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Abstract

One of the major issues in the analysis of unemployment durations concerns the distinction between state dependence (i.e., duration dependence of the exit rate out of unemployment for a given individual) and unobserved heterogeneity. Empirical studies rely heavily on functional form restrictions, which may be hazardous. This paper presents nonparametric eyeball tests for both phenomena. By examining features of graphs and tables of aggregate duration data, one can detect in a simple way whether there is evidence for state dependence and/or unobserved heterogeneity, without the need to make parametric assumptions. The tests are applied to data from several European countries.

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1. Introduction.

In the past decade, the econometric analysis of unemployment durations has become widespread. One of the major issues in this literature concerns the distinction between state dependence of the hazard rate (i.e. dependence of the exit rate out of unemployment for a given individual on his elapsed duration of unemployment) and unobserved heterogeneity (for surveys, see for example Devine & Kiefer (1991)). Often, there is reason to believe that for a given individual the hazard rate decreases as a function of duration. For example, there may be stigma effects reducing the number of job opportunities for the long–term unemployed (see e.g. Vishwanath (1989) and Van den Berg (1991)). On the other hand, the presence of unobserved heterogeneity in the distribution of the duration variable causes the hazard rate of the distribution of observed durations to decrease as well. This follows from the fact that on average individuals with the largest hazard rate leave unemployment first. Obviously, from a policy point of view, it is important to know the relative importance of state dependence (also called genuine duration dependence) on the one hand, and unobserved heterogeneity on the other. For example, if state dependence is the dominant factor, then efforts may be concentrated on the long–term unemployed, while otherwise it may be useful to screen short–term unemployed and concentrate efforts on those with bad characteristics. Also, the degree of state dependence is of importance for macro analyses of the labour market (see e.g. Layard, Nickell & Jackman (1991) and Jackman & Savouri (1992)). However, since both factors affect the hazard rate in a similar way, it seems to be hard to distinguish empirically between them.

In the present paper we present nonparametric eyeball tests that can be used to detect whether there is state dependence and/or unobserved heterogeneity. So–called eyeball tests are based on easy–to–detect characteristics of data summarized in graphs and tables. By examining particular characteristics, one can detect in a simple way whether there is evidence for (one of) these phenomena, without the need to make parametric assumptions. The tests are designed to be applicable to discrete–time time–series data on aggregate outflows from different unemployment duration classes. Aggregate (or gross, or macro) data have the advantage that they provide the exact values of the exit probabilities for the different duration classes considered (averaged over unobserved heterogeneity). It should be noted from the outset that eyeball tests are not tests in the usual statistical sense. Rather, they are useful as preliminary informal checks.

We examine in detail one of the two eyeball tests for state dependence proposed by Jackman & Layard (1991), and an eyeball test for unobserved heterogeneity based on Van den Berg & Van Ours (1993). We present assumptions under which they are sensible tests and characterize cases in which they are likely to result in rejection of the null hypothesis. Further, we show that another test for state dependence in Jackman & Layard (1991), which was first proposed by Budd, Levine & Smith (1988), and which is rather popular, has unattractive properties. We also outline how the ideas underlying the eyeball tests could be transferred to a continuous-time micro-econometric framework.

The eyeball tests do not rely on parametric assumptions on the determinants of the hazard. This is a marked advantage. Intuitively, it is clear that in parametric analyses the results on the degree of state dependence and unobserved heterogeneity may be extremely sensitive with respect to misspecification of the corresponding parts of the model (see e.g. Ridder (1987) for some evidence).

Section 2 presents the model framework and the type of data needed to perform the eyeball tests. Basically, the model is a Mixed Proportional Hazard (MPH) model in which calendar time replaces the role of the observed explanatory (x) variables. In Section 3 the tests are examined. Section 4 contains empirical illustrations. We use data from France, the UK and The Netherlands. Section 5 concludes.

2. The model.

In this section we present the unemployment duration model and the underlying assumptions. We use two measures of time. The variable t denotes the duration of unemployment, as measured from the moment the individual becomes unemployed. The variable τ denotes calendar time, which has its origin somewhere in the past. For simplicity we take t and τ to have the same measurement scale, apart from the difference in origin. Both t and τ are discrete variables. As an example, consider an individual who is unemployed for t periods at calendar time τ. If he fails to leave unemployment in period t, he will be unemployed for t+1 periods at calendar time τ+1.

Ideally, aggregate data give the total numbers of individuals in the labor market who are unemployed for t periods of time (t=0,1,2,...) at calendar times τ, τ+1, τ+2, etc.. By comparing the number of individuals who are unemployed for t periods of time at τ to the number unemployed for t+1 periods at τ+1, we observe the exit probability out of unemployment at calendar time τ

for duration t. In other words, we observe the conditional probability that an individual leaves unemployment when being unemployed for t periods, when calendar time equals τ at the moment of exit, for different values of t and τ.

In the model, t is endogenous, whereas τ is an explanatory variable, in the sense that the exit probability out of unemployment for individuals with duration t may vary over calendar time. Thus, calendar time is assumed to capture cyclical macro effects on individual exit probabilities out of unemployment as well as structural changes influencing these probabilities.

The model explains variations in unemployment duration distributions in terms of observed and unobserved individual characteristics, calendar time, and the state dependence pattern. Usually, gross data do not contain information on individual characteristics that could be used as explanatory variables. At best, gross figures are collected separately for a few different groups of individuals. In the sequel we will therefore suppress the conditioning on the prevailing value of observed explanatory variables.

We assume that all variation in the exit probabilities out of unemployment can be explained by the prevailing unemployment duration t and calendar time τ and by unobserved heterogeneity across individuals. We denote the unobserved heterogeneity variable by v. Consider an individual with unobserved characteristics v who is unemployed for t periods when calendar time equals τ. We denote the conditional probability that this individual leaves unemployment after t periods of unemployment by $\theta(t|\tau,v)$. By definition, this is the exit probability out of unemployment (or hazard) at t conditional on τ and v. The unemployment duration density conditional on calendar time and conditional on v can be constructed from these exit probabilities. For example, the probability that unemployment duration equals t, when calendar time was $\tau-t$ at the moment of inflow into unemployment, conditional on v, equals

$$\theta(t|\tau,v) \cdot \prod_{i=0}^{t-1} (1 - \theta(i|\tau-t+i,v)) \tag{2.1}$$

for all $t\in\{0,1,..\}$. We take the product term to be one if $t=0$.

We make the following assumptions.

Assumptions

1. MPH: $\theta(t|\tau,v)$ has a mixed proportional hazard specification, i.e. there are functions ψ_1 and ψ_2 such that

$$\theta(t|\tau,v) = \psi_1(t).\psi_2(\tau).v \tag{2.2}$$

with ψ_1 and ψ_2 positive and uniformly bounded from above. Further, the distribution of v is such that, for every t and τ, $Pr(0<\theta(t|\tau,v)<1) = 1$.

2. Independence of v and τ: v does not depend on the moment of inflow into unemployment and does not change during unemployment.

3. Variation over calendar time: the function ψ_2 is not constant.

If ψ_1 is constant then there is no state dependence. If the distribution of v is degenerate then there is no unobserved heterogeneity.

Assumption 1 is similar to the standard MPH assumption in reduced-form duration models for micro duration data (see Lancaster (1990) for an extensive survey of such models). In models for micro duration data, dependence on calendar time is usually ignored, and the role of τ in the model above is replaced by the role of observed explanatory variables x. Elbers & Ridder (1982) prove that the latter type of models are nonparametrically identified if assumptions similar to above are satisfied. In particular, state dependence and unobserved heterogeneity can be distinguished empirically.

An important difference between the present model and these MPH models for micro duration data is that here we have discrete time, whereas in micro studies time is usually treated as continuous. Because of this, we had to introduce the last line of Assumption 1. Note that it implies that the support of v is bounded. This in turn implies that all moments of v exist.

The present model should be regarded as a flexible accounting device for discrete time data, with an appealing interpretation. Later on we show how the eyeball tests can be used to infer whether the model specification is correct.

The first part of Assumption 2 states that the distribution of v in the inflow is the same whatever the moment of inflow. So, it rules out that there are cohort effects in the distribution of the unobserved heterogeneity term. There is abundant evidence that the season at the moment of inflow influences the unemployment duration (see e.g. Van den Berg & Van Ours (1994)). A possible explanation for this is that the composition of the inflow, as far as unobserved characteristics are concerned, varies over the seasons. This would violate Assumption 2. We ignore this issue in the theoretical analysis in this paper, while in the empirical illustration we seasonally adjust the data.

Assumption 3 is similar to the assumption in Elbers & Ridder (1982) that there is dispersion of observed explanatory variables. Note that sufficient for Assumption 3 is that there is a data point τ_0 somewhere in the time series such that $\psi_2(\tau)=a_1$ for $\tau<\tau_0$ and $\psi_2(\tau)=a_2$ for $\tau\geq\tau_0$. In that case we have two steady states.

As mentioned above, the data provide observations on the conditional probabilities that individuals leave unemployment when being unemployed for t periods, when calendar time equals τ at the moment of exit, for different values of t and τ. These probabilities are unconditional on the unobserved heterogeneity term v, and will be denoted by $\theta(t|\tau)$. To express these observed exit probabilities $\theta(t|\tau)$ in terms of the exit probabilities $\theta(t|\tau,v)$, we have to integrate v out of the latter. Let $\boldsymbol{t}$ denote the random unemployment duration, and t its realization. We have that

$$\theta(t|\tau) \equiv \frac{Pr(\boldsymbol{t}=t|\tau)}{Pr(\boldsymbol{t}\geq t|\tau)} \equiv \frac{\mathrm{E}_v(Pr(\boldsymbol{t}=t|\tau,v))}{\mathrm{E}_v(Pr(\boldsymbol{t}\geq t|\tau,v))} \tag{2.3}$$

in which $Pr(\boldsymbol{t}=t|\tau,v)$ and $Pr(\boldsymbol{t}\geq t|\tau,v)$ can be expressed in terms of $\theta(t|\tau,v)$ (note that equation (2.1) gives $Pr(\boldsymbol{t}=t|\tau,v)$). By doing this, and by substituting equation (2.2), we get

$$\theta(t|\tau) = \frac{\psi_1(t).\psi_2(\tau).\ \mathrm{E}_v\left[v.\prod_{i=0}^{t-1}\left[1-\psi_1(i).\psi_2(\tau-t+i).v\right]\right]}{\mathrm{E}_v\left[\prod_{i=0}^{t-1}\left[1-\psi_1(i).\psi_2(\tau-t+i).v\right]\right]} \tag{2.4}$$

To avoid confusion, note that even though t enters the argument of ψ_2 in (2.4), ψ_2 is a function of calendar time only. From (2.4) it follows that $\theta(t|\tau)$ can be expressed in terms of the "structural functions" ψ_1, ψ_2 and the distribution function $G(v)$ of v. We denote $\mathrm{E}_v(v^i)$ by μ_i.

3. Eyeball tests.

3.1. *Jackman & Layard* (1991)*'s "stock/flow" eyeball test for state dependence.*

It should be noted from the outset that the exposition below is more formal than in Jackman & Layard (1991). For the eyeball test to be sensible we have to strengthen Assumption 3 and add another assumption,

3'. Steady states: there are at least two calendar time intervals T_i such that $\psi_2(\tau)=a_i$ for $\tau\in T_i$ and $a_i\neq a_j$ for $i\neq j$. T_i is so large that it contains points such that all individuals who are unemployed at such a point have entered unemployment during T_i and will exit unemployment during T_i. For the cohorts entering and leaving unemployment in T_i the mean unemployment duration is finite.

4. Constant inflow: the inflow rate into unemployment for individuals with characteristics v does not depend on calendar time except possibly for the value of $\psi_2(\tau)$.

Thus, we have (at least) two steady states. If T_1 and T_2 are sufficiently large and if Assumptions 1 and 2 are valid then it is not difficult to check whether the first part of Assumption 3' is true. It implies that for each cohort entering unemployment just after the beginning of T_i the observed unemployment distribution must be the same, while the distribution for cohorts entering just after the beginning of T_1 must differ from the distribution for cohorts entering just after the beginning of T_2.

Assumption 4 is similar to the standard "constant inflow rate" assumption made in empirical analyses of duration data from stock samples (see e.g. Heckman & Singer (1984) and Ridder (1984)). Note that it refers to the size of the inflow, whereas Assumption 2 refers to the composition of the inflow. Assumption 4 implies that within a steady state the inflow rate is constant. So, if data on the inflow size are available then this assumption can be checked as well.

The eyeball test is based on the ratio of the observed exit probability for the newly unemployed and the observed over-all exit probability in the stock of unemployed. Specifically, it compares this ratio for one steady state to the ratio for others. Consider a steady state. Because of Assumption 3', we may replace τ in the exit probabilities at τ by the value a of ψ_2 in that steady state, if τ is a point of time satisfying the second part of Assumption 3'. (So, for example, we may replace $\theta(t|\tau,v)$ by $\theta(t|a,v)$. Note however that τ is observed while a is unobserved. The last part of Assumption 3' can now be written as $E(t|a)<\infty$.) From (2.4), the observed exit probability for the newly unemployed equals

$$\theta(0|a) = \psi_1(0).a.\mu_1 \tag{3.1}$$

Let p denote the elapsed duration of unemployment of individuals in the stock of unemployed at a certain point of time. Consider a point of time satisfying the second part of Assumption 3'. We want to know the observed over-all exit probability in the stock of unemployed at that point of time. We denote this probability by $\overline{\theta}(a)$. It equals the proportion of individuals unemployed at that point who leave unemployment within one time period. This in turn equals the sum over all values of p of the probability that the elapsed duration of unemployment $\boldsymbol{p}$ equals p and the residual duration $\boldsymbol{r}$ equals zero, conditional on presence in the stock of unemployed at that particular point of time. From results in for example Heckman & Singer (1984) and Ridder (1984) it can be inferred that under Assumptions 1, 2, 3' and 4 this equals

$$\bar{\theta}(a) = \sum_{p=0}^{\infty} Pr(\boldsymbol{p}=p,\boldsymbol{r}=0|a) = \frac{\sum_{p=0}^{\infty} \mathrm{E}_v\left[\psi_1(p).a.v. \prod_{i=0}^{p-1} \left[1-\psi_1(i).a.v\right] \right]}{\sum_{p=0}^{\infty} \mathrm{E}_v\left[\prod_{i=0}^{p-1} \left[1-\psi_1(i).a.v\right] \right]} \tag{3.2}$$

in which the product terms are one if p=0. By comparing equations (2.3), (2.4) and (3.2) it follows that the numerator in the r.h.s. of (3.2) equals one. Let $\boldsymbol{t}$ be the random variable denoting the duration of unemployment of individuals sampled in the inflow. The denominator in the r.h.s. of (3.2) equals the sum over all $t \geq 0$ of $Pr(\boldsymbol{t} \geq t|a)$. It is easy to show that for a discrete nonnegative random variable $\boldsymbol{t}$ there holds that

$$\sum_{t=1}^{\infty} Pr(\boldsymbol{t} \geq t) = \sum_{t=0}^{\infty} t.Pr(\boldsymbol{t}=t) \equiv \mathrm{E}(\boldsymbol{t})$$

Thus,

$$\bar{\theta}(a) = 1/(\mathrm{E}(\boldsymbol{t}|a)+1) \tag{3.3}$$

which is positive by virtue of Assumption 3'.

Proposition 1. *Let Assumptions* 1, 2, 3' *and* 4 *be satisfied. If there is no state dependence then* $\theta(0|a)/\bar{\theta}(a)$ *does not depend on* a. *In that case this ratio is larger than or equal to one.*

Proof. If there is no state dependence then $\boldsymbol{t}|a,v$ has a geometric distribution with parameter $\psi_1(0).a.v$. Consequently, $\mathrm{E}(\boldsymbol{t}|a) = \mathrm{E}_v(\mathrm{E}(\boldsymbol{t}|a,v)) = \mathrm{E}_v((1-\psi_1(0).a.v)/(\psi_1(0).a.v))$, so $\bar{\theta}(a) = \psi_1(0).a/\mathrm{E}_v(1/v)$. (Note that $\mathrm{E}_v(1/v)<\infty$ by virtue of Assumption 3'.) Combining this with (3.1) gives

$$\frac{\theta(0|a)}{\bar{\theta}(a)} = \mathrm{E}_v(v).\mathrm{E}_v(1/v)$$

which does not depend on a and which by virtue of Jensen's inequality is larger than or equal to one. □

Jackman & Layard (1991) were the first to argue that in the absence of state dependence $\theta(0|a)/\overline{\theta}(a)$ does not depend on a. Their exposition is more intuitive than above. For example, Assumption 4 is not mentioned. Also, they do not state the link between $\overline{\theta}(a)$ and $E(t|a)$. This link will turn out to be useful for the extensions in Subsection 3.4.

As Jackman & Layard (1991) argue, from the result above it follows that if $\theta(0|a)/\overline{\theta}(a)$ does depend on a (i.e. varies between steady states) then there must be state dependence. It is clear that the attractiveness of a test based on this depends on the extent to which the reverse holds as well. We will examine this in detail. In the general model, the ratio $\theta(0|a)/\overline{\theta}(a)$ does not depend on a if and only if $a(E(t|a)+1)$ does not depend on a. Recall that $E(t|a)+1$ equals the denominator of the r.h.s. of (3.2). The derivative of $a(E(t|a)+1)$ w.r.t. a equals

$$E(t+1|a) \quad + \quad a.E_v\left[\sum_{t=1}^{\infty} \sum_{k=0}^{t-1} -\psi_1(k).v \,.\prod_{\substack{i=0 \\ i\neq k}}^{t-1} \left[1 - \psi_1(i).a.v\right] \right]$$

which can be rewritten as

$$E_v \sum_{t=0}^{\infty} \left\{ \left[(t+1).\psi_1(t).a.v - \sum_{k=0}^{t-1} \frac{\psi_1(k).a.v}{1-\psi_1(k).a.v} \right] \prod_{i=0}^{t-1} \left[1 - \psi_1(i).a.v\right] \right\} \tag{3.4}$$

in which the second summation is zero if t=0. If the term in square brackets in (3.4) is positive (negative) for every t>0 and every possible v and a then the whole expression (3.4) is positive (negative). In that case $\theta(0|a)/\overline{\theta}(a)$ is strictly increasing (decreasing) in a. The conditions on the term in square brackets can be translated into conditions on the ψ_1 function, given v and a. If the length of the unit time period approaches zero then these conditions become more transparent. For the limiting case we obtain the strong results listed in Proposition 2 below. In this proposition, IFRA and DFRA stand for Increasing Failure Rate Average and Decreasing Failure Rate Average, respectively (see e.g. Hollander & Proschan (1984); their definitions are stated in the proof of Proposition 2).

Proposition 2. *Consider the continuous-time analogue of the model satisfying Assumptions* 1, 2, 3' *and* 4 *in which* $\theta(0|a)$ *exists and is positive.*

(i) *The ratio* $\theta(0|a)/\overline{\theta}(a)$ *does not change with* a *on* $<0,\infty>$ *if and only if there is no state dependence.*

(ii) *If* $\psi_1(t)$ *as a hazard rate is strictly IFRA (for which it is sufficient that* $\psi_1(t)$ *is strictly increasing) then* $\theta(0|a)/\overline{\theta}(a)$ *is strictly increasing in* a.

(iii) *If* $\psi_1(t)$ *as a hazard rate is strictly DFRA (for which it is sufficient that* $\psi_1(t)$ *is strictly decreasing) then* $\theta(0|a)/\overline{\theta}(a)$ *is strictly decreasing in* a.

Proof. In continuous time, $\theta(t|\tau,v)$, $\theta(t|\tau)$ etc. are rates rather than probabilities. Equation (3.1) still holds while (3.3) is replaced by

$$\overline{\theta}(a) = 1/\mathrm{E}(t|a) \qquad (3.5)$$

(this can be inferred from for example Heckman & Singer (1984) and Ridder (1984)). According to Lancaster (1990), the only continuous-time MPH model for which $\mathrm{E}(t|a)$ is multiplicative in a is the model in which the baseline hazard ψ_1 follows a Weibull specification: $\psi_1(t) = \alpha t^{\alpha-1}$, with $0<\alpha<\infty$. The only α for which $a\mathrm{E}(t|a)$ does not depend on a is $\alpha=1$ (which incidentally is the only α for which $0<\theta(0|a)<\infty$). This means that ψ_1 is constant, so there is no state dependence.

It can be shown that the continuous-time equivalent of (3.4) (which is the derivative of $a\mathrm{E}(t|a)$) equals

$$\int_0^\infty \left[t\psi_1(t) - \int_0^t \psi_1(u)\mathrm{d}u\right] .\mathrm{E}_v \left\{ av.\exp\left\{-av.\int_0^t \psi_1(u)\mathrm{d}u\right\} \right\} \mathrm{d}t$$

If the term in square brackets is positive (negative) for every $t>0$ then the expression above is positive (negative) for every a. In that case $\theta(0|a)/\overline{\theta}(a)$ is strictly increasing (decreasing) in a. The term in square brackets is positive for every $t>0$ if and only if the distribution of $t|a,v$ has strictly IFRA, or, in other words, if $\psi_1(t)$ as a hazard rate has the "strictly IFRA" property. Sufficient for this is that the distribution of $t|a,v$ has strictly IFR (increasing failure rate), or, in other words, that $\psi_1(t)$ is strictly increasing. Similarly, the term in square brackets is negative for every $t>0$ if and only if the distribution of $t|a,v$ has strictly DFRA. For the latter it suffices that $\psi_1(t)$ is strictly decreasing. □

Somewhat loosely, one might translate IFRA as "a failure rate (here: $\psi_1(t)$) which is increasing in most points and never strongly decreasing" and DFRA analogously. Proposition 2 suggests the following practical guidelines. First, if one does not want to rule out that $\psi_1(t)$ is strongly increasing at some t and strongly decreasing at other t, then one should use as many steady states a as possible when examining the behaviour of $\theta(0|a)/\overline{\theta}(a)$. Secondly, if it is known that state dependence is almost monotone, then comparing two steady states suffices, because then for any $a_1 \neq a_2$ there holds that $\theta(0|a_1)/\overline{\theta}(a_1) \neq \theta(0|a_2)/\overline{\theta}(a_2)$. So, the eyeball test discussed in this subsection has very high power against monotone state dependence. Thirdly, if state dependence is known to be monotone, then the eyeball test is informative on the sign of the state dependence. For example, if $\theta(0|a)/\overline{\theta}(a)$ for the steady state with high $\theta(0|a)$ is smaller than $\theta(0|a)/\overline{\theta}(a)$ for the steady state with low $\theta(0|a)$, then there is negative state dependence (meaning that the exit probability out of unemployment for a given individual decreases as a function of unemployment duration). Instead of using $\theta(0|a)$ one may also use $\overline{\theta}(a)$ in this relationship, since both are increasing in the value of a (for $\overline{\theta}(a)$ this follows from equations (3.2) and (3.3)).

We finish this subsection by examining intuitively the fundamental idea underlying the eyeball test on state dependence. For ease of exposition, let time be continuous. If there is no state dependence then $t|a,v$ has an exponential distribution with constant exit rate $\psi_1(0).av$, so the expectation of t conditional on a and v is proportional to $1/a$ and to $1/v$. Thus, the expectation conditional on a only is proportional to $1/a$. This is exploited in the test (by multiplying the expectation with a term $\theta(0|a)$ that is always proportional to a). Equation (3.5) expresses $E(t|a)$ in terms of a quantity $\overline{\theta}(a)$ that can readily be observed in aggregate data even if $\theta(t|a)$ is not observed for every t separately. (See Jackman & Layard (1991) for a direct intuitive explanation of the fact that $\overline{\theta}(a)$ is proportional to a if there is no state dependence.) In discrete time, the argument is analogous. The test exploits the fact that $E(t|a,v)+1$ is proportional to $1/a$ and $1/v$ if there is no state dependence.

3.2. *Examining the cross effect of t and a in the logarithm of the observed exit probability*, $\log \theta(t|a)$.

Budd, Levine & Smith (1988) and Jackman & Layard (1991) present and use another eyeball test for state dependence (see also Layard, Nickell & Jackman

(1991)). This test is based on the behaviour of the observed exit probability as a function of duration in different steady states. Specifically, as shown below, it examines the sign of the cross effect of t and a in the observed log exit probability $\log \theta(t|a)$. Or, in terms of a continuous-time model, it examines the sign of the cross-derivative of $\log \theta(t|a)$ w.r.t. t and a. In this subsection we show that these statistics are not informative on the presence of state dependence. By implication, the corresponding eyeball test has unattractive properties. In contrast, we will argue that the behaviour of the cross effect is informative on the presence of unobserved heterogeneity. In Subsection 3.3 we present an eyeball test on unobserved heterogeneity exploiting this.

Like in Subsection 3.2 we have to adopt Assumptions 1, 2, 3' and 4. Budd, Levine & Smith (1988) in addition assume that v has a discrete distribution with a finite number of points of support.

Budd, Levine & Smith (1988) and Jackman & Layard (1991) state that if there is no state dependence then, when comparing a steady state with a small a to one with a large a, the proportionate difference between the exit probabilities is smaller for a high duration than it is for a low duration. Given the statements in those papers that a large a implies large observed exit probabilities at all durations, the main statement can be formalized as follows: if there is no state dependence then

$$\frac{\mathrm{d}\log \theta(t_2|a)}{\mathrm{d}a} < \frac{\mathrm{d}\log \theta(t_1|a)}{\mathrm{d}a} \qquad \text{for all } a \text{ and all } t_2>t_1>0 \qquad (3.6)$$

So, if for particular $a_1<a_2$ and $t_1<t_2$ it is found empirically that $\theta(t_2|a_2)/\theta(t_2|a_1) > \theta(t_1|a_2)/\theta(t_1|a_1)$ then the argument goes that there must be state dependence.

Note that (3.6) basically states that the cross effect of t and a in $\log \theta(t|a)$ is negative, given that both derivatives appearing in (3.6) are positive. In a continuous-time model, (3.6) is virtually the same as stating that the cross-derivative of $\log \theta(t|a)$ w.r.t. t and a is negative everywhere (i.e. $\mathrm{d}^2\log\theta(t|a)/\mathrm{d}a\mathrm{d}t < 0$), given that both derivatives appearing in (3.6) are positive.

The easiest way to show that (3.6) and related statements on the signs of derivatives are incorrect is by giving counter-examples.

Discrete distributions of unobserved heterogeneity with two points of support. From equation (3.1) it follows that $\mathrm{d}\log\theta(t|a)/\mathrm{d}a$ at $t=0$ equals $1/a$. We assume that $G(v)$ is such that $Pr(v=v_1) = p = 1-Pr(v=v_2)$ with $0<p<1$ and $0<v_1<v_2$. There

is no state dependence and we normalize $\psi_1(t)$ to one. Thus, $\theta(t|a,v) = av$ and we need $v_2<1/a$. By substituting $\psi_2(\tau)=a$, $\psi_1(t)=1$ and $G(v)$ into (2.4) and differentiating log $\theta(t|a)$ w.r.t. a we obtain

$$\frac{1}{a} - \frac{p(1-p)(v_1-v_2)^2 \cdot t.(1-av_1)^{t-1}(1-av_2)^{t-1}}{[pv_1(1-av_1)^t+(1-p)v_2(1-av_2)^t]\cdot[p(1-av_1)^t+(1-p)(1-av_2)^t]} \quad (3.7)$$

First note that for every $t>0$ this is smaller than $1/a$. By working out the product of the terms in square brackets in (3.7) and by dividing numerator and denominator by $(1-av_1)^{t-1}(1-av_2)^{t-1}$, it follows that when $t\to\infty$ then (3.7) goes to $1/a$. Thus, the value of $d\log\theta(t|a)/da$ as $t\to\infty$ equals the value at $t=0$, implying that this derivative is not a monotone function of t.

Intuitively, it is plausible that when $G(v)$ is discrete then, as $t\to\infty$, $d\log\theta(t|a)/da$ goes to its value at $t=0$. When t increases, the group of unemployed becomes increasingly more homogeneous, since the individuals with large v leave unemployment on average earlier than the individuals with small v. In the limit, the group is homogeneous (all remaining individuals have the smallest v) so the value of $d\log\theta(t|a)/da$ equals the value in a model without unobserved heterogeneity. This in turn equals the value in a model with unobserved heterogeneity at $t=0$, because at $t=0$ the selection due to heterogeneity has not yet taken place.

Suppose $a=1$, $p=1/2$, $v_1=1/5$ and $v_2=3/5$. The solid line in Figure 1 depicts $d\log\theta(t|a)/da$ as a function of t. For clarity we plot this function for integer as well as non-integer values of t. First note that this derivative is positive for every $t\geq0$. Thus, in the neighbourhood of $a=1$, larger a imply larger observed exit probabilities at all durations. Secondly, this derivative is decreasing in t only for $t<2.6$. So, when comparing a steady state with a relatively small a to one with a relatively large a, the proportionate difference between the exit probabilities is larger for large t than it is for t close to 3. Figure 2 plots $\theta(t|a)$ for two different values of a, namely $a=1$ and $a=1.35$. Clearly, the proportionate difference gets larger when t increases from 2 onwards. If $a=1.35$ then the initial exit probability is high, causing most individuals with $v=v_2$ to leave unemployment before $t=2$. As a result, the group of individuals who are unemployed at $t=2$ is almost homogeneous, so the exit probability is almost constant afterwards. If $a=1$ then at $t=2$ there are still many unemployed individuals with $v=v_2$, so the dynamic selection due to heterogeneity continues after $t=2$.

Figure 3 shows the same phenomenon in a style similar to the figures in Jackman & Layard (1991). It plots log $\theta(t|a)$ separately for $t=0,1,..,5$, as continuous functions of a. According to the previously mentioned papers, the distances between the lines should increase as a increases. Clearly, that is not what happens.

In the examples considered so far, $d\log\theta(t|a)/da$ is positive for every $t\geq 0$. However, contrary to what is stated in the previously mentioned papers, this is not a general property of the model. Consider again the discrete distribution for v with $p=1/2$, $v_1=1/5$ and $v_2=3/5$. Take $a=1.5$ (note that $v_2<1/a$ so this is an admissible value for a). The dashed line in Figure 1 depicts $d\log\theta(t|a)/da$ as a function of t. It is negative for $t=1$. Intuitively, a is so large that most individuals with $v=v_2$ leave unemployment at $t=0$. The drop in the mean value of v among the unemployed when going from $t=0$ to $t=1$ is so large that its negative effect on $\theta(1|a)$ is not offset by the positive effect of the increase of a. This means that in a model without state dependence, the lines in graphs like Figure 2 can actually cross. In sum, it is not true for all durations that in a "good" steady state (i.e. with a large a) the observed exit probability is larger than in a "bad" steady state. □

Clearly, because of this, the "cross effect" eyeball test for state dependence proposed and used by Budd, Levine & Smith (1988) and Jackman & Layard (1991) has unattractive properties. Incidentally, it should however be noted that in the empirical analysis in Jackman & Layard (1991) both eyeball tests for state dependence give the same result.

The topic of this subsection can be related to a statement in Lancaster (1979) on cross effects in MPH models. Lancaster (1979) analyzes continuous-time models in which a term x depending on fixed observed explanatory variables replaces a (so $\theta(t|x,v) = \psi_1(t).x.v$). He proves that if $G(v)$ belongs to the Gamma family then, for any function $\psi_1(t)$, the cross-derivative of log $\theta(t|x)$ w.r.t. t and x is always negative. In this result, the Gamma condition cannot be generalized to include all possible $G(v)$. If v has the discrete distribution with two points of support used in the numerical example above, and $\psi_1(t)\equiv 1$, then the cross-derivative of log $\theta(t|x)$ w.r.t. t and x equals zero if $t.x = 4.61$ and it is positive if and only if $t.x > 4.61$. Other counterexamples can be constructed by using discrete distributions with more points of support or uniform distributions with support $[c_0,c_1]$ with $0<c_0<c_1<\infty$. In any case, the general result for the Gamma heterogeneity model implies that even if there is strong and monotone state dependence then the cross-derivative of log $\theta(t|x)$ w.r.t. t and x can be negative for every $t>0$ and $x>0$. This strengthens the conclusion above that the "cross effect" eyeball test for state dependence has unattractive properties.

We will now argue intuitively that examining the cross effect is informative on the presence of unobserved heterogeneity (rather than state dependence). For ease of exposition, let time be continuous. Suppose that there is unobserved heterogeneity and that there are no cross effects of t and a in the observed log exit probability $\log \theta(t|a)$. Then $\theta(t|a)$ is multiplicative in t and a. But then the model is observationally equivalent to a model without unobserved heterogeneity (with $\theta(t|a,v) = \psi_1(t).a$), which violates the nonparametric identifiability of the MPH model (see Elbers & Ridder (1982) and Van den Berg (1992)). Consequently, if there is unobserved heterogeneity then there are cross effects of t and a in $\log \theta(t|a)$ and vice versa, for any type of state dependence, and indeed $G(v)$ is identified from these cross effects. In the next subsection we exploit this.

3.3. *Van den Berg & Van Ours (1993)'s "long-term/short-term" eyeball test for unobserved heterogeneity.*

For this eyeball test there is no gain in adopting Assumptions 3' or 4. The test is based on examining the cross effect of t and calendar time τ on the observed log exit probability $\log \theta(t|\tau)$. For practical reasons it turns out to be more convenient to describe the test as a comparison of values of ratios $\theta(t|\tau)/\theta(t-1|\tau)$ for one calendar time point to the values for other calendar time points. We use the name in the heading of this subsection to distinguish the test from the "cross effect" eyeball test of the previous subsection.

It is clear that, for a test based on cross effects to be sensible, $\theta(t|\tau)$ has to vary over calendar time. This is guaranteed by Assumption 3. Note that it is straightforward to verify whether $\psi_2(\tau_1) \neq \psi_2(\tau_2)$ for $\tau_1 \neq \tau_2$, since equation (2.4) implies that $\psi_2(\tau_1)/\psi_2(\tau_2) = \theta(0|\tau_1)/\theta(0|\tau_2)$, and the latter ratio can be observed.

Proposition 3. *Let Assumptions* 1, 2 *and* 3 *be satisfied. If there is no unobserved heterogeneity then* $\theta(t|\tau)/\theta(t-1|\tau)$ *does not depend on* τ *for any* $t \in \{1,2,..\}$. *Further, if there is unobserved heterogeneity then* $\theta(1|\tau)/\theta(0|\tau)$ *depends on* τ *in the sense that whenever* $\psi_2(\tau_1-1) \neq \psi_2(\tau_2-1)$ *(which is true for at least some* τ_1 *and* τ_2*) then* $\theta(1|\tau_1)/\theta(0|\tau_1) \neq \theta(1|\tau_2)/\theta(0|\tau_2)$.

Proof. If there is no unobserved heterogeneity then $Pr(v=\mu_1)=1$ and $\theta(t|\tau)=\theta(t|\tau,v)=\psi_1(t).\psi_2(\tau).\mu_1$. Consequently, $\theta(t|\tau)/\theta(t-1|\tau) = \psi_1(t)/\psi_1(t-1)$ which does not depend on τ. It remains to prove that if there is unobserved heterogeneity then $\theta(1|\tau)/\theta(0|\tau)$ does depend on τ in the sense described above. From equation (2.4) it follows that

$$\frac{\theta(1|\tau)}{\theta(0|\tau)} = \frac{\psi_1(1)}{\psi_1(0)} \cdot \frac{1-\psi_1(0).\psi_2(\tau-1).\mu_2/\mu_1}{1-\psi_1(0).\psi_2(\tau-1).\mu_1} \tag{3.8}$$

The derivative of the r.h.s. of (3.8) w.r.t. $\psi_2(\tau-1)$ is proportional to $\mu_1^2-\mu_2$. There is unobserved heterogeneity if and only if Var(v)>0, i.e. if and only if $\mu_2>\mu_1^2$. So, $\theta(1|\tau)/\theta(0|\tau)$ is a non-constant monotone decreasing function of $\psi_2(\tau-1)$ if and only if there is unobserved heterogeneity. Recall that by Assumption 3, $\psi_2(\tau-1)$ varies over τ. (As we have seen, it is easy to detect τ_1 and τ_2 for which $\psi_2(\tau_1)\neq\psi_2(\tau_2)$.) □

From Proposition 3 it follows that if there is a $t\in\{1,2,..\}$ such that $\theta(t|\tau)/\theta(t-1|\tau)$ does depend on τ (i.e. varies over τ) then there must be unobserved heterogeneity. Further, for t=1 the reverse is also true, i.e. if $\theta(1|\tau)/\theta(0|\tau)$ does not depend on τ then there is no unobserved heterogeneity. These insights were first developed and used by Van den Berg & Van Ours (1993).

From Proposition 3 it follows that in principle it suffices to examine $\theta(1|\tau)/\theta(0|\tau)$. In practice one may prefer examining ratios of other observed exit probabilities as well. Unfortunately, for $t\in\{2,3,..\}$ it is not true for all possible structural functions $\psi_1(t)$, $\psi_2(\tau)$ and $G(v)$ that if $\theta(t|\tau_1)/\theta(t-1|\tau_1) = \theta(t|\tau_2)/\theta(t-1|\tau_2)$ for some τ_1,τ_2 with $\theta(i|\tau_1-t+i)\neq\theta(i|\tau_2-t+i)$ for all $i\in\{0,1,..,t\}$, that then there is no unobserved heterogeneity. Suppose we observe steady states. The logarithm of the ratio $\theta(t|a)/\theta(t-1|a)$ can be thought as the discrete-time equivalent of the derivative of log $\theta(t|a)$ w.r.t. t. It follows from Subsection 3.2 that it is possible to have a situation in which there is unobserved heterogeneity and in which the derivative of log $\theta(t|a)$ w.r.t. t is a non-monotone function of a. Thus, for a given t, there may be different values of a generating the same value of $\theta(t|a)/\theta(t-1|a)$. For example, if $G(v)$ is discrete with $Pr(v=1/5) = 1/2 = 1-Pr(v=3/5)$, then $\theta(2|a{=}1.15)/\theta(1|a{=}1.15) = \theta(2|a{=}1.448)/\theta(1|a{=}1.448)$. However, if additional values of a or t are used then there is no ambiguity anymore.

Indeed, from extensive numerical analyses based on particular $\psi_1(t)$, $\psi_2(\tau)$ and $G(v)$ it follows that if there is unobserved heterogeneity then in most cases $\theta(t|\tau_1)/\theta(t-1|\tau_1) \neq \theta(t|\tau_2)/\theta(t-1|\tau_2)$ when $\theta(i|\tau_1-t+i)\neq\theta(i|\tau_2-t+i)$ for all $i\in\{0,1,..,t\}$. Thus, it seems that in practice one may safely use ratios $\theta(t|\tau)/\theta(t-1|\tau)$ with $t\in\{2,3,..\}$ for eyeball tests as well.

Van den Berg (1992) addresses the issue of the previous paragraphs for the continuous-time analogue of the model satisfying Assumptions 1, 2 and 3' (i.e. the case in which there are steady states characterized by $\psi_2(\tau)=a$) under the assumption that the data contain a continuum of different values of a. Unlike Proposition 2 above, it is not obvious to what extent the results in Van den Berg (1992) can be regarded as good approximations of results for the discrete-time model with a small unit time period. Therefore they are not discussed in detail here. Basically, it turns out that it is very unlikely to have a situation in which there is unobserved heterogeneity and in which there are $t_1 \neq t_2$ and $a_1 \neq a_2$ for which $\theta(t_1|a_1)/\theta(t_2|a_1) = \theta(t_1|a_2)/\theta(t_2|a_2)$, if a_1 and a_2 can be (and are) chosen randomly from an interval of real numbers.

As noted before, we are able to identify the presence of unobserved heterogeneity from the cross effects of t and τ in $\log \theta(t|\tau)$. State dependence and calendar time dependence are identified from the separate additive effects of t and τ in $\log \theta(t|\tau)$ (or, in other words, multiplicative effects in $\theta(t|\tau)$). From this it is clear that the MPH assumption is crucial for identification of the structural functions. The proof of Proposition 3 provides a test on this assumption. From this proof it follows that $\theta(1|\tau)/\theta(0|\tau)$ cannot be increasing as a function of $\psi_2(\tau-1)$. Thus, if the calendar time effect causes the exit probabilities to be small (e.g. in times of recession) then the aggregate exit probability falls less sharply when going from $t=0$ to $t=1$ than if the opposite case holds. This is because in a recession the initial weeding out of individuals with a high quality (i.e. a large v) cannot occur as fast as in the other case. Now suppose one observes that, for some τ_1 and τ_2, $\theta(0|\tau_1-1)>\theta(0|\tau_2-1)$. This is equivalent to $\psi_2(\tau_1-1)>\psi_2(\tau_2-1)$. If it is also observed that $\theta(1|\tau_1)/\theta(0|\tau_1) > \theta(1|\tau_2)/\theta(0|\tau_2)$ then that is evidence that the MPH assumption is violated. It can be shown that a wide range of alternative (i.e. non-MPH) models generate such observations (see Van den Berg & Van Ours (1993), Blanchard & Diamond (1990) and Layard, Nickell & Jackman (1991)).

Finally, the model specification can also be tested by examining whether the results of different eyeball tests of the same hypothesis are in accordance to each other.

3.4. *Extensions.*

We start this subsection by examining whether alternative eyeball tests for state dependence can be derived under Assumptions 1, 2, 3' and 4. First of all, note that if $\theta(t|a)$ is observed for every $t \in \{0,1,...\}$ then $E(t|a)$ can be calculated. Consequently, when applying Proposition 1, $\overline{\theta}(a)$ can be replaced by

$1/(E(t|a)+1)$. Along the lines of for example Heckman & Singer (1984) and Ridder (1984) it can be shown that in the stock of unemployed individuals the probability that the elapsed duration p is zero is equal to $1/(E(t|a)+1)$ as well (so $Pr(p=0|a) = 1/(E(t|a)+1)$). Consequently, if the distribution of $p|a$ in the stock of unemployed is observed then, when applying Proposition 1, $\bar{\theta}(a)$ can also be replaced by $Pr(p=0|a)$.

For ease of exposition we will for the moment take time to be continuous. Remember from Subsection 3.1 that the fundamental idea underlying Jackman & Layard (1991)'s "stock/flow" eyeball test is that $E(t|a,v)$ is proportional to $1/a$ and $1/v$ if there is no state dependence. In the latter case there also holds that $E(t^k|a,v)$ (if existent) is proportional to $1/a^k$ and $1/v^k$. This suggests that by using higher moments of $t|a,v$ other eyeball tests can be derived. Consider $E(t^2|a,v)$. If there is no state dependence then this equals $2/(a^2v^2)$. So, in that case, $E(t^2|a) = 2E_v(1/v^2)/a^2$ and

$$\frac{1}{2}\cdot\frac{E(t^2|a)}{E(t|a)} = \frac{1}{a}\cdot\frac{E_v(1/v^2)}{E_v(1/v)} \tag{3.9}$$

which is proportional to $1/a$. Thus, this can be used for an eyeball test. For example, if there is no state dependence then $E(t^2|a)/E^2(t|a)$ does not depend on a. According to Heckman & Singer (1984), the l.h.s. of (3.9) is equal to $E(p|a)$ in the stock of unemployed. So, an alternative formulation of the test examines $E(p|a)/E(t|a)$. Unfortunately, the test derived in this paragraph does not satisfy the analogue of Proposition 2. In particular, if there is Weibull state dependence ($\psi_1(t) = \alpha t^{\alpha-1}$) then $E(t^2|a)/E^2(t|a)$ does not depend on a. So it seems that the test of this paragraph is less attractive than Jackman & Layard (1991)'s "stock/flow" test.

We now outline how the eyeball tests could be transformed to formal statistical tests for continuous-time models to be estimated with micro duration data. Suppose there is no calendar time dependence but that instead there are fixed explanatory variables x. In such analysis, $\theta(t|x)$ is not observed and must be estimated. The estimation method determines the asymptotic distributions of test statistics like

$$\theta(0|x_1)E(t|x_1) - \theta(0|x_2)E(t|x_2)$$

for a test on state dependence (note that $E(t|x)$ depends on the whole function $\theta(t|x)$ as a function of t) or

$$\theta(t_1|x_1)/\theta(t_2|x_1) - \theta(t_1|x_2)/\theta(t_2|x_2)$$

for a test on unobserved heterogeneity. In practice it may be hard to estimate $\theta(t|x)$ nonparametrically for t close to zero, since zero is the boundary of the support of t.

4. Empirical illustration.

4.1. *The data.*

In this section we apply the eyeball tests presented in Subsections 3.1 and 3.3 to data from France, the UK and The Netherlands. The primary aim is here to illustrate the use of these tests in an empirical setting, rather than to give detailed analyses of the labour markets in these countries.

For the eyeball tests two indicators are important, a state dependence and an unobserved heterogeneity indicator. The state dependence indicator $\theta(0|a)/\overline{\theta}(a)$ from Subsection 3.1 refers to steady state periods a. Below we discuss the characteristics of these periods and how to find them empirically. For the moment we define the ***state dependence indicator*** for every calendar time point τ as the ratio of the exit probability for the first duration class and the over-all exit probability, both at time τ: $\theta(0|\tau)/\overline{\theta}(\tau)$. The term $\overline{\theta}(\tau)$ can be expressed in a way similar to $\overline{\theta}(a)$ in equation (3.2).

From Subsection 3.3 it follows that ratios of exit probabilities for different duration classes are informative on the presence of unobserved heterogeneity. For sake of brevity we mainly restrict attention to exit probabilities for the first two duration classes. (Recall the attractive properties of $\theta(1|\tau)/\theta(0|\tau)$. Also, data on exit probabilities for the first duration classes are in general more reliable than those for high duration classes.) So, we define the ***unobserved heterogeneity indicator*** as the ratio $\theta(1|\tau)/\theta(0|\tau)$ of the exit probabilities for the second and first duration class, both at time τ.

To investigate the behaviour of both indicators, we need observations on exit probabilities. As noted above, we need time series information on aggregate numbers of unemployed individuals, distinguished by their elapsed duration (class). The frequency in which calendar time information is available has to be the same as the measurement scale in which the duration classes are available. Let $U(t|\tau)$ denote the number of unemployed in duration class t at time τ. The exit probability for individuals in duration class t at time τ equals

$$\theta(t|\tau)= [\ U(t|\tau)\ -\ U(t+1|\tau+1)\ ]\ /\ U(t|\tau)$$

For our analyses we use quarterly data. We have information on the number of unemployed in the first three quarterly duration classes and on the total number of unemployed at time τ ($U(\tau)$). We measure the over-all exit probability from unemployment at time τ as:

$$\bar{\theta}(\tau)= [\ U(\tau)\ -\ U(\tau+1)\ +\ U(0|\tau+1)\ ]\ /\ U(\tau)$$

So, we implicitly assume that the inflow into unemployment at time $\tau+1$ is equal to the number of unemployed in the first duration class at time $\tau+1$. In a discrete-time framework, this seems to be the most straightforward way to estimate the inflow size. See Jackman & Layard (1991) for an alternative approach. To eliminate seasonal fluctuations we use four quarterly moving averages of exit probabilities.

Ideally, aggregate data provide exact values of exit probabilities. In reality, observations may differ from the values as predicted from the model because of measurement and specification errors. We assume somewhat loosely that the latter kind of errors do not dominate and that they are unsystematic. An advantage of eyeball tests is that they can be used in situations in which it is not aimed to construct and estimate formal models for these errors, or in situations in which informal data analyses are desired prior to more formal analyses.

We use data from three countries: France, the UK and The Netherlands. (It turned out to be impossible to obtain data meeting our requirements from a number of other European countries.) For each country we distinguish between male and female unemployed workers. The calendar time periods for which we have information cover a large part of the 1980s and the beginning of the 1990s (see Table 1). The French and Dutch data are collected by public employment offices and refer to registered unemployment. The UK data refer to benefit claimants.

Table 1 also lists averages of the over-all quarterly exit probability as well as the exit probabilities for the first and second quarterly duration class. Comparing the over-all exit probability with the exit probabilities for the first two duration classes is informative about the way the exit probability changes over the duration of unemployment. For example, if the over-all exit probability is smaller than those for the first two duration classes, then the exit probability declines after the first two quarters of unemployment.

The state dependence eyeball test compares different steady states. So, we have to find (at least) two steady state periods. In a steady state of unemployment, the stock, inflow and outflow are constant over a period of time. This means that we have to find two periods of time during which at least the over-all exit probability out of unemployment is more or less constant. The length of the period of time during which stability is required depends on the average unemployment duration. If the average unemployment duration is short, the length of the required stability period is short too.

Taking the inverse of the over-all exit probability as an indicator for the average duration of unemployment, we find substantial differences between countries. The average duration of unemployment in The Netherlands is 5 quarters and in France and the UK 3 quarters.

Figure 4 shows the development of the over-all exit probabilities out of unemployment. Note that the range of values on the horizontal axis and the length of the range of values on the vertical axis are the same in each graph. On the basis of these graphs we define the steady state periods indicated in Table 2. In general, the length of the steady state periods is about 2-4 times as long as the average unemployment duration, which seems reasonable. It should be noted that the results are insensitive with respect to the exact location of the steady state periods.

4.2. *Eyeball tests on state dependence.*

The eyeball test for state dependence compares the values of the state dependence indicators for different steady states. Figure 5 shows the development of the state dependence indicator for each country and gender. Here, as well as in Figure 6, we use the rule that the length of the range of values on the vertical axis is the same for each of the six groups, and is equal to the largest of the six observed ranges. In addition, we merged the graphs for both genders per country. As a result, the length of the range on the vertical axis for The Netherlands exceeds the length for the other two countries. The range on the horizontal axis is the same for each picture.

For British females the difference between the values of the indicator in different steady states is small, while for the other groups the opposite is true. We conclude that state dependence occurs for British males and French and Dutch males and females.

Proposition 2 in Subsection 3.1 suggests that if state dependence is known to be monotone, then a negative (positive) relationship between the state dependence indicator and the over-all exit probability implies that there is negative (positive) state dependence. From Figures 4 and 5 we infer a positive relation for the Dutch and French unemployed, indicating positive state dependence. For the British male unemployed we find a negative relation, indicating negative duration dependence. For British females the state dependence indicator is fairly constant over calendar time, so there is no obvious relation to the level of the over-all exit probability. The results are summarized in Table 2.

We have also investigated the sign of the relation more formally by calculating the correlation coefficients between the state dependence indicators and the over-all exit probabilities, using data from the steady state periods only. The results are shown in the first two columns of Table 3. They are in accordance to those in Table 2. For British females, however, the correlation coefficient is significantly negative. It may be that for this group there is negative state dependence, but that the decrease of the exit probability over the duration of unemployment is small. In a way, this inconclusiveness illustrates the limitations of the eyeball tests. In the absence of a formal statistical framework it is not possible to infer the presence of small but significant effects.

It should be stressed again that the results on the sign of the state dependence are conditional on the assumption of monotone state dependence. There are theoretical as well as empirical indications that under certain circumstances non-monotone state dependence may be important (see Devine & Kiefer (1991), Van den Berg (1991) and Van den Berg & Van Ours (1994) and the references in those papers).

4.3. *Eyeball tests on unobserved heterogeneity*

The eyeball test for unobserved heterogeneity examines the behaviour of the heterogeneity indicator over calendar time. This test does not depend on steady state assumptions. From Figure 6 it is obvious that unobserved heterogeneity is important in the French and Dutch unemployment duration data. In the British data, unobserved heterogeneity does not seem to be important. It turns out that for France and The Netherlands the indicators $\theta(t|\tau)/\theta(t-1|\tau)$ with $t\geq 2$ (not reported here) lead to the same conclusion as the indicator graphed in Figure 6. For the UK, however, the former indicators suggest the presence of unobserved heterogeneity. This difference in outcome between eyeball tests for the same phenomenon suggests that the model specification may be incorrect for UK unemployment durations. This is confirmed below.

As shown in Subsection 3.3, the heterogeneity indicator can also be used for testing the MPH assumption. In particular, a positive relation between this indicator and the – one period lagged – exit probability for the first duration class suggests that the MPH assumption is violated.

We have investigated this by calculating the correlation coefficient over the sample period between the heterogeneity indicator and the lagged exit probability for the first duration class. (For sake of brevity we dot not present the corresponding graphs.) Columns 3 and 4 of Table 3 show the results of these calculations. For France and The Netherlands we find a significant negative relationship, confirming the model specification. For the UK the correlation is significantly positive. Therefore, for the UK the MPH assumption is violated. This casts doubt on the results of the other eyeball tests in this paper for the UK.

In a companion paper, Van den Berg & Van Ours (1994) formally estimate models for exit probabilities for different duration classes, using similar data. Almost all of their results are in agreement to those presented here. For The Netherlands they find that state dependence is non–monotonic.

5. Conclusion.

In this paper we have examined eyeball tests for state dependence and unobserved heterogeneity in aggregate duration data. We have shown that the test on state dependence proposed by Jackman & Layard (1991) and the test on unobserved heterogeneity based on Van den Berg & Van Ours (1993) have excellent properties in terms of the power against the corresponding alternative hypothesis.

The eyeball tests do not rely on parametric assumptions on the determinants of the hazard. This is a marked advantage. It makes the test for state dependence (unobserved heterogeneity) insensitive to the shape of the unobserved heterogeneity distribution (state dependence). The tests are based on the facts that (*i*) if there is no state dependence then the mean duration is inversely proportional to the steady–state impact parameter in the exit probabilities, and (*ii*) if there is no unobserved heterogeneity then the observed exit probabilities are multiplicative in terms of duration and calendar time.

We also showed that another eyeball test for state dependence, proposed by

Budd, Levine & Smith (1988) and Jackman & Layard (1991), may well reject the null hypothesis of no state dependence even if there is no state dependence, and vice versa. This is because this eyeball test is based on cross effects of unemployment duration and calendar time in the observed log exit probability, and these cross effects are informative on the presence of unobserved heterogeneity rather than state dependence.

As an empirical illustration, we applied the eyeball tests to aggregate unemployment duration data from three European countries: France, the UK and The Netherlands. The data are quarterly and distinguish between males and females. The results indicate that in general there is state dependence in unemployment. Under the assumption of monotonicity of state dependence, we find positive state dependence for the French and Dutch unemployed and negative state dependence for British male unemployed. There is no strong indication of state dependence for the British female unemployed. Furthermore, unobserved heterogeneity is important in the French and Dutch unemployment data, while it is not an important phenomenon in the British data.

Finally, our paper provides eyeball tests of the Mixed Proportional Hazard model framework. For British data this test results in rejection, which casts doubt on the results of the other eyeball tests for the UK.

References

Blanchard, O.J. and P. Diamond (1990), Ranking, unemployment duration, and wages, Working Paper (NBER).

Budd, A., P. Levine and P. Smith (1988), Unemployment, vacancies and the long-term unemployed, *Economic Journal* 98, 1071–1091.

Devine, T.J. and N.M. Kiefer (1991), *Empirical Labor Economics*, Oxford University Press, New York.

Elbers, C. and G. Ridder (1982), True and spurious duration dependence: the identifiability of the proportional hazard model, *Review of Economic Studies* 49, 403–410.

Heckman, J. and B. Singer (1984), Econometric duration analysis, *Journal of Econometrics* 24, 63–132.

Hollander, M. and F. Proschan (1984), Nonparametric concepts and methods in reliability, in: P.R. Krishnaiah and P.K. Sen, eds., *Handbook of statistics* 4, North-Holland, Amsterdam.

Jackman, R. and R. Layard (1991), Does long-term unemployment reduce a person's chance of a job? A time-series test, *Economica* 58, 93–106.

Jackman, R. and S. Savouri (1992), Regional migration in Britain: an analysis of gross flows using NHS central register data, *Economic Journal* 102, 1433–1450.

Lancaster, T. (1979), Econometric methods for the duration of unemployment, *Econometrica* 47, 939–956.

Lancaster, T. (1990), *The econometric analysis of transition data*, Cambridge University Press, Cambridge.

Layard, R., S. Nickell and R. Jackman (1991), *Unemployment*, Oxford University Press, Oxford.

Ridder, G. (1984), The distribution of single-spell duration data, in: G.R. Neumann and N. Westergård-Nielsen, eds., *Studies in labor market analysis*, Springer Verlag, Berlin.

Ridder, G. (1987), The sensitivity of duration models to misspecified unobserved heterogeneity and duration dependence, Working Paper (Groningen University).

Van den Berg, G.J. (1991), The effect of an increase of the rate of arrival of job offers on the duration of unemployment, Working Paper (Groningen University).

Van den Berg, G.J. (1992), Nonparametric tests for unobserved heterogeneity in duration data, Working Paper (Groningen University).

Van den Berg, G.J. (1994), The effects of changes of the job offer arrival rate on the duration of unemployment, *Journal of Labor Economics*, forthcoming.

Van den Berg, G.J. and J.C. van Ours (1993), Unemployment dynamics and duration dependence, Working Paper (Free University Amsterdam).

Van den Berg, G.J. and J.C. van Ours (1994), Unemployment dynamics and duration dependence in France, The Netherlands and the UK, *Economic Journal* 104, 432–443.

Vishwanath, T. (1989), Job search, stigma effect, and escape rate from unemployment, *Journal of Labor Economics* 7, 487–502.

Table 1. Data periods and statistics.

		Exit probabilities (%)					
		Males			*Females*		
Country	*Data period*	$\bar{\theta}(\tau)$	$\theta(0\|\tau)$	$\theta(1\|\tau)$	$\bar{\theta}(\tau)$	$\theta(0\|\tau)$	$\theta(1\|\tau)$
France	1983.4–92.1	31	38	34	26	32	30
UK	1984.3–92.2	27	42	32	34	46	34
Netherlands	1982.1–91.4	20	32	32	20	26	28

Table 2. Steady state periods and results of the eyeball test on state dependence.

Country	*Steady state with low exit prob.*	*Steady state with high exit prob.*	*Eyeball test on state dependence* Presence		Sign (monotone)	
			Males	*Females*	*Males*	*Females*
France	1984.2–86.4	1987.4–90.4	*yes*	*yes*	+	+
UK	1984.3–87.1	1989.4–90.4	*yes*	*no*	–	0
Netherlands	1985.2–87.2	1988.4–91.4	*yes*	*yes*	+	+

Table 3. Correlation coefficients.

	Informative on sign (monotone) state dependence[a)]		*Informative on MPH assumption*[b)]	
	Males	*Females*	*Males*	*Females*
France	0.90	0.76	–0.85	–0.83
UK	–0.98	–0.98	0.75	0.65
Netherlands	0.87	0.95	–0.93	–0.97

Note: all correlation coefficients differ significantly from zero at 1% level.

a) Correlation between the state dependence indicator and the over-all exit probability in steady state periods. The sign corresponds to the sign of the monotone state dependence.

b) Correlation between the heterogeneity indicator and the lagged exit probability for the first duration class in the total data period. A positive sign corresponds to a misspecified model.

Figure 1. The derivative of log $\theta(t|a)$ w.r.t. a as a function of t, for a=1 and a=1.5, in the example of Subsection 3.2.

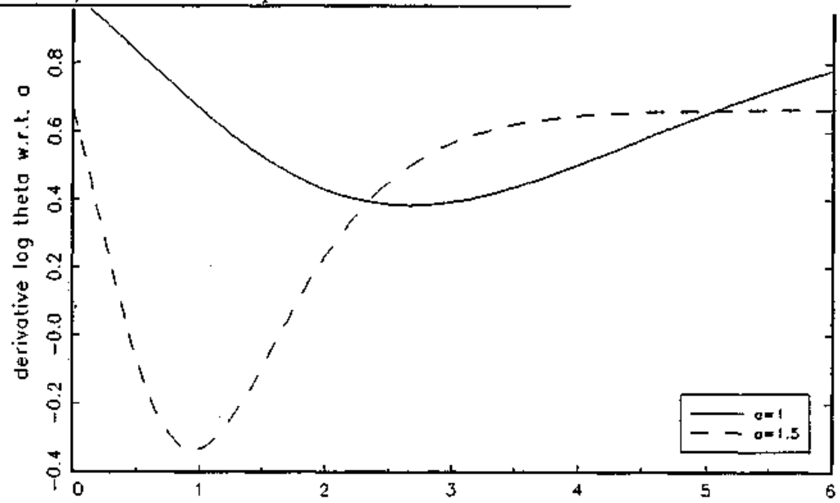


Figure 2. The observed exit probability $\theta(t|a)$ as a function of t, for a=1 and a=1.35, in the example of Subsection 3.2.

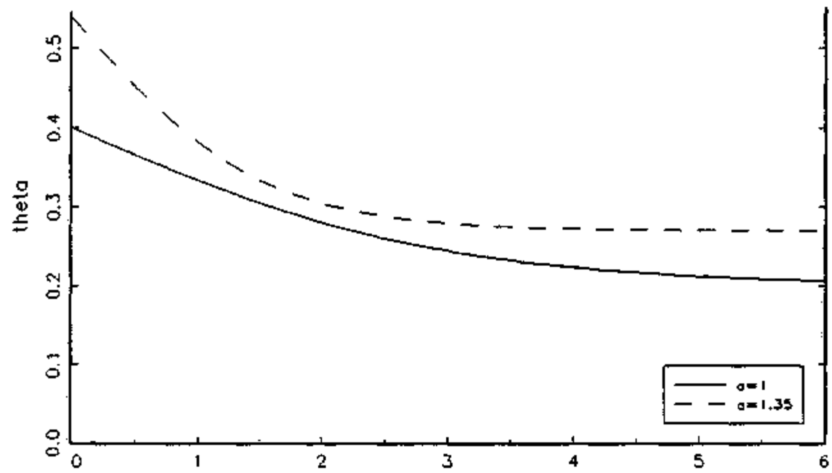


Figure 3. The observed exit probability $\theta(t|a)$ as a function of a, for t=0, t=1, t=2, t=3, t=4 and t=5, in the example of Subsection 3.2.

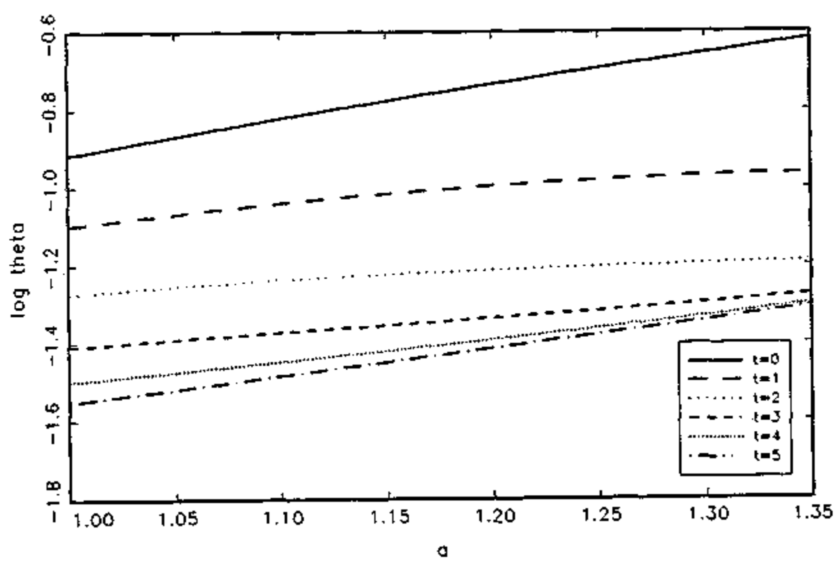
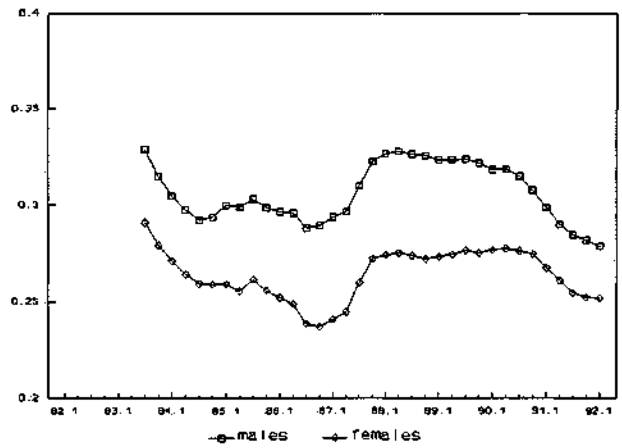
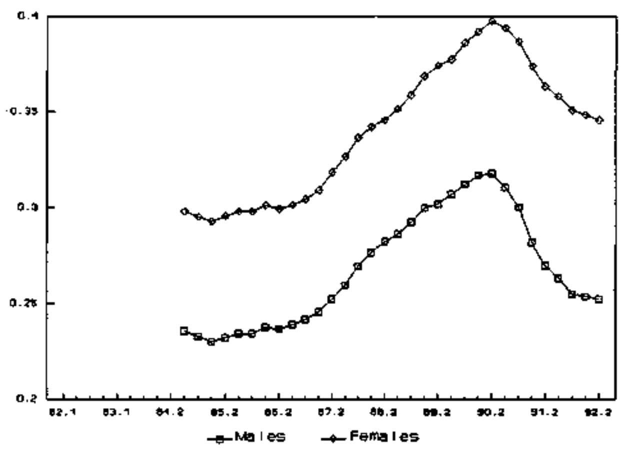


Figure 4. Over-all exit probabilities; 3 countries.

France



UK



The Netherlands

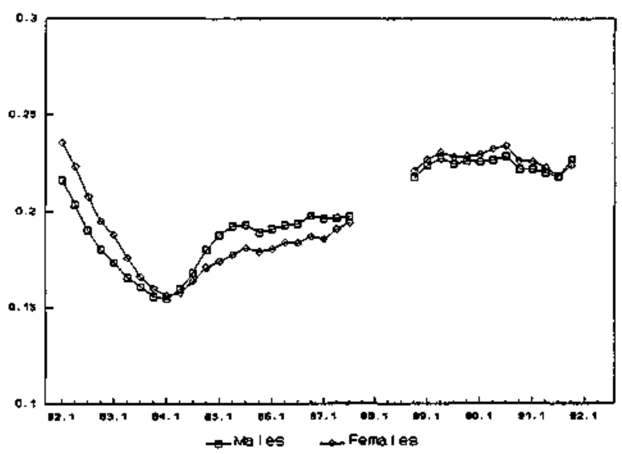
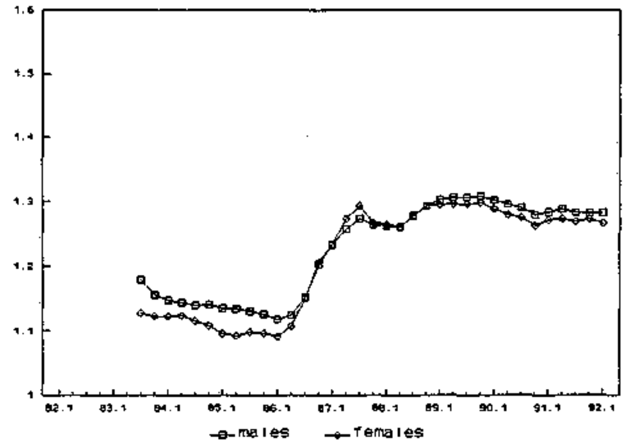
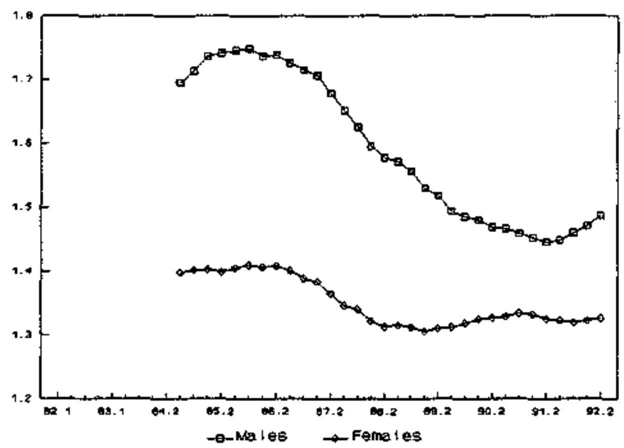


Figure 5. Indicators state dependence; 3 countries.

France



UK



The Netherlands

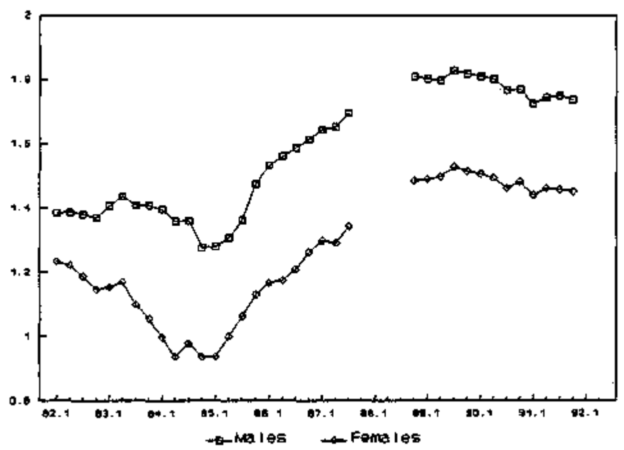
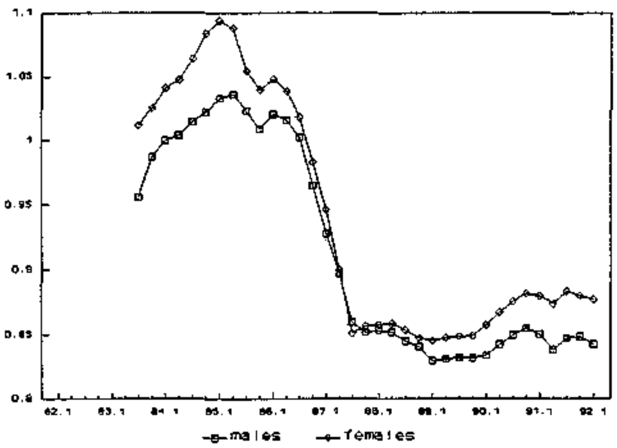
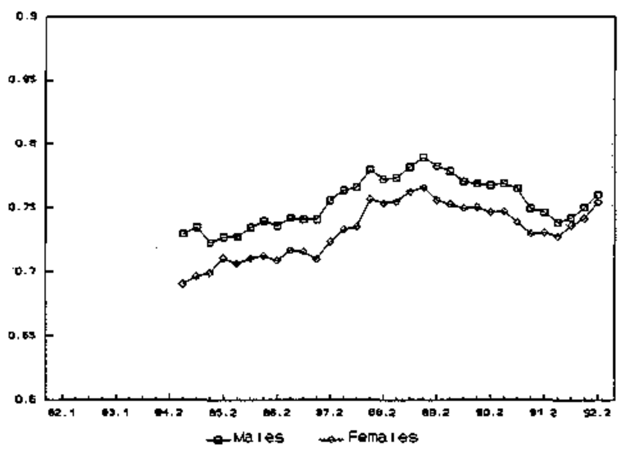


Figure 6. <u>Indicators unobserved heterogeneity; 3 countries.</u>

France



UK



The Netherlands