DISTRIBUTIVE POLITICS AND ECONOMIC GROWTH: THE MARKOVIAN STACKELBERG SOLUTION*

INES LINDNER and HOLGER STRULIK†

Department of Economics, University of Hamburg,

Von Melle Park 5, 20146 Hamburg, Germany

May 2000, Revised October 2002

Abstract We generalize the result of Alesina and Rodrik (1994) by showing that their static solution is also a time consistent Stackelberg solution of a differential game between the government and the median voter.

Keywords: Economic Growth, Inequality, Differential Game, Time-Consistency

JEL classification: O15, O40, E61, C73

*We would like to thank Claus Peter Ortlieb, Nikolaus A. Siegfried, and two anonymous referees for useful comments. The first version of this work was written while Holger Strulik was Marie Curie Research Fellow at the Department of Applied Economics, University of Cambridge, U.K. A research grant of the European Commission is gratefully acknowledged.

†Corresponding author. Email: strulik@hermes1.econ.uni-hamburg.de

1. Introduction

The model developed by Alesina and Rodrik (1994) has become a basic reference to explain the empirically evident negative correlation between income inequality and economic growth. Alesina and Rodrik found that a government that maximizes utility of the median voter does not maximize growth of the economy and that the growth rate is the lower the more unequal the income distribution. Their approach, however, has been criticized for two reasons. First, taxes are voted upon only at time zero, and, second, the tax rate is required to be constant over time. Especially, Krusell et al. (1997) argue that the equilibrium in Alesina and Rodrik is not time consistent.

In this note, we relax the assumption that the government credibly commits to constant tax rates and generalize the model of Alesina and Rodrik towards a dynamic game. We show that the solution obtained by Alesina and Rodrik is a time consistent Markovian Stackelberg equilibrium in a differential game between the government and the median voter.

2. The Model

We consider the model established by Alesina and Rodrik and follow their notation. The aggregate production function of the economy is of the Cobb-Douglas type and linear homogenous in the privately provided factors capital, k, and labor, l, as well as in the private capital and productive government spending, g. The economy is thus capable of long-run growth.

The government runs a balanced budget and finances productive spending by a tax rate τ on private capital. In contrast to Alesina and Rodrik, we do not assume that the government credibly commits itself to a constant tax rate. In order to derive a time consistent solution, we require instead that the government makes its current (possibly non-linear) choice of the tax rate dependent on the state of the system, namely the median voter's current stock of capital, i.e. it uses a Markovian (feedback) strategy.

Output is produced by a large number of identical competitive firms so that wages and gross interest rates are obtained as

$$r = \alpha A \tau^{1-\alpha} \ , \tag{1}$$

$$w = (1-\alpha) A \tau^{1-\alpha} k \equiv \omega k \ , \tag{2}$$

where α is the capital share and $A > 0$ denotes general productivity. The economy is populated by a large number of individuals, which are indexed by their relative factor endowment. Relative factor endowment of the i-th individual is then given by

$$\sigma^i = \frac{l^i}{k^i/k} \quad , \sigma^i \in [0, \infty) \quad . \tag{3}$$

Labor is supplied inelastically and aggregate labor supply is normalized to one. Each individual i maximizes intertemporal utility from consumption taking the time paths of taxes and aggregate capital as given, i.e. he solves

$$\max_{c^i} \int_0^\infty \log c^i \mathrm{e}^{-\rho t} \mathrm{d}t \tag{4}$$

subject to his budget constraint

$$\dot{k}^i = w l^i + (r - \tau) k^i - c^i \tag{5}$$

and the no-Ponzi-game rule

$$\lim_{t \to \infty} k^i \mathrm{e}^{\int_0^t (r-\tau) \mathrm{d}s} \geq 0 \quad , \tag{6}$$

with state variable $k^i \geq 0$ and control variable $c^i \geq 0$.

Using the tax rate τ, the government maximizes intertemporal utility of the median voter m subject to his reaction function $c^m(\tau)$ and budget constraint:

$$\max_{\tau} \int_0^\infty \log c^m(\tau) \mathrm{e}^{-\rho t} \mathrm{d}t \quad , \tag{7}$$

$$\dot{k}^m(\tau) = \omega(\tau) l^m k(\tau) + (r(\tau) - \tau) k^m(\tau) - c^m(\tau) \quad , \tag{8}$$

$$0 \leq \lim_{t \to \infty} k^m \mathrm{e}^{\int_0^t (r-\tau) \mathrm{d}s} \quad . \tag{9}$$

Equations (1) to (9) define a differential game for which we assume the government as the Stackelberg leader and the median voter as the follower.

3. The Markovian-Stackelberg Equilibrium

Lemma 3.1. *The Markovian consumption strategy for individual i is given by*

$$c^i(k^i, t) = \rho \left[k^i + \tilde{w}(t) \right] , \tag{10a}$$

where

$$\tilde{w}(t) = \int_t^\infty \omega(s) k(s) l^i \mathrm{e}^{-(z(s)s - z(t)t)} \mathrm{d}s, \tag{10b}$$

and

$$z(t) = 1/t \int_0^t [r(s) - \tau(s)]\mathrm{d}s \tag{10c}$$

define the present value of wage income and the average net interest rate, respectively.

Proof. The i-th individual maximizes the Hamiltonian

$$H^i = \log(c^i) + \lambda \left[wl^i + (r-\tau)k^i - c^i \right] \quad . \tag{11}$$

The equilibrium fulfills the first order conditions (12) and (13) and the transversality condition (14):

$$1/c^i = \lambda \ , \tag{12}$$

$$\dot{\lambda} = -\lambda(r - \tau - \rho) \ , \tag{13}$$

$$0 = \lim_{t\to\infty} \lambda k^i \mathrm{e}^{-\rho t} \quad . \tag{14}$$

From (12) and (13) we obtain the Ramsey rule

$$\dot{c}^i = (r - \tau - \rho)c^i \ , \tag{15}$$

which applies for all individuals.[1] Multiplying (5) by $\exp[-z(s)s]$ and integrating provides for any $T \geq t$

$$k^i(T)\mathrm{e}^{-z(T)T} - k^i(t)\mathrm{e}^{-z(t)t} = \int_t^T w(s)l^i\mathrm{e}^{-z(s)s}\mathrm{d}s - \int_t^T c^i(s)\mathrm{e}^{-z(s)s}\mathrm{d}s. \tag{16}$$

Inserting $c^i(s)$ from (15) and applying the transversality condition one obtains for $T \to \infty$

$$-k^i(t)\mathrm{e}^{-z(t)t} = \int_t^\infty w(s)l^i\mathrm{e}^{-z(s)s}\mathrm{d}s - c^i(t)\mathrm{e}^{-z(t)t}\int_t^\infty \mathrm{e}^{-\rho(s-t)}\mathrm{d}s. \tag{17}$$

Multiplying by $\exp[z(t)t]$ provides (10). □

Since consumption depends only on the current value of the household's state variable and current time it constitutes a time-consistent Markovian strategy.[2]

[1] Following Kemp et al. (1993, p. 422), the equilibrium should more accurately be called a partial-feedback equilibrium. If households took into account an influence of their consumption choice on taxes, the Ramsey rule would read $\dot{c}^i = (r - \tau - \tau' - \rho)c^i$.

[2] It can be verified that

$$V(k^i, t) = (1/\rho)\log[\rho + \tilde{w}(t)] + \int_t^\infty [(z(s) - \rho)s - (z(t) - \rho)t]\exp[-\rho(s-t)]\mathrm{d}s$$

constitutes a value function such that (10) follows from the corresponding Hamilton-Jacobi-Bellman equation.

Generally, the government, as a Stackelberg leader could use $c(k^i, t)$ as a reaction function to obtain a time-consistent tax strategy. The appearance of τ in integrals of (10), however, prevents an analytic solution. Krusell et al. (1997) present numerical solutions. Here we follow an alternative route. We assume that households expect tax rates to be constant over time. Note, however, an important difference to Alesina and Rodrik's original contribution. There, the government decides once and for all on a constant tax rate (τ_{AR}). An approach that has been criticized by Krusell et al. for requiring commitment or, alternatively, for rendering time-inconsistency when re-optimization is possible. In the following we show that when the government re-optimizes in favor of the median voter at any point in time it sticks to the initially chosen tax policy τ_{AR} i.e. we provide the missing proof of time-consistency of Alesina and Rodrik's solution. The necessary assumption that enables this result is that households suppose that tax rates are constant over time. This assumption, however, appears to be justified since it turns out that households' expectations are correct.

THEOREM 3.1. *The constant tax rate $\tau \equiv \tau_{AR}$ and the linear consumption strategy*

$$c^m \equiv (\omega\sigma^m + \rho)k^m \tag{18a}$$

constitute a time consistent Stackelberg equilibrium, where τ_{AR} fulfills

$$\frac{\omega(\tau_{AR})\sigma^m + \rho}{\sigma^m} \frac{\tau - \alpha\omega(\tau_{AR})}{(1-\alpha)\omega(\tau_{AR})} - \rho = 0 \ . \tag{18b}$$

Proof. The proof consists of three steps: Step 1 and Step 2 obtain the optimal strategy of the follower and the leader, respectively, Step 3 analyzes the equilibrium.

Step 1: Inserting $c^i = (\omega\sigma^i + \rho)k^i$ into (5) yields $\dot{k}^i/k^i = r - \tau - \rho$, independent from i. Since all individuals accumulate capital with the same rate, $\dot{k}/k = r - \tau - \rho = zt - \rho$, and σ^i remains constant. Hence, aggregate capital evolves according to

$$k(t) = \rho e^{z(t)t} \int_t^\infty k(s) e^{-z(s)s} ds. \tag{19}$$

Inserting this into $c^i = (\omega\sigma^i + \rho)k^i$ and using the fact that $\omega\sigma^i$ remains constant, c^i can be written as in (10) for all i and in particular for $i = m$.

Step 2: The corresponding current value Hamiltonian for the government's maximization problem reads

$$H^G = \log\left[(\omega\sigma^m + \rho)k^m\right] + \mu\left[r - \tau - \rho\right] k^m \ , \tag{20}$$

with first order conditions and transversality condition:

$$0 = \frac{1}{c^m}\frac{\partial \omega}{\partial \tau}\sigma^m k^m + \mu\left[\frac{\partial r}{\partial \tau} - 1\right] k^m \ , \tag{21}$$

$$\dot{\mu} = \mu\rho - \mu\left[r - \tau - \rho\right] - 1/k^m \ , \tag{22}$$

$$0 = \lim_{t\to\infty} \mu k^m \mathrm{e}^{-\rho t} = 0 \ . \tag{23}$$

Inserting the derivatives $\partial\omega/\partial\tau = (1-\alpha)\omega/\tau$ and $\partial r/\partial\tau = \alpha\omega/\tau$ obtained from (1) and (2) into (21) the condition can be expressed as:

$$\mu = \frac{(1-\alpha)\omega\sigma^m}{(\tau - \alpha\omega)c^m} \ . \tag{24}$$

Differentiation with respect to time yields

$$\dot{\mu} = -(1-\alpha)\sigma^m \left\{ \frac{\omega - \tau(\partial\omega/\partial\tau)}{(\tau-\alpha\omega)^2}\frac{1}{c^m}\dot{\tau} + \frac{\omega}{\tau - \alpha\omega}\left(\frac{1}{c^m}\right)^2 \frac{\partial c^m}{\partial k^m}\dot{k}^m \right\} \ . \tag{25}$$

Inserting $\partial\omega/\partial\tau$ and using $\partial c^m/\partial k^m = c^m/k^m$ and $\dot{k}^m = \gamma k^m$ the differential equation simplifies to

$$-\dot{\mu} = \mu\gamma + \frac{(1-\alpha)\alpha\omega\sigma^m\gamma}{(\tau-\alpha\omega)^2}\frac{1}{c^m}\dot{\tau} \ . \tag{26}$$

And substituting this into (22), dividing by μ and substituting c^m yields the optimal tax strategy:

$$\dot{\tau} = \frac{\tau - \alpha\omega}{\alpha}\left[\frac{\omega\sigma^m + \rho}{\sigma^m}\frac{\tau - \alpha\omega}{(1-\alpha)\omega} - \rho\right] \ . \tag{27}$$

Since $\dot{\tau}$ is independent from the state of the system, the Markovian solution coincides with the open-loop solution if τ_0 is independent from the initial state.

Step 3: The differential equation (27) has two equilibrium solutions: the capitalist's ideal tax rate $\tau_C = \alpha\omega = [\alpha(1-\alpha)A]^{1/\alpha}$ and the equilibrium which fulfills (18b). This equilibrium is the one obtained by Alesina and Rodrik and is labelled τ_{AR}, $\tau_{AR} > \tau_C$. The derivative of $\dot{\tau}$ with respect to τ is

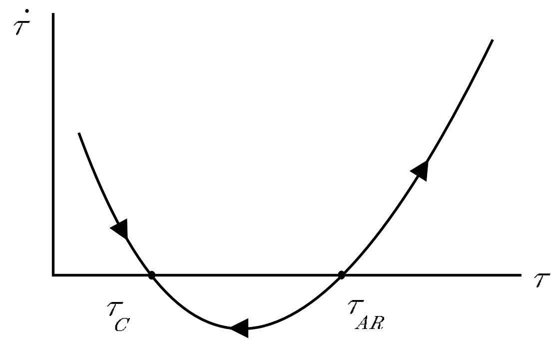
$$\frac{\partial\dot{\tau}}{\partial\tau} = -\rho\left[1 - \alpha(1-\alpha)\omega(\tau_C)/\tau_C\right]/\alpha = -\rho < 0 \tag{28}$$

at τ_C, and

$$\frac{\partial\dot{\tau}}{\partial\tau} = \rho + \left[\tau_{AR} - \alpha\omega(\tau_{AR})\right]^2/(\alpha\tau_{AR}) > 0 \tag{29}$$

at τ_{AR}. Hence, the equilibrium at τ_C is stable and the equilibrium at τ_{AR} is unstable. Figure 1 shows the possible tax dynamics according to (27). Any policy starting with a tax rate smaller than τ_{AR} converges towards the capitalistic equilibrium τ_C.

Figure 1: Tax Dynamics According to (27)



Inserting c^m and (24) in (23) one obtains

$$\lim_{t\to\infty}\left[\frac{(1-\alpha)\omega\sigma^m}{(\tau-\alpha\omega)(\sigma^m\omega+\rho)}\right]\mathrm{e}^{-\rho t}=0\ . \tag{30}$$

For the constant policy τ_{AR} the term in brackets is finite and constant and the transversality condition is fulfilled. Thus, the strategy $\tau(k^m) = \tau_{AR}$ for all k^m is an optimal Markovian strategy for the government.

Since the first term of the Hamilitonian (20) is strictly concave in k^m and τ and the second term is concave in k^m and τ, the optimal solution trajectory $k^{m*}(t) = k^m(0)$ $\mathrm{e}^{[r(\tau_{AR})-\tau_{AR}-\rho]t}$ is unique (See e.g. Theorem 10.1. in Takayama, 1993). And since the trajectory $k^{m*}(t)$ can only be realized by the constant policy τ_{AR}, the policy τ_{AR} constitutes the unique Markovian Stackelberg strategy for the government. The choice of τ_{AR} is independent from time, implying that the Stackelberg solution is time consistent. □

References

Alesina, A., and Rodrik, D., Distributive politics and economic growth, Quarterly Journal of Economics 109, 465-49 (1994)

Krusell, P., Quadrini, V., and Rios-Rull, J.V., Politico-economic equilibrium and economic growth, Journal of Economic Dynamics and Control, 21, 243-72 (1997)

Kemp, M.C., Long, N.V., and Shimomura, K., Cyclical and noncyclical redistributive taxation, International Economic Review 34, 415-29 (1993)

Takayama, A., Analytical methods in economics, Ann Arbor: The University of Michigan Press (1993)