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SOFT SPATIAL ECONOMETRIC CAUSALITY MODELS

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Abstract

The paper attempts to develop a methodological and operational framework for analyzing soft (ordinal, categorical or nominal) data in causal (spatial) econometric models. After an introduction into causality analysis and an exposition of causality principles, the attention is focussed on soft data in causal (regional) economic models.
Several soft data techniques will be discussed, inter alia multidimensional scaling techniques and logit techniques. Then it will be shown that graph theory provides an appropriate framework for causality analysis in soft spatial econometric modelling. The analysis will be illustrated by means of some numerical applications to a causal regional income formation model.

Contents:

1. Causality Analysis: An Introduction

The majority of (spatial) economic models takes for granted the existence of well defined relationships based on a logical causal foundation for the quantitative impact of cause variables on effect variables. Causality analysis has attracted much attention as a cornerstone for model building in economics. Assuming a simple linear model of the following type:

$$A\underline{y} + B\underline{x} = \underline{c} \,, \qquad (1)$$

where $\underline{y}$ is a vector of N endogenous variables, $\underline{x}$ a vector of K exogenous variables, $\underline{c}$ a vector of constant parameters, and where A and B are matrices of coefficients, then the causality structure of this model is determined by the structure of matrix A (see also Fox et al., 1966, Rietveld, 1981, and Simon, 1953). For instance, if A is a (block-) triangular matrix, the model is (block-) recursive and it reflects an unambiguous causality pattern.

In case of non-linear models, a similar approach can be adopted by analyzing the causality structure via the existence of potential (block-) triangular impact patterns (see Rietveld, 1981). A causality analysis is also an extremely important element in the analysis of (multi-) regional models, especially as far as the presence of a top-down or bottom-up impact structure is concerned. Especially in complex regional models, a closer examination of the causality structure may shed more light on the impact mechanism of exogenous (cause) variables on endogenous (effect) variables.

Usually , causality analysis (cf. Harvey, 1969, and Wold, 1954) presupposes the following conditions to be fulfilled in a causality system:

- the existence of a functional (non-reflexive, asymmetric and transitive) relationship between stimulus (cause) variables and response (effect) variables, based on a theoretical foundation and a consistent dependence structure.

- a predictability of the effects, after a stimulus has taken place (based on controlled or non-controlled experiments), apart from random or disturbance factors.
- the availability of suitable statistical techniques (correlation analysis, significance tests, e.g.) which may justify the assumption of a certain cause-effect relationship (see Blommestein, 1981a).

In case of an integrated model, very often the condition of a recursive structure is imposed to warrant causality (see Wold, 1954).

In consequence, a causal relationship does not only assume a functional relationship among variables, but also a more precise presentation of the kind and direction of impacts, so that a testable causality relationship is obtained. It should be noted, however, that such a causality relationship is not necessarily deterministic, but it may also be probabilistic. In addition, a causality relationship is hard "to prove": statistical techniques (such as correlation and association analysis) indicate only the existence of a statistical relationship among variables. Consequently, a theoretical foundation of such causal relationships is necessary, while the plausibility or justification of such a theoretical framework must be based among others on appropriate statistical and econometric methods. In respect to this, Lazarsfeld (1954) and Kendall (1955) have specified 3 necessary conditions for the existence of a causal relationship (see also Leitner and Wohlschlägl, 1980; Blommestein, 1981a; and section 3):

- causal order: the cause variables are realized prior to the effect variables, so that the direction of impacts is irreversible.
- association: a testable (statistical) relationship does exist among the causal variables and the effect variables.
- lack of spuriousness: the causal relationship among cause and effect variables does not vanish, when the (partial) impact of other variables on this relationship is exactly determined.

It should be added, that causality is a property to be studied in the specific framework of a <u>model</u>; it does not necessarily refer to real-world impact patterns (see also Simon, 1957). In a spatial context, the statistical test of spatial causality patterns is even more complicated due to the existence of spatial spillover effects. (see also section 3). These phenomena can be studied by means of spatial (auto)correlation analysis, although especially in the framework of a simultaneous equations system still much research work needs to be done (see Cliff and Ord, 1973, Hordijk, 1979, Nijkamp, 1979 and Blommestein, 1971c). The next section will be devoted to a more formal discussion of causality relationships in structural models, while in section 3 spatiotemporal causality problems will be dealt with.

2. Causal and Qualitative Representations of Structural Models

A model can formally be represented by means of N structural relationships[1] as follows:

$$\underline{h}\ (\underline{y},\ \underline{x}) = \underline{0}\ , \qquad (2)$$

where $\underline{y}$ is a (NX1) vector with endogenous variables, $\underline{x}$ a (KX1) vector with exogenous variables, and $\underline{h}$ an implicit vector function.

System (2) represents a particular structure of the pair of sets $\{\underline{z}, H\}$; Gilli (1980) notes that the causal structure of model (2) is already given, if the set of hypotheses H defines the partition[2] $\underline{z} = \underline{y} \cup \underline{x}$, as well as a set of binary relationships of the form $h_i R z_j$, $h_i \epsilon H = \{h_1, h_2, \ldots, h_I\}$ and $z_j \epsilon \underline{z} = \{z_1, z_2, \ldots, z_N, z_{N+1}, \ldots, z_{N+K}\}$; R stands for "the i^{th} relation contains variable z_j". Alternatively, the causal structure can be defined by the zero entries in the matrix of first-order derivatives (cf. Rietveld, 1981), i.e.,

1) The term "structural relationships" is used here in the well-known traditional setting of simultaneous equation models.
2) It is assumed that there is sufficient prior information - for example from spatial economic theory, institutional knowledge, previous empirical studies, etc. - to define this partition.

$$D = \left[\frac{\partial \underline{h}}{\partial \underline{y}'} \;\vdots\; \frac{\partial \underline{h}}{\partial \underline{x}'} \right] \tag{3}$$

Further insight into causality structures implied by (3) can be gathered by examining the sign of the non-zero elements. The information obtained in this way constitutes a so-called calculus of qualitative relations (Samuelson, 1947). It should be noted that the zero and non-zero entries of (3) can be derived independent from a specific quantification of relationship $\underline{h}$. Despite the limited information regarding the specific quantification of the set of hypotheses H, substantial prior information must be available for defining partitions like $\underline{z} = \underline{y} \cup \underline{x}$. If the latter information is not available, it is necessary to conduct inter alia more empirical research or to gather additional institutional information, etc.

Causality analysis can be carried out at different levels of measurement. Consider, for example, the following simple structural model with 3 endogenous variables, viz. income (W), investment (I) and consumption (C), and 3 exogenous variables, viz. government expenditures (G), taxes (T) and the interest rate (d):

$$\left.\begin{aligned} h_1\ (C, W, T, d) &= 0 \\ h_2\ (W, C, I, G) &= 0 \\ h_3\ (I, W, d) &= 0 \end{aligned}\right\} \tag{4}$$

where these equations represent the consumption relationship, the income identity relationship, and the investment relationship, respectively. Putting the non-zero elements of the matrix with the first-order derivatives of each endogenous variable of system (4) with respect to a cause variable equal to one, yields a causal structure in the form of the adjacency matrix A for the successive variables W, I, C, G, T and d:

$$A = \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \tag{5}$$

The qualitative structure of model (4) can be expressed in the form of a so-called sign matrix S with elements +, - or 0.

$$S = \begin{bmatrix} 0 & + & 0 & \vdots & 0 & - & - \\ + & 0 & + & \vdots & + & 0 & 0 \\ 0 & + & 0 & \vdots & 0 & 0 & - \end{bmatrix} \tag{6}$$

In (3) and (4), it was necessary to assume that all variables were measured on a metric scale. The alternative assumption that some or all variables are measured on a non-metric scale requires at least information about the direction of the impacts, before the causality analysis - based on the calculus of qualitative relations - can be applied. For example, the first relationship of system (4) can then be expressed as follows (Blommestein, 1981a):

$$h_1\ (f^o_C\ (C),\ f^o_W\ (W),\ f^o_T\ (T),\ f^o_r\ (d)) = 0 \tag{7}$$

in which f^o_x (x) is an (arbitrary, general) order preserving function of an ordinal variable x with properties ($r_i \varepsilon \{1, \ldots, R\}$ indicates the i^{th} rank order of variable x) :

$$\begin{aligned} &f^o_x\ (r_i) \quad > f^o_x\ (r_{i'}),\ \text{if}\ r_i > r_{i'} \\ &f^o_x\ (r_i) \quad = f^o_x\ (r_{i'}),\ \text{if}\ r_i = r_{i'} \\ &f^o_x\ (r_1 = 1) = 1;\ f^o_x\ (r_i = R) = R \end{aligned} \tag{8}$$

If one wants to go further than the calculus of qualitative <u>relations</u> by also calculating the <u>weights</u> which measure the relative strength of the relationship(s) between the variables, it is necessary to construct causal structural models for dealing with <u>non-metric variables</u>. This leads us into the area of soft econometrics, which will be discussed in section 4. First, however, some more explicit attention will be paid to causality analysis in a space-time context (see section 3).

3. Causality Analysis in a Spatiotemporal Context

The notion of causal relationships in spatial and/or dynamic systems has led to the intriguing question as to whether there exists a basic difference between causality in spatial and temporal models (see Blommestein, 1981a).

It has been stressed by many authors (cf. Bennett and Chorley, 1978; Harvey, 1969; Blalock, 1972), that the notion of time is crucial for analyzing, understanding and interpreting causal orderings. Time-based systems share usually the following properties:

- asymmetry: $E_1 = T_1(E_2) \rightarrow E_2 \neq T_2(E_1)$;
- transitivity: $E_1 = T_1(E_2) \wedge E_2 = T_2(E_3) \rightarrow E_1 = T_3(E_3)$,

in which E_i denotes an event i, and T_i a (time-based) transformation operator. Especially the first property plays an important role in the solution of many causal inference problems; for example, in Simon's method of drawing causal inferences from correlation data[1].
Consider the following system with 3 variables v_1, v_2 and v_3 (see also Simon, 1954):

$$A\,\underline{v} = \underline{u} \tag{9}$$

in which A is a (3x3) matrix with unknown parameters α_{ij} ($\alpha_{ii} = 1$); $\underline{v} = (v_1, v_2, v_3)$; and $\underline{u} = (u_1, u_2, u_3)'$ is a vector with error terms. System (9) contains 9 unknown elements (6 coefficients α_{ij}, and 3 unobservable error terms u_i), and only 3 equations. To infer causal information from this system, it is necessary to make 2 kinds of a priori assumptions (Simon, 1954), viz.:

a) Certain variables are not directly mutually dependent. An important source for this kind of a priori information is the time precedence of variables. For example, if v_2 precedes v_1 in time, then $\alpha_{21} = 0$, i.e. v_1 does not directly influence v_2.

b) The error terms are uncorrelated. In the case of system (9), three more relationships are obtained by assuming that:

$$E(u_1u_2) = 0;\ E(u_1u_3) = 0;\ E(u_2u_3) = 0, \tag{10}$$

where E represents the mathematical expectation symbol.

1) To avoid misunderstandings, Simon (1953) also notes that there is no necessary connection between the asymmetry of the (causal) relation and asymmetry in time.

For purely space-based systems, often the properties of asymmetry and transitivity do not hold: The so-called spatial simultaneity problem - i.e. spatial events are associated but cannot be ordered - is a typical illustration of this proposition. The problem of spatial simultaneity arises very clearly in, among others, the following statistical problem: the log-likelihood function corresponding to the linear regression model, $\underline{y} = X\underline{\beta} + \underline{u}$[1] with autocorrelated errors $\underline{u} = \rho W\underline{u} + \underline{e}$; $\underline{e} \sim NID\ (0, \sigma_e^2)$, can be written in a concentrated form as (see Hepple, 1976) :

$$L_c = -\frac{R}{2}\ (\ln 2\Pi + 1) - \frac{R}{2}\ \ln \left[\frac{\underline{u}'V^{-1}\underline{u}}{|P|^{2/R}}\right] \tag{10}$$

in which $P = I - \rho W$; $\underline{u} = P^{-1}\underline{e}$; $V^{-1} = P'P$; and R is the sample size.

In the time-series case, W denotes a temporal lag-operator matrix which contains zeros, apart from ones on the first subdiagonal, while $|P| = (1 - \rho^2)^{\frac{1}{2}}$. Maximizing L_c is equivalent to minimizing the expression in square brackets in equation (10). For $R \to \infty$, this expression becomes equivalent to $\underline{u}'V^{-1}\underline{u}$, since the Jacobian term $|P|^{2/R}$ is asymptotically equivalent to unity (P can, therefore, asymptotically be considered as a triangular matrix).

In the spatial case, W denotes a spatial lag-operator - or contiguity matrix -, with typical elements $w_{rr'}$ $(w_{rr'} \geq 0)$. Since both $w_{rr'}$ and $w_{r'r}$ will probably be nonzero, especially if the spatial units r and r' are adjacent, P will, in general, not be a triangular matrix $(\lim_{R \to \infty} |P|^{2/R} \neq 1)$.

Assume a set of spatial phenomena A, B and C (for example regional unemployment levels, regional activity rates, etc.). The spatial configuration as such of the observations on these phenomena does not allow us

1) ρ denotes an autocorrelation parameter; W a temporal or spatial lag-operator, and $\underline{e}$ a vector with white-noise error terms.

to differentiate between, for example, the following 3 causal orderings of these spatial phenomena (no matter the number of realisations):

A = f(B);

B = f(A);

A,B = f(C);

etc.

Bennett (1979) discusses 3 methods of resolving the question of the chain of dependence or, alternatively, the causal ordering of spatial phenomena, viz.,

a) the use of exogenous *a priori* information (behavioural or institutional information, e.g.);
b) the estimation of the simultaneous structure of the spatial field;
c) the use of Markov properties for spatial equilibrium fields.

The first approach is successful, if it is possible to postulate the hypothesis that A causes B, or B causes A. Actually, the formulation and 'solution' of this problem is equivalent to the one presented in section 2, viz.: given a structural model, a set of hypotheses H which define the partition $\underline{z} = \underline{y} \cup \underline{x}$ and binary relationships of the form $h_i R z_j$, the causal structure is given as well. Clearly, in a purely inductive system, this is not a feasible approach.

If one wishes to test (in the statistical sense) the set of hypotheses H, it is necessary to turn to the second approach, i.e. to estimate the simultaneous structure of the observed spatial phenomena. However, compared to temporal data this is a rather complex undertaking (cf. Blommestein, 1981a for more details).

The third approach deals with the application of the Markov property in the case of spatial equilibrium situations. This is a highly restrictive approach, as it implies essentially the assumption of time reversibility (cf. Preston, 1974). Furthermore, this assumption is very unlikely to be fulfilled in spatial behavioural processes (cf. Blommestein, 1981a).

From the foregoing discussion, one can conclude that there is no fundamental difference between the notion of causality in temporal and in spatial systems. Both notions fit into the 'classical' concept of causality, as this concept is independent of both the explicit time pattern and the functional form of the relationships (see for a discussion of this concept also Basman, 1963). However, the operationalization of this concept is - due to the problem of spatial simultaneity - more complex in purely spatial systems. At this stage, it is also interesting to point out some (dis)similarities between models for the estimation of simultaneous spatial systems (the second approach) proposed among others by Ord (1975) and Hepple (1976), and the traditional time-based simultaneous models from econometrics. Like in spatial models, the use of OLS (ordinary least squares) in simultaneous - equation models may be invalid, because the Jacobian may be nonvanishing. In this context, Hepple (1976) points out that the OLS estimators tend to overestimate the absolute value of the spatial autocorrelation parameter ρ. Furthermore it is also interesting to mention a dissimilarity between the performance of OLS in spatial regression models and in the structural equations from recursive (temporal) simultaneous - equation models. In the case of recursive (temporal) causal systems, OLS has optimal statistical properties, i.e. setting the partial derivatives of the (log-) likelihood function equal to zero gives the standard OLS expression for each structural equation. This result is not necessarily valid for spatial systems. For example, in the following spatial interaction model in regression equation form - which may be part of a (block-) recursive spatial system - ($\underline{e} \sim NID(0, \sigma_e^2)$),

$$\underline{y} = \rho W\underline{y} + X\underline{\beta} + \underline{e} \qquad (11)$$

the use of OLS is inappropriate. The simultaneous structure (i.e., the spatial simultaneity problem) of model (11), requires the estimation of the parameters with the help of a maximum likelihood procedure, as proposed by Ord (1975).

Finally, in mixed (i.e., spatiotemporal) systems the asymmetry and transitivity properties of time-based systems often do also hold (see for an overview of identification, estimation and forecasting problems of mixed models also Bennett, 1979).

The analysis of qualitative relations and qualitative variables in structural spatial systems, can be pursued in the same way as described in section 2. In the next section, more explicit attention will be paid to such soft information problems.

4. Soft Spatial Econometric Models

Traditional spatial econometric causal models are (implicitly or explicitly) based on the following (non-exhaustive) set of assumptions:

- the existence of a well defined set of variables that can be measured by means of a cardinal metric.
- the complex relationships among these variables can be quantified by means of an operational economic model that describes the various relevant causal impacts.
- the technical, institutional, social and economic side-conditions of the system concerned are precisely known and can be specified in an operational way.
- the spatial spillover effects (horizontal or hierarchical) can precisely be estimated via a spatial distributional model.
- in case of uncertainty concerning the state of the system (for instance, due to stochastic variables), the probability distribution of the stochastic elements is assumed to be known.
- in case of a dynamic model, the time trajectory of all variables of the spatial system at hand can be computed precisely.

The conclusion may be that full and precise information is a basic ingredient for traditional spatial econometric model building. In recent years, however, several authors have argued that many phenomena can hardly be quantified by means of a cardinal metric system (for instance, the social climate in a city) (see, among others, Nijkamp, 1979, 1980a, Blommestein and Van Deth, 1981).

Others have indicated that even cardinal information may imply a serious bias, so that only a pseudo-reliability can be claimed (see, among others, Adelman and Morris, 1974). Therefore, it may be meaningful to discuss measurement scales for variables, which are usually distinguished in social sciences:

- nominal scale: a classification into distinct groups (green or blue, e.g.) or into distinct size classes (small or large impacts, e.g.)
- ordinal scale: a ranking of events or effects in order of magnitude (1, 2, 3, 4, ..., e.g.)
- interval scale: a measurement system which allows a calculation of distances up to a constant
- ratio scale: a measurement system which allows a calculation of distances in an absolute sense.

Usually, the ratio scale and the interval scale are called cardinal scales, while the nominal and the ordinal scales are called categorical scales.

Soft econometric models are models dealing with non-metric variables, normally measured on a categorical scale.

Very often ordinal data or categorical data are being used in such models. During the last decade, several techniques have been developed for taking account of soft information in econometric models. The following (non-exhaustive) set of techniques for soft (often ordinal) data in econometric models may be mentioned:

- ordinal correlation analysis based on rank correlation coefficients for pairwise series of ordinal data (for instance, the Spearman coefficient)
- dummy variables methods especially related to non-metric explanatory variables measured on a nominal scale.
- path models based on proxy variables related to a certain quantitative attribute of the original variables (see Blalock, 1964). Two useful techniques for path models are:
  - Lisrel: a maximum likelihood approach for latent variables (see Folmer, 1980, and Jöreskog, 1977).
  - Partial Least Squares: an iterative regression analysis for identifying a block structure among the latent variables (see Wold, 1975).
- multidimensional scaling techniques transforming ordinal explanatory variables into a smaller number of cardinal explanatory variables (see Nijkamp, 1980b). This method will be employed in the empirical illustration in section 6.

- categorical data analysis employing nominal or ordinal information on objects (based on survey questionnaires, e.g.) in order to transform the proportion of certain categorial data into a linear logit form for an explanatory model (see Wrigley, 1980).
- logit and probit analysis for discrete data on (mainly) qualitative attributes of objects. This analysis is based on a probabilistic approach in which usually the frequencies of the occurrence of a phenomenon (or the shares of a variable in a whole set) are used as data input (see Domencich and McFadden, 1975, Van Lierop and Nijkamp, 1980, and Theil, 1971).
- ordinal regression analysis based on various modes of stochastic orderings of an ordinality structure, for instance by means of a proportional odds model or a proportional hazards model via a logit transformation (see McCullagh, 1980); or non-metric regression analysis based on optimal scaling methods (see Blommestein, 1981b, 1981d).
- contingency table analysis in combination with chi-square methods, dummy variable regression, and analysis - of - variance methods (see Grizzle et al., 1969, and Lehen and Koch, 1974).
- pairwise logit transformation of ordinal data on response and covariate variables via a pairwise dominance analysis by means of so-called regimes leading to probability statements regarding the effect of a specific regime of explanatory variables on the occurrence of a dominance of dependent variables (see Nijkamp and Rietveld, 1980).

Finally, it has to be noted that the abovementioned measurement problems of variables are different from uncertainty problems emerging from the stochastic nature of variables; probability distributions of stochastic elements can be specified for any measurement scale. Obviously, these measurement problems may be related to the fuzzy nature of some variables, because fuzziness implies that the boundaries of the measurement space of variabels are not exactly known, but have to be approximated by means of (cardinal) membership grades (see Zadeh et al., 1975). Fuzzy set problems, however, fall beyond the scope of this paper.

After this discussion of soft information problems in spatial econometric and statistical analyses, it is worth while to examine the possibility of studying causality patterns of soft spatial models by means of appropriate techniques. It will be shown in the next section that graph theory provides an extremely useful and also operational tool for soft spatial causality problems.

5. Graph Theory and Causality Analysis

The complex relationships in a spatial system can be represented in many forms, for instance, mathematical models, qualitative relationships, path diagrams etc. Causal models (including non-metric models) are increasingly analyzed by means of path and graph analyses. Especially graph theory provides an attractive uniform multilevel language for dealing with complex problems (see also Riguet, 1980).

In regard to soft information, the proof of complex casuality structures is fraught with statistical problems. A meaningful way of formally analyzing a causality structure based on soft information is the use of graph theory. Graph theory is a mathematical tool for reducing the structural and functional complexity in a system toward a set of systematic linkages that allow a further mathematical treatment (see among others Andrasfai, 1977, Behzad and Chartrand, 1971, Beineke and Wilson, 1978, Christofides, 1975, Lane, 1970, and Marshall, 1971).

For the analysis of causality relationships especially the directed graphs (di-graphs) are important; a di-graph is a finite non-empty set of vertices V together with a set E (distinct from V) of ordered pairs of individual elements of V. The elements of E are called arcs. For instance, (a,b) is an arc means: there is a relationship from a to b, or

$$a \longrightarrow b$$

A causality relationship between a and b presupposes formally the existence of an arc. The degree of causality can be measured via the degree of the associated vertex (a vertex of degree n is n-valent), where the degree is only related to the (unweighted) number of oriented graphs. The algebraic representation of such a causality analysis may proceed via an oriented adjacency matrix.

In this way, graphs may act as a mathematical representation of structural models. Suppose, for instance, a structural model with endogenous variables $\underline{y} \in R^N$ and exogenous variables $\underline{x}$:

$$\underline{h}(\underline{y}, \underline{x}) = \underline{0} \tag{12}$$

This model is not necessarily a fully specified mathematical model, while the sets of variables are not necessarily measured on a cardinal metric. A causal structure of (12) presupposes a set $\underline{h}$ of cause-effect relationships which partitions a set of variables $\underline{z}$ into $\underline{y} \cup \underline{x}$. A set of relationships can then be represented by a bi-partite graph G (see also Harary et al., 1965, and Gilli, 1980):

$$G = (\underline{h}, \underline{z}, \underline{q}), \tag{13}$$

where $\underline{q} = \{(h_i, x_j) | h_i R x_j\}$ is a set of edges. Clearly, the information contained in (13) can also be transformed into a related adjacency matrix. Next, a causal structure for a set of relationships can be represented by means of an oriented graph G_0:

$$G_0 = (\underline{h}, \underline{z}, \underline{s}) \tag{14}$$

where $\underline{s}$ is a set of arcs.

In addition, one may also distinguish a directed graph G_d which is based on a well-defined contraction of the vertices.

On the basis of the above mentioned notions of graphs various types of causality may be defined (see Gilli, 1980):

- direct causality:

  $$z_j D z_{j'} \leftrightarrow G_d = \{\text{arc } (z_j, z_{j'})\} , \tag{15}$$

  where D represents a relationship based on a directed graph.
- immediate causality:

  $$z_j C z_{j'} \leftrightarrow G_d = \{\text{path } (z_j, z_{j'})\} , \tag{16}$$

  where C defines a quasi-ordering with the following properties:

z_jCz_j (reflexivity) $\forall j$
$(z_jCz_{j'}) \wedge (z_{j'}Cz_{j''}) \rightarrow z_jCz_{j''}$ (transitivity)
- mutual causality (for endogenous variables)[1]:

$y_nEy_{n'} \leftrightarrow \{(y_nCy_{n'}) \wedge (y_{n'}Cy_n)$, (17)

where E is a reflexive, transitive and symmetric relationship. It should be noted, however, that a symmetric causality structure may be questionable, as this may be due to the neglect or improper treatment (e.g. enforced aggregation) of the time dimension. In those cases, a recursive system may be more appropriate (see Wold, 1954).

Finally, some simple topological measures of network or graph structures will be discussed, together with a possible causal interpretation. These measures are based on so-called gross characteristics of networks (cf. Garrison and Marble, 1962; Kansky, 1963).

The cyclomatic number is defined as :

$$\mu = E - V + G_S \qquad (18)$$

where E is the number of edges (links) in the network, V the number of vertices (nodes), and G_s the number of subgraphs.
The cyclomatic number μ defines the number of fundamental circuits in a network. A pairwise comparison of networks with the same number of variables (vertices) may reveal approximately the differences in the causal (recursive/independent) structure of networks.
The alpha - or redundancy - index gives additional information about the connectivity of networks. It is defined as follows:

Planar graphs : $\alpha = \frac{\mu}{2V-5}$

Non-planar graphs: $\alpha = \frac{2\mu}{(V-1)(V-2)}$ (19)

1) Formally, a new graph for endogenous variables can be obtained by eliminating all vertices with in-degree smaller than 1 from the original graph (exogenous variables have vertices with in-degree zero).

Since the α-index is defined as the ratio between the observed number of circuits and the maximum number of circuits, its value provides relatively sensitive information about the causal form of networks, both in the case of a single network and pairwise comparisons of networks.

The beta-index, defined as

$$\beta = E/V \ , \tag{20}$$

can be considered as a simple measure of the complexity of causal networks. The β-index differentiates between simple topological structures (with low β-values) and complicated structures (with high β-values).

The gamma-index is calculated by dividing the number of edges by the maximum number of edges, i.e.:

$$\begin{aligned} &\text{Planar graphs} &&: \ \gamma = \frac{E}{3(V-2)} \\ &\text{Non-Planar graphs} &&: \ \gamma = \frac{E}{V(V-1)/2} \end{aligned} \tag{21}$$

Like the α-index,its value provides information about the connectivity of the network.

Finally it can be noted that it is possible that different networks yield the same values for α-, β-, γ- and μ-indices.
Therefore, it might be of interest to compare 'restricted' networks, for example, by considering only a limited number of variables; by deleting identities; by differentiating between directed graphs (G_d), homogeneous graphs (G_h) and p-graphs (G_p); by focussing on well-defined aspects of the causal structure of models such as the relationship(s) between national and regional variables, final demand and supply of production factors (see Rietveld, 1980); etc.

It should be noted that the abovementioned causality analysis can be used in a spatial system by making a distinction between intra-causality and inter-causality of a regional system. Suppose a spatial system is composed of R regions, while each region is described by means of a set of structural relations reflecting also interregional linkages. Next, the intra-causality structure of a region refers to the causality of the variables and relationships within the region at hand, whereas the inter-causality refers to the causality of the whole system including interregional causal linkages. It is clear that a poorly interwoven spatial system will have a much lower causality degree than a single region. In this way, causality analysis for modules of a spatial system can also be used to obtain more insight into the connectivity of the system at hand.

6. Empirical Application

In the foregoing sections three basic elements have been distinguished: causal relationships, soft data and graph theory. Graph theory appeared to provide a useful framework for causality analysis in case of soft data. This approach will be illustrated by means of a simplified static causal model for regional (i.e. provincial) economic information in the Netherlands (1970). This model, partly based on hard information and partly based on soft information, can be represented means of the following diagram:

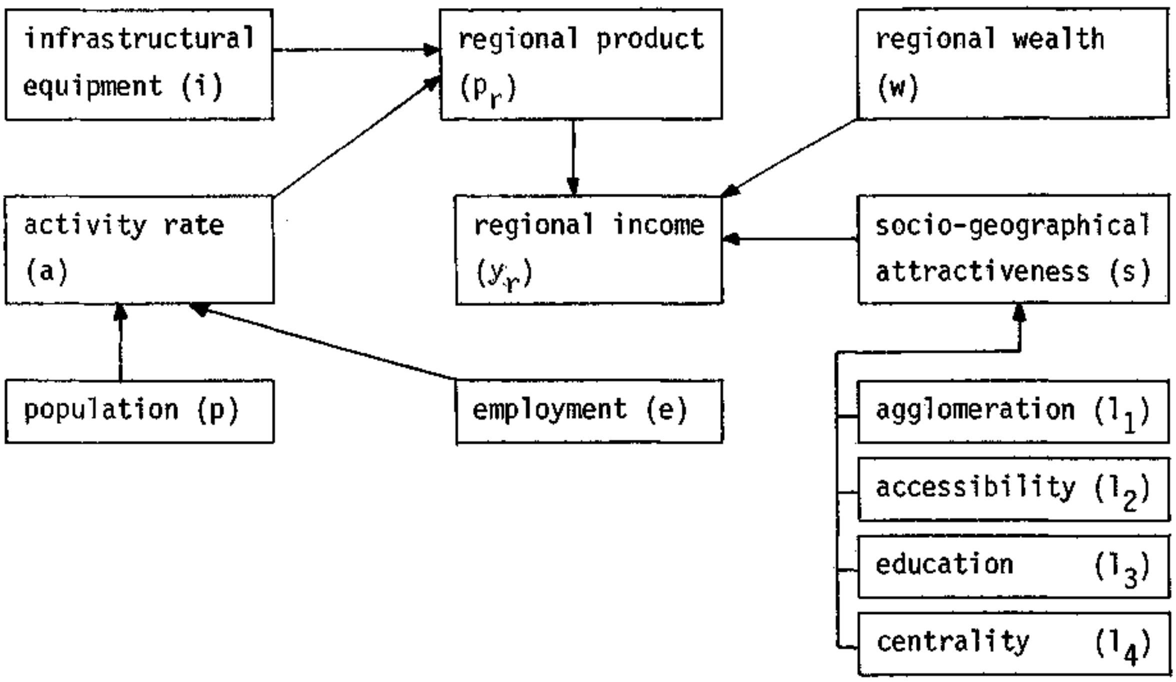

Fig. 1: Causal flow diagram of a simple regional income information model

The model represented by the arrow scheme in Fig. 1 can be described by the following equations:

- a definitional relationship for the activity rate (on a metric scale):

$$a = e/p \qquad (22)$$

- an explanatory relationship for socio-geographical attractiveness defined by regional agglomeration, accessibility, educational facilities and centrality (the y explanatory variables are measured on an ordinal scale):

$$s = f(l_1, l_2, l_3, l_4) \qquad (23)$$

- a so-called quasi-production function that explains regional product from infrastructure capital and the average rate (on a metric scale;see Nijkamp, 1981):

$$p_r = \alpha_0 i^{\alpha_1} a^{\alpha_2} \qquad (24)$$

- an income formation relationship that relates average regional income to average gross regional product, average regional wealth and the socio-geographical attractiveness (on a metric scale; see Nijkamp, 1980b).

$$y_r = \kappa_0 + \kappa_1 p_r + \kappa_2 w + \kappa_3 s \qquad (25)$$

First, the attention will be focused on the causality structure of this model. Figure 1 can be conceived of as a directed graph $G_d = (\underline{z}, \underline{t})$ with a set of vertices $\underline{z} = \{a, s, p_r, y_r\} \cup \{p, e, w, i, l_1, l_2, l_3, l_4\}$ and edges (the arrows in figure 1) $\underline{t} = \{(z_j, z_{j'}) | (z_j, h_i) \wedge (h_j, z_{j'}) \varepsilon \underline{m}\}$. Given the partition $\underline{z} = \underline{y} \cup \underline{x}$, as well as a set of binary relations of the form $h_i R z_j$, $h_i \varepsilon H = \{h_1, h_2, h_3, h_4\}$, the <u>causal structure</u> of the spatio-economic system (22) - (25) can be expressed in the form of the adjacency matrix A (cf. section 2):

$$A = \left[\begin{array}{cccc|cccccccc} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Replacement of the non-zero elements of A by the 'expected'sign of the coefficients in system (22) - (25), may easily yield the sign matrix S (see section 2). This is straightforward and will not be presented here.

The next step of the empirical application will be a calculation of the causality degree, a discussion of the kinds of causality in Fig. 1, and a calculation of the α-, β-, γ-, and μ-indices (see section 5). This analysis can be carried out on the basis of the information contained in G_d, so that even in the case of soft data input this causality analysis is a relevant approach.

According to the approach set out in section 5, the causality degree between the successive variables can be calculated via the degree of the associated vertex between each pair of variables. The number of edges incident on a vertex z_j is called the degree (valency) of the vertex, and is denoted by $d(z_j)$. This can easily be incorporated in the following table, where the entries indicate the causality degree between each pair of variables.

	a	s	p_r	y_r	p	e	w	i	l_1	l_2	l_3	l_4	$d(z_j)$
a	-	0	1	0	1	1	0	0	0	0	0	0	3
s	0	-	0	1	0	0	0	0	1	1	1	1	5
p_r	1	0	-	1	0	0	0	1	0	0	0	0	3
y_r	0	1	1	-	0	0	1	0	0	0	0	0	3
p	1	0	0	0	-	0	0	0	0	0	0	0	1
e	1	0	0	0	0	-	0	0	0	0	0	0	1
w	0	0	0	1	0	0	-	0	0	0	0	0	1
i	0	0	1	0	0	0	0	-	0	0	0	0	1
l_1	0	1	0	0	0	0	0	0	-	0	0	0	1
l_2	0	1	0	0	0	0	0	0	0	-	0	0	1
l_3	0	1	0	0	0	0	0	0	0	0	-	0	1
l_4	0	1	0	0	0	0	0	0	0	0	0	-	1

Table 1: Table with causality degrees.

Concerning the kinds of causality (direct, immediate and mutual) the following comments can be made.

Direct causality relationships, $z_j D z_{j'}$, are defined by pairs of edges $\{z_j, z_{j'}\} \varepsilon \underline{t}$ of G_d viz., $\{a, p_r\}$, $\{i, p_r\}$, $\{w, y_r\}$, $\{p_r, y_r\}$, etc.

Immediate causality relationships, $z_j C z_{j'}$, are defined by open edge trains in which all the vertices are distinct, i.e. by paths of the graph G_d viz., $\{(p, a), (a, p_r), (p_r, y_r)\}$, $\{(e,a), (a, y_r), (p_r, y_r)\}$, $\{(l_1, s), (s, y_r)\}$, etc.

Mutual causality relationships, $y_n D y_{n'}$, do not occur in system (22) - (25), since the regional development model possesses a recursive causal structure [compare also the adjacency matrix A, and equation (26)].

The topological measures for a graph structure can also be calculated according to the indices discussed in section 5.
The cyclomatic number μ corresponding to the network shown in figure 1, is equal to zero. This means that the recursive causal system (22) - (25), can be conceived of as a branching network. From definition (19), it can be seen that : $\mu=0 \rightarrow \alpha=0$. This indicates a 'minimal spanning tree' in which the removal of any edge $(z_j \; , \; z_{ji}) \in \underline{t}$ would break G_d into two unnconnected subgraphs.
Calculation of the beta-index yields the value β = .92, indicating a very simple network structure (the lower portion of the β-scale - i.e. from zero to one - differentiates between different types of branching networks). The low value of the gamma-index, γ =.37 , confirms the fact that the causal structure of the regional development model is 'minimal' connected (for example, in the sense of a minimal spanning tree),indicating a poorly interwoven spatial system.
The foregoing causality analysis demonstrates that - even without the use of a metric system for the variables- meaningful conclusions can be inferred regarding the causal structure of an interrelated economic model by using notions from graph theory.

The second part of the empirical application concerns the estimation of equations (22) - (25), taking into account the non-metric nature of (23).
The regional structural model can be expressed in the following stochastic form:

$$Y\Gamma + XB = U \qquad (26)$$

where $Y = [\underline{y}_1 \; \underline{y}_2 \; \ldots \; \underline{y}_r \; \ldots \; \underline{y}_R]'$ is a (RxN) matrix with as elements the (1xN) vectors with data $\underline{y}_r$ on all N endogenous variables in all regions r; Γ an(NXN) matrix with coefficients for endogenous variables; X a (RXK) matrix with as elements the (1XK) vectors with data $\underline{x}_r$ on all K exogenous variables in all regions r; B a (KXN) matrix with coefficients of exogenous variables; and U the (RXN) matrix, each row of which is the (1XN) vector $\underline{u}_r \sim$ NID (0, Σ) with error terms.

Since Γ is a triangular matrix and the variance-covariance matrix $\Sigma = E(\underline{u}_r' \underline{u}_r)$ has a diagonal form, the simultaneous-equation system (26) is a recursive equation system. This means that - in principle - each equation of system (26) may be estimated by OLS. Equation (22) is a definition and can directly be calculated on the basis of available (metric) regional cross-section data. Equation (24) is a (metric) quasi-production function of a Cobb-Douglas type and can easily be estimated, given the availability of data on p_r, i and a (see Nijkamp, 1980b). The results for the coefficients are[1]:

$$\ln \hat{\alpha}_0 = -0.973 \ (0.750)$$
$$\hat{\alpha}_1 = 0.620 \ (0.240) \qquad R^2 = 0.515$$
$$\hat{\alpha}_2 = 1.017 \ (0.578)$$

Equation (25) is a linear relationship which can readily be estimated via OLS (see Nijkamp 1981). The results are[2]:

$$\hat{\kappa}_0 = 2947.30 \ [5.46]$$
$$\hat{\kappa}_1 = 0.02 \ [2.90]$$
$$\hat{\kappa}_2 = 0.28 \ [4.84] \qquad R^2 = 0.928$$
$$\hat{\kappa}_3 = 137.07 \ [1.29]$$

The assumption of (25) was that s is a metric variable. In reality, however, the socio-geographical attractiveness s is a qualitative variable which cannot be directly measured on cardinal scale. It is composed of soft data on l_1, l_2, l_3 and l_4. These regional indicators have been measured as regional ordinal data. In order to use s in equation (25), it was necessary to transform the ordinal information on l_1, l_2, l_3 and l_4 into cardinal information via a multidimensional scaling analysis (see section 4). It turned out that a one-dimensional configuration led already to a fairly high goodness-of-fit for the transformation of the ordinal information into a cardinal configuration. Thus, the sequence of operations to be carried out was: ordinal data on $l_1, \ldots, l_4$ → multidimensional scaling of ordinal data → cardinal one-dimensional configuration for regional socio-geographical attractiveness → use of provincial configuration of s in (25) (see also Nijkamp, 1980b).
The latter part of this analysis shows that methods for dealing with non-metric variables may form a useful complement to traditional techniques.

So the conclusion can be drawn that soft econometric techniques may be useful tools for dealing with qualitative information in causal regional modelling. Of course, it should be kept in mind that the foregoing model has only an illustrative meaning and that more extensive research work on soft causal models has to be done.

1) Figures in brackets are standard deviations.
2) Figures in square brackets are t-values.

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