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Abstract

Power is a core concept in the analysis and design of organizations. One of the problems with the extant literature on positional power in hierarchies is that it is mainly restricted to the analysis of power in terms of the bare positions of the actors. While such an analysis informs us about the authority structure within an organization, it ignores the decision-making mechanisms completely. The few studies which take into account the decision-making mechanisms make all use of adaptations of well-established approaches for the analysis of power in non-hierarchical organizations such as the Banzhaf measure; and thus they are all based on the structure of a simple game, i.e. they are 'membership-based'. In van den Brink and Steffen (2008) it is demonstrated that such an approach is in general inappropriate for characterizing power in hierarchies as it cannot be extended to a class of decision-making mechanisms which allow certain actors to terminate a decision before all other members have been involved. As this kind of sequential decision-making mechanism turns out to be particularly relevant for hierarchies, we suggested an action-based approach - represented by an extensive game form - which can take the features of such mechanisms into account. Based on this approach we introduced a power score and power measure that can be applied to ascribe positional power to actors in sequential decision making mechanisms. In this paper we provide axiomatizations of this power score and power measure for one of the most studied decision models, namely that of binary voting.

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1 Introduction

Positional power is what results from the interplay of two components of an organization's architecture: the arrangement of positions in the organization and the decision-making mechanisms in use. The extant literature on positional power in hierarchies is mainly restricted to the analysis of power only in terms of the arrangement of the positions which informs us about the authority structure within the hierarchy[1]. The few studies which also take into account the decision-making mechanisms make all use of adaptations of well-established approaches for the analysis of power in non-hierarchical organizations such as the Banzhaf (1965) measure; and thus they are all based on the structure of a simple game, i.e. they are 'membership-based'.[2] In van den Brink and Steffen (2008) we demonstrated that such an approach is in general inappropriate for sequential decision-making mechanisms which allow certain actors to terminate a decision before all other members have been involved. As this kind of decision-making mechanism is particularly relevant for hierarchies, we suggested an action-based approach - represented by an extensive game form - which can take the features of such mechanisms into account. Based on this approach we introduced a power score and measure which can be applied to ascribe positional power to actors in sequential decision making mechanisms.

Hierarchies form a certain subclass of organizational architectures. Following van den Brink (1994) they distinguish themselves from other organizational architectures by the arrangement of its members being connected via directed relations, which we interpret as dominance (or superior to) relations. Loosely speaking, we can say that an actor i in a dominating position has an influence on the 'powers' of other actors who are in positions that are dominated by i. Domination can be either indirect or direct, i.e. with or without intermediate actors. Note, that if we just make use of the term 'domination' without further specification, we allow for indirect and direct domination.

Actors in dominating positions are called superiors (or principals) - bosses or managers in common parlance -, while the actors in dominated positions are called subordinates (or agents). If we refer to a superior who directly dominates another actor, the dominating actor is called a predecessor, and if we refer to a subordinate who is directly dominated by another actor, the dominated actor is called a successor.

The second component of an organizational architecture we refer to in our analysis are the decision-making mechanisms (DMMs) in use. A decision-making mechanism

[1]See, for instance, Copeland (1951), Russett (1968), Grofman and Owen (1982), Daudi (1986), Brams (1968), van den Brink (1994, 2002), van den Brink and Gilles (2000), Mizruchi and Potts (1998), Hu and Shapley (2003), Herings et al. (2005), and the references therein.

[2]See van den Brink (1994, 1997, 1999, 2001), Gilles et al. (1992), Gilles and Owen (1994), van den Brink and Gilles (1996), Berg and Paroush (1999), Shapley and Palamara (2000a,b), and Steffen (2002).

consists of a decision rule together with a decision-making procedure. A decision-making procedure provides the course of actions of the actors for a collective decision and determines the actions to be counted, i.e. which actions go into the domain of the decision rule. The sets of actions from which an individual actor can choose are created by the proposals submitted to the organization. The choices within the organization are made by one or more actors where each of these actors has to perform an action to make its individual choice effective. How those actions and the outcomes are linked is given by the decision rule. In other words: a decision rule is a function which maps ordered sets of individually chosen actions into outcomes.

In this paper we propose an axiomatization of the power score and measure for positional power in hierarchies introduced in van den Brink and Steffen (2008) for one of the most discussed decision-making situations in the voting power literature, namely that of binary voting. The paper is organized as follows. In Section 2 we discuss some preliminaries on collective decision making in hierarchies, power and directed graphs. Section 3 describes the model and the power score and measure. In Section 4 we provide axiomatizations of the power score, while Section 5 contains an axiomatization of the power measure. Finally, Section 6 contains concluding remarks on the relation of our model to Sah and Stiligtz's (1985, 1986) work on hierarchies.

2 Preliminaries

2.1 Collective decision making in hierarchies

There exist two DMMs, the One Desk Conjunctive and the One Desk Disjunctive DMM which can be regarded to be the elementary bottom-up DMMs for hierarchies as they form the root of the other DMMs discussed in the literature[3]. They are the simplest DMMs that take into account the existence of the dominance structure which distinguishes a hierarchy from any other organizational architecture. Both DMMs are characterized by a set of nine joint assumptions. These assumptions are:

1. Proposals submitted to the hierarchy are exogenous: it is the task of the hierarchy either to accept or to reject the proposal, i.e. we have a binary outcome set: $\{acceptance, rejection\}$.

2. A proposal can be submitted to the hierarchy only once.

3. A hierarchy contains a finite set of members N, $|N| \geq 2$, whose actions bring about the decision of the hierarchy.

[3]See van den Brink and Steffen (2008) for an overview.

4. Each actor $i \in N$ has a binary action set, $\{yes, no\}$, to chose from where 'yes' means that i supports the proposal and 'no' that i rejects the proposal.

5. The direction of the decision-making procedure through the hierarchy is bottom-up.

6. New proposals entering the organization can only be received by bottom actors being actors in positions with no successors.

7. A new proposal can only be received by one bottom actor at the same time (One Desk Model).

8. The choice of the 'yes'-action results (i) in a final approval if actor i is the top actor, i.e. if i has no predecessor, or (ii) in forwarding the proposal to one or more predecessors if i is not the top actor.

9. The choice of the 'no'-action results in a final rejection of the proposal, if (i) actor i is a bottom actor, or (ii) if for the actor who has forwarded the proposal to i there is no other predecessor left to ask for an approval of the proposal whose individual approval contains the potential of a final approval. If such predecessor exists it results in forwarding the proposal to this actor.

Assumptions 1-4 are also common in the analysis of power in non-hierarchical organizations. The other assumptions are additional for hierarchies. Both DMM's that we referred to above satisfy assumptions 1-9. They differ with respect to the following two assumptions, where each satisfies either one of these two assumptions.

10. (One Desk Conjunctive DMM) For the approval of a new proposal received by bottom actor i, the consent of all superiors of i is necessary and sufficient.

11. (One Desk Disjunctive DMM) For the approval of a new proposal received by bottom actor i, the consent of all superiors of i along one path up to the top is necessary and sufficient.

These two DMM's respect the following two basic principles:

P1. Reduction of the breadth of the hierarchy involved in a particular decision by a truncation of the hierarchy.

P2. Permission for non-top actors to make certain types of final decisions on behalf of the whole hierarchy even before one of their superiors has been involved.

Especially the second principle is an essential characteristic of DMMs in hierarchies which is often overlooked in the literature.

In this paper we restrict our attention to *strict* hierarchies that are represented by (rooted) directed trees (see Section 2.3), see also e.g. Radner (1992). For such strict hierarchies the Conjunctive and Disjunctive DMM's coincide and therefore we will speak only about the One Desk Conjunctive DMM.

2.2 Power

Our understanding of 'power' is based on Harré (1970) and Morriss (1987/2002) who define power as a concept that always refers to a generic ability or capacity of an object. In a social context this object is an actor and a power ascription refers to its ability: what the actor is able to do against the resistance of at least some other actor. Following Braham (2008) we say that an actor i has power with respect to a certain outcome if i has an action (or sequence of actions) such that the performance of the action under the stated or implied conditions will result in that outcome despite the actual or possible resistance of at least some other actor. That is, power is a claim about what i is able to do against some resistance of others irrespective of the actual occurrence of the resistance. Thus, power is a capacity or potential which exists whether it is exercised or not. In our context, this capacity is based on the positions of the actors in an organizational architecture. The measurement of power involves the following steps:

(i) The identification of the action profiles within the organization that are sufficient for bringing about an outcome.

(ii) The ascription of power to an individual actor in these action profiles by determining if the actor has an action that if performed will, ceteris paribus, alter the outcome of the collective action.

(iii) The aggregation of the individual power ascriptions of each actor, giving us a bare power score.

(iv) The weighting of the aggregated power ascriptions of each actor yielding a power measure.

We will make use of these four steps in Section 3 explaining our score and measure.

2.3 Directed graphs

We end the preliminaries by describing some basic concepts of directed graphs. For a finite set of actors $N \subset \mathbb{N}$ we model a hierarchical structure by a directed graph (N, D) where

$D \subseteq N \times N$ is a binary relation on N. Given a digraph (N, D) we denote $S_D(i) = \{j \in N \mid (i,j) \in D\}$ as the set of *successors* of i, and the set $S_D^{-1}(i) = \{j \in N \mid i \in S_D(j)\}$ as the set of *predecessors* of i. (We will omit the subscript D if this leads to no confusion.) Further we denote by $\widehat{S}_D(i)$ the successors of i in the *transitive closure* of D, i.e. $j \in \widehat{S}_D(i)$ if and only if there exists a sequence of actors $(h_1, \ldots, h_t)$ such that $h_1 = i$, $h_{k+1} \in S(h_k)$ for all $1 \leq k \leq t-1$, and $h_t = j$. The actors in $\widehat{S}_D(i)$ are called the *subordinates* of i, and the players in $\widehat{S}_D^{-1}(i) = \{j \in N \mid i \in \widehat{S}_D(j)\}$ are called the *superiors* of i. In this paper we restrict our attention to hierarchies with a tree structure, i.e. we consider acyclic structures with a unique actor at the top having no predecessors and being superior of all other actors, while all other actors have exactly one predecessor. These structures are also called *strict hierarchies*. Since in this paper we only consider such strict hierarchies we will often refer to them just as hierarchies. We denote the *top actor* by i_0, i.e. $|S^{-1}(i_0)| = 0$ and $|S^{-1}(j)| = 1$ for all $j \in N \setminus \{i_0\}$. Note that every directed tree has at least one actor that has no successors. We call these actors in the set $K(D) = \{i \in N \mid S_D(i) = \emptyset\}$ the *bottom actors*. For a subset $T \subset N$ we denote by $(T, D(T))$ with $D(T) = \{(i,j) \in D \mid \{i,j\} \subseteq T\}$, the subdigraph restricted to T. Finally, we denote the class of all digraphs by $\mathcal{D}$.

3 The model

3.1 Power in sequential voting

Presuming that only bottom actors may receive a proposal on their desk, we presuppose that only those have contact with the outside world. Based on this let us assume a voting procedure that combines hierarchical and polyarchical features similar to those of Sah and Stiglitz (1985, 1986), but in the context of binary voting[4].

Assume an organization described by a tree (N, D) on the set of actors N. As mentioned above, new proposals enter the organization at one desk of a bottom actor, i.e. an actor in the set $K(D)$. Let us first consider the most simple case, namely that of a line structure. Without loss of generality we consider the line given by $D = \{(i, i+1) \mid i \in \{1, \ldots, n-1\}\}$. So, $S(i) = i+1$ for all $i \in \{1, \ldots, n-1\}$ and $S(n) = \emptyset$. We consider this line as a hierarchy in the sense of Sah and Stiglitz (1986) meaning that the approval of all actors in the line is necessary for acceptance. So, in this case the proposal enters the organization at position n. If the actor in this position rejects then the proposal is rejected by the organization, and if it accepts then the proposal goes to actor $n-1$. If that actor $n-1$ rejects then the proposal is rejected and if it accepts then it goes to actor

[4]Thus, the 'correct' choice is endogenously determined by the preference profile and social welfare function, and not exogenously given as in Sah and Stiglitz (1985, 1986). For a discussion of the relation of our model to Sah and Stiglitz's work see Section 6.

$n-2$, and so on until it (possibly) reaches actor 1. Acceptance or rejection of actor 1 leads to acceptance or rejection, respectively, by the organization. Clearly, in this case the proposal is accepted if and only if all actors accept it. Note that this also implies that the proposal has reached all actors. Clearly then, in this specific circumstance all actors seem equally powerful. However, that is not so obvious if one of the actors rejects the proposal since by the sequential nature of the decision making process the proposal will not reach all superior actors who, thus, cannot cast their vote. As mentioned before, the established power measures in the literature do not take account of this sequential feature (Principle P2).

Besides the choice whether to take account of the sequential nature of the decision making process, we must make more assumptions in case the tree is not a line. In that case there exists more than one actor who might receive the proposal from the outside world, and we must make an assumption about which bottom actor receives the proposal under the One Desk Model. Applying the principle of insufficient reason of classical probability theory[5], we assume that each bottom actor receives the proposal with equal probability[6]. Then a sequential hierarchical decision process in the line from the corresponding bottom position to the top will start[7]. In this way, when measuring power we can focus on truncated hierarchies, i.e. we consider a tree as a union of lines.

3.2 Power score and measure

In van den Brink and Steffen (2008) a power score and measure are proposed for general DMMs in hierarchies. There we argue, in a more general setting, that step (i), i.e. identifying action profiles, requires a set-up that allows for an *action-based* representation of a DMM in an organizational architecture. Similar as with models of simultaneous voting, for the One Desk Conjunctive DMM in a binary voting model, the score of an actor is determined by its number of swings. In traditional (simultaneous) voting, a swing of an actor is an action profile where the outcome changes if the actor changes its vote, given that the other actors do not change their vote. This works well in a simultaneous voting situation

[5]In the absence of any information about the outside world the application of the principle of insufficient reason appears to be legitimate here as we fulfill the condition that we have a finite probability space consisting of finitely many clearly distinguished indivisible 'atomic events' (Felsenthal et al. 2003).

[6]Of course, other probability distributions than this uniform distribution may be considered.

[7]Alternatively, and in line with Sah and Stiglitz's (1985, 1986) idea of a polyarchy, one may, for instance apply an All Desk Model stating that the proposal enters at all bottom positions, and that one full hierarchical approval in one of the lines is sufficient for approval by the organization, see the concluding remarks at the end of this paper. Another intermediate version would be a Multi Desk Model assuming that a new proposal enters the organization at more than one but less than all desks belonging to bottom actors.

where every action profile consists of n votes, one for each actor. Usually these profiles are represented by *simple games*, i.e. sets of coalitions voting 'yes', while the complement consists of the actors that vote 'no'. Therefore, this is referred to as a *membership-based* approach. However, in a sequential DMM as considered here, not all actors vote in every action profile. This creates several problems in step (ii), for example with respect to the *ceteris paribus* clause. It might be that with a certain action of actor i, another actor j will not vote, but when i changes its action, then j gets to vote and has an influence on the outcome. Then i changing its action might lead to a different outcome, but not necessarily (depending on the voting behavior of j and possibly other actors). On one hand we cannot consider this to be a 'full' swing for actor i since it cannot, for sure, change the outcome, given that the other actors vote the same (because in the original action profile j did not vote). On the other hand, by changing its vote i can create a situation where the outcome might change, so we can still consider this to be some kind of swing. Therefore, we distinguish between *strong* and *weak* swings. In our case of a binary outcome set actor i has a strong swing if, by changing its action it changes the outcome for sure (i.e. turns a rejection into an acceptance or vice versa). Actor i has a weak swing, if by changing its action it allows other outcomes, but also the original outcome is still possible.

We illustrate this with an example.

Example 3.1 Consider the hierarchy (N, D) with $N = \{1, 2\}$ and $D = \{(2, 1)\}$. In a simultaneous voting situation we would consider four action (or voting) profiles:

$$(yes, yes),\ (yes, no),\ (no, yes) \text{ and } (no, no).$$

However, in our DMM, if actor 1 votes 'no' then the voting stops and the proposal is rejected. So, we have only three action profiles: (yes, yes), (yes, no) and (no). Actor 2 has a strong swing in the first two profiles: by changing its vote the outcome changes from acceptance to rejection, respectively the other way round. Actor 1 has a strong swing in profile (yes, yes) since changing its vote would change an acceptance into a definite rejection. However, considering the profile (no) which yields rejection, by changing its vote the outcome might still be rejection, but also acceptance is possible, depending on the vote of actor 2. Therefore we refer to this as a weak swing. □

It is usually said that an actor i has a positive swing, if i by switching from a 'no'- to a 'yes'-action can alter the outcome from rejection to acceptance and has a negative swing, if by switching from a 'yes'- to a 'no'-action can alter the outcome from acceptance to rejection.

To formally measure power, taking account of the sequential nature of the DMM, van den Brink and Steffen (2008) introduced a power score and measure as follows[8]. In the power score (step iii) for every actor its expected number of strong swings are fully counted while its expected number of weak swings are counted only for a fraction $\epsilon \in [0,1]$.[9] Thus, if $\epsilon = 1$ weak swings are counted the same as strong swings, while weak swings are not counted at all if $\epsilon = 0$.

For every bottom subordinate of actor i the number of strong and weak swings containing that particular bottom subordinate is the same. Therefore, by $|SW_i^s|$ we denote the number of strong swings of actor i containing a particular bottom subordinate of i, and by $|SW_i^w|$ we denote the number of weak swings of actor i containing a particular bottom subordinate of i. Since we assumed the proposal to enter every bottom desk with equal probability, the power score $\eta^\epsilon(N,D)$ of actor i is given by

$$\eta_i^\epsilon(N,D) = \frac{|\overline{S}(i)|}{|K(D)|}(|SW_i^s| + \epsilon|SW_i^w|), \tag{3.1}$$

where $\overline{S}(i) = (\widehat{S}(i) \cup \{i\}) \cap K(D)$ (and, thus, $\frac{|\overline{S}(i)|}{|K(D)|}$ is the probability that the proposal enters at the desk of one of the bottom subordinates of i or at i's desk if i is a bottom actor itself).

Consider a bottom actor $i \in K(D)$. This actor has one strong and one weak swing. The strong swing results from the action profile where every actor votes 'yes'. Clearly, this action profile yields acceptance, while actor i switching its vote from 'yes' to 'no' yields rejection of the proposal. Bottom actor i also has one weak swing, namely the action profile where it votes 'no'. This leads to a rejection, which might change if the actor votes 'yes', but not necessarily changes. Since all bottom actors receive the proposal with equal probability, i's swings occur with probability $\frac{1}{|K(D)|}$. Considering the actors $i \notin K(D) \cup \{i_0\}$ that are not at the bottom nor at the top, the same reasoning can be followed for each of their subordinates at the bottom. Hence, for each truncated hierarchy 'middle' actor i is a member of, i has one weak and one strong swing, each occurring with probability $\frac{1}{|K(D)|}$. Finally, the top actor i_0 has two strong swings for every bottom actor.

Since (i) every nontop actor has one strong and one weak swing associated to each of it subordinate bottom actors, and (ii) the top-actor has two strong swings associated to

[8]The binary setup in this paper allows us to represent our action-based score and measure by the digraph of the dominance structure. For a more general representation using extensive game forms we refer to van den Brink and Steffen (2008).

[9]Note that in van den Brink and Steffen (2008) we assume $\epsilon \in (0,1)$ as it appears conceptually reasonable to exclude $\epsilon = 0$ and $\epsilon = 1$. However, for the axiomatization of the score and measure it turns out to be useful to formally allow for both extremes.

every bottom actor, the power score $\eta^\epsilon(N, D)$ of actor $i \neq i_0$ given by (3.1) is equal to

$$\eta_i^\epsilon(N, D) = \begin{cases} \frac{2|\overline{S}(i_0)|}{|K(D)|} = 2 & \text{if } i = i_0 \\ \\ \frac{|\overline{S}(i)|}{|K(D)|}(1+\epsilon) & \text{if } i \neq i_0. \end{cases} \tag{3.2}$$

To define the power measure (step iv), the power score of every actor is divided by the expected number of action profiles it is a member of [10]. Every bottom actor (who can receive the proposal from the outside world), and all its superiors are member of a 'positive' profile where they all vote 'yes', which yields acceptance. Further, in this truncated hierarchy the bottom actor is member of the 'negative' profile where it votes 'no'. Each of its superior actors is member of a 'negative' profile where every subordinate on the path from this actor to the bottom actor votes 'yes', and this actor votes 'no', yielding rejection. (Note that every such action profile consists of a number of consecutive 'yes' votes, and ends with a 'no' vote.) Clearly, there are $|\widehat{S}^{-1}(i)|+1$ of such action profiles for bottom actor $i \in K(D)$. So, for each of its subordinate bottom actors, every actor i is member of $|\widehat{S}^{-1}(i)|+2$ action profiles containing this bottom subordinate, and the probability that nature chooses an action profile containing one of i's bottom actors or i itself if i is a bottom actor is $\frac{|\overline{S}(i)|}{|K(D)|}$. This yields the $\hat{\beta}^\epsilon$- power measure

$$\hat{\beta}_i^\epsilon(N, D) = \frac{\eta_i^\epsilon(N, D)}{\frac{|\overline{S}(i)|}{|K(D)|}\left(|\widehat{S}^{-1}(i)|+2\right)} = \begin{cases} \frac{2}{1(0+2)} = 1 & \text{if } i = i_0 \\ \\ \frac{\frac{|\overline{S}(i)|}{|K(D)|}(1+\epsilon)}{\frac{|\overline{S}(i)|}{|K(D)|}\left(|\widehat{S}^{-1}(i)|+2\right)} = \frac{(1+\epsilon)}{\left(|\widehat{S}^{-1}(i)|+2\right)} & \text{if } i \neq i_0. \end{cases} \tag{3.3}$$

Note, that the composition of the denominator of this power measure shows a significant difference compared to the denominator of the well known Banzhaf measure for simultaneous voting situations. To obtain the Banzhaf measure from the Banzhaf score for an actor i its score is divided by 2^{n-1}. Representing the voting situation by a simple game being a 'one-sided' model based upon so-called 'winning coalitions' this number corresponds to the number of coalitions actor i is a member of and in which all members vote 'yes'. In terms of action profiles this is equal to the number of all action profiles containing actor i and in which i votes 'yes'. However, in principle, 2^{n-1} could also be interpreted to be

[10] In the more general setting considered in van den Brink and Steffen (2008) the denominator is the expected number of alternative actions in action profiles actor i is a member of. Since in the binary voting model considered here an actor can choose only between two actions, this denominator boils down to the expected number of action profiles i is a member of. In order to obtain this expected number the action profiles of each truncated hierarchy i is a member of are weighted by the probability that this truncated hierarchy occurs, i.e. by the probability that a proposal enters at the desk of the bottom actor of this truncated hierarchy.

the number of coalitions actor i is a member of and in which all members vote 'no' or, in terms of action profiles, as the number of all action profiles containing actor i and in which i votes 'no'. Which interpretation is used makes no difference to the power ascribed by the Banzhaf measure as under a simultaneous voting situation the number of both types of coalitions (and action profiles) is identical. However, this equality does no longer hold for sequential voting situations. We showed above that in sequential voting, for each of its subordinate bottom actors an actor i has one action profile in which it votes 'no' (in which, moreover, i has a positive weak swing), and $|\widehat{S}^{-1}(i)| + 1$ action profiles in which i votes 'yes' (in one of those i has a positive strong swing). Thus, following the idea of the Banzhaf measure and modifying the denominator in (3.3) by dividing our power score $\eta_i^\epsilon(N, D)$ by the number of action profiles in which i votes 'yes' or i votes 'no' yields two different power measures. If we would define the measure by dividing the power score to the expected number of action profiles in which an actor votes 'no', then the measure is equal to the power score, since for every subordinate bottom actor there is only one such an action profile. However, the β^ϵ-measure that is obtained by dividing the power score to the expected number of action profiles in which an actor votes 'yes' is not equal to the power score, and is given by

$$\beta_i^\epsilon(N, D) = \frac{\eta_i^\epsilon(N, D)}{\frac{|\overline{S}(i)|}{|K(D)|}\left(|\widehat{S}^{-1}(i)| + 1\right)} = \begin{cases} \frac{2}{1} = 2 & \text{if } i = i_0 \\ \frac{\frac{|\overline{S}(i)|}{|K(D)|}(1+\epsilon)}{\frac{|\overline{S}(i)|}{|K(D)|}\left(|\widehat{S}^{-1}(i)|+1\right)} = \frac{(1+\epsilon)}{\left(|\widehat{S}^{-1}(i)|+1\right)} & \text{if } i \neq i_0. \end{cases} \tag{3.4}$$

Example 3.2 For the hierarchy of Example 3.1 the power score and measures, respectively, are given by $\eta^\epsilon(N, D) = (1 + \epsilon, 2)$, $\hat{\beta}^\epsilon(N, D) = (\frac{1}{3}(1 + \epsilon), 1)$ and $\beta^\epsilon(N, D) = (\frac{1}{2}(1 + \epsilon), 2)$. □

In the next sections we present axiomatic characterizations of the η^ϵ-score and β^ϵ-measure of power under the One Desk assumption.

4 Axiomatizations of the power score

In this section we provide axiomatizations of the power score η^ϵ given by (3.2). We first discuss the extreme cases $\epsilon \in \{0, 1\}$, from which the general result follows.

4.1 The case $\epsilon = 1$

We begin with the special case where strong and weak swings are assigned equal weight. For $\epsilon = 1$ the power score (3.2) equals

$$\eta_i^1(N, D) = \frac{2|\overline{S}(i)|}{|K(D)|} \text{ for all } i \in N. \tag{4.5}$$

Next, we provide two axiomatizations of the power score η^1. The first one uses three axioms. As mentioned earlier, our view of power is based on a definition of power as an ability or capacity which exists whether it is exercised or not. What counts is what an actor is able to do in its position in the hierarchy if a proposal reaches him. Thus, its power is independent from the choices of his successors (in the dominance structure). Furthermore, according to our definition of a swing the top has two (strong) swings. Hence, we postulate that the top should have a power score of two having in mind that the score of an actor is the result of the aggregation of its individual power ascription (step (iii) in Section 2.2). In the following we denote by f a generic power score or measure.

Axiom 4.1 (Normalization) $f_{i_0}(N, D) = 2$.

Note, that due to the fact that we truncate all strict hierarchies which are not already a line into overlapping lines with the top being the unique actor who is member of all truncated hierarchies, the probabilities determining which bottom actor receives a new proposal under the One Desk Model do not affect the power score of the top.

The principle of truncation of hierarchies is formulated in the following axiom which states that the power score of an actor in a hierarchy is equal to the average of its power scores in all truncated hierarchies. To every bottom actor $j \in K(D)$ is associated a truncated hierarchy $(\widehat{S}^{-1}(j) \cup \{j\}, D(\widehat{S}^{-1}(j) \cup \{j\}))$ which consists of this bottom actor with all its superiors and the dominance relations between them. We denote by $\mathcal{T}_D = \{(\widehat{S}^{-1}(j) \cup \{j\}, D(\widehat{S}^{-1}(j) \cup \{j\}))\}_{j \in K(D)}$ the set of all truncated hierarchies in (N, D). For $i \in N$ we denote by $\mathcal{T}_D^i = \{(T, D(T)) \in \mathcal{T}_D \mid i \in T\} = \{(\widehat{S}^{-1}(j) \cup \{j\}, D(\widehat{S}^{-1}(j) \cup \{j\})\}_{j \in \overline{S}(i))}$ the set of all truncated hierarchies that contain actor $i \in N$.[11]

Axiom 4.2 (Truncation) *For every hierarchy* (N, D) *it holds that*
$f_i(N, D) = \frac{1}{|K(D)|} \sum_{(T, D(T)) \in \mathcal{T}_D^i} f(T, D(T))$.

For firms with constant span of control (meaning that every actor that is not a bottom actor has the same number of successors) and identical workers, Williamson (1967) states that the ratio between the wage of a manager and a successor lies between one and the span

[11]Recall from Section 2.3 that $D(T)$ denotes the subgraph of D restricted to $T \subset N$.

of control (i.e. the number of the manager's successors). In van den Brink (2008) a (co-operative) game theoretic model and corresponding class of wage functions are introduced that satisfy these bounds in a more general context, where the span of control does not have to be constant[12]. This result states that in a firm with a strict hierarchical structure (i) the wage (to be replaced by power in our case) of a manager is always at least as high as the wage of each of its successors (and thus by repeated application of this property as the wage of each of its subordinates), and (ii) the wage of a manager never exceeds the sum of the wages of its successors. Applying these bounds in developing a power score for hierarchies we obtain the following property.

Axiom 4.3 (Bound property) *For every $i \in N \setminus K(D)$ it holds that*

(i) $f_i(N,D) \geq f_j(N,D)$ *for all* $j \in S(i)$, *and*

(ii) $f_i(N,D) \leq \sum_{j \in S(i)} f_j(N,D)$.

In case of a line-hierarchy (N,D) (i.e. $|S(i)| = 1$ for all $i \in N \setminus K(D)$) the bound property determines that all scores are equal, and by normalization all scores are determined to be equal to 2. Then truncation determines the power scores for arbitrary hierarchies. Since it is straightforward to verify that η^1 satisfies these three axioms, we state the following axiomatization without further proof[13].

Theorem 4.4 *A power score f is equal to η^1 if and only if it satisfies normalization, truncation and the bound property.*

It seems that truncation and the bound property are rather strong axioms to require from a power score. Therefore we suggest the following three alternative axioms. The first one is the axiom of symmetry which is standard.

Axiom 4.5 (Symmetry) *If $S(i) = S(j)$ and $S^{-1}(i) = S^{-1}(j)$ then $f_i(N,D) = f_j(N,D)$.*

The second axiom is an independence axiom. This follows Arrow's famous 'independence of irrelevant alternatives' axiom in social choice theory. Here we assume that given that we are organizing the same set of bottom actors, the power of an actor only depends on the structure of its superiors and subordinates. With this we mean that as long as i and all its subordinates have the same successors, and i and all its superiors have the same predecessors, the power score of i does not change.

[12]The production functions are supermodular and satisfy the zero-property meaning that nothing is produced with zero labor effort.

[13]Moreover, the uniqueness part of this theorem also follows from Theorem 4.9 which proof we give later.

Axiom 4.6 (Independence of outside organization) *Consider two hierarchies* $(N, D), (N', D')$ *with* $i \in N \cap N'$. *If* $S_D(j) = S_{D'}(j)$ *for all* $j \in \widehat{S}_D(i) \cup \{i\}$, $S_D^{-1}(j) = S_{D'}^{-1}(j)$ *for all* $j \in \widehat{S}_D^{-1}(i) \cup \{i\}$, *and* $K(D) = K(D')$, *then* $f_i(N, D) = f_i(N', D')$.

Finally, we reflect the principle of truncation of the hierarchy (with overlapping memberships) by requiring that the power score of a non-bottom actor equals the sum of the powers of its successors.

Axiom 4.7 (Successor-sum property) *For every* $i \in N \backslash K(D)$ *it holds that* $f_i(N, D) = \sum_{j \in S(i)} f_j(N, D)$.

So, the successor-sum property relates the power score of an actor i to those of its successors in the hierarchy, while truncation (Axiom 4.2) relates i's score to its own scores in all truncated hierarchies.

Since the changes made in the hierarchy according to the independence of outside organization axiom do not change the structure of the truncated hierarchies that involve agent i, it is obvious that truncation implies independence of outside organization. Further, truncation and the bound property together imply the successor-sum property.

Proposition 4.8 *If power score* f *satisfies truncation and the bound property then it satisfies the successor-sum property.*

Proof

Suppose that power score f satisfies truncation and the bound property. Consider a hierarchy (N, D) and an actor $h \in N$ with $S(h) \neq \emptyset$.

For every truncated hierarchy $(T, D(T)) \in \mathcal{T}_D^h$ the bound property implies that all actors in T have the same power score. In particular actors h and $j \in S(h) \cap T$ have the same power score. Since $\{\overline{S}(j)\}_{j \in S(h)}$ is a partition of $\overline{S}(h)$, the truncation property implies that $f_h(N, D) = \sum_{j \in S(h)} f_j(N, D)$. So, f satisfies the successor-sum property. □

Replacing in Theorem 4.4 the truncation and bound properties by symmetry, independence of outside organization and the successor-sum property, also yields an axiomatization of the power score η^1.

Theorem 4.9 *A power score* f *is equal to* η^1 *if and only if it satisfies normalization, symmetry, independence of outside organization and the successor-sum property.*

Proof

It is straightforward to verify that η^1 satisfies normalization, symmetry and independence of outside organization. The successor-sum property follows since by $\{\overline{S}(j)\}_{j \in S(i)}$ being a partition of $\overline{S}(i)$, we have that $\sum_{j \in S(i)} \eta_j^1(N, D) = \sum_{j \in S(i)} \frac{2|\overline{S}(j)|}{|K(D)|} = \frac{2|\overline{S}(i)|}{|K(D)|} = \eta_i^1(N, D)$.

To show uniqueness assume that the power score f satisfies the four axioms, and consider hierarchy (N, D). Define a reduced hierarchy (N^0, S^0) by $N^0 = \{i_0\} \cup K(D)$, $S^0(i_0) = K(D)$ and $S^0(j) = \emptyset$ for all $j \in K(D)$, see Figures 1 and 2 as an illustration of deleting middle actors. Normalization implies that $f_{i_0}(N^0, S^0) = 2$. Symmetry implies that all actors in $K(D)$ have the same power score in (N^0, S^0). Since the successor-sum property implies that the sum of the powers of all actors in $K(D)$ should add up to the power score of the top i_0, it holds that $f_j(N^0, S^0) = \frac{2}{|K(D)|}$ for all $j \in K(D)$.
Next take an $i \in K(D)$. Define hierarchy (N^i, S^i) by $N^i = K(D) \cup \widehat{S}^{-1}(i)$ (note that this implies that $i_0 \in N^i$) and

$$S^i(j) = \begin{cases} (K(D) \setminus \{i\}) \cup (S(i_0) \cap (\widehat{S}^{-1}(i) \cup \{i\})) & \text{if } j = i_0 \\ \\ S(j) \cap (\widehat{S}^{-1}(i) \cup \{i\}) & \text{if } j \in \widehat{S}^{-1}(i) \setminus \{i_0\} \\ \\ \emptyset & \text{otherwise,} \end{cases}$$

see Figure 3 for an example. Normalization implies that $f_{i_0}(N^i, S^i) = 2$.
Independence of outside organization implies that $f_j(N^i, S^i) = f_j(N^0, S^0) = \frac{2}{|K(D)|}$ for all $j \in K(D) \setminus \{i\}$.
Since the successor-sum property implies that the sum of the powers of all actors in $K(D) \setminus \{i\}$ and that of the actor in $S(i_0) \cap (\widehat{S}^{-1}(i) \cup \{i\})$ should add up to the given power of 2 for the top i_0, it follows that $f_j(N^i, S^i) = \frac{2}{|K(D)|}$ for $j \in S(i_0) \cap (\widehat{S}^{-1}(i) \cup \{i\})$. Repeated application of the successor-sum property further determines that $f_j(N^i, S^i) = \frac{2}{|K(D)|}$ for all $j \in \widehat{S}^{-1}(i)$, and thus eventually $f_i(N^i, S^i) = \frac{2}{|K(D)|}$ is determined.
Independence of outside organization then implies that $f_i(N, D) = f_i(N^i, S^i) = \frac{2}{|K(D)|} = \eta_i^1(N, D)$.
Since all $f_i(N, D)$ are thus determined for $i \in K(D)$, the successor-sum property determines the power score for all other actors, and thus $f(N, D)$ is uniquely determined. $\square$

4.2 The case $\epsilon = 0$

Next we consider the other extreme case where weak swings are assigned weight zero. Compared to the case $\epsilon = 1$ discussed above, all non-top actors $i \neq i_0$ 'lose' their weak

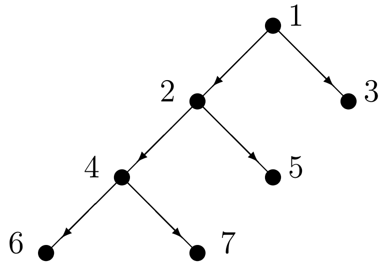

Figure 1: A hierarchy (N, D)

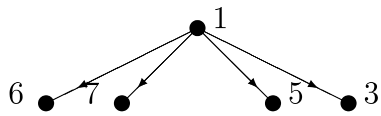

Figure 2: Hierarchy (N^0, S^0)

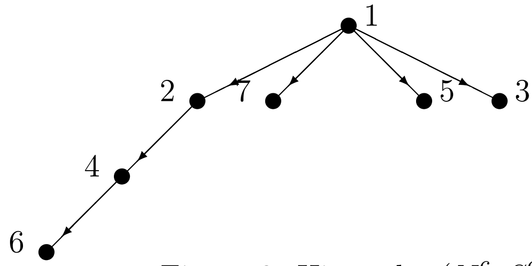

Figure 3: Hierarchy (N^6, S^6)

swings, i.e. the swings where they vote 'no' and their subordinates who receive the proposal on their desk (if they exist) voted 'yes'. But they still have their strong swings, where every actor involved voted 'yes'. Since for the top actor i_0 all swings are strong, for $\epsilon = 0$ the power score given by (3.2) is equal to

$$\eta_i^0(N,D) = \begin{cases} \eta_i^1(N,D) & \text{if } i = i_0 \\ \frac{|\overline{S}(i)|}{|K(D)|} = \frac{1}{2}\eta_i^1(N,D) & \text{for all } i \neq i_0. \end{cases}$$

For $\epsilon = 0$ the axioms of normalization, symmetry and independence of outside organization still hold. The successor-sum property only holds outside the top.

Axiom 4.10 (Outside the top successor-sum property) *For every $i \in N \setminus (K(D) \cup \{i_0\})$ it holds that $f_i(N,D) = \sum_{j \in S(i)} f_j(N,D)$.*

The possibility of the top to finalize the decision to a 'yes' (if it gets the opportunity to vote) is now reflected in a doubling of the sum of the powers of its subordinates.

Axiom 4.11 (Top successor-sum property) *$f_{i_0}(N,D) = 2\sum_{j \in S(i_0)} f_j(N,D)$.*

Replacing the successor-sum property by these two axioms in Theorem 4.9 characterizes η^0.

Theorem 4.12 *A power score f is equal to η^0 if and only if it satisfies normalization, symmetry, independence of outside organization, the outside the top successor-sum property and the top successor-sum property.*

The proof goes similar as the proof of Theorem 4.9 (the only difference is that where the successor-sum property is used concerning the top actor, now the top successor-sum property has to be used) and is therefore omitted.

It is straightforward to verify that the power score η^0 also satisfies truncation. However, the bound property must be adapted, taking account of the 'special' top position.

Axiom 4.13 (Outside the top bound property) *For every $i \in N \setminus (K(D) \cup \{i_0\})$ it holds that*

(i) *$f_i(N,D) \geq f_j(N,D)$ for all $j \in S(i)$, and*

(ii) *$f_i(N,D) \leq \sum_{j \in S(i)} f_j(N,D)$.*

Axiom 4.14 (Top bound property) *For top actor i_0 in hierarchy (N,D) it holds that*

(i) $f_{i_0}(N,D) \geq 2f_j(N,D)$ *for all* $j \in S(i_0)$, *and*

(ii) $f_{i_0}(N,D) \leq 2\sum_{j\in S(i_0)} f_j(N,D)$.

We state the following result without proof.

Theorem 4.15 *A power score f is equal to η^0 if and only if it satisfies normalization, truncation, the outside the top bound property and the top bound property.*

4.3 The case $\epsilon \in [0,1]$

Finally, let us consider the general case. From the two extreme cases considered in the previous two subsections we can derive that for $\epsilon \in [0,1]$ the power score is given by

$$\eta^\epsilon(N,D) = \eta^0(N,D) + \epsilon(\eta^1(N,D) - \eta^0(N,D)) = \epsilon\eta^1(N,D) + (1-\epsilon)\eta^0(N,D),$$

and, thus, is a convex combination of η^0 and η^1. Taking account of $\epsilon \in [0,1]$ with respect to the Top successor-sum property in a straightforward way, we obtain the following axiom.

Axiom 4.16 (ϵ-top successor-sum property) $f_{i_0}(N,D) = \frac{2}{1+\epsilon}\sum_{j\in S(i_0)} f_j(N,D)$.

Replacing the top successor-sum property by this axiom in Theorem 4.12 characterizes η^ϵ.

Theorem 4.17 *A power score f is equal to η^ϵ, $\epsilon \in [0,1]$, if and only if it satisfies normalization, symmetry, independence of outside organization, the outside the top successor-sum property and the ϵ-top successor-sum property.*

The proof goes similar as the proofs of Theorems 4.9 and 4.4, and is therefore omitted. In a similar way the top bound property can be generalized.

Axiom 4.18 (ϵ-top bound property) (i) $f_{i_0}(N,D) \geq \frac{2}{1+\epsilon} f_j(N,D)$ *for all* $j \in S(i_0)$, *and*

(ii) $f_{i_0}(N,D) \leq \frac{2}{1+\epsilon}\sum_{j\in S(i_0)} f_j(N,D)$.

We state the following without proof.

Theorem 4.19 *A power score f is equal to η^ϵ if and only if it satisfies normalization, truncation, the outside the top bound property and the ϵ-top bound property.*

5 An axiomatization of the power measure

Next we adapt the axioms of the previous section in order to obtain an axiomatization of a measure of power under the One-Desk assumption. We focus our attention on the β^ϵ-measure, see (3.4). Again, we first consider the case $\epsilon = 1$ where all weak swings are fully counted. Substituting the power score η^1 as given by (4.5) yields the power measure

$$\beta_i^1(N,D) = \frac{\eta_i^1(N,D)}{\frac{|\overline{S}(i)|}{|K(D)|}\left(|\widehat{S}^{-1}(i)|+1\right)} = \frac{2}{|\widehat{S}^{-1}(i)|+1} \text{ for all } i \in N.$$

To characterize this power measure we first consider what properties of the power score it satisfies. It is easy to verify that it satisfies normalization, symmetry and independence of outside organization[14]. It does not satisfy the successor-sum property.

With respect to the independence axiom, β^1 even satisfies the stronger independence axiom where we do not require the set of bottom actors to be the same. Moreover, it does not depend on the successors of subordinates.

Axiom 5.1 (Strong independence of outside organization) *Consider two hierarchies* $(N,D),(N',D')$ *with* $i \in N \cap N'$. *If* $S_D^{-1}(j) = S_{D'}^{-1}(j)$ *for all* $j \in \widehat{S}_D^{-1}(i)$, *then* $f_i(N,D) = f_i(N',D')$.

Note, that deleting the requirement $K(D) = K(D')$ can be interpreted as the power measure not depending on the probabilities with which bottom actors are chosen to receive the proposal.

For the axiomatization we add another axiom that discusses a special type of regular hierarchies. We call a hierarchy a regular hierarchy if the top actor has two successors and in every level (except the bottom level) the number of successors of the actors in that level is one more than that of the direct superior level. So, a hierarchy (N,D) is *regular* if $|S(i)| = |\widehat{S}^{-1}(i)| + 2$ for all $i \in N \setminus K(D)$.

We define an *office* as the set of successors of the same predecessor, i.e. every $S(i),\ i \in N \setminus K(D)$, is an office. Here we also consider the singleton $\{i_0\}$ as an office. The next axiom says that the total power in any office in a regular hierarchy is the same. We denote the set of all offices in hierarchy (N,D) by $\mathcal{P}(D) = \{S(i) \mid i \in N \setminus K(D)\} \cup \{\{i_0\}\}$

Axiom 5.2 (Regularity) *If* (N,D) *is a regular hierarchy, then* $\sum_{i\in P} f_i(N,D) = \sum_{i\in P'} f_i(N,D)$ *for every pair of offices* $P, P' \in \mathcal{P}(D)$.

These axioms characterize the power measure β^1.

[14]It does satisfy another truncation property where to determine the power measure for an actor i the average is taken over all truncated hierarchies containing i and not all truncated hierarchies as done for the power score, i.e. $f_i(N,D) = \frac{1}{|\overline{S}(i)|}\sum_{j\in\overline{S}(i)} f(\widehat{S}^{-1}(j)\cup\{j\}, D(\widehat{S}^{-1}(j)\cup\{j\}))$.

Theorem 5.3 *A power measure f is equal to β^1 if and only if it satisfies normalization, symmetry, strong independence of outside organization and regularity.*

Proof

It is straightforward to verify that β^1 satisfies normalization, symmetry and strong independence of outside organization. Consider a regular hierarchy (N, D) with $i \in P \in \mathcal{P}(D)$. By regularity, $|P| = |\widehat{S}^{-1}(i)| + 1$, and thus $\sum_{i \in P} \beta^1_i(N, D) = \sum_{i \in P} \frac{2}{|\widehat{S}^{-1}(i)|+1} = (|\widehat{S}^{-1}(i)| + 1) \cdot \frac{2}{|\widehat{S}^{-1}(i)|+1} = 2 = \beta^1_{i_0}(N, D)$, showing that β^1 satisfies regularity.

To show uniqueness assume that the power measure f satisfies the four axioms, and consider hierarchy (N, D).

Next, consider actor $i \in N \setminus \{i_0\}$, and define the reduced hierarchy (N^i, D^i) by $N^i = \{i\} \cup \widehat{S}^{-1}(i)$ and $D^i = D(N^i)$. Next consider a set of actors $\overline{N}^i \supset N^i$ with $|\overline{N}^i| = 1 + \sum_{k=1}^{|\widehat{S}^{-1}(i)|} k(k+1)$, and $\overline{D}^i$ such that $(\overline{N}^i, \overline{D}^i)$ is a regular hierarchy with $S_{\overline{D}^i}(j) \supseteq S_{D^i}(j)$ for all $j \in \widehat{S}^{-1}_{D^i}(i)$. (Note that we choose $|\overline{N}^i|$ such that a regular hierarchy exists.) Normalization implies that $f_{i_0}(\overline{N}^i, \overline{D}^i) = 2$. Let $S^{-1}(i) = \{h\}$. Regularity implies that $\sum_{j \in S_{\overline{D}^i}(h)} f_j(\overline{N}^i, \overline{D}^i) = 2$. Since by regularity of $(\overline{N}^i, \overline{D}^i)$ we have $|S_{\overline{D}^i}(h)| = |\widehat{S}^{-1}(i)| + 1$, symmetry then requires that $f_i(\overline{N}^i, \overline{D}^i) = \frac{2}{|\widehat{S}^{-1}(i)|+1}$. Independence of outside organization finally implies that $f_i(N, D) = f_i(\overline{N}^i, \overline{D}^i) = \frac{2}{|\widehat{S}^{-1}(i)|+1}$. □

Considering the case that weak swings are not counted at all ($\epsilon = 0$), we obtain the power measure β^0 given by

$$\beta^0_i(N, D) = \frac{\eta^0_i(N, D)}{\frac{|\overline{S}(i)|}{|K(D)|}\left(|\widehat{S}^{-1}(i)| + 1\right)} = \begin{cases} 2 & \text{if } i = i_0 \\ \frac{1}{|\widehat{S}^{-1}(i)|+1} & \text{for all } i \neq i_0 \end{cases}$$

This power measure satisfies the axioms of Theorem 5.3 except regularity. It still satisfies regularity as long as we do not consider the singleton $\{i_0\}$ as an office.

Hence, for the top actor we need a separate regularity condition. We denote by $\overline{\mathcal{P}}(D) = \mathcal{P}(D) \setminus \{\{i_0\}\}$ the set of all offices except the top-office.

Axiom 5.4 (Top-regularity) *If (N, D) is a regular hierarchy, then $f_{i_0}(N, D) = 2 \sum_{i \in P} f_i(N, D)$ for every office $P \in \overline{\mathcal{P}}(D)$.*

Replacing regularity in Theorem 5.3 by top regularity characterizes β^0.

Theorem 5.5 *A power measure f is equal to β^0 if and only if it satisfies normalization, symmetry, strong independence of outside organization, and top-regularity.*

Proof

It is straightforward to verify that β^0 satisfies these axioms. With respect to uniqueness, normalization fixes the power measure for i_0. The power measures for the other actors then are determined by top-regularity and symmetry. Uniqueness for the power measure of the other actors follows similar as the uniqueness proof of Theorem 5.3 $\square$

Finally, we can obtain the power measures $\beta^\epsilon,\ \epsilon \in [0,1]$, as

$$\beta^\epsilon(N,D) = \epsilon\beta^1(N,D) + (1-\epsilon)\beta^0(N,D).$$

For $\epsilon \in [0,1]$ we generalize top-regularity in a similar way as we generalized the top successor-sum property for power scores.

Axiom 5.6 (ϵ-Top regularity) *If (N,D) is a regular hierarchy, then $f_{i_0}(N,D) = \frac{2}{1+\epsilon}\sum_{i\in P} f_i(N,D)$ for every office $P \in \overline{\mathcal{P}}(D)$.*

Replacing top-regularity in Theorem 5.5 by this axiom characterizes β^ϵ.

Theorem 5.7 *A power measure f is equal to β^ϵ if and only if it satisfies satisfies normalization, symmetry, strong independence of outside organization, and ϵ-Top regularity.*

The proof goes similar as the proof of the previous theorems and is therefore omitted.

6 Concluding remarks

In this paper we applied the power score and measure introduced in van den Brink and Steffen (2008) for general sequential decision making mechanisms in hierarchies to the special class of binary voting problems. We provided axiomatizations of both a power score and power measure, which can be seen as analogues of the Banzhaf score and measure for simple games (modeling simultaneous voting situations).

We want to wind up this paper with some remarks on the indicated relation of our model to Sah and Stiglitz's (1985, 1986) work on hierarchies (see Section 3)[15]. Their work focuses on the relation between the architecture of an organization and its members' collective competence to detect the truth of a proposition, i.e. they have to decide whether a proposition is true or false.

In general, a competence is defined as the state of being adequately qualified to do something, which implies that there exists a special type of ability containing an evaluative

[15]While in Sah and Stiglitz (1986) the analysis is restricted to polyarchies and hierarchies with a line structure only, in Sah and Stiglitz (1985) also more complex architectures as investigated in the underlying paper are taken into account.

component in addition to the bare ability to make a decision, i.e. to choose a non-empty proper subset of elements out of an outcome set (see van den Brink and Steffen 2008)[16]. In Sah and Stiglitz's case this evaluative component is the ability to detect the truth in addition to the bare ability to choose between elements of the action set containing the elements 'true' and 'false'. Hence, like in our case the action and outcome set is binary. Sah and Stiglitz commence with the assumption that each member of an organization possesses an exogenously given individual competence (ability) to detect the truth (see footnote 4). Via the architecture of the organization, i.e. the dominance structure and the DMM in use, these individual competences (abilities) are then aggregated into a collective competence (ability) of the organization. However, by inserting the members of an organization into their positions, i.e. by the aggregation mechanism via the organizational architecture, their individual bare ability to make a decision is affected by the ability of their positions to make a decision. The latter ability is the subject matter of the underlying paper: positional power. Both abilities together establish the collective competence of the organization[17]. Thus, in their analysis Sah and Stiglitz implicitly integrate individual and positional abilities without neither distinguishing between the components nor discussing whether their composition is appropriate or not. However, doing so they analyze the effect of different organizational architectures on the collective competence of an organization, i.e. they investigate how different distributions of positional power affect the aggregated extent of the exogenously given individual competences of these actors.

References

Banzhaf, J.F. (1965), "Weighted Voting Doesn't Work: A Mathematical Analysis", *Rutgers Law Review*, 19, 317-343.

Berg, S. and J. Paroush (1998), "Collective Decision Making in Hierarchies", *Mathematical Social Sciences*, 35, 233-244.

Braham, M. (2008), "Social Power and Social Causation: Towards a Formal Synthesis", in M. Braham and F. Steffen (eds), Power, Freedom, and Voting, Springer, 1-21.

[16]In a recent paper on the relation of Sah and Stiglitz's work to theory of voting power, Kaniovski (2008) has called this special type of ability 'problem solving power'. Unfortunately, Kaniovski fails to provide a rigorous analysis whether this terminology withstands a deeper conceptual examination. Therefore, we refrain from making use of it here.

[17]Note, that following Kaniovski's (2008) terminology it could be said that the problem solving power of an organization (collective problem solving power) is based upon two ingredients: the power resulting from the bare positions in the organizational architecture (positional power) and the power rooted in the individual actors which are placed into these positions (individual problem solving power).

Brams, S. (1968), “Measuring the Concentration of Power in Political Systems”, *American Political Science Review*, 62, 461-475.

Brink, R. van den (1994), Relational Power in Hierarchical Organizations, PhD dissertation, Tilburg University.

Brink, R. van den (1997), “An Axiomatization of the Disjunctive Permission Value for Games with a Permission Structure”, *International Journal of Game Theory*, 26, 27-43.

Brink, R. van den (2002), “Apex Games as Relational Power Measures for Directed Graphs”, *Social Choice and Welfare*, 19, 845-867.

Brink, R. van den (2008), “’Vertical Wage Differences in Hierarchically Structured Firms”, *Social Choice and Welfare*, 30, 225-243.

Brink, R. van den, and G. van der Laan (1998), “Axiomatizations of the Normalized Banzhaf Value and the Shapley Value”, *Social Choice and Welfare*, 15, 567-582.

Brink R. van den, and R.P. Gilles (1996), “Axiomatizations of the Conjunctive Permission Value for Games with Permission Structures”, *Games and Economic Behavior*, 12, 113-126.

Brink, R. van den, and F. Steffen (2008), ”Positional Power in Hierarchies”, in M. Braham and F. Steffen (eds), Power, Freedom, and Voting, Springer, 57-81.

Copeland, A.H. (1951), A Reasonable Social Welfare Function, mimeo, Seminar on Applications of Mathematics to Social Sciences, University of Michigan.

Daudi, P. (1986), Power in the Organization, Basil Blackwell.

Dubey, P., and L.S. Shapley (1979), “Mathematical Properties of the Banzhaf Power Index”, *Mathematics of Operations Research*, 4, 99-131.

Felsenthal, D.S. D. Leech, C. List, and M. Machover (2003), “In Defence of Voting Power Analysis: Responses to Albert”, *European Union Politics*, 4, 473-497.

Gilles, R.P., and G. Owen (1999), “Cooperative Games and Disjunctive Permission Structures,” *CentER Discussion Paper* 9920, CentER for Economic Research, Tilburg University, Tilburg.

Gilles R.P., G. Owen, and R. van den Brink (1992), “Games with Permission Structures: The Conjunctive Approach”, *International Journal of Game Theory*, 20, 277-293.

GROFMAN, B. AND G. OWEN (1982), "A Game Theoretic Approach to Measuring Degree of Centrality in Social Networks", *Social Networks*, 4, 213-224.

HARRÉ, R. (1970), "Powers", *British Journal of the Philosophy of Science*, 21, 81-101.

HERINGS, P.J.J., G. VAN DER LAAN, AND A.J.J. TALMAN (2005), "The Positional Power of Nodes in Digraphs", *Social Choice and Welfare*, 24, 439-454.

HU, X. AND L.S. SHAPLEY (2003), "On Authority Distributions in Organisations: Controls", *Games and Economic Behavior*, 45, 153-170.

KANIOVSKI, S. (2008), "Straffin Meets Condorcet: What can a Voting Power Theorist Learn from a Jury Theorist?", *Homo Oeconomicus*, 25, 1-22.

MIZRUCHI, M.S. AND B.B. POTTS (1998) "Centrality and Power Revisited: Actor Success in Group Decision Making", *Social Networks*, 20, 353-387.

MORRISS, P. (1987/2002) Power: A Philosophical Analysis, Manchester University Press.

OWEN, G. (1975), "Multilinear Extensions and the Banzhaf Value", *Naval Research Logistics Quarterly*, 22, 741-750.

RADNER, R. (1992), "Hierarchy: The Economics of Managing", *Journal of Economic Literature*, 30, 1382-1415.

RUSSETT, B.M. (1968), "Probalism and the Number of Units: Measuring Influence Concentration", *American Political Science Review*, 62, 476-480.

SAH, R.K. AND J.E. STIGLITZ (1985), "The Theory of Economic Organizations", *AEA Papers and Proceedings* 75, 292-297.

SAH, R.K. AND J.E. STIGLITZ (1986) "The Architecture of Economic Systems: Hierarchies and Polyarchies", *American Economic Review*, 76, 716-727.

SHAPLEY, L.S. AND J.R. PALAMARA (2000a), Control Games and Organizations, UCLA Working Paper 795.

SHAPLEY, L.S. AND J.R. PALAMARA (2000b), Simple Games and Authority Structure, UCLA Working Paper 796.

STEFFEN, F. (2002), Essays in the Theory of Voting Power, PhD Dissertation, University of Hamburg.

WILLIAMSON, O.E. (1967), "Hierarchical Control and Optimum Firm Size", *Journal of Political Economy*, 75, 123-138.