DYNAMIC EFFECTS OF EXTERNAL AND PRIVATE TRANSPORT COSTS ON URBAN SHAPE: A MORPHOGENETIC PERSPECTIVE

Francesca Medda
CUSP/CTS
University College London

Peter Nijkamp
Department of Spatial Economics
Free University Amsterdam

Piet Rietveld
Department of Spatial Economics
Free University Amsterdam

Abstract:

When we examine urban growth we often consider the dual relationship between variables that induce growth and variables that halt it. In this paper, we assume a mutual dependence between transportation costs and urban form, and by applying the morphogenetic algorithm, we determine the dynamic processes that this relationship induces to spatial urban changes. The objective of our model is to be able to describe the spillover and cumulative effects present in the urban growth process which have been missing in other studies. The model is developed within a dynamic framework and with the introduction of two specific elements: an accumulative trend of the variables and a diffusion process in their variation. The numerical simulation of an illustrative case study depicts how the entire urban shape can be modified in different ways by a transport system's improvement.

1. Introduction

A modern city is a complex entity characterized by a pluriformity of behaviour, volatility of interactions, and mobility of residents. It is in a permanent state of flux due to a dynamic force field that impacts on its functional structure and its spatial configuration (see Batty, 2007; Ingram, 1998). Urban dynamics mirrors often fundamental changes in a transportation system and its spatial spillovers (see also Crane, 2000; Handy, 1996). The externality dimensions of urban growth often relate to congestion and detrimental environmental effects due to car usage (air pollution, noise, accidents); so for this reason, a proper investigation of evolving urban forms – and their change patterns – could potentially be a means of understanding and combating urban sprawl, reducing automobile dependence, increasing the use of alternative transport modes, and supporting pedestrian mobility.

In the literature we see that the relationship between transport and urban form has been studied extensively. A number of analyses (Cervero and Gorham, 1995; Friedman et al., 1994; Newman and Kenworthy, 1989) investigate the relationship between urban form and transport by using aggregate indicators or measurements such as urban density or urban land rent in relation to trip frequency or average trip lengths. These approaches bring to light significant results between an urban transportation system and a general characterization of urban form and may therefore, support land use policies which might effectively lead to different overall travel patterns in the city, and in particular reduce car travel. Nonetheless, they neither convincingly address the problem of how specific characteristics of urban forms correlate with different travel patterns nor illustrate how urban form influences individual decisions. For example, multivariate regression in disaggregate models (Boarnet and Crane, 2001), which considers socio-economic and travel characteristics of individuals, yields mixed results on the relationships between urban form and transport, implying that modification of the urban form (pre-WWII traditional communities and post-WWII dispersed communities) does not always significantly correspond with realized or anticipated changes in travel behaviour. And Mohring (1993), investigating whether there are possible benefits a city derives from

improvement in urban transportation systems in relation to land rents, concludes *"regrettably, the answer is very little"*.

The relationship between urban form and transport, and in particular travel behaviour, is markedly complex, because it depends on the characteristics of the urban form (functional-geographic structure of the city, activity-based zoning, etc.) and the characteristics and purposes of the travel under scrutiny (working, shopping, by car, by mass transit system, etc.). The objective of this paper is therefore to analyze the relationship between urban form and transport by considering the antagonistic behaviour of two types of transport costs: the external cost of transport borne by the city and the private transport cost borne by the user. Both costs influence individual choices of citizens or actors in relation to location, and thus ultimately have an impact on the morphologic structure and dynamics of the city and its shape.

The methodological-conceptual approach we propose in this paper applies the essentials of the morphogenetic algorithm based on Turing (1954), which we will deploy in order to study the effects of transportation costs on city shape changes. The morphogenetic algorithm analyses the formation of spatial concentration patterns which occur due to different diffusion rates of considered 'substances'. The interesting aspect of such a formulation is that, contrary to our intuition, diffusion is no longer associated with smooth processes, but instead is related to the creation of peaks of concentrated 'substances'. Our purpose is that through this algorithm we may be able to describe the spillover and accumulative effects present in the urban growth process which had been missing in other studies. We consider in our model the spatial spillover effects of the transport system's variables over the urban space as well as the cumulative nature of the related growth processes. Spatial spillover effects have been analyzed in various conventional urban economic models, in particular, in the study of Yinger (1993), where the spread effects of congestion in urban areas are examined. Our approach, however, differs from Yinger in that we offer a dynamic formulation and specification of this space-time phenomenon. This dynamic approach aims to model more precisely the spatial diffusion process inherent in the effects – negative or positive – of changes in urban forms. The second element introduced here

is the existence of an accumulative effect in the variation of the variables. Like, for instance, multiplier accelerator theory in macroeconomic growth theory, the variables gain momentum in their increasing and decreasing patterns of movement, and we examine this aspect by assuming an increased rate of the variables that induce and inhibit urban growth.

The Turing algorithm has in the past decades been applied successfully in various fields of study in the natural sciences and has recently also drawn the attention of economic scholars after the definition of Krugman's edge city model (1996). In this model Krugman examines how the concentration of businesses in various urban locations can be ascribed to fluctuations at different frequencies of "economic centrifugal forces" and "economic centripetal forces". Krugman's application does, however, not explicitly use the analytical formulation of the algorithm, but rather employs only one part of the operational solution of the algorithm, i.e. the Fourier analysis. It is therefore prudent that we use the application of the morphogenetic algorithm in Krugman's model, which thus remains more as background to our analysis rather than as a key reference frame in our analysis. In our approach we will address the Turing approach more precisely by applying its associated morphogenetic algorithm, while we will assess in our model the implications of the dynamic processes inherent in the relationship between transport costs and spatial urban morphology.

The subsequent analysis is subdivided into two main parts: we first propose the urban dynamic model based on the morphogenetic algorithm, and next we develop an illustrative simulation of urban shape formation.

2.   Interaction between transport costs and urban morphology

We will start our analysis from conventional urban economics and consider a standard monocentric city with a circular central business district (CBD). We will focus – without loss of generality - our attention only on the boundary of the city of which the distance from the CBD is given by the outer radius L. The reason is that – with a given density (per uniform resident or per economic activity) – only the boundary will

be affected by the underlying morphologic changes in the underlying urban area. Thus, we only analyse the 'top of the iceberg'. The choice to reduce the city to a simple circle in motion arises from the theoretical attempt in our approach to focus on the macro-dynamic relationship between transport costs and city shape. If we assume a fixed urban space occupancy per person, i.e. a given urban density, then urban shape variations are essentially examples of boundary 'remodelling'; by this we mean that the modification of the boundary reflects at an aggregate level the spatial alteration in the micro-based structure of the city. Clearly, alternative shapes (rectangles, hexagons, or other patterns) could be used as well. For the sake of illustration, a circular model will suffice to illustrate our exposition.

Our city is thus a circle with a perimeter equal to $2\pi L$. We subdivide now the boundary of the city into p distinct districts where the location of each district on the boundary is indicated by i. The maximum total population living on the city boundary is supposed to be equal to N. Each district is then characterized by the maximum number of people living in it, which is equal to N/p at the initial situation. We assume – as mentioned – that the maximum density in all districts is fixed and equals D. If at some stage a district would attract and hence have to accommodate a number of people greater that N/p, the district would require a larger area for its residents in order to maintain the same density level R, and thus it would need to expand. The growth of the city is assumed to be only outward-oriented from the relevant city boundary i onward.

Next, we assume that each district positioned at the edge of the city is connected to a collective spatial transport infrastructure system (buses, metro's, trams, taxis, shared vehicles etc.) which allows residents to move from the district to, e.g., the CBD (see Figure 1). We assume a mutual dependence between transportation cost and population distribution and will now discuss the composition of these transportation costs.
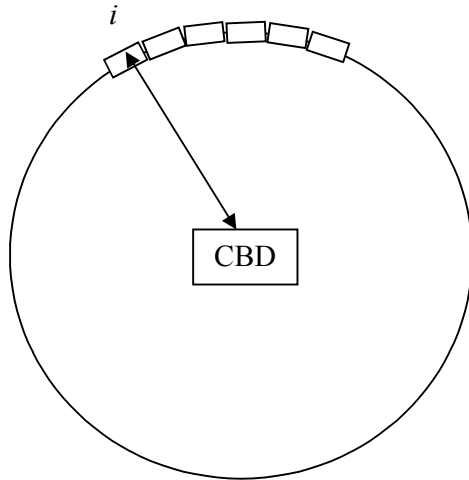
Figure 1   Urban districts on the city boundary

The Total Transport Cost (TTC) caused by the entire urban transport system is then composed of the sum of two terms: the External Transport Cost (congestion, environmental costs, safety costs) to be borne by all actors in the city as a whole (ETC) and the Private Transport Cost (in terms of time and money) (PTC) to be borne by the individual users of the collective transport system in the city.

The External Transport Cost for district i is then equal to:

$ETC_{i,t} = K_i + f(n_{i,t})$

where:

K_i = a fixed external sunk cost related to externalities, in particular air and noise pollution caused by the operation of the transport system;

$f(n_i)$ = the congestion cost; this cost comprises the variable travel cost related to using the infrastructure when the number of people living in a particular district i is n_i at time t. This travel cost increases, if the number of people in the district would increase; it therefore, represents a congestion cost for urban travellers.

The Private Transport Cost for district i is equal to:

$PTC_{i,t} = H_i + c(F(n_{i,t}))$

where:

H_i = the fixed costs (e.g., fare or tax) related to the use of the collective transport system.

5

c (F $(n_{i,t})$) = the cost, without congestion, of the total travel time for the number of people living in a given district i, including waiting time. This cost is an indirect function of the standard travel time or frequency F of the transport service, offered by the urban transportation system. We assume a supply response system, which means that the higher the number of people living in a particular district i at time t, the higher the supply of infrastructure or the frequency of transport services will be. This implies that, as the infrastructure supply or the frequency of transport increases, total travel time will be lower and thus the total cost related to travel time will decrease.

In other words, the variation of the two transport costs is *ceteris paribus* a function of the number of people living in the district. ETC increases as the number of people living in the district increases, and PTC increases when the number of people in the district decreases, which leads to a mutually contrasting force field.

Next, we assume that all families living in districts on the urban boundary are identical from an economic perspective (following conventional urban land rent theory). They each have an income Y, and they choose a quantity of housing space of which the rent R is an aggregate compound function of the number of people living in the district. We assume that each household in a specific district i will minimize the total transport cost under the income constraint as follows:

Min $TTC_{i,t} = ETC_{i,t} + PTC_{i,t}$
subject to:
$Y_{i,t} < TTC_{i,t} + R (n_{i,t})$,

where $R(n_{i,t})$ is the rent value, which is a direct function of the households living in the given district i at time t. The higher the number of people living in the district, the higher the rent cost for the household will be.

We assume the urban transportation system to be initially in equilibrium. Thus, we may impose the equilibrium conditions that at $t = 0$, the Total Transport Cost (TTC) is known and identical for all districts and the rent value is equal across all districts. Next, we want to examine the effects of exogenous changes in the transportation

system in the city and hence, we assume an external shock in the urban spatial system. Without loss of generality, this shock is supposed to be a specific environmental improvement in the transport system due to the use of bio-fuel technology which is introduced exclusively in district i. Consequently, at $t = 1$, the fixed external transport cost (sunk cost) K_i in district i is assumed to decrease. This change has a twofold consequence:

- Due to the specific significance of the External Transport Cost, ETC_i decreases due to the decrease of K_i ; this will induce a movement of households from the other districts to district i, given their cost minimization behaviour.
- An increase in population number in district i will subsequently engender two simultaneous effects in order to maintain the same level of population density: an increase in the External Transport Cost in district i, and – since we assume the city population to be constant – a decrease in population in the other districts, with the consequent changes in the shares of the two transport cost categories.

Therefore, our urban system tends to move away from the original equilibrium states, while the two transport costs will respond in mutually opposing ways. For instance, in district i, as the population increases, the External Transport Cost will increase and the Private Transport Cost will decrease (see Figure 2).
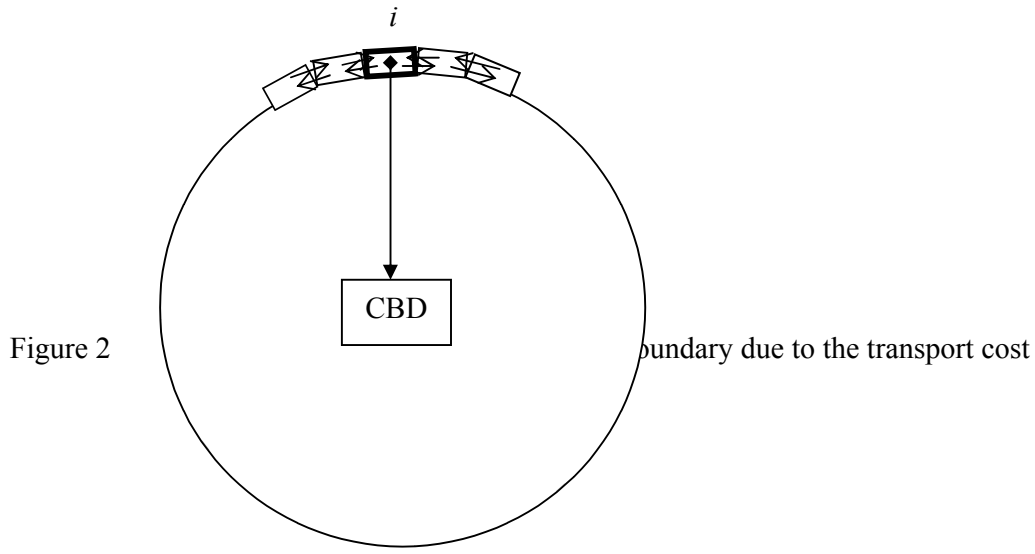
Figure 2   boundary due to the transport cost

In our model the associated interactions between transport cost and urban rent will now induce a change in the location of households along the city edge until we reach a new equilibrium. Our aim is now to model this change in urban structure. This interaction, as we have indicated above, can be modelled by a system of partial differential equations that are essentially encapsulated by the more general morphogenetic algorithm (see for details Appendix 1) which, in our case, depicts the relationship between transportation costs and urban morphology changes. We apply now here concisely the general analytical form of the morphogenetic algorithm in order to examine the dynamic effects of the two transport cost categories:

$$\frac{\partial T_1}{\partial t} = S_{kj}\, h\left(T_1, T_2\right) + D_1\, \nabla^2 T_1 \qquad \text{where k, j = 1,2}$$

and

$$\frac{\partial T_2}{\partial t} = S_{jk} g\left(T_1, T_2\right) + D_2\, \nabla^2 T_2 \qquad \text{where k, j = 1,2}$$

where:

T_1 = the External Transport Cost (ETC)

T_2 = the Private Transport Cost (PTC)

S_{ij} = are the slopes of $h(T_1, T_2)$ with respect to T_1 and T_2 at t=0;

S_{ji} = are the slopes of $g(T_1, T_2)$ with respect to T_1 and T_2 at t=0.

Francesca 1:

- Please specify in more detail how h and g related to page 5 and 6, as this should be consistent
- Index i has been replaced by k, as i refers already to districts
- Define D_1 and D_2

The two general functions $h(T_1,T_2)$ and $g(T_1,T_2)$ represent in our urban transportation case the mutual dependencies between these two expressions and their related transport costs as described by us above, while their change value is calculated by the first-order conditions of the minimization functions as described above. These changes do not only concern the single district *i*, but have also consequences for the entire city boundary. Consequently, D_1 and D_2 are the diffusion constants which account for the spatial effects of the transport costs up to the city boundary. The Laplacian operator, $\nabla^2 \equiv \partial^2/\partial x^2$, describes the processes of diffusion in space. As Kauffman (1993) observes, the exchange by diffusion is represented by a Laplacian operator, because what we want to examine is not the change of concentration, but rather the rate of change of concentration at each point *x* along a certain line.

The above presented system of partial differential equations describes for our urban case the interdependence between urban population and transport costs. Such a system may appear at first glance to be very similar to the system defined by Solow (1973)[1], in which the author uses a system of first-order differential equations for n(x) and w(x), where x is the distance from the CBD, i.e. the radial spatial distance. In our case, however, we consider both a circular location around the city boundary and a temporal distance. In particular, we examine here the interdependence between the transport costs and the number of people living in the district within a dynamic complexity paradigm, and we assume that this relationship instigates spillover effects in urban space. Clearly, it is noteworthy that the above analysis bears some resemblance to predator-prey dynamic models in spatial evolution (see for an illustration Nijkamp and Reggiani, 1999).

---

[1] The system of differential equations defined in the Solow model is:

$$N'(x) = -\frac{2\pi_1}{ky}b(y)xw(x)^{1/k-1}$$

$$w'(x) = -\frac{t_0}{y} - \frac{a}{2\pi y}\frac{N(x)}{x[1-b(x)]}$$

3.   Numerical solutions

 It is clear that the above model cannot be solved analytically. The best way to understand the mechanisms of the previous model is to experiment with computer simulations by changing parameters and conditions and then observe the outcome for the urban shape. In our experimental case we assume – as mentioned − a random shock; that is, in random locations at the city boundary we assume a decrease of the fixed cost K_i of the external transport cost. The simulations are conducted through the use of the software programme SP.

Francesca 2:

This numerical illustration cannot be well understood, if there is no information on the numerical forms of the equations and of the parameters, which lead to the results of Figures 3-5; please add (perhaps in Appendix II)

We assume now for the sake of illustration random shocks at various districts on the city boundary (see Figures 3 and 4). Figure 3 illustrates the variation in time and space of the population. In the first graph the variation of the population around the city is due to the decrease of the External Transport Cost. The second graph depicts the distribution of the population when we consider the increase in the Private Transport Cost, which acts as a barrier to residential relocation. Due to the relationship between these two costs and the number of people living in the district, we can also interpret the darker line as representing the number of people induced by the decrease of the External Transport Cost. The light line represents the number of people generated by the increase of the Private Transport Cost. Since the two costs are changing in opposite directions, we have in both graphs (Figures 3 and 4) a similar variation of population along the city boundary.

In Figure 4 we do not depict the foregoing dynamic aspects of the process (which, however, the computer programme can show in order to reach Figure 3). The darker line is the distribution of the population due to the External Transport Cost; the light line is the distribution of the population due to the Private Transport Cost. When we

map out the two effects added together, then we can observe the distribution of the population around the city after it has reached a stable pattern. Then, in Figure 4, we can identify the new urban shape created by the relocation of people.
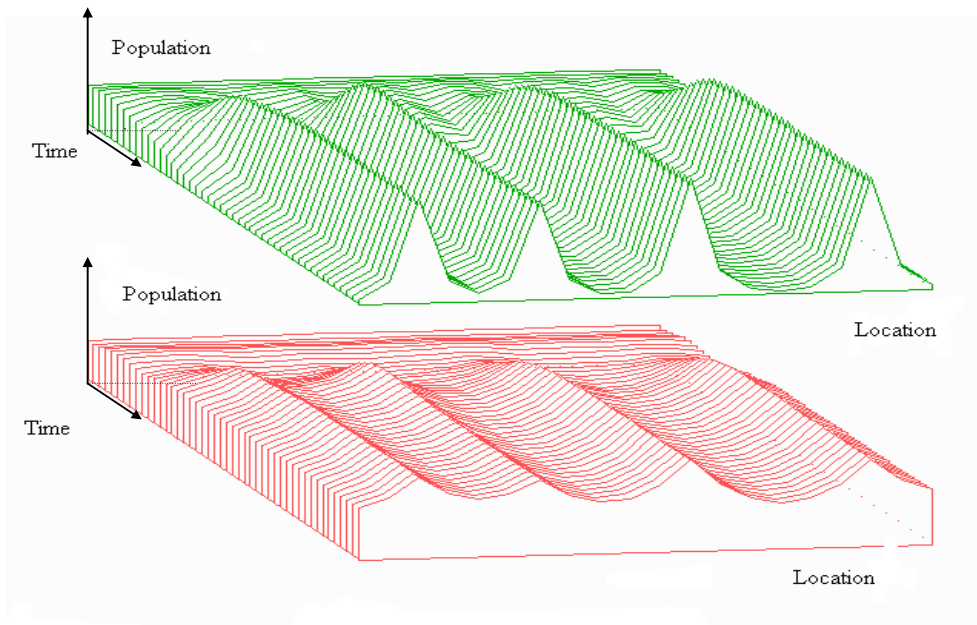

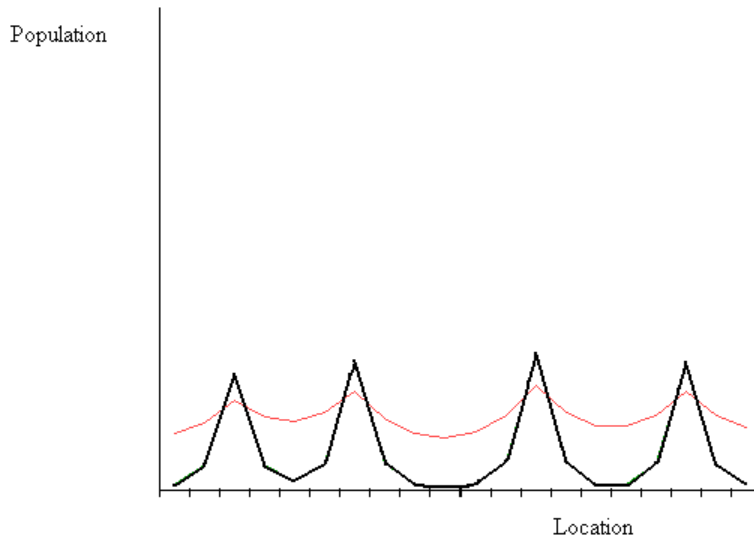Figure 3   Dynamics of urban pattern formation

Figure 4   Stable pattern condition of urban population

The final equilibrium at the stable condition shows that when the two effects are added together, we obtain an urban growth scenario which is diffused along the city boundary. The External Transport Cost (darker line) determines four peaks in the city boundary. The sharpness of these peaks is reduced by the population growth and thus represents the urban growth which has been determined by the Private Transport Cost (lighter line) (see also Figure 5).

Figure 5   Dynamics of urban shape formation

In summary: this illustrative example of the approach developed in Section 3 shows how a transport improvement can determine a direct impact upon an entire urban shape. Since we analyse a variation in the number of people living in the district, we

are assuming a consequent change in urban land use. A transport-oriented environmental improvement can, according to the hypotheses of our model, determine effects not only in the area where the improvement is located, but also through spillover effects in distant areas. Two consequences are implied in this particular circumstance. First, a transport improvement, acting as a shock in the equilibrium pattern of a city, can structurally determine the formation of a new urban shape. The second consequence we derive from the model is that transport improvements in different locations in the urban boundary can determine variations of the initial urban shape.

5. Conclusion

The model we have developed by using the morphogenetic algorithm has aimed to depict the urban shape changes under the impact of transport costs: External and Private costs. We have analysed only the outer boundary of the city, which we have subdivided into distinct districts. Each district has been characterized by the type of transport node and number of people. Since all the districts are equidistant, we have defined transport costs based on the temporal distance, i.e. the time needed to cover a spatial distance. Such costs are functions of the number of households living in the district.

Since a shock occurs in a stable condition in, for example, an environmental improvement of a transport node, two effects act simultaneously but with opposite trends. The decrease of External Transport Cost induces people to move to the location where the decrease has occurred. The rise of number of people in a certain location determines a consequent increase of Private Transport Cost, which then acts as a barrier for subsequent relocation of other people at the location. The two forces act as activator and inhibitor of urban growth, which consequently impacts upon the variation of urban shape.

The model has been developed within a dynamic framework, while two specific elements have been introduced: an accumulative trend of the variables and a diffusion process in their variation. These two elements assume a fundamental role when we consider the impact that a transport environmental improvement can generate, not

only in the area surrounding where an improvement takes place, but also in areas distant from its point of origin. The impact area of a transport-oriented environmental improvement is therefore not limited to a defined area calculated by iso-transport cost curves, but actually encompasses the entire city. Certainly, we can observe that, by reaching a stable state, not all points in the boundary will have changed their spatial positions.

Our approach is in line with the rationale of complexity theory, in which it is assumed that a common principle may apply to subjects with very different details. The fact that we use a principle applied in many – mainly physical science – disciplines may represent a major limitation in our model, where conversely, standard urban economics has defined models having self-contained structures. Our model explains pattern formations in a simple analytical form that has heretofore required very restricted assumptions in its application. This suggests that the application of our model to a real urban pattern formation may prove to be difficult. However, despite this limitation, our approach is not meant to be merely an analytical exercise, but rather it has as its objective to extend the approach of urban economics towards the inclusion of urban pattern formation. As mentioned, other regular urban shapes – and also irregular shapes – can be handled in an analogous manner.

Appendix 1

The urban transportation system's model presented in Section 2 is a specific case of the morphonenis principles. This will be outlined in this Appendix.

Brief description of Turing's morphogenetic algorithm

Turing (1952) assumes that a dynamic spatial pattern is formed by two components: the inhibitor X and the inhibitor Y. The process is based on X, which activates the formation of itself and of Y, and in turn Y, which inhibits the formation of X and also of Y. Both X and Y diffuse in the tissue but Y can diffuse more rapidly than X. The two chemical components are identified according to their position in the tissue and the time. The two components are synthesized and destroyed at the rates f(X,Y) and g(X,Y) (Kauffman, 1993). The partial differential equations which describe the diffusion process in the tissue are given by:

$$\frac{\delta X}{\delta t} = f(X,Y) + D_x \nabla^2 X$$

$$\frac{\delta Y}{\delta t} = g(X,Y) + D_y \nabla^2 Y$$

where:

D_x and D_y are diffusion constants.

Francesca 3: additions necessary:

- Show that the 2 chemical components are similar to the interpretation of the 2 cost categories

- Show the similarity between these diffusion processes and the cost equations from Section 2; in particular, show how h and g are related to the 2 cost equations in dynamic form.

We assume a steady state X_0, Y_0, where for X_0, Y_0, f (X,Y) = 0 and g(X,Y) = 0.

The linearized equations are:

$$\frac{\delta X}{\delta t} = K_{11}X + K_{12}Y + D_x \nabla^2 X$$

$$\frac{\delta Y}{\delta t} = K_{21}X + K_{22}Y + D_y \nabla^2 Y$$

The two equations, after sinusoidal perturbations of wavenumber k, are evaluated by the determinant of the following matrix

$$\begin{vmatrix} K_{11} - k^2 D_x - \lambda & K_{12} \\ K_{21} & K_{22} - k^2 D_y - \lambda \end{vmatrix} = 0$$

where λ is one of the two eigenvalues. If an eigenvalue is positive, the spatial pattern will grow in amplitude; if an eigenvalue is negative, the associated spatial pattern will decay. In this way, change patterns after a perturbation of an initial equilibrium state of the system can be analyzed.

References

Batty, M., 2007. "Model Cities", *Town Planning Review* **78**(2): 125-151.

Boarnett, M., Crane R., 2001. *Travel by Design: The Influence of Urban Form on Travel,* Oxford: Oxford University Press.

Cervero, R., Gorham, R., 1995. "Commuting in transit versus automobile neighborhoods", *Journal of the American Planning Association* **61**(2): 210–225.

Crane, R., 2000. "The Influence of Urban Form on Travel: an Interpretive Review", *Journal of Planning Literature* **15**(1): 3-23.

Friedman, B, S.P. Gordon, S.P., Peers, J.B., 1994. "Effect of neo-traditional neighborhood design on travel characteristics", *Transportation Research Record* **1466:** 63–70.

Handy, S., 1996. "Methodologies for Exploring the Link between Urban Form and Travel Behaviour", *Transportation Research* D 1(2): 151-165.

Ingram, G., 1998. "Patterns of Metropolitan Development: What Have We Learned?", *Urban Studies* **35** (7): 1019.

Kauffman, S.A., 1993. *The Origins of Order: Self-organization and Selection in Evolution,* New York: Oxford University Press.

Krugman, P., 1996. *The Self-Organizing Economy,* Cambridge MA: Blackwell

Mohring, H., 1993. "Land Rents and Transport Improvements: Some Urban Parables", *Transportation* **20**: 267–283.

Newman, P., Kenworthy, J., 1989. *Cities and Automobile Dependence: An International Sourcebook,* London: Gower.

Nijkamp, P., Reggiani, A., 1999. *The Economics of Complex Spatial Systems*, Amsterdam: Elsevier, Amsterdam.

Solow, R.M., 1973. *"Congestion Cost and the Use of Land for Streets",* Bell Journal of Economics, **4**(2): 602-618.

Turing, A.M., 1952. "The Chemical Basis of Morphogenesis". In: P.T. Saunders (ed.), 1992. *Morphogenesis: The collected works of A.M. Turin,* Amsterdam: North-Holland.

Yinger, J., 1993. "Around the Block: Urban Models with a Street Grid", *Journal of Urban Economics,* Spring: 305-330.