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Abstract

This paper is focused on interdependencies between criteria in multicriteria decision analyses. Such interdependencies are usually ignored in sensitivity analyses. After a discussion on the nature of interdependencies, methods are presented to deal with interdependencies in the context of Monte Carlo experiments. These methods are applied in the context of soil pollution treatment alternatives. It is shown that ignoring interdependencies may have a distorting effect on the results of sensitivity analysis on rankings of alternatives.

Key words: sensitivity analysis, interdependent criteria, multicriteria decision making, soil pollution treatment.

1. Introduction

The complexity of many environmental decision problems calls for a multidimensional analytical framework in order to capture a wide range of relevant aspects. Two major scores of uncertainty can be mentioned in this respect. Firstly, the various aspects to be taken into account are often difficult to compare; it is hard to arrive at quantitative figures to trade them off against each other. This raises the issue of uncertainty on the trade-offs in multicriteria decision methods. Secondly, for some relevant criteria there may be a considerable degree of uncertainty regarding the precise values attained. This may relate both to environmental impacts and to other relevant aspects, such as cost of pollution abatement, duration of treatment, etc.

A possible way of dealing with uncertainty is to use Monte Carlo simulation techniques. This involves the formulation of a statistical distribution of one or more parameters and the use of a corresponding random generator. There are several ways to formulate such a statistical distribution. One way is to use intervals and to assume a uniform distribution on these intervals. Another way is to formulate a certain statistical distribution (for example a normal one) with a certain value for the mean and the variance. The broader the intervals and the larger the variances, the less certain the outcome of multicriteria decision analysis will be.

The way this uncertainty can be analyzed varies among particular multicriteria methods. In the case one-dimensional utility functions are used, the uncertainty can be analyzed immediately by inspecting ranges and variances of the utility scores of the respective alternatives. A strong overlap of the intervals of utility scores for two alternatives means that there is little certainty about their ranking. Stochastic dominance is one of the possible tools for analyzing the robustness of rankings in this case (cf. Rietveld and Ouwersloot, 1992).

When multicriteria methods are used which are not based on a one-dimensional utility function, such a procedure is no longer applicable. In this case one may use a rank-probability matrix P as a vehicle to represent the results of the sensitivity analysis. Such a rank probability matrix is a square matrix with J rows and elements p_{nj} where p_{nj} denotes the probability that alternative j achieves rank n. The sum of the elements in each row and column in P is by definition equal to 1. There is complete certainty about the ranking of alternatives if a permutation of alternatives exists such that the diagonal elements of the matrix are equal to 1. The other extreme of complete uncertainty occurs when all elements of the matrix are equal to 1/J.

In the standard way to address uncertainty statistical interdependencies among criteria are ignored. However, as we will indicate in this paper, such interdependencies often play an important role. They may have a considerable impact on the outcome of the sensitivity analysis. This means that ignoring these interdependencies may give a distorted view of the relative attractiveness of alternatives.

This paper is organized as follows. In section 2 we discuss the issue of interdependencies in multicriteria decision analysis. In section 3 methods will be presented to generate random numbers in the case of interdependencies. Section 4 contains a method to decompose uncertainty of utility scores. In section 5 the methods developed will be applied to an evaluation problem in the context of soil pollution. Concluding remarks are given in section 6.

2. Interdependencies of criteria

Consider an evaluation matrix X with elements x_{ij}, where x_{ij} denotes the score for criterion i (i=1,...,I) of alternative j (j=1,..,J). An analysis of the correlation coefficients among the criteria often reveals that some of the criteria are strongly correlated. For example, in an eight criterion case on impacts of industrial sites (cf. Rietveld, 1980) it appears that there are five pairs of criteria with an absolute value of the correlation coefficient higher than .70. For one pair of criteria the correlation coefficient is as high as .977.

When applying multicriteria evaluation methods the occurrence of high correlations is often used (implicitly or explicitly) to reduce the cost of implementation of the method. For example, when studying the negative impacts of road construction on the fauna one might in principle have to consider a very large number of species. However, effects on several types of birds may be very similar, so that when one has studied the impact on one type, one can easily extrapolate what will be the effects on other birds. The loss of relevant information due to ignoring all individual species may be very small in this case. (Of course, this use of one criterion for a particular species to represent the impact for a larger set of species has to be taken into account at the phase of formulating the relative importance of the pertaining criterion.)

An investigation of the correlation coefficients among criteria may also be helpful to understand the basic conflicts involved. A strong negative correlation between two criteria means that an improvement in one direction almost certainly will lead to a worsening in the other one. On the other hand, when criteria have a high positive correlation, one may infer that there is a small degree of conflict among them. Aiming at the selection of an alternative with a good performance according to one criterion will in this case usually also lead to a good performance according to the other criterion. Correlation analysis may also be helpful for analyzing probabilities of coalition formation in the case of multi-actor problems. When different actors attach a high priority to criteria which have strong negative correlations, the probability of the forming of a coalition is low.

When the criterion values concerned are uncertain, such interdependencies have to be taken into account when sensitivity analyses are carried out. Consider for example a regional government that has to chose between different infrastructure improvement projects. Among the criteria to be taken into account are regional employment growth and growth in regional production. The **direction** of the response of the private sector with respect to the infrastructure improvement is most probably positive: infrastructure improvement will induce existing firms to expand and new firms to locate in the region. One may expect that the two criteria are positively correlated, accordingly. The **size** of the response is uncertain, however. The positive correlation between the two criteria implies that when the employment impact will be higher than expected, also the impact on production will be higher. When in a sensitivity analysis such an interdependence is ignored, one may arrive at a distorted view of the range of possible outcomes of a utility score. In the case of two criteria which are positively correlated ignoring interdependencies leads to an underestimate of the variance in the utility score.

The way the elements of the evaluation matrix have been measured has an impact on the treatment of interdependencies in sensitivity analysis. We distinguish three different cases: direct measurement of criterion scores, subjective estimates of experts, and estimates based on scientific models.

First, the measurement of the criteria considered in the evaluation matrix may take place in an immediate way without the use of models. For example, when one searches a dwelling there are simple ways of measuring the performance of alternatives in terms of size of the dwelling, last year's consumption of gas and electricity, its distance to the shopping centre, etc. In this case uncertainty (for example in the form of measurement errors) usually plays a rather small role, and this also holds true for interdependencies.

Uncertainties become more important when criteria are involved that cannot be measured as easily. This may occur for example when one wants to take into account the future values of the relevant criteria. In such a case one may distinguish subjective strategies where use is made of expert judgements and strategies where use is made of scientific models.

In the case use is made of subjective estimates, experts may be asked to formulate the distribution (for example by means of the mean value and the variance) of criterion scores. In addition, they may indicate to which extent they think criterion scores are correlated. One of the ways to arrive at such formulations of distributions is to base them on experiences in the past about similar cases.

In the case of completely specified models, criterion scores can be generated more or less automatically. One should be aware, however, that here again several subjective elements will play a role, for example in the choice of the type and specification of the model. Also in the case of models one has to face the problem of uncertainty, since models are not necessarily directly applicable to the decision problem at hand. Model parameters have to be estimated or to be guessed. In both cases one ends up with formulating statistical distributions of parameters. By using Monte Carlo techniques one can then generate the relevant information on the shape of the distributions of all elements of the evaluation matrix X, as well as on the interdependencies among these elements of the matrix.

From this description it becomes clear that in both latter cases information is needed on the statistical distribution of a number of parameters. The difference is that in the case of subjective estimates, the statistical information directly relates to the criterion scores, whereas when models are used, the statistical information relates to the parameters of the underlying model. An advantage of using models is that they may help to ensure consistency in the estimation of the impacts. In terms of interdependencies there is also a difference between the approaches. If one wants to take into account interdependencies between criterion scores in the case of subjective direct estimates of the criterion scores, one has to formulate these interdependencies explicitly. In the case of model based estimates such interdependencies follow automatically given the interrelationships specified in the model. We note in passing that it is not impossible that in the model approach one may also have to take into account interdependencies between model parameters, but this is not necessary to arrive at non zero correlations between criterion scores.

It remains a question how experts can express their knowledge about interdependencies in the case of subjective estimates. Especially when the number of criteria is large this may become a problem, since the number of interdependencies increases in a quadratic way with the number of criteria. In addition, it is by definition more complex to indicate a quantitative measure for the degree of interrelation between **two** criteria, than that it is to indicate a mean and a range of uncertainty for **one** criterion. A much easier approach would be to use the correlation coefficients between the mean criterion values in the evaluation matrix X as a proxy for the intensity of interrelation between the measurement errors. These correlation coefficients can be shown to the experts carrying out the sensitivity analysis; if they feel that some correlation coefficients do not give an appropriate indication of the interdependencies between the errors, they may adjust them according to their own insight.

3. Methods for generating interdependent random normal criterion scores

When interdependencies between criterion scores are ignored, it is not difficult to generate random values. Given the mean and the variance of a normal distribution, one can immediately make use of a standard random normal generator. In the case of interdependencies the situation is more complex. Let x_{nj} be a random number drawn from a normal distribution with mean μ_{nj} and variance σ^2_{nj} so that μ_{nj} is the expected value of criterion n for alternative j.

The correlation coefficient between criterion n and m is equal to ρ_{nm} so that the covariance between criteria n and m for alternative j is:

$$\sigma_{nmj} = \rho_{nm}\, \sigma_{nj}\, \sigma_{mj}$$

Then, in order to generate random values for the criterion scores one may proceed as follows.

First, one may use **conditional distributions** as follows:

x_{1j} is generated from the normal distribution with mean μ_{1j} and variance σ^2_{1j}

x_{2j} is generated from a normal distribution conditional on the previously generated criterion x_{1j}

x_{3j} is generated from a normal distribution conditional on the previously generated criteria x_{1j} and x_{2j}

etc.

This approach requires the explicit formulation of all conditional distributions. Note that this involves the inversion of the variance-covariance matrix in each step in order to compute the conditional variances (cf. Mood, Graybill and Boes, 1974).

The second approach, is due to Scheuer and Stoller (1962) and makes use of the fact that the covariance matrix Σ of the normally distributed variables can be (Choleski-) decomposed as $\Sigma = CC^T$ where C is a lower triangular matrix, since the covariance matrix is symmetric and positive definite. It is not difficult to compute the matrix C. Once C is given, the interdependent values of X_{ij} can be obtained as weighted summations of independently distributed random variates with the elements of C as the weights.

4. Decomposition of uncertainty

Sensitivity analysis usually addresses the robustness of rankings of alternatives. In the present section we discuss a particular approach in which a decomposition of the uncertainty of the outcomes of alternatives is given. Our point of departure is a linear utility function

$$Z_j = \sum_{i=1}^{I} w_i x_{ij} \tag{1}$$

with weights w_i for the criteria i=1,...,I, and criterion scores x_{ij} for alternatives j=1,...,J.

In sensitivity analysis, the values of x_{ij} are no longer assumed to be known with certainty. This leads to uncertainty on the utility scores Z_j, and thus the question arises how uncertainty in the utility scores can be decomposed: what is the contribution of various criteria to the uncertainty in the utility score. There is an easy solution to this question when we use variances (and covariances) to formulate uncertainties.

Let Var and Cov denote the variance and covariance operators respectively. Then

$$Var(Z_j) = \sum_{i'} \sum_{i} w_i w_{i'} \, Cov(x_{ij}, x_{i'j}) \tag{2}$$

so that

$$\frac{\sum_{i'} \sum_{i} w_i w_i Cov(x_{ij}, x_{i'j})}{Var(Z_j)} = 1 \tag{3}$$

Then, the relative contribution of criterion i to the uncertainty in Z_j can be written as:

$$a_{ij} = \frac{\sum_{i'=1}^{I} w_i w_{i'} \, Cov\,(x_{ij}, x_{i'j})}{Var(Z_j)} \tag{4}$$

Note that in this equation interdependencies between the criterion scores play a role via the covariances. If errors in the criterion scores would be **independent**, i.e. $Cov(x_{ij}, x_{i'j})=0$, the above equation reduces to:

$$a_{ij} = \frac{w_i^2 Var(x_{ij})}{Var(Z_j)} \tag{5}$$

Since Cov(x,x) = Var(x)

From (5) we infer that the contribution of a criterion to uncertainty in Z is large when the product of weight squared and variance is relatively large. Equation (4) indicates that with interdependencies a more complex picture emerges: negative correlations among criteria will have a mitigating effect on the contribution of a particular criterion, positive correlations an amplifying effect.

A limitation of this decomposition method is that it can only be used in the case of an additive function. Note, that the method does not depend on assumptions about the form of statistical distributions. It can be used for both normal and other distributions. Further, an attractive property of the method is that it yields results in an analytical way: there is no need to carry out Monte Carlo simulations. This method is illustrated in the next section.

5. Case study: soil pollution treatment

During the last decade, concern about soil pollution has grown rapidly in many industrialized countries. In the Netherlands it has been estimated that six thousand contaminated areas require urgent remedial action. The selection of the most appropriate sanitation strategy is not easy, due to the large number of contaminants involved, the limited budget available and several other complicating factors. In order to formulate the main points of a sanitation policy, Dutch legislation provides a framework for the evaluation of contaminated soils and for establishing priorities for clean-up operations (Soil clean-up guideline. 1983). A decision support system has been developed to select the most appropriate alternatives (Herwijnen et al.. 1992, and Beinat and Janssen, 1992). The present application will focus on sensitivity tests and the role of interdependencies using linear utility functions. The application concerns a polluted site in Nieuwerkerk aan de IJssel, a village in The Netherlands. Eleven alternatives were developed for a clean-up operation. The number of criteria to be taken into account was nineteen. Table 1 shows the relevant criteria with the respective weights. The weights have been determined on the basis of expert judgements of officials from the Ministry of Environment by means of the expected value method described in Rietveld and Ouwersloot (1992). Relatively high weights are attached to the residual concentrations of pollutants in the cleaned soil and the residuals of treatment materials on the cleaned soil. Also, emissions to air and groundwater due to the treatment receive high weights. For criteria of type "+" higher values are preferred above lower values. For criteria of type "-" the reverse holds true.

Table 1. Criteria in soil sanitation decision problem.

		unit of measurement	weight	type of criterion
Time				
st	sanitation time	days	0.066	-
Amounts of soil produced				
cs	cleaned soil	tons	0.005	+
ss	silt and sediments	tons	0.009	-
ug	uncleaned soil	tons	0.018	-
Residual concentrations of pollutants in cleaned soil				
rcc	cadmium	mg/kg	0.148	-
rcz	zinc	mg/kg	0.148	-
Treatment residuals in cleaned soil				
trc	cadmium	mg/kg	0.098	-
trz	zinc	mg/kg	0.098	-
Quality cleaned soil				
pom	percentage of organic matter	%	0.076	+
poc	percentage of clay	%	0.009	+
psos	preserved structure of soil	qualitative index 0-10	0.039	+
pl	preserved life	qualitative index 0-10	0.021	+
Emissions to air				
ea	emission matter	mg/kg	0.111	-
Emissions to ground water				
egw	emission matter	mg/kg	0.086	-
Company reliability				
cre	experience of company	years	0.016	+
cmfp	number of finished projects	number	0.005	+
Absence of nuisance				
ns	stench	qualitative index 0-10	0.029	+
nnav	noise and vibrations	qualitative index 0-10	0.005	+
nsa	nuisance to surrounding activities	qualitative index 0-10	0.013	+

Table 2. Evaluation Matrix

	BIOREST. alt 1	LUCHTV. alt 2	STOOMST. alt 3	THERM.1 alt 4	THERM.2 alt 5	EXTR.1 alt 6	EXTR.2 alt 7	FLOTATIE alt 8	LANDF.1 alt 9	LANDF.2 alt 10	BIOREAKT alt 11
st	-713.00	-543.00	-355.50	-127.00	-136.50	-153.00	-174.50	-177.00	-1627.00	-1091.00	-596.50
cs	14598.50	14763.00	14316.00	11209.50	11138.00	82940.00	10576.00	11527.00	14744.00	14625.00	13906.00
ss	-308.00	-117.00	-785.00	-200.50	-177.00	-9482.00	-6320.50	-3575.50	-89.50	-254.00	-1325.50
ug	-111.38	-44.55	-126.22	-393.52	-373.48	-430.65	-460.35	-408.37	-460.35	-415.80	-452.92
rcc	-22.00	-24.60	-23.00	-22.20	-19.75	-1.50	-1.75	-1.20	-24.60	-24.10	-19.75
rcz	-1774.15	-1967.55	-1895.05	-1975.35	-1975.35	-248.00	-248.00	-138.85	-1975.35	-1965.40	-1835.90
trc	-20.50	-23.50	-17.50	-24.50	-25.50	-1.50	-2.00	-1.50	-17.50	-17.00	-18.50
trz	-1325.00	-1673.00	-1490.00	-840.00	-840.00	-206.50	-219.00	-125.00	-1763.00	-1755.50	-1825.00
pom	23.28	23.82	21.60	1.08	0.48	3.00	9.60	9.00	23.88	23.28	19.20
poc	24.38	24.87	24.12	23.75	23.88	2.50	11.25	18.13	24.69	24.50	23.75
psos	8.25	8.50	6.75	2.50	2.50	3.00	4.50	3.50	7.50	8.00	5.50
pl	7.50	8.00	6.00	0.50	0.50	0.50	0.50	0.50	8.00	8.00	7.00
ea	-14.00	-262.50	-500.00	-3250.00	-3250.00	-3250.00	-3250.00	-3250.00	-3250.00	-3250.00	-3250.00
egw	-0.50	-0.04	-0.63	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04
cre	3.00	4.00	4.00	8.00	8.00	8.00	1.50	8.50	6.00	4.50	3.50
crnfp	2.50	15.00	4.00	175.00	350.00	85.00	3.00	35.00	35.00	10.00	5.00
ns	9.25	8.50	7.50	5.50	5.50	5.50	5.50	5.50	5.50	5.50	5.50
nnav	9.25	9.50	7.50	5.50	5.50	5.50	5.50	5.50	5.50	5.50	5.50
nsa	9.25	9.63	8.50	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00

Table 2 shows the evaluation matrix for 11 sanitation alternatives including biological, thermal and chemical techniques. Using an additive utility function as indicated in Janssen (1992) yields the result that alternative 9 is the most attractive alternative, followed by alternative 10. Alternatives 6, 7 and 8 are least attractive (see Table 3).

Table 3. Results of weighted nummation

alternative	utility score
alt 9	.84
alt 10	.81
alt 11	.73
alt 3	.73
alt 2	.72
alt 1	.72
alt 4	.63
alt 5	.62
alt 7	.28
alt 8	.27
alt 6	.26

The correlation coefficients between each pair of criteria are shown in Table 4. A correlation value close to 1 or -1 implies a strong relationship between the criteria. A considerable number of high correlations (absolute value higher than 0.9) is found.

Table 4. Correlation Table

	cs	ss	ug	rcc	rcz	trc	trz	pom	poc	psos	pl	ea	egw	cre	crnfp	ns	nnav	nsa
st	-0.7054	-0.4455	0.0490	0.5473	0.4701	0.2550	0.7165	-0.7427	-0.4440	-0.7229	-0.7878	-0.0390	-0.0048	0.2523	0.3884	-0.0713	-0.0667	-0.0380
cs		0.7933	0.4907	-0.7860	-0.7113	-0.5671	-0.8920	0.9062	0.8265	0.8771	0.9227	0.5359	-0.3825	-0.5005	-0.4535	0.5230	0.5231	0.5371
ss			0.3864	-0.8947	-0.8892	-0.8698	-0.7482	0.4653	0.9929	0.4683	0.5820	0.3398	-0.2314	-0.1069	0.1227	0.3352	0.3368	0.3415
ug				-0.4373	-0.3743	-0.4282	-0.3454	0.4616	0.3832	0.5533	0.4420	0.9760	-0.6664	-0.3253	-0.2075	0.9478	0.9580	0.9832
rcc					0.9878	0.9190	0.8918	-0.5474	-0.8669	-0.5806	-0.7219	-0.4119	0.2852	0.1948	-0.0582	-0.3938	-0.3995	-0.4154
rcz						0.9539	0.8563	-0.4398	-0.8582	-0.4687	-0.6392	-0.3365	0.2369	0.1419	-0.1815	-0.3185	-0.3234	-0.3394
trc							0.7005	-0.2430	-0.8273	-0.3026	-0.4578	-0.3563	0.1800	0.0425	-0.3656	-0.3609	-0.3755	-0.3650
trz								-0.7901	-0.7591	-0.7637	-0.9193	-0.3734	0.2322	0.4067	0.2777	-0.3513	-0.3658	-0.3810
pom									0.5108	0.9699	0.9524	0.5582	-0.3971	-0.6447	-0.7301	0.5465	0.5463	0.5594
poc										0.4917	0.6065	0.3467	-0.2499	-0.1613	0.0487	0.3362	0.3365	0.3475
psos											0.9385	0.6331	-0.3902	-0.6487	-0.6605	0.6409	0.6483	0.6391
pl												0.5138	-0.3202	-0.5390	-0.5565	0.5158	0.5226	0.5190
ea													-0.7514	-0.4525	-0.3464	0.9794	0.9681	0.9966
egw														0.3624	0.2824	-0.6829	-0.5794	-0.6989
cre															0.6385	-0.4533	-0.4381	-0.4462
crnfp																-0.3338	-0.3262	-0.3431
ns																	0.9868	0.9765
nnav																		0.9785

As discussed in Section 2, Monte Carlo techniques can be used to generate random scores according to the distribution for each method (independence and interdependence). Variances of the scores are based on expert judgements of the same experts mentioned above about intervals within which the criterion scores will most probably be found (cf. Janssen and Herwijnen, 1993). For correlations between the scores we use the correlation matrix in Table 4. The following tables represent the results of the Monte Carlo analysis after 1000 trials. Table 5 presents the probabilities that alternatives achieve certain ranks of these 1000 trials under the assumption of independence and Table 6 represents these probabilities under the assumption of interdependence.

Table 5. Rank Probability Table (Independence Assumed).

	alt9	alt10	alt11	alt3	alt2	alt1	alt4	alt5	alt7	alt8	alt6
rank 1	0.73	0.26	0	0	0	0	0	0	0	0	0
rank 2	0.26	0.67	0.05	0.01	0	0.01	0	0	0	0	0
rank 3	0.01	0.06	0.44	0.25	0.11	0.14	0	0	0	0	0
rank 4	0	0.01	0.16	0.34	0.25	0.23	0.01	0	0	0	0
rank 5	0	0	0.14	0.24	0.33	0.28	0.01	0	0	0	0
rank 6	0	0	0.19	0.16	0.29	0.33	0.03	0	0	0	0
rank 7	0	0	0.01	0.01	0.02	0.02	0.55	0.40	0	0	0
rank 8	0	0	0	0	0	0	0.40	0.60	0	0	0
rank 9	0	0	0	0	0	0	0	0	0.55	0.28	0.18
rank 10	0	0	0	0	0	0	0	0	0.30	0.38	0.32
rank 11	0	0	0	0	0	0	0	0	0.16	0.34	0.50

Table 6. Rank Probability Table (Interdependence Assumed).

	alt9	alt10	alt11	alt3	alt2	alt1	alt4	alt5	alt7	alt8	alt6
rank 1	0.29	0.20	0.13	0.10	0.10	0.10	0.04	0.03	0	0	0
rank 2	0.19	0.18	0.13	0.12	0.11	0.14	0.07	0.07	0	0	0
rank 3	0.16	0.18	0.12	0.15	0.13	0.12	0.07	0.08	0	0	0
rank 4	0.12	0.15	0.12	0.13	0.13	0.14	0.10	0.10	0	0	0
rank 5	0.09	0.09	0.14	0.14	0.12	0.13	0.13	0.13	0.01	0.01	0.01
rank 6	0.07	0.09	0.12	0.13	0.14	0.13	0.15	0.12	0.02	0.01	0.01
rank 7	0.04	0.06	0.12	0.11	0.12	0.12	0.16	0.17	0.04	0.03	0.04
rank 8	0.04	0.03	0.07	0.09	0.09	0.08	0.17	0.17	0.10	0.11	0.08
rank 9	0.01	0.02	0.03	0.02	0.05	0.04	0.08	0.08	0.24	0.23	0.20
rank 10	0	0.01	0.01	0.01	0.01	0.01	0.03	0.04	0.28	0.29	0.31
rank 11	0	0	0	0	0	0	0.01	0.01	0.30	0.32	0.35

In the case of independence (Table 5) one observes an approximately block diagonal structure in Table 5 based on the following groups of alternatives {9,10}, {11,3,2,1}, {4,5} and {7,8,6}. Within these groups there is sometimes a considerable degree of uncertainty on the ranking of alternatives, but the ranking of alternatives between the groups is very certain. This clear result disappears when interdependencies are taken into account. Only a small part of the cells in Table 6 consists of zero's. The distribution of the possible rank an alternative may attain is for most alternatives very broad.

A Chi-Square test can be used to test whether a significant difference exists between the two methods. In Mood, Graybill and Boes (1974) a procedure is presented to calculate sums that converge to the chi-square distribution. These sums can then be derived from probability tables 5 and 6.

Table 7. Chi square sums.

alt1	alt2	alt3	alt4	alt5	alt6	alt7	alt8	alt9	alt10	alt11
290.58	307.66	274.70	426.96	494.69	86.07	167.95	88.49	356.24	370.63	271.38

The value of chi square with 10 degrees of freedom at the 0.995 level is 25.2. As a result, it is found that for all alternatives the two probability tables are different. Taking into account interdependencies indeed has a significant impact on rankings of alternatives in this case study. In the present case the degree of uncertainty about the ranking is much larger when interdependencies are taken into account. Or formulated in the reverse way: ignoring interdependencies may lead to a strong underestimation of uncertainties in the ranking of alternatives.

The formulas in section 4 are now used to demonstrate a decomposition of uncertainty in utility scores using variances of each criterion.

Table 8 represents the decomposition of the variance per criterion for each soil clean up method.

Table 8. Total average share of variance per criterion.

	average share independence assumed	average share interdependence assumed
st	0.02	0.01
cs	0.00	0.00
ss	0.00	0.00
ug	0.02	0.00
rcc	0.02	0.01
rcz	0.02	0.01
trc	0.10	0.10
trz	0.07	0.07
pom	0.01	0.01
poc	0.00	0.00
psos	0.01	0.01
pl	0.00	0.01
ea	0.56	0.62
egw	0.13	0.15
cre	0.00	0.00
crnfp	0.00	0.00
ns	0.03	0.01
nnav	0.00	0.00
nsa	0.01	0.00
total variance	1.00	1.00

The average shares indicate the contribution of each criterion to the overall uncertainty (across all alternatives). For instance emissions to air (ea) has the greatest contribution. The percentages of the two methods are similar: the chi square test does show a significant difference between the two methods.

If one wants to improve one's confidence about the ranking produced by the multi-criterion decision method, the first thing to be considered is to collect additional information on the air emission criterion. Other important sources of uncertainty are the residual cadmium (trc) and groundwater emissions (egw).

6. Concluding remarks

Interdependencies between criterion scores are usually ignored in sensitivity analyses in multicriteria evaluation. Our case study in the field of soil pollution treatment shows that interdependencies may be quite high and wide spread. They appear to have a significant impact on the probabilities that certain ranks are achieved by the alternatives. Ignoring these interdependencies may have a distorting effect on the results of sensitivity analysis in multicriteria evaluation.

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