } C^1(N_{n,x} + N_{n,y}) \geq C^1(N_{p,y} + N_{p,x}^1);$$
- *expected link travel costs decrease due to information:*  

$$(1-p)C^0(N_{n,x} + N_{n,y}) + pC^1(N_{n,x} + N_{n,y}) \geq (1-p)C^0(N_{p,y} + N_{p,x}^0) + pC^1(N_{p,y} + N_{p,x}^1);$$
- *number of y-travellers increases due to information:*  

$$N_{n,y} \leq N_{p,y};$$
- *expected number of x-travellers increases due to information:*  

$$N_{p,x}^1 \leq N_{n,x} \leq (1-p)N_{p,x}^0 + pN_{p,x}^1 \leq N_{p,x}^0;$$
- *System welfare in model P exceeds system welfare in model N.*

The proposition has an interesting interpretation. It tells us that information increases the expected road use for both the drivers with and without information. However, at the same time the expected travel costs in the network will decrease. Hence, an increase in expected road use is achieved while expected network travel costs have decreased. Therefore, system welfare, measured as the sum of individual benefits minus the sum of individual costs, increases due to information (ignoring the cost of providing the information). This result stems from the fact that - when provided with information - more road users will use the network when it is relatively cheap (state 0 occurs), while less informed drivers will use it when it is relatively expensive (state 1 occurs). Moreover, knowing that informed drivers behave in this fashion, more uninformed drivers will also find it profitable to use the network, because the informed drivers will relieve part of the congestion under high cost circumstances (state 1 occurs). However, it is important to notice that link travel costs in state 0 are higher with than without information, which is a direct consequence of the second point in the proposition. Nevertheless, the expected link travel costs are smaller when information is provided.

The proposition also shows the relevance of using elastic rather than fixed demand patterns. As shown in the proposition, information does in fact alter the system performance even in a one-link network. Under fixed (inelastic) demand, however, it is clear that information does not affect the performance of the system since, independent of the prevailing link travel cost function, the same number of drivers will always use the network.

Even though the above results are appealing, the merits of information provision for governments' policy purposes should be based on changes in social welfare rather than on some (derived) performance indicator, such as expected road use or expected network travel costs. Social welfare, measured by the total system benefits minus the total (expected) system costs, is the most appropriate criterion on which to judge the network's

performance in terms of efficiency. Even though the proposition shows that information provision increases system welfare in our simple model, it is worthwhile to analyse: (1) the relative size of this welfare improvement; and (2) which travellers benefit most from the information. The latter issue, also referred to as an equity issue, is highly relevant when analysing the political feasibility of policy measures. Policies that have a strong effect on the current equity situation are likely to provoke resistance. In Section 4 these issues are addressed.

#### 4. Efficiency and Equity Issues of Information

The welfare economic aspects of information provision are studied here using the previously specified models  $P$  and  $N$ . A link travel cost function  $C$  without superscript denotes the expected link travel costs, that is,  $C = (1-p)C^0 + pC^1$ . By substituting the results obtained from the proposition in the respective link travel costs functions, the following relationship for the equilibrium link travel costs is derived:

$$C_n^0 \leq C_p^0 \leq C_p \leq C_n \leq C_p^1 \leq C_n^1 \quad (9)$$

These link travel cost functions obviously have to be evaluated at their relevant equilibrium levels of trip demand, for example  $C_n^0$  denotes  $C_n^0(N_{n,y} + N_{n,x})$ .

In subsection 4.1, the question of which travellers benefit most from the information is analysed (the equity issue). In subsection 4.2, the relative size of the welfare improvement due to information provision is studied (the efficiency issue).

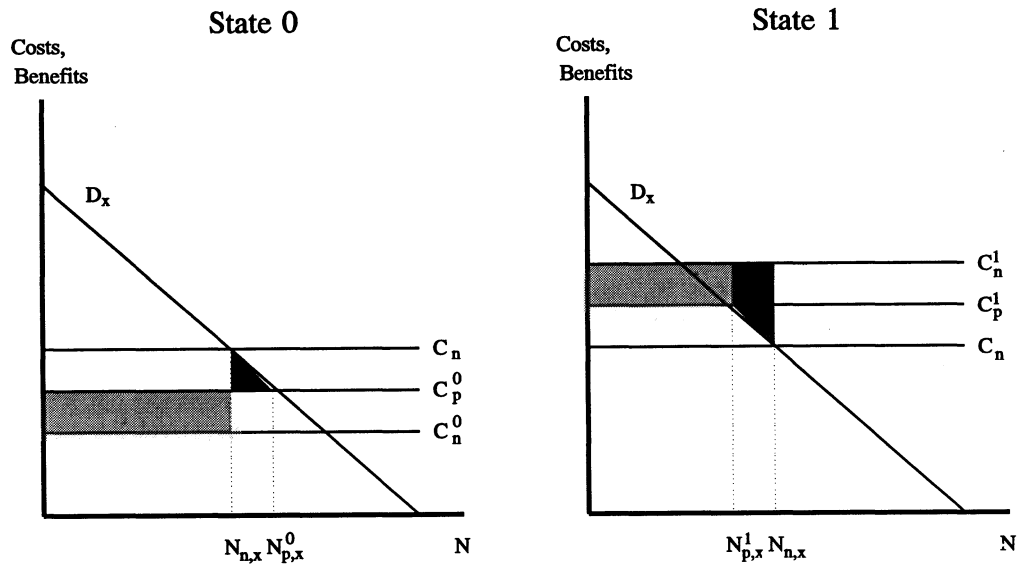
#### 4.1 Information and equity

##### 4.1.1 Informed drivers ( $x$ -travellers)

The situation for the informed drivers is schematically depicted in Figure 2. In this figure,  $D_x$  gives the demand curve, while the horizontal lines denoted  $C$  give equilibrium values of costs, and hence should not mistakenly be seen as cost curves.

When state 0 occurs, then  $N_{n,x}$  is less than or equal to  $N_{p,x}^0$  and  $C_n^0$  is less than or equal to  $C_p^0$ . This situation is depicted in the left-hand panel of Figure 2. The drivers on the left-hand side of  $N_{n,x}$  will always use the network under state 0. With information provision, their link travel costs will be larger than in the absence of information. Hence, in state 0, these drivers suffer a cost disadvantage that is equal to the size of  $C_p^0$  minus  $C_n^0$ , and is given by the shaded rectangle. It is interesting to note that this cost disadvantage is an *increasing congestion externality*, since it is caused by the fact that *other* road-users are informed. The size of this negative external effect decreases as fewer drivers have access to the information, since the difference between  $C_p^0$  and  $C_n^0$  will then decrease.

For the drivers between  $N_{n,x}$  and  $N_{p,x}^0$ , information on the actual occurrence of state 0 induces them to change their behaviour. Without information they will not use the network because expected costs exceed their benefits, whereas they will use the network when they are provided with the information that low costs prevail. The size of the total welfare improvement for drivers between  $N_{n,x}$  and  $N_{p,x}^0$  is equal to  $\frac{1}{2}(N_{p,x}^0 - N_{n,x})(C_n - C_p^0)$  and is given by the shaded area in the left-hand panel of Figure 2. It is important to note that these welfare gains are *internal* in nature, since these arise from better decision-making by the



**Figure 2**  
Welfare Effects for *x*-travellers

informed drivers themselves. Therefore, we call these information benefits *internal decision-making benefits*. The size of the internal decision-making benefits decreases as more drivers are informed; with more informed drivers the difference between  $C_p^0$  and  $C_n^0$  will increase, thereby (other things being equal) decreasing the difference between  $C_n$  and  $C_p^0$ . This negative effect for already informed drivers of equipping an additional driver is clearly external in nature. In this state, the marginally equipped driver will gain benefits from the information, while the information benefits for the already equipped drivers will dwindle. (See also Emmerink, Nijkamp, Rietveld and Axhausen, 1994, where the same phenomenon is discussed.)

If state 1 occurs, then  $N_{n,x}$  is greater than or equal to  $N_{p,x}^1$ , and  $C^1(N_{n,x} + N_{n,y})$  is greater than or equal to  $C^1(N_{p,y} + N_{p,x}^0)$ . The situation is depicted in the right-hand panel of Figure 2. First, drivers on the left-hand side of  $N_{p,x}^1$  will always use the network. Owing to the information provision, these will incur benefits equal to the difference in link travel costs  $C_n^1$  minus  $C_p^1$ . This cost advantage is a *decreasing congestion externality*, since it arises from the fact that *other* road users are provided with information and they reduce link travel demand when state 1 occurs. In the right-hand panel of Figure 2 this external beneficial effect is shown by the large shaded rectangular area. Second, drivers between  $N_{p,x}^1$  and  $N_{n,x}$  will, knowing that state 1 occurs, change their travel decision and avoid the network. As a consequence, these drivers will benefit from a cost advantage equal to  $C_n^1$  minus  $C_p^1$ , and in addition, from a decision-making advantage equal to the size of  $\frac{1}{2}(N_{n,x} - N_{p,x}^1)(C_p^1 - C_n)$ . Notice that the decision-making advantage is an *internal* effect, while the cost advantage is *external* in nature. The former arises from the fact that the

driver himself is informed of the prevailing traffic conditions, not from the fact that other drivers are informed. These two beneficial effects are illustrated in the right-hand panel of Figure 2 by the black rectangular area (decreasing congestion externality) and the shaded triangular area (decision-making benefits).

In summary, drivers on the left-hand side of  $N_{p,x}^1$  (that is, drivers who always use the network independent of the occurring state) will suffer from an external cost disadvantage if state 0 occurs and an external cost advantage if state 1 occurs. Drivers between  $N_{n,x}$  and  $N_{p,x}^0$  benefit from an internal decision advantage if state 0 occurs. Drivers between  $N_{p,x}^1$  and  $N_{n,x}$  incur an external cost increase if state 0 occurs, and an external cost and internal decision-making advantage if state 1 prevails. Finally, drivers to the right of  $N_{p,x}^0$  never use the network and are therefore indifferent about whether they obtain information or not.

When we consider the equity aspects of information provision in our model, we can conclude that no informed individual is worse off due to information provision. Above, it was noticed that informed individuals on the left-hand side of  $N_{p,x}^1$  are worse off in state 0 and better off in state 1. In terms of expected individual welfare (net private benefits), however, these drivers are at least as well off as without information, since  $C_n$  is larger than or equal to  $C_p$  as stated in the proposition. Therefore,  $p(C_n^1 - C_p^1) \geq (1-p)(C_p^0 - C_n^0)$ . Using the same argument, it follows that individuals between  $N_{p,x}^1$  and  $N_{n,x}$  are also better off as they incur the same external cost advantage as drivers to the left-hand side of  $N_{p,x}^1$  and in addition benefit from an internal decision-making advantage when state 1 occurs. Informed individuals between  $N_{n,x}$  and  $N_{p,x}^0$  are also individually better off as they gain when state 0 occurs and are indifferent when state 1 occurs. Finally, individuals to the right-hand side of  $N_{p,x}^0$  never use the network and are therefore indifferent about whether they obtain information or not. *Therefore, in our model the provision of information will always lead to a welfare improvement for the group of informed drivers.* A typical individual (expected) welfare pattern (net private benefits) as generated by the model is shown in the left-hand panel of Figure 3. The shaded area under the bold curve (being equal to the difference between the net expected private benefits under model  $P$  and model  $N$ ) shows the expected welfare gains due to information.

The left-hand panel of Figure 3 indicates that individuals close to  $N_{n,x}$  gain most from information provision. This is an intuitively appealing result, since individuals close to  $N_{n,x}$  are exactly those who are most doubtful about whether or not to use the network. For these individuals, information provision will enhance their knowledge and will affect their travel decisions. On the other hand, individuals on the left-hand side of  $N_{p,x}^1$  will never change their travel decisions regardless of the kind of information provided. Thus it is clear that the information benefits for these drivers are external in nature, because of an improved network efficiency due to information provision to other individuals. Finally, individuals who never use the network have obviously nothing to gain (or lose) from information provision.

#### 4.1.2 Uninformed drivers (y-travellers)

The situation for the y-travellers is schematically depicted in Figure 4. As before, the horizontal lines denote equilibrium values for costs. First of all it is important to note that

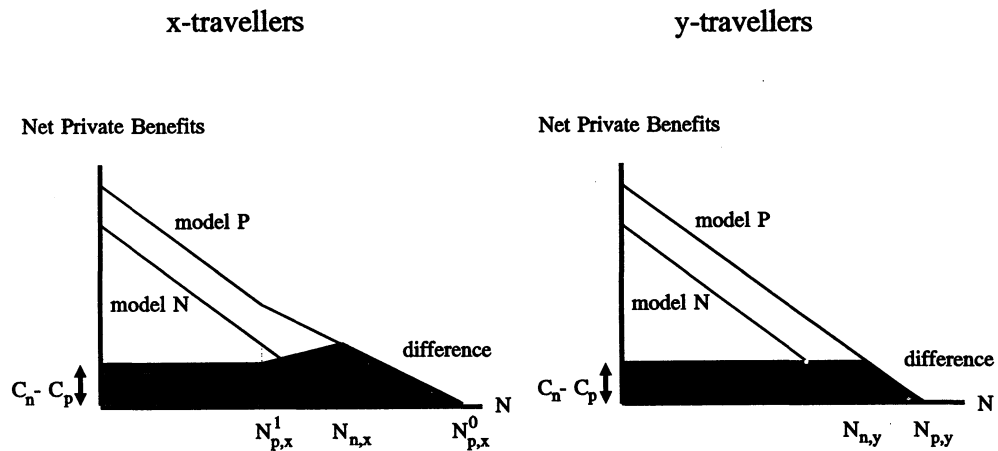


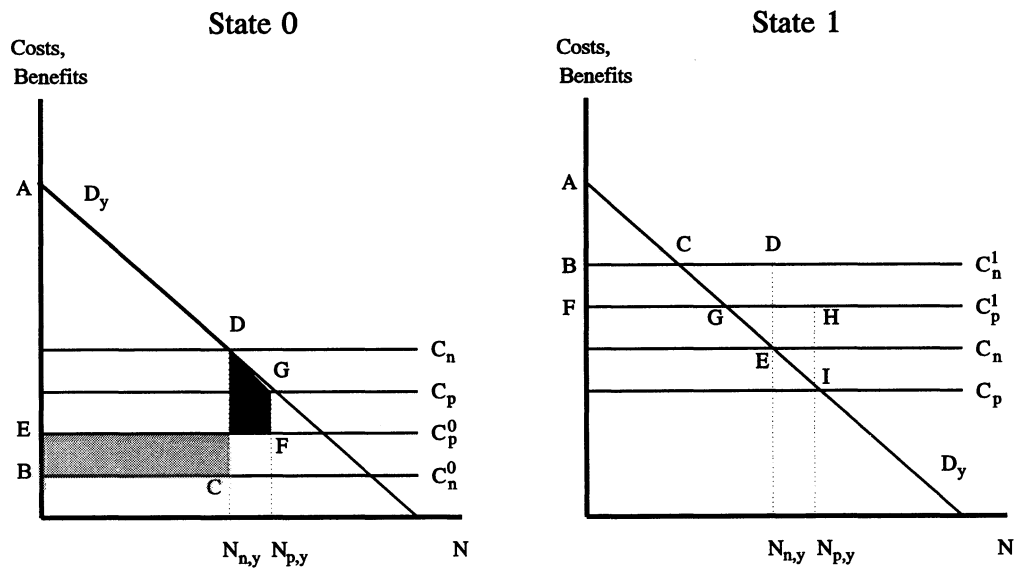
Figure 3

*Expected Net Private Benefits for x- and y-travellers; Model P – Model N*

(following the proposition)  $N_{n,y}$  is smaller than or equal to  $N_{p,y}$ , that is, the number of uninformed drivers will increase when information is provided to others. Then in state 0, the total benefits minus the total costs for uninformed individuals when no information is provided is shown by the polygon ABCD in the left-hand panel of Figure 4. When information is provided to informed individuals, then the total net benefits for uninformed drivers are given by the polygon AEF G. Hence, in state 0, the change in total welfare for uninformed drivers, due to information provision to informed individuals, is equal to the surface of the shaded polygon minus the surface of the shaded rectangle in the left-hand panel of Figure 4.

When the prevailing network condition is state 1, changes in total welfare are as depicted in the right-hand panel of Figure 4. Total welfare of the uninformed drivers when no information is available is given by the area ABC minus the area CDE. When information is provided, total welfare becomes the area AFG minus area GHI.

As uninformed individuals between  $N_{n,y}$  and  $N_{p,y}$  decide to use the network when information is provided to informed individuals, they will experience individual expected benefits; if this were not the case, they would not decide to use the network in the first place. Also, the uninformed drivers on the left-hand side of  $N_{n,y}$  benefit from the information provided to informed drivers. This is because expected link travel costs decrease when information is provided ( $C_p$  is smaller than  $C_n$ ; see the proposition). Therefore, *information provision to informed drivers will also lead to a welfare improvement for uninformed drivers* (see also Emmerink, Nijkamp, Rietveld and Axhausen, 1994). Clearly, these beneficial effects to uninformed drivers are external in nature; they are induced by behavioural responses from other (informed) road users. A typical expected welfare pattern for uninformed drivers is shown in the right-hand panel of Figure 3.



**Figure 4**  
*Welfare Effects for y-travellers*

**4.1.3 Summary**

In the model presented so far information provision will lead to a *strict* Pareto improvement (ignoring the costs of information provision): both the informed and uninformed drivers are at least as well off. Furthermore, due to the information provided the expected level of road use will increase, while the expected link travel costs will decrease. Finally, it is worth noting that these beneficial effects can in theory be reached by providing a very limited number of drivers with information: only drivers between  $N_{p,x}^1$  and  $N_{p,x}^0$  have to receive information to obtain the results discussed, because these are the informed drivers who might change their travel behaviour because of the information on the prevailing link travel cost function. In practice, it is of course hard to identify this group. In a free market system with perfect information on the costs and benefits of being provided with traffic information this group of drivers would identify themselves. In this way, the number of informed and uninformed drivers would be endogenised (Emmerink, Verhoef, Nijkamp and Rietveld, 1995).

**4.2 Information and efficiency**

In the previous section the important result was obtained that information provision in a one-link network leads to a strict Pareto improvement (ignoring the costs of information provision). In this section we address the size of this efficiency improvement on the basis of some experiments. We do so by comparing the effects on total (expected) welfare of the following three regimes:

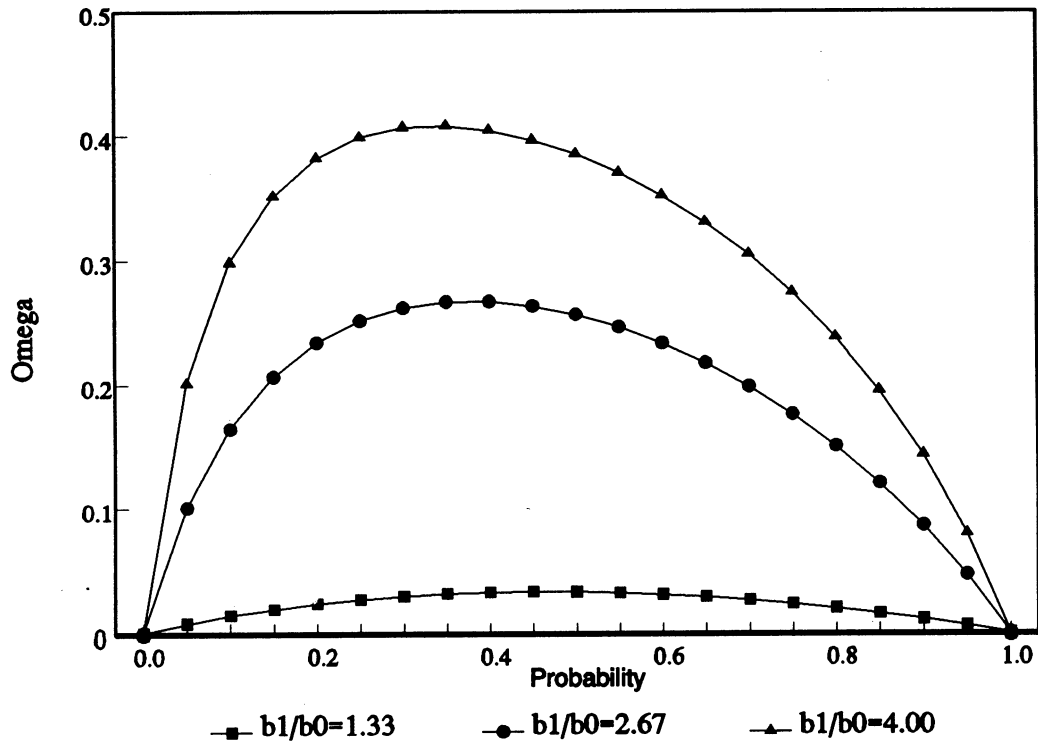


Figure 5  
Varying Probabilities of Low Capacity

- information provision as studied in model *P*,
- no information provision as studied in model *N*, and
- system optimal behaviour.

Under system optimal behaviour, the number of individuals using the network is derived in such a manner that total expected welfare, as measured by total system benefits minus total expected system costs, is maximised. It is well known that this can (in theory) be implemented by means of a fluctuating congestion-pricing scheme.

The effects of these three regimes on expected welfare are captured in the performance indicator  $\omega$  (see Arnott *et al.*, 1991; Verhoef *et al.*, 1995), which in the present paper indicates the relative welfare improvement of providing information to a group of drivers. The index  $\omega$  is defined as:

$$\omega = \frac{\text{Welfare}(\text{model } P) - \text{Welfare}(\text{model } N)}{\text{Welfare}(\text{System Optimum}) - \text{Welfare}(\text{model } N)} \quad (10)$$

Hence,  $\omega$  gives the achievable welfare gains as a proportion of the theoretically possible welfare gains. Clearly,  $\omega$  cannot exceed the value one. In addition,  $\omega$  cannot be smaller than zero, since it was shown in the previous section that information provision leads to a strict Pareto improvement, implying that the numerator of expression (10) cannot take on negative values.

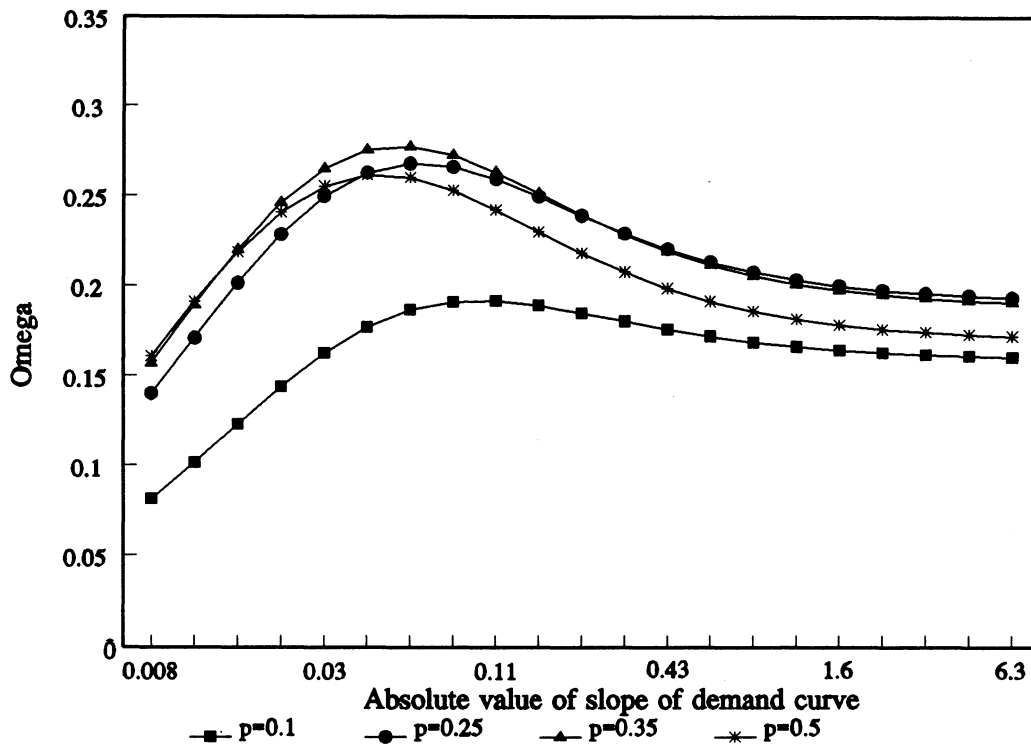


Figure 6  
Impact of Demand Elasticity on the Relative Efficiency Indicator  $\omega$

For reasons of space, only the most interesting model experiments are presented. The experiments have been conducted with linear inverse demand  $D_j(N) = d_j - a_j N$  ( $j=x, y$ ) and cost  $C^m(N) = k^m + b^m N$  ( $m=0,1$ ) functions. Given the satisfactory results obtained with previously conducted experiments in Verhoef *et al.* (1994) and Verhoef *et al.* (1996), the base case parameters were set equal to  $d_j=50$ ,  $a_j=0.03$  ( $j=x, y$ ),  $k^m=20$  ( $m=0,1$ ),  $b^0=0.015$ ,  $b^1=0.04$  and  $p=0.25$ .

Figure 5 shows the impact of changes in the probability of having low capacity on  $\omega$ . Clearly, if there is complete certainty on the link travel cost function, then  $\omega$  falls to zero ( $p=0$  and  $p=1$ ). For values in between,  $\omega$  reaches a maximum of 0.4, depending on the size of the low capacity congestion cost parameter  $b^1$ . It should be noted that the probability for which  $\omega$  takes on a maximum is dependent on  $b^1$ . In addition, the value of  $\omega$  is surprisingly stable for large ranges of  $p$ . For example, for the case that  $b^1$  is equal to 0.04,  $\omega$  falls in between 0.2 and 0.3 when  $p$  is in the interval ranging from 0.15 to 0.70.

Next, the impact of the elasticity of the demand functions on  $\omega$  is analysed. To do so, the two demand curves were simultaneously *tilted* around the original intersection of model  $N$ , varying from high elasticities on the left-hand side to almost perfect inelasticity on the right-hand side. The reason for changing both  $a$  and  $d$  parameters (of the demand



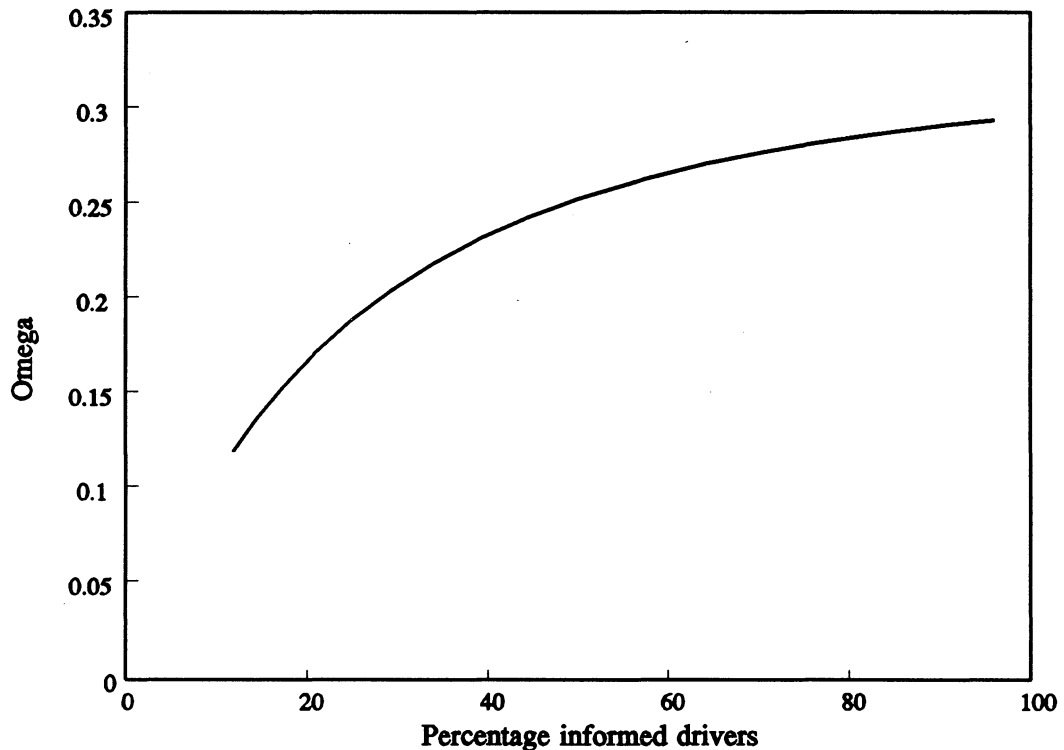


Figure 7

*Impact of Market Penetration on Efficiency Indicator  $\omega$*

function) simultaneously is to avoid very (small) large levels of road use when demand approaches complete (in)elasticity.

In Figure 6 the  $x$ -axis follows a logarithmic scaling with base number 1.4. The results shown in Figure 6 indicate that as demand becomes less elastic, the welfare-improving properties of information provision first increase and then decrease. Apparently, there exists some level of elasticity for which  $\omega$  is maximised. Although with almost inelastic demand  $\omega$  is still significantly different from zero, the available welfare improvement relative to the total welfare is in these circumstances very small; the equilibrium values for the three different models contained in the  $\omega$  index are practically the same. It can be noted, however, that with more routes available,  $\omega$  may approach unity at inelastic demand due to the beneficial impact of information on route choice (Verhoef *et al.*, 1996).

Finally, some experiments that focus on the impact of the group size on the relative efficiency indicator  $\omega$  were conducted. In order to do so, the total demand curve was kept at the same position, while the respective demand curves of the informed and uninformed drivers were varied from few to many informed drivers. For the base case parameters, and the total inverse demand curve equal to  $D(N) = 50 - 0.015N$ , the results are depicted in Figure 7. The  $x$ -axis of this figure presents the percentage of informed drivers using the

network; in the literature this is known as the level of market penetration.<sup>3</sup> Figure 7 indicates that the relative efficiency indicator  $\omega$  increases as the expected number of informed drivers increases (hence, as the demand curve of the informed drivers rotates outwards). However, the concavity of the curve demonstrates that this increase takes place at a decreasing rate. Therefore, from an efficiency point of view (taking the costs of implementing the technology into account), it might not be optimal to supply all drivers with the information.

The above observations have shown that the size of the welfare-improving properties of information provision depend on a number of complex interactions between the probability of having low capacity ( $p$ ), the impact of such an incident ( $b^1$ ), the elasticity of demand ( $a_j, j=x,y$ ), and the respective group sizes. Our experiments suggest that for a linear system the maximum achievable welfare gains, expressed as a proportion of the theoretically possible efficiency gains, will most likely not exceed 0.4. For achieving larger values of  $\omega$ , some form of congestion pricing is inevitable. The combination of congestion-pricing and information is analysed in El Sanhoury (1994), De Palma and Lindsey (1994) and Verhoef *et al.* (1996).

### 5. Concluding Comments

This paper has studied the welfare economic effects of information provision to a group of drivers. For that purpose an equilibrium model with elastic demand for road use was used. The model was introduced in a one-link network with stochastic capacity and two groups of (potential) users, informed and uninformed ones. Informed users base their decisions on *actual* prevailing traffic conditions, while uninformed drivers use *expected* traffic conditions.

The applied methodology explicitly models the link travel costs as stochastic variables. This provides a natural means to supply informed drivers with realisations of these stochastic variables, while uninformed drivers will base their behaviour on expectations of the stochastic travel costs. This contrasts with most of the prevailing literature, where it is assumed that the discrepancy between the traditional deterministic and stochastic user equilibrium is an indicator for the impact of information provision.

It was found that information provision is welfare-improving for both the informed and uninformed drivers. Hence, information leads in our model to a strict Pareto improvement (ignoring the costs of provision). With information provision, the user equilibrium is nearer to the system optimum. Information provision will, however, not close the gap between the two concepts. Even when all road users are well informed, the user equilibrium will still be different from the system optimum. Another interesting result of the analysis is that information increases expected road use (generating more traffic), while at the same time decreasing expected link travel costs.

Furthermore, the analysis showed that many of the beneficial effects (and some of the adverse effects) of information provision are external in nature; they arise from changes

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<sup>3</sup> Observations below 10 per cent market penetration are not available, because these would violate the group-regularity condition; that is, the network is not marginally used by both  $x$ - and  $y$ -travellers.

in trip-making decisions by others. For example, the beneficial effects of information provision to uninformed drivers are clearly external in nature. They arise from behavioural adaptations by the informed drivers, rather than from changes in travel decisions by the uninformed road users. The existence of these external effects raises an interesting question about governments' role in introducing these technologies. It is well known that without proper government intervention, external effects distort the market mechanism, and result in an inefficient allocation of scarce resources.

In contrast to most of the literature on the impacts of information provision, our equilibrium model allows for elastic demand. In doing this, we acknowledge the important economic relationship between demand and supply. The results in this paper indicate that the elasticity of demand is an important factor in determining the welfare-improving properties of information provision. Information was found to be less useful at both low and high levels of demand elasticity.

The analysis in this paper was confined to perfect information to informed drivers and a one-link network. Clearly, these are serious restrictions. In future research, the issue of imperfect information and more general networks deserves more attention. It is important to obtain greater understanding of the influence of the quality of the information on network efficiency, since it is unrealistic to assume that route guidance devices will be able to provide perfect information. Finally, more general networks would allow us to study so-called route-split effects (in addition to mode split effects).

## References

- Al-Deek, H. and A. Kanafani (1993): "Modeling the Benefits of Advanced Traveler Information Systems in Corridors with Incidents". *Transportation Research*, 1C, 4, pp.303-24.
- Arnott, R., A. de Palma and R. Lindsey (1990): "Economics of a Bottleneck". *Journal of Urban Economics*, 27, pp.11-30.
- Arnott, R., A. de Palma and R. Lindsey (1991): "Does Providing Information to Drivers Reduce Traffic Congestion?" *Transportation Research*, 25A, 5, pp.309-18.
- Arnott, R., A. de Palma and R. Lindsey (1992): "Route Choice with Heterogeneous Drivers and Group-Specific Congestion Costs". *Regional Science and Urban Economics*, 22, pp.71-102.
- Arnott, R., A. de Palma and R. Lindsey (1993): "A Structural Model of Peak-Period Congestion: A Traffic Bottleneck with Elastic Demand". *American Economic Review*, 83, 1, pp.161-79.
- Arnott, R., A. de Palma and R. Lindsey (1994): "The Welfare Effects of Congestion Tolls with Heterogeneous Commuters". *Journal of Transport Economics and Policy*, 28, 2, pp.139-61.
- Ben-Akiva, M., A. de Palma and P. Kanaroglou (1986): "Dynamic Model of Peak Period Traffic Congestion with Elastic Arrival Rates". *Transportation Science*, 20, 2, pp.164-81.
- Ben-Akiva, M., A. de Palma and I. Kaysi (1991): "Dynamic Network Models and Driver Information Systems". *Transportation Research*, 25A, 5, pp.251-66.
- Bonsall, P.W. (1992): "The Influence of Route Guidance Advice on Route Choice in Urban Networks". *Transportation*, 19, 1, pp.1-23.
- Caplice, C. and H.S. Mahmassani (1992): "Aspects of Commuting Behavior: Preferred Arrival Time, Use of Information and Switching Propensity". *Transportation Research*, 26A, 5, pp.409-18.
- Conquest, L., J. Spyridakis, M. Haselkorn and W. Barfield (1993): "The Effect of Motorist Information on Commuter Behavior: Classification of Drivers into Commuter Groups". *Transportation Research*, 1C, 2, pp.183-201.
- Daganzo, C. and Y. Sheffi (1977): "On Stochastic Models of Traffic Assignment". *Transportation Science*, 11, 3, pp.253-74.
- De Palma, A. and R. Lindsey (1994): "The Potential Benefits and Synergy of Advanced Driver Information Systems and Road Pricing". Working paper 1994.6, University of Geneva.

- El Sanhoury, I.M. (1994): *Evaluating the Joint Implementation of Congestion Pricing and Driver Information Systems*. PhD thesis, Massachusetts Institute of Technology.
- Emmerink, R.H.M., K.W. Axhausen, P. Nijkamp and P. Rietveld (1995a): "Concentration, Overreaction, Market Penetration and Wardrop's Principles in an ATIS environment". *International Journal of Transport Economics*, 22, pp.123-41.
- Emmerink, R.H.M., K.W. Axhausen, P. Nijkamp and P. Rietveld (1995b): "Effects of Information in Road Transport Networks with Recurrent Congestion". *Transportation*, 22, 1, pp.21-53.
- Emmerink, R.H.M., P. Nijkamp, P. Rietveld and K.W. Axhausen (1994): "The Economics of Motorist Information Systems Revisited". *Transport Reviews*, 14, 4, pp.363-88.
- Emmerink, R.H.M., P. Nijkamp, P. Rietveld and J.N. Van Ommeren (1994): "Variable Message Signs and Radio Traffic Information: An Integrated Empirical Analysis of Drivers' Route Choice Behaviour". Tinbergen Institute Discussion Paper, TI 94-127. Forthcoming in *Transportation Research A*.
- Emmerink, R.H.M., E.T. Verhoef, P. Nijkamp and P. Rietveld (1995): "Endogenising Demand for Information in Road Transport". Tinbergen Institute Discussion Paper, TI 95-38.
- Hicks, J.E., D.E. Boyce and A. Sen (1992): "Static Network Equilibrium Models and Analyses for the Design of Dynamic Route Guidance Systems". Technical Report in Support of the Design Phase of the ADVANCE Project, Illinois Department of Transportation.
- Mahmassani, H.S. and R. Jayakrishnan (1991): "System Performance and User Response under Real-Time Information in a Congested Traffic Corridor". *Transportation Research*, 25A, 5, pp.293-308.
- Mannering, F., S.-G. Kim, W. Barfield and L. Ng (1994): "Statistical Analysis of Commuters' Route, Mode, and Departure Time Flexibility". *Transportation Research*, 2C, 1, pp.35-47.
- Mirchandani, P. and H. Soroush (1987): "Generalized Traffic Equilibrium With Probabilistic Travel Times and Perceptions". *Transportation Science*, 12, pp.133-52.
- Ortúzar, J. de D. and L.G. Willumsen (1994): *Modelling Transport*. Chichester: John Wiley & Sons.
- Ran, B. and D.E. Boyce (1994): *Dynamic Urban Transportation Network Models: Theory and Implications for Intelligent Vehicle-Highway Systems*. Berlin: Springer-Verlag.
- Sheffi, Y. (1985): *Urban Transportation Networks: Equilibrium Analysis with Mathematical Programming Methods*. Englewood Cliffs, N.J.: Prentice-Hall.
- Sheffi, Y. and W.B. Powell (1981): "A Comparison of Stochastic and Deterministic Traffic Assignment over Congested Networks". *Transportation Research*, 15B, 1, pp.53-64.
- Spyridakis, J., W. Barfield, L. Conquest, M. Haselkorn and C. Isakson (1991): "Surveying Commuter Behavior: Designing Motorist Information Systems". *Transportation Research*, 25A, 1, pp.17-30.
- Tsuji, H., R. Takahashi, H. Kawashima and Y. Yamamoto (1985): "A Stochastic Approach for Estimating the Effectiveness of a Route Guidance System and its Related Parameters". *Transportation Science*, 19, 4, pp.333-51.
- Van Berkum, E.C. and P.H.J. Van Der Mede (1993): "The Impact of Traffic Information". PhD thesis, Universiteitsdrukkerij, Delft.
- Van Vuren, T. and D. Van Vliet (1992): *Route Choice and Signal Control*. Aldershot: Avebury.
- Verhoef, E.T., R.H.M. Emmerink, P. Nijkamp and P. Rietveld (1996): "Information Provision, Flat- and Fine-Congestion Tolling and the Efficiency of Road Usage". Tinbergen Institute Discussion Paper, TI 94-157. Forthcoming in *Regional Science and Urban Economics*.
- Verhoef, E.T., P. Nijkamp and P. Rietveld (1994): "Second-best Congestion Pricing: The Case of an Untolled Alternative". Tinbergen Institute Discussion Paper, TI 94-129. Forthcoming in *Journal of Urban Economics*.
- Verhoef, E.T., P. Nijkamp and P. Rietveld (1995): "Second-best Regulation of Road Transport Externalities". *Journal of Transport Economics and Policy*, 29, 2, pp.147-67.
- Vickrey, W.S. (1969): "Congestion Theory and Transport Investment". *American Economic Review*, 59 (Papers and Proceedings), pp.251-61.
- Wardrop, J. (1952): "Some Theoretical Aspects of Road Traffic Research". In *Proceedings of the Institute of Civil Engineers*, 1 (Part II), pp.325-78.

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