Why Not Africa? – On Growth and Welfare Effects of Secure Property Rights

Ines Lindner and Holger Strulik*
Hamburg University

October 2002

The paper presents the long-run equilibrium and development dynamics in the neoclassical growth model and a simple model of endogenous growth when property rights are absent. The results are compared to the outcome in a corresponding model economy with secure property rights. The main findings are that there exists a considerable gain in level and growth of consumption from establishing secure property rights, that economic performance without property rights worsens with increasing number of competing groups, and that the existence, or absence of property rights explains conditional convergence.

Keywords: Differential Games, Economic Growth, Property Rights

JEL Classification: C73, O40

*Hamburg University, Economics Department, Von Melle Park 5, 20146 Hamburg, Germany. Email: ilindner@hermes1.econ.uni-hamburg.de, strulik@econ.uni-hamburg.de. We would like to thank Sean Holly, Kai Konrad, Andreas Pick, Koji Shimomura, Nikolaus Siegfried, and participants at the 2002 Conference of the European Association for Law and Economics for helpful comments. This research was supported through a Marie Curie Research Fellowship of the European Commission.

1. Introduction

Standard growth literature assumes secure property rights. A large number of individuals supply capital and labor and in exchange receive an income according to the marginal product of these factors. Secure factor income is guaranteed by secure property rights, a reasonable assumption for a fully developed economy situated in a democratic, constitutional state. A less developed economy which is characterized by a high degree of social conflict, a low degree of institutionalized or enforceable laws, or a high degree of political instability may be better approximated by the assumption that secure property rights are absent. In this paper we introduce missing property rights into two popular models of economic growth and compare the results with the corresponding results from standard models. The comparison provides an assessment of the importance of property rights for economic development and of the possible gain from establishing secure property rights.

Empirical studies which include a proxy for insecure property rights usually find it negatively correlated with economic growth, see e.g. Sala-i-Martin (1997), Scully (1988), Goldsmith (1995) and Keefer and Knack (1997). In their survey article "Why not Africa?" Freeman and Lindauer (1999) argue that missing property rights can be identified as the one major obstacle to growth for Sub Saharan Africa. They conclude: "There is no single recipe for achieving economic growth, but there is one way to prevent growth: through instability and absence of property rights." (p. 22).

In this paper we present a theoretical framework which supports the empirical evidence and explains conditional convergence in simple models of economic growth. We show that an economy without property rights converges towards a different long-run state than an otherwise identical economy with secure property rights. Individuals in an economy without property rights choose a lower investment ratio and – depending on the given production technology – approach either a lower level of consumption or a lower long-run growth rate than individuals in an economy with secure property rights. Growth is conditioned on investment but investment in turn is conditioned on the existence of property rights.

We model missing property rights by the assumption of a society of different groups in which all groups have the right to invest and the right to expropriate. The groups play a dynamic game of capital accumulation and expropriation. For a low number of groups we think of powerful political or ethnic groups and for a high number of groups we think of a society close to anarchy.

In Section 2 we set up the framework with a general production function and calculate Markovian (sometime also called feedback-) Nash-strategies. In the remaining sections we introduce specific forms of the production function and discuss the results.

The Nash-equilibrium is not the only available solution. Benhabib and Rustichini (1996) argue that social norms may develop which enable two players to reach a Pareto-optimal equilibrium using trigger strategies. This way they calculate an upper bound for growth without property rights. Their approach can be understood as a complement to this paper where the Markovian Nash-equilibrium characterizes the lower bound of what can go wrong when property rights are absent.

In Section 3 we introduce missing property rights to the neoclassical growth model (Cass, 1965). A special two-player case of this game has a long history in the economic literature. It is the game of capitalism as developed in Lancaster (1973) where one group has the right to invest and bears the risk to be (partly) expropriated by the second group. The second group in turn lacks the right to invest. Shimomura (1991) shows how to solve the game in feedback strategies with nonlinear utility functions and convex technologies, which relates the game of capitalism close to the neoclassical growth model without property rights. Hence, Section 3 can also be understood as a generalization of Shimomura's game of capitalism. We show that the existence or absence of property rights explains conditional convergence in levels. Starting at the same initial state, individuals in an economy without property rights choose a lower investment ratio and converge towards a lower steady-state consumption level. We combine Shimomura's analytical solution technique with the numerical solution technique of backward integration (Brunner and Strulik, 2002). This enables us to identify explicitly the Markovian Nash equilibrium in a calibrated growth model and to compute the welfare effects of establishing secure property rights.

In the fourth section we calculate development dynamics when missing property rights are introduced in a simple model of endogenous growth (Jones and Manuelli, 1990). First we consider the effect of the number of competing groups on equilibrium growth and show that an increase in the number of groups reduces the growth rate. Moreover, long-run growth in a Markovian equilibrium could easily become negative while an otherwise identical economy with secure property rights would grow at a positive rate. The result supports Freeman and Lindauers finding that income per capita retrogressed in many Sub-Saharan countries with insecure property rights.

We then demonstrate conditional convergence of growth rates: an economy without property rights adjusts towards a steady-state of lower growth than an otherwise identical economy with secure property rights. Conditional convergence arises because individuals in the economy without property rights choose a lower investment ratio for any given capital productivity along the adjustment path. Finally we numerically compute the growth and welfare effects of establishing secure property rights.

The notation follows the presentation of the models with secure property rights in Barro and Sala-i-Martin (1995). In order to be brief, our analysis frequently refers to results displayed in this widespread textbook of growth theory.

2. The General Framework

The economy is populated by $n \geq 2$ homogenous groups. Each group $i = 1, 2, \ldots, n$ consists of a continuum $[0, 1]$ of agents with intertemporal utility of consumption, c_i, according to

$$\int_0^\infty \frac{c_i^{1-\theta} - 1}{1-\theta} \mathrm{e}^{-\rho t} \mathrm{d}t \quad . \tag{1}$$

In (1) $\rho > 0$ denotes the time preference rate and $1/\theta > 0$ the intertemporal elasticity of substitution.

Using capital $k \geq 0$ a single output is produced via a production function f. There exist no property rights so that agents of each group are free to choose consumption and the evolution of k is given by

$$\dot{k} = f(k) - \delta k - \sum_{i=1}^{n} c_i \quad , \tag{2}$$

where $\delta \geq 0$ denotes the rate of depreciation.

The production function f is twice continuously differentiable with $f(0) = 0$, $f' > 0$, $f'' \leq 0$.

Given $k(0) = k_0 \geq 0$ agents maximize (1) with respect to (2) using a feedback Nash-strategy, $c_i(k)$. Since all agents share symmetric utility functions and the same state equation, we confine the analysis to Nash-equilibria in symmetric strategies:

$$c_i = c \quad \text{for} \quad i = 1, \ldots, n \quad . \tag{3}$$

We begin with constructing the Hamilton-Jacobi-Bellman equations for our differential game by applying Theorem 6.16 of Basar and Olsder (1995).

Theorem 2.1. *If a continuously differentiable function $V(k)$ can be found that satisfies*

$$\rho V(k) = (V'(k)^{(\theta-1)/\theta} - 1)/(1-\theta) + V'(k)[f(k) - \delta k - nV'(k)^{-1/\theta}] \ , \tag{4}$$

subject to the boundary condition

$$\lim_{t\to\infty} V(k(t)) \exp[-\rho t] = 0 \ , \tag{5}$$

where $k(t)$ is the nonnegative solution to $\dot{k} = f(k) - \delta k - nc(k)$, with $k(0) = k_0$ and

$$c(k) = V'(k)^{-1/\theta} \ , \tag{6}$$

then it generates a symmetric Markovian (feedback) Nash-equilibrium with the strategy of each player defined by (6).

Proof. Using the Hamiltonian functions defined by

$$H_i(k, c_1, ..., c_n, \lambda_i, t) := \frac{c_i^{1-\theta} - 1}{1-\theta} \exp[-\rho t] + \lambda_i[f(k) - \delta k - c_1 - ... - c_n] \tag{7}$$

the Hamilton-Jacobi-Bellman equations can be written as:

$$\begin{aligned} -\frac{\partial S_i(k,t)}{\partial t} &= \max_{c_i} H_i(k, c_1(k,t), ..., c_i, ..., c_n(k,t), \frac{\partial}{\partial k} S_i(k,t), t) \ , \\ c_j(k,t) &= \arg\max_{c_j} H_j(k, c_1(k,t), , , , , c_j, ..., c_n(k,t), \frac{\partial}{\partial k} S_j(k,t), t) \end{aligned} \tag{8}$$

for $i, j = 1, ..., n$ with boundary conditions

$$\lim_{t\to\infty} S_i(k(t), t) = 0, \qquad i = 1, ..., n \ , \tag{9}$$

and $k(t) \geq 0$ solves $\dot{k} = f(k) - \delta k - c_1(t) - ... - c_n(t)$ with $k(0) = k_0$.

If there are C^1-functions $S_1(k,t), ..., S_n(k,t)$ which satisfy (8) and (9), then they generate a feedback Nash-equilibrium by maximizing the Hamiltonians (8).

By setting $S(k,t) = V(k) \exp[-\rho t]$ solving eqs. (8) and (9) simplifies to solving the following system of ordinary differential equations:

$$\rho V_i(k) = \max_{c_i} H_i(k, c_1(k), ..., c_i, ..., c_n(k), V_i'(k), 0), \tag{10}$$

$$c_j(k) = \arg\max_{c_j} H_j(k, c_1(k), ..., c_j, ..., c_n(k), V_j'(k), 0) \ , \tag{11}$$

with boundary conditions

(12) $$\lim_{t\to\infty} V_i(k(t))\exp[-\rho t] = 0 \ .$$

Maximization of the Hamiltonians provides

(13) $$c_i^{-\theta} = V_i'(k) \Leftrightarrow c_i = V_i'(k)^{-1/\theta} \ ,$$

and (10) can be rewritten as

(14) $$\rho V_i(k) = H_i(k, V_1'(k)^{-1/\theta}, ..., V_n'(k)^{-1/\theta}, V_i'(k), 0) \ .$$

For symmetric solutions, (12) and (14) simplify to

(15) $$\rho V(k) = H(k, V'(k)^{-1/\theta}, ..., V'(k)^{-1/\theta}, V'(k), 0) \ ,$$

(16) $$\lim_{t\to\infty} V(k(t))\exp[-\rho t] = 0 \ ,$$

which equals (4) and (5), and applying (13) provides (6). □

Theorem 2.2. *Let c be a solution of*

(17) $$c'(k) = \frac{[f'(k) - \delta - \rho]c(k)}{\theta[f(k) - \delta k - nc(k)] + (n-1)c(k)} \ ,$$

with boundary condition

(18) $$\lim_{t\to\infty} \int_{k0}^{k(t)} u'(c(y))\mathrm{d}y \exp[-\rho t] = 0 \ ,$$

where $k(t)$ is the corresponding non-negative state-trajectory of

$$\dot{k} = f(k) - \delta k - nc(k), \qquad k(0) = k_0.$$

Then $(c_1, ..., c_n) = (c, ..., c)$ constitutes a symmetric feedback Nash-equilibrium.

Proof. Let $V(k)$ be defined as

(19) $$V(k) := \int_{k0}^{k} u'(c(y))\mathrm{d}y + V(k_0) \ ,$$

with $V(k_0)$ given by

$$V(k_0) = (1/\rho)\{u(c_0) + u'(c_0)[f(k_0) - \delta k_0 - nc_0]\} \ .$$

We verify that $V(k)$ is a solution to (4): Differentiating (4) with respect to k and substituting $u'(c) = V'(k)$ and $u''(c)c'(k) = V''(k)$ provides (17). Equation (19) is obtained by integrating (6). Insertion of $V(k)$ into (4) using $V'(k_0) = u'(c_0)$ yields the initial value $V(k_0)$ and (18) ensures that the transversality condition (5) holds. □

Since $c'(k) = \dot{c}/\dot{k}$, (17) can be decomposed into

$$\dot{c} = (f'(k) - \delta - \rho)c/\theta \ , \tag{20}$$

$$\dot{k} = f(k) - \delta k - nc + (n-1)c/\theta \ . \tag{21}$$

The ordinary differential equation system (20) and (21) bear a striking resemblance to the solution of the standard growth model. The decomposition provides two advantages which are exploited throughout the remainder of the paper: The problem can be solved for $c(k)$ with standard methods and it can be easily compared to the solution of the corresponding model with secure property rights. For these purposes, however, we have to specify the production function.

3. Property Rights and Growth: The Neoclassical Case

The neoclassical production function is assumed to be of the Cobb-Douglas type:

$$f(k) = Ak^{\alpha} \ , \ \ 0 < \alpha < 1 \ , \ \ A > 0 \ . \tag{22}$$

Insertion of (22) in (20) and (21) provides

$$\dot{c} = (\alpha A k^{\alpha-1} - \delta - \rho)c/\theta \ , \tag{23}$$

$$\dot{k} = Ak^{\alpha} - \delta k - nc + (n-1)c/\theta \ . \tag{24}$$

To assess the advantage of property rights we introduce an otherwise identical economy with secure property rights. This is an economy with a large number of firms operating on competitive markets and a continuum $[0, n]$ of price-taking consumers which follow the Ramsey rule

(23). The aggregate budget constraint is obtained after inserting (22) into (2).

THEOREM 3.1. *An economy with secure property rights converges towards the equilibrium*

$$k_p = k^\star = \left(\frac{\rho+\delta}{\alpha A}\right)^{1/(\alpha-1)} \quad , \quad c_p = \frac{Ak^{\star\alpha} - \delta k^\star}{n} \tag{25}$$

The proof is in Cass (1965) and in Barro and Sala-i-Martin (1995, Ch. 2).

We confine the analysis to the case where the economy without property rights is initially situated below the long-run equilibrium of an otherwise identical economy with secure property rights: $k(0) < k^*$.

THEOREM 3.2. *If $\theta > (n-1)/n$, then an economy without property rights converges along a unique path towards an equilibrium level of consumption which falls short of the consumption level of an otherwise identical economy with secure property rights.*

Proof. Step 1: For $\theta > (n-1)/n$ system (23) and (24) has a unique positive equilibrium at

$$k^\star = \left(\frac{\rho+\delta}{\alpha A}\right)^{1/(\alpha-1)} \quad , \quad c^\star = \frac{Ak^{\star\alpha} - \delta k^\star}{n-(n-1)/\theta} \quad . \tag{26}$$

The Jacobian determinant evaluated at $c^\star$, $k^\star$ is $\det J = (n-(n-1)/\theta)\alpha(\alpha-1)Ak^{\star\alpha-2}c^\star/\theta$ and negative for $\theta > (n-1)/n$. The equilibrium is a saddlepoint.

Step 2: Figure 1 displays the phase diagram, where the $\dot{k} = 0$ locus of (24) is given by $(Ak^\alpha - \delta k)/(n-(n-1)/\theta)$ and the $\dot{c} = 0$ locus is the vertical line at $k^\star$. All integral curves except the stable manifold can be excluded for violating the transversality condition.

Let k_1 denote the intersection of the stable manifold with the abscissa. Assume a capital stock $0 < k^c < k_1$ exists with $c(k^c) = 0$. Let the point in time when this happens be denoted by $\tau > 0$. Then from (13) it follows that $V_i'(k(\tau)) = \infty$ and $(\mathrm{d}/\mathrm{d}t)(V_i'(k)) = V_i''(k)\dot{k} < 0$ for $k < k^*$ so that $V_i'(k(t)) > V_i'(k(\tau))$ for all $t < \tau$ which is a contradiction to $V_i'(k(\tau))$ being infinite. Therefore, the stable manifold goes through the origin.

Step 3: The equilibrium of the economy is situated where the stable manifold intersects the *real* $\dot{k} = 0$ locus obtained from (2) with (22) as $\tilde{c}(k) = (Ak^\alpha - \delta k)/n$ with

$$c' = \frac{\alpha A k^{\alpha-1} - \delta - \rho}{n-1} \quad , \tag{27}$$

at the intersection. Since $c^\star > c_p$, consumption at an intersection $k^{\star\star}$ falls short of c_p, $c^{\star\star} < c_p$.

Suppose there are multiple intersections $k^\star > k_1 > k_2 > \ldots$. The stable manifold is located above the $\tilde{c}(k)$-curve at $k^\star$ so that at k_1:

$$c'(k_1) > \tilde{c}'(k_1) \quad \Leftrightarrow \quad \frac{\alpha A k_1^{\alpha-1} - \delta - \rho}{n-1} > \frac{\alpha A k_1^{\alpha-1} - \delta}{n} \quad \Leftrightarrow \quad \alpha A k_1^{\alpha-1} > \delta + n\rho \ ,$$

Hence, at k_2:

$$c'(k_2) < \tilde{c}'(k_2) \quad \Leftrightarrow \quad \frac{\alpha A k_2^{\alpha-1} - \delta - \rho}{n-1} < \frac{\alpha A k_2^{\alpha-1} - \delta}{n} \quad \Leftrightarrow \quad \alpha A k_2^{\alpha-1} < \delta + n\rho \ ,$$

and therefore $\alpha A k_2^{\alpha-1} < \delta + n\rho < \alpha A k_1^{\alpha-1}$, which contradicts the assumption $k_1 > k_2$. Hence, if an equilibrium exists, it is unique.

It remains to prove that a positive intersection exists. Since $c(k)$ is situated below the $\dot{k} = 0$ locus of (24), $(Ak^\alpha - \delta k - nc) + (n-1)c/\theta > 0$ for $k \in (0, k^\star)$ and hence

$$c' = \frac{(\alpha A k^{\alpha-1} - \delta - \rho)c}{\theta(Ak^\alpha - \delta k - nc) + (n-1)c} < \frac{(\alpha A k^{\alpha-1} - \delta)c}{\theta(Ak^\alpha - \delta k - nc) + (n-1)c} \ . \tag{28}$$

for $k \in (0, k^\star)$. Suppose $c(k) > \tilde{c}(k)$ for $k \in (0, k^\star)$. Then there exists an $\epsilon > 0$, $\epsilon < k^\star$ so that

$$c' = \frac{(\alpha A k^{\alpha-1} - \delta - \rho)c}{\theta(Ak^\alpha - \delta k - nc) + (n-1)c} > \tilde{c}' = \frac{\alpha A k^{\alpha-1} - \delta}{n}$$

for $k \in (0, \epsilon)$ and (28) implies

$$\frac{(\alpha A k^{\alpha-1} - \delta)c}{\theta(Ak^\alpha - \delta k - nc) + (n-1)c} > \frac{\alpha A k^{\alpha-1} - \delta}{n}$$

for $k \in (0, \epsilon)$. Since $\alpha A k^{\alpha-1} - \delta > 0$ for $k \in (0, \epsilon)$ it follows that

$$\frac{c}{\theta(Ak^\alpha - \delta k - nc) + (n-1)c} > \frac{1}{n}$$

for $k \in (0, \epsilon)$, and taking the limit

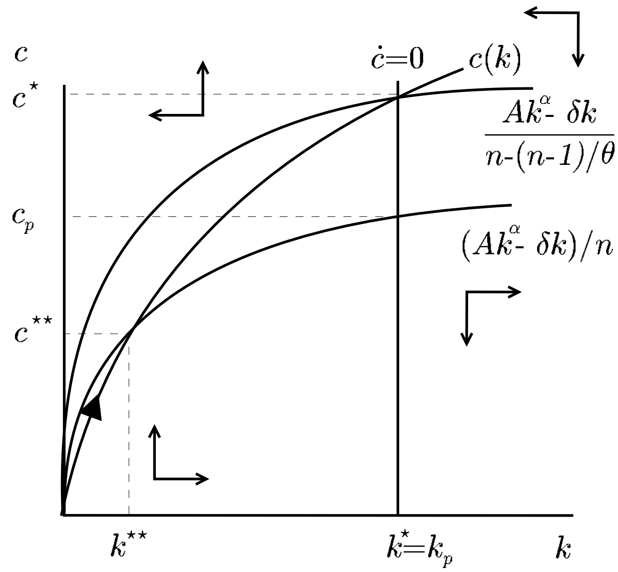
$$\lim_{k \to 0} \frac{c}{\theta(Ak^\alpha - \delta k - nc) + (n-1)c} = \frac{1}{n(1-\theta) - 1} \geq \frac{1}{n} \ ,$$

which is a contradiction since the left hand side of the inequality condition is negative for $\theta > (n-1)/n$.

Hence, a unique positive equilibrium $k^{\star\star} < k^\star$ exists.

Step 4: Because $c^{\star\star}$ and $k^{\star\star}$ are positive and constant, the transversality condition (18) is fulfilled. □

Figure 1: Phase Diagram – Neoclassical Growth



The condition for existence of a Markovian Nash-equilibrium requires that θ exceeds one half when there are two competing groups and one for $n \to \infty$. Since estimation results as well as rules of thumb suggest that the intertemporal elasticity of substitution, $\sigma = 1/\theta$, is well below one, the condition is a very mild one.[1]

The question remains whether the absence of property rights does *significantly* affect the performance of a developing economy. This question can only be answered numerically. Therefore, we parameterize the model and determine adjustment dynamics and steady-state consumption by means of backward integration (Brunner and Strulik, 2002).

In the first step we compute the stable manifold of (23) and (24) and obtain a numerical solution for the Nash-strategy $c(k)$. Let ϵ denote a small positive number close to the smallest computable number on the computer. Starting in $k^\star - \epsilon$, $c^\star - \epsilon$ we integrate the parameterized system (23) and (24) backwards in time using $k = k_0$ as termination criterion. Hence, we replace the inherently unstable boundary value problem by an inherently stable initial value problem which can be solved easily and accurately with standard methods[2]. With M denoting the number of executed integration steps the procedure provides a list of values for k and c and after reverting them we get the forward looking list of values $((k_j)_{j=1}^M, (c_j)_{j=1}^M)$ with $(k_M, c_M) \approx (k^\star, c^\star)$. From

[1]See Hall (1988) and Ogaki and Reinhart (1998), see Ogaki et el. (1996) for estimates of σ for developing countries. One could argue that interpreting powerful groups as rich oligarchs shifts the range of possible values upwards, since it has been shown that σ increases in wealth levels. Evidence for elasticities above one, however, is lacking. In a panel analysis Atkeson and Ogaki (1996) estimate a σ around 0.8 for the richest Indian households.

[2]We have used MATLAB's ODE45.

this list we use the first m elements, $((k_j)_{j=1}^m, (c_j)_{j=1}^m)$ with $f(k_m) - \delta k_m - nc_m \approx 0$ so that (k_m, c_m) is an approximation for the equilibrium of (2).

In the second step we use the *real* equation of motion obtained from (2) and (22) as $g(k) = Ak^\alpha - \delta k - nc$ and calculate the *real* time paths by setting $t = 0$ at k_0 and integrating

$$t_{j+1} - t_j = \int_{k(j)}^{k(j+1)} 1/g(k)\mathrm{d}k$$

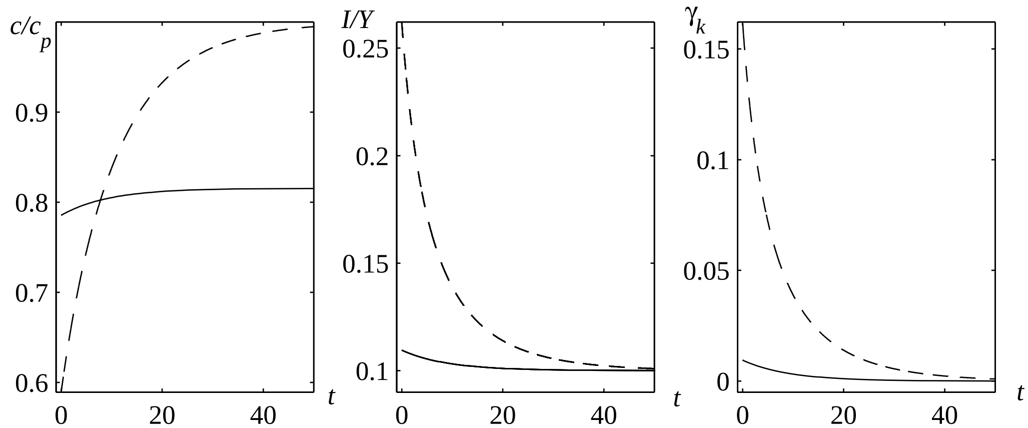
for the $i = 1, \ldots, m-1$. For that purpose we employ the trapezoidal rule.

We compare the result with the outcome in an otherwise identical economy with secure property rights, for which the time paths are also obtained by means of backward integration.

Figure 2 shows the result for some frequently used parameters in calibration of growth models (as described below the figure). Solid lines represent the economy without property rights and dashed lines are the standard result for an economy with secure property rights. Both economies begin their development at the same initial state where $k_0 = 0.3k_p$. The first panel shows consumption measured in terms of steady–state consumption under secure property rights. Without property rights people consume more in the first years as compared to people in an economy with secure property rights. Since the capital stock is far below its steady-state level individuals in the economy with secure property choose high investment ratios at the beginning of the transition (I/Y in the middle panel). Without property rights, however, people bear the risk of being expropriated and investment ratios are comparatively low even when capital productivity is high. Low investment implies slow growth along the adjustment path (capital growth rate γ_k in the right panel) and convergence towards a lower long-run consumption level of about 80 percent of an economy with secure property rights.

Table 1 compares consumption and welfare for a variety of alternative parameters of the model. The first row shows steady-state consumption per capita without property rights relative to consumption in an otherwise identical economy with secure property rights. Relative consumption decreases sharply in the number of competing groups. If there are only two competing groups people may end up with about 80 percent of consumption of an economy with secure property rights. The arrival of a third group, however, reduces this ratio to about 65 percent. If we consider $n \to \infty$ as anarchy, population converges to starvation when society converges towards anarchy. This can also be seen from Figure 1 and (26) –(27). An increasing

Figure 2: Adjustment Dynamics: Neoclassical Growth



Solid Lines: Without Property Rights, Dashed Lines: With Property Rights
$n = 2$, $\alpha = 0.3$, $\delta = 0.05$, $\rho = 0.02$, $\theta = 4$, $A = 1$

number of competing groups shifts $c^\star$ downwards and increases c' at the intersection point $k^{\star\star}$ thereby reducing $c^{\star\star}$.

TABLE 1
Relative Economic Performance Without Property Rights:
Neoclassical Growth Model

	Basic Scenario[a]	$\rho = 0.03$	$\theta = 1$	$\theta = 4$	$\alpha = 0.4$	$\delta = 0.05$	$n = 3$	$n = 5$	n=10
Consumption	81	78	65	88	79	76	65	45	25
Welfare	23	27	54	14	26	31	54	120	296
Welfare Gain	13	13	37	6	14	13	34	81	205

Consumption is the steady-state level of consumption without property rights relative to steady-state consumption with secure property rights, Welfare is the corresponding steady-state welfare differential, Welfare Gain is the gain from establishing secure property rights. Numbers in percent and rounded.
[a] $\alpha = 0.36$, $A = 1$, $\rho = 0.02$, $\theta = 2$, $\delta = 0.1$.

The intuition for the result is that the market share (i.e. power) of a group decreases in n. An increasing number of competing groups increases the possibility of being exploited and hence reduces the incentive to invest. This can be seen by comparing capital productivities. For the basic parameterization the steady-state interest rate is 2 percent in an economy with secure property rights and equates the net marginal productivity of capital. In contrast, net marginal capital productivity is 7.9 percent in an economy without property rights and two competing groups. Generally, the result can be used to explain why investment is low although the capital stock is underdeveloped and capital productivity is high. Starting at the same initial state, individuals in an economy without property rights choose a lower initial investment ratio and converge towards a lower steady-state consumption level. Hence the model explains conditional convergence *in levels*, where the condition is the existence of secure property rights.

The second row shows the implied difference in welfare in percent between both economies at the steady-state and the last row shows the welfare gain of establishing secure property rights. It consists of the steady-state welfare difference and a temporary welfare loss since people invest more and consume less during the first phase of transition towards the new equilibrium. For an assessment it is helpful to recall some other welfare effects presented in the economics literature. For example, the welfare gain of replacing distortionary capital taxation (Lucas, 1990) and the welfare gain of reducing inflation from ten percent to zero (Lucas, 2000) are estimated to be around one percent. The welfare gain of eliminating business cycles is calculated to be 0.05 percent (Lucas, 1987). Against these magnitudes, the welfare gain of establishing secure property rights is huge. It is around 13 percent in the basic scenario for two groups and increases sharply in the number of competing groups.

4. Growth and Development Dynamics

The easiest way to introduce the potential for perpetual growth would be the assumption of a linear production technology. As it is well-known from the corresponding model with secure property rights, under such a technology growth is constant at all times. Insecure property rights, however, are usually considered as a phenomenon of developing countries and it seems more appropriate to analyze them in a growth model capable of displaying adjustment dynamics. In this section we therefore combine the Ak model with the neoclassical growth model by introducing a convex growth technology:[3]

$$f(k) = Ak + Bk^{\alpha}, \quad A, B > 0 \ , 0 < \alpha < \theta. \tag{29}$$

The model for a competitive economy with secure property rights has first been presented in Jones and Manuelli (1990). Our discussion is related to the textbook presentation in Barro and Sala-i-Martin (1995). The advantage of having transitional dynamics is that we can derive conditional convergence of growth rates, where the condition is the existence of secure property rights.

[3]Lane and Tornell (1996) have investigated missing property rights in a linear growth model with a special focus on the so-called voracity effect i.e. parameter constellations where positive exogenous shocks lead to lower long-run growth. In the discussion below a sufficiently small intertemporal elasticity of substitution $1/\theta$ according to condition (33) excludes the voracity effect.

Since the growing economy has no steady-state in c and k, we first introduce the consumption capital ratio $\chi = c/k$ as control-like variable and the output capital ratio as $z = f(k)/k$ as state-like variable. Note that $z \geq A$ for $k \geq 0$.

Again, we first summarize the behavior of an economy with secure property rights populated by a continuum $[0, n]$ of price taking consumers.

THEOREM 4.1. *If*

$$\varphi = \left[\frac{\theta - 1}{\theta}(A - \delta) + \frac{\rho}{\theta}\right] > 0 \ , \tag{30}$$

$$A - \delta > \varphi \Leftrightarrow A - \delta > \rho \ , \tag{31}$$

then an economy with secure property rights develops along a unique adjustment path towards the saddlepoint equilibrium at $\chi = \varphi/n$, $z = A$ *with constant positive growth at rate*

$$\frac{\dot{c}}{c} = \frac{\dot{k}}{k} = (1/\theta)(A - \delta - \rho) \ . \tag{32}$$

The proof is in Barro and Sala-i-Martin (1995), Ch. 4.5.1.

THEOREM 4.2. *If an economy with secure property rights has a unique path of positive growth, and if*

$$\theta > \frac{n-1}{n} \ , \tag{33}$$

$$A - \delta > \varphi \frac{\theta n}{n\theta - (n-1)} \Leftrightarrow \frac{A - \delta}{n} > \rho \ , \tag{34}$$

then an otherwise identical economy without property rights has a unique adjustment path towards the equilibrium

$$\chi^\star = \varphi \frac{\theta}{n\theta - (n-1)} \ , \tag{35}$$

$$z^\star = A \ . \tag{36}$$

where the economy grows at constant rate

$$\frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \frac{A - \delta - \rho n}{n\theta - (n-1)} \quad . \tag{37}$$

Proof. Step 1: Using (29) system (20) and (21) can be rewritten in state-like-control-like notation as

$$\dot{z} = (\alpha - 1)(z - A)\left[z - \delta - \frac{n\theta - (n-1)}{\theta}\chi\right] \quad , \tag{38}$$

$$\dot{\chi} = \chi\left[-\frac{\theta - \alpha}{\theta}(z - A) - \varphi + \frac{n\theta - (n-1)}{\theta}\chi\right] \quad . \tag{39}$$

Since we have assumed that $\theta > \alpha$, conditions (33) and (34) ensure that a unique positive equilibrium for $z \geq A$ exists. It is located at $\chi^\star, z^\star$. The Jacobian determinant of (38) and (39) evaluated at the equilibrium is $\varphi(\alpha - 1)(A - \delta - \varphi)$ and hence negative from (33) and (34). The equilibrium is a saddlepoint. Figure 3 shows the (relevant part of the) phase diagram.

We next show that the transversality condition (18) is fulfilled. Using $\chi = c/k$, it follows that

$$\int_{k0}^{k} u'(c(y))\mathrm{d}y = \int_{k0}^{k} u'(y\chi(y))\mathrm{d}y.$$

Since χ converges, it is bounded and with $\dot{\chi} < 0$ the minimum can be written as

$$\chi^* = \min\{\chi(t), t \in [0, \infty)\} > 0.$$

With $u'' < 0$ it follows that

$$\int_{k0}^{k} u'(y\chi(y))\mathrm{d}y \quad \leq \quad \int_{k0}^{k} u'(y\chi^*)\mathrm{d}y = (\chi^*)^{-\theta} \int_{k0}^{k} y^{-\theta}\mathrm{d}y.$$

Integration by substitution provides

$$\int_{k(0)}^{k(t)} y^{-\theta}\mathrm{d}y \quad = \quad \int_0^t k(s)^{-\theta}\dot{k}(s)ds \leq \int_0^t k(s)^{-\theta-1}(z_0 - \delta - n\chi^*)ds,$$

since $\dot{z} < 0$.

From $\gamma_k := \dot{k}/k = A + Bk^{\alpha-1} - \delta - n\chi$ it follows that $\dot{\gamma_k} = B(\alpha - 1)k^{\alpha-2} - n\dot{\chi} < 0$ and therefore $k(t) \geq k_0 \exp(\gamma^* t)$ with $\gamma^* = A - \delta - n\chi^*$.

Equation (18) can now be rewritten as

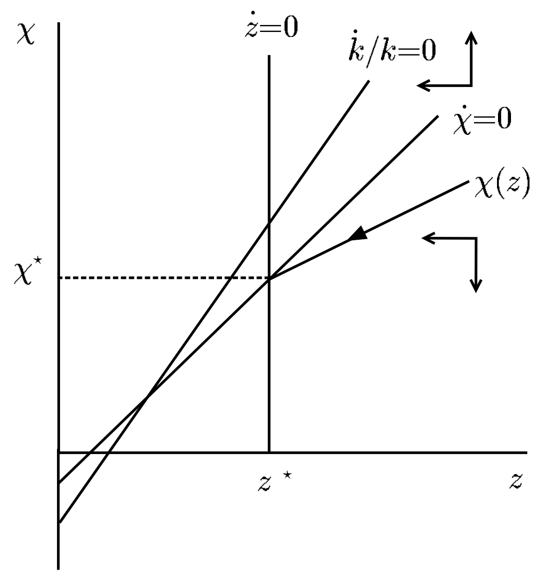
$$\begin{aligned} & \lim_{t\to\infty}\int_{k0}^{k} u'(c(y))\mathrm{d}y\exp[-\rho t] \leq const * \lim_{t\to\infty}\int_0^t k(s)^{-\theta-1}\mathrm{d}s\exp[-\rho t]. \\ \leq \quad & const * \lim_{t\to\infty}\int_0^t \{k_0\exp[(A-\delta-\varphi\frac{n\theta}{n\theta-(n-1)})s]\}^{-(\theta+1)}\mathrm{d}s\exp[-\rho t] \\ = \quad & const * \lim_{t\to\infty}\exp[-((A-\delta-\varphi\frac{n\theta}{n\theta-(n-1)})(\theta+1)-\rho)t], \end{aligned}$$

which equals zero because (34) guarantees that

$$-(A-\delta-\varphi\frac{n\theta}{n\theta-(n-1)})(\theta+1)-\rho<0.$$

Hence, $\lim_{t\to\infty}\int_{k0}^{k} u'(c(y))\mathrm{d}y\exp[-\rho t] = 0$ because with increasing $k(t)$, the integral cannot become negative.

Figure 3: Phase Diagram: Endogenous Growth Without Property Rights



Step 2: It remains to prove that the consumption strategy $\chi(z)$ is compatible with positive long-run growth. After insertion of (29) into (2) the *real* growth rate of the economy is obtained in state-like control-like notation as $\gamma_k = z - \delta - n\chi$. For positive growth the $\chi(z)$ curve must be situated below the *real* $\dot{k}/k = 0$-locus

(40) $$\tilde{\chi}(z) = (z-\delta)/n \ .$$

Since $\tilde{\chi}(z^\star) = (A-\delta)/n$, condition (34) ensures that χ lies below the $\dot{k}/k = 0$-locus at the equilibrium. The $\dot{k}/k = 0$-curve is linear with slope $1/n$.

Assume $\chi(z)$ has intersections with the $\dot{k}/k = 0$-locus for $z > A$. With (38), (39) and (40), the slope $\chi'(z) = \dot{\chi}/\dot{z}$ at the intersection points is given by

$$\chi'(z) = \frac{-\frac{\theta-\alpha}{\theta}(z-A) - \varphi + \frac{n\theta-(n-1)}{\theta n}(z-\delta)}{(\alpha-1)(z-A)\frac{n-1}{\theta}} \quad .$$

Let the intersection point closest to $z^\star$ for $z > A$ be denoted by z_1. Then the slope of $\chi(z)$ in z_1 has to be larger than $1/n$: $\chi'(z_1) > 1/n$. This leads to the inequality

$$z_1 < \frac{A\left[(1-\alpha)\frac{n-1}{n\theta} - \frac{\theta-\alpha}{\theta}\right] + \varphi + \left[\frac{n\theta-(n-1)}{n\theta}\right]\delta}{\alpha/(n\theta)} \quad .$$

Inserting φ this simplifies to

$$z_1 < \frac{A(\alpha-1)+\delta+n\rho}{\alpha} < A \ ,$$

where the last inequality follows from (34) . This is a contradiction to the assumption $z_1 > A$.

Finally, insertion of (29) and (35) in (2) provides (37) and condition (34) has to hold for positive growth. □

Let us first consider the steady-state growth path.

THEOREM 4.3. *There exists a set of feasible parameter specifications for $A, \delta, \rho, \theta, n$ which enables an economy with secure property rights to grow forever but not an otherwise identical economy without property rights. The possibility for this scenario increases with the number of groups.*

Proof. Conditions (31) and (34) have to hold for positive growth with secure property rights and without property rights, respectively. Condition (34) can always be violated by a sufficiently large n, whereas (31) is independent from n. □

In an economy with secure property rights net capital productivity $A-\delta$ has to be larger than the time-preference rate for long-run growth to be possible. This is not enough for positive growth without property rights. There, net capital productivity divided by the number of competing groups must exceed the time-preference rate. In other words, if there are two competing groups and no property rights capital productivity has to be twice as large as under secure property rights to trigger sufficient investment for positive growth in the long-run.

THEOREM 4.4. *If an economy is capable of long-run growth without property rights then growth in an otherwise identical economy with secure property rights is higher. The difference in growth rates is given by*

$$\Delta\gamma = \varphi \frac{n-1}{n\theta - (n-1)} \ , \tag{41}$$

which is increasing in the number of competing groups.

Proof. Substraction of (37) from (32) provides (41). The difference increases in n since $\partial(\Delta\gamma)/\partial n = \theta\,[1 - n/1 - \theta)]^{-2} > 0$. □

TABLE 2
Relative Economic Performance Without Property Rights:
Endogenous Growth Model

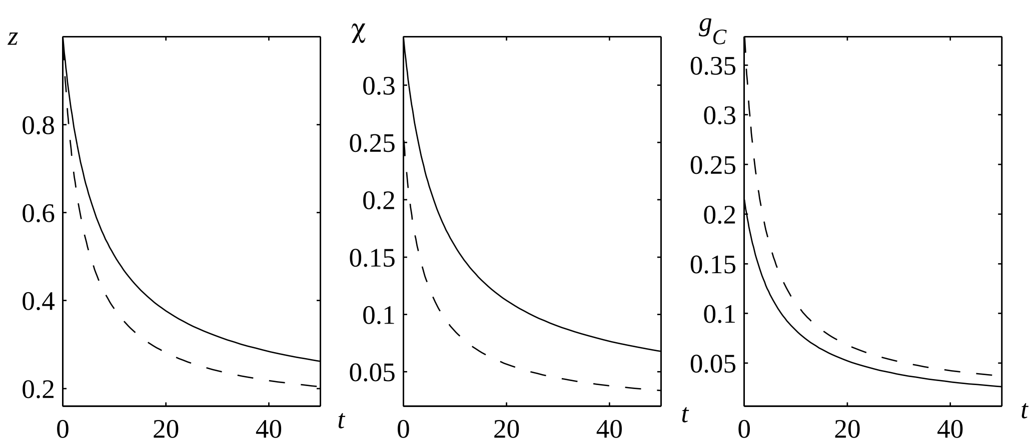
	Basic Scenario[a]	$\theta = 1$	$\theta = 4$	$A = 0.18$[b]	$\rho = 0.03$	$n = 3$	$n = 4$	n=5
Growth ($\Delta\gamma$)	1.33	2.00	0.70	2.00	3.00	4.00	6.00	8.00
c/k	133	198	114	133	133	150	160	167
Welfare Gain	12	35	6	12	12	33	–	–

$\Delta\gamma$ is the steady-state difference of growth rates according to (41), c/k is the steady-state consumption capital ratio relative to secure property rights, Welfare Gain is the gain from establishing secure property rights. Numbers in percent and rounded.
[a] $A = 0.16$, $\rho = 0.02$, $\theta = 2$, $\delta = 0.1$. These parameters imply equilibrium growth $g_C{}^* = 0.02$ under secure property rights, [a] implies $g_C{}^* = 0.03$

The question remains whether the absence of property rights lowers economic growth *significantly.* For that purpose we calculate growth rates for parameterized economies with and without property rights. We take the parameter values from Table 1 and adjust A so that the economy with secure property rights generates an equilibrium growth rate of two percent. The results are presented in Table 2. The first row shows the difference of growth rates according to (41). In the basic scenario the economy with secure property rights grows at two percent while the economy without property rights grows at a rate of only 0.67 percent. Generally, we observe that the relative performance of an economy without property rights is worse than in the neoclassical economy from the previous section. If only two competing groups exist, the growth rate is about one third of an economy with secure property rights. If n rises up to three the economy without property rights stagnates. A slightly further increase of n may already produce disaster. The – Symbol in Table 2 indicates that although a Markovian equilibrium exists it implies a negative growth rate. Hence these economies converge towards the origin and the welfare gain of establishing secure property rights is infinitely large.

The second row shows that the inferior performance without property rights results from an excessively high consumption ratio. (in the basic scenario 133 percent of the consumption ratio under secure property rights). In other words, although the capital stock is much lower, and capital productivity is much higher, investment is too low to generate satisfying economic growth. If the number of groups exceeds three, investment is too low to support long-run existence of the economy. The welfare gain from establishing secure property rights is presented in the last row. Again, the welfare gain is huge and – when it is computable – in order of magnitude comparable to the welfare gain computed for the neoclassical economy.

Figure 4: Development Dynamics: Endogenous Growth



Solid Lines: Without Property Rights, Dashed Lines: With Property Rights
$n = 2$, $\alpha = 0.3$, $A = 0.25$, $B = 1$, $\theta = 4$, $\rho = 0.02$.

Now consider development dynamics with and without property rights. We use the basic scenario from Table 2 (and $B = 1$, $\alpha = 0.36$) and employ the method of backward integration. The integration is terminated at $z = 1$, i.e. at a capital output ratio of one, which may represent a less developed country. After having obtained the stable manifold and reverting the solution sequence to forward looking we calculate the *real* adjustment path by employing the trapezoidal rule and the *real* growth rate $\gamma_k = z - \delta - 2\chi$ (see Section 3 for details). We compare the development path to the solution for an otherwise identical economy with secure property rights.

In Figure 4 solid lines show adjustment dynamics without property rights and dashed lines the corresponding development path with secure property rights. Without property rights people choose a higher consumption capital ratio (χ) during the adjustment process and in the long–run. As a result, they arrive at a steady-state of permanently lower growth in capital and consumption. Although both economies show the same (gross) capital productivity ($A - \delta$), the investment ratio ($I/K = z - n\chi$) is higher in the economy with secure property rights where the consumption ratio χ is lower.

In conclusion, the simple convex model of growth can be employed to explain conditional convergence. The condition is the existence of secure property rights. If preferences and technology allow for long-run growth, insecure property rights lead to a lower rate of investment and adjustment dynamics towards a steady-state of lower growth compared to an otherwise identical economy with secure property rights. However, the analysis has also shown that long-run growth is not necessarily positive. For a wide range of parameter values income per capita retrogresses without secure property rights while it would grow perpetually if property rights were secure.

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