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G.J. van den Berg
J.C. van Ours

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UNEMPLOYMENT DYNAMICS AND DURATION DEPENDENCE

Gerard J. van den Berg
Jan C. van Ours

Dept. of Economics, Free University Amsterdam
De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands

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Abstract

One of the major issues in the analysis of unemployment durations concerns the distinction between duration dependence of the exit rate out of unemployment and unobserved heterogeneity. Empirical studies rely heavily on functional form restrictions, which may be hazardous. We present a method for the nonparametric estimation of both phenomena. This method is designed to be applicable to aggregate time-series data on outflows from different duration classes. The model and estimation method explicitly take into account that individual exit rates are affected by the business cycle and by seasonal and cohort effects. The method is applied to US gross data on unemployment durations. It turns out that, except for white males, duration dependence is dominated by unobserved heterogeneity.

1. Introduction

In this paper we present a method to distinguish between duration dependence and unobserved heterogeneity in time-series duration data. In particular, the effects of both phenomena can be estimated nonparametrically. The method is applied to US gross data on unemployment durations.

In the past decade, the econometric analysis of unemployment durations has become widespread. One of the major issues in this literature concerns the distinction between duration dependence of the hazard rate (or exit rate out of unemployment) and unobserved heterogeneity (for surveys, see for example Lancaster (1990) and Devine & Kiefer (1991)). Often, there is reason to believe that for a given individual the hazard rate decreases as a function of duration. For example, there may be stigma effects reducing the number of job opportunities for the long-term unemployed (see e.g. Vishwanath (1989) and Van den Berg (1990b)). On the other hand, the presence of unobserved heterogeneity in the distribution of the duration variable causes the hazard rate of the distribution of observed durations to decrease as well. This follows from the fact that on average individuals with the largest hazard rate leave unemployment first. Obviously, from a policy point of view, it is important to know the relative importance of genuine duration dependence (also called state dependence) on the one hand, and unobserved heterogeneity on the other. For example, if duration dependence is the dominant factor, then efforts may be concentrated on the long-term unemployed, while otherwise it may be useful to screen short-term unemployed and concentrate efforts on those with bad characteristics. However, since both factors affect the hazard rate in a similar way, it seems to be hard to distinguish empirically between them.

It is known that in the class of Mixed Proportional Hazard (MPH) models, both the shape of the duration dependence of the hazard and the distribution of the unobserved heterogeneity are nonparametrically identified (see Elbers and Ridder (1982)). However, it is generally believed that in practice it is next to impossible to distinguish between these elements if no strong prior information is present on the shape of the duration dependence or the heterogeneity distribution. In any case, up to now no nonparametric estimation strategy has been developed.

Therefore, in reduced-form empirical analysis of unemployment durations, it has been common to make functional form assumptions on (*i*) the shape of the duration dependence, (*ii*) the distribution of unobserved heterogeneity, and (*iii*) the way the observed explanatory variables enter the model. For example, typical choices are: (*i*) Weibull duration dependence (*ii*) Gamma distributed unobserved heterogeneity, and (*iii*) loglinear dependence of the hazard on observed explanatory variables (see the surveys mentioned above and the references therein). Sometimes more flexible forms are chosen, or semiparametric approaches are followed in which only part of the assumptions mentioned above are made. In any case, the results are conditional on the particular parametric parts of the specification. Intuitively, it is clear that the results on the degree of duration dependence and unobserved heterogeneity may be extremely sensitive with respect to misspecification of the corresponding parts of the model. As an example, Ridder (1987) proves that estimates may be heavily biased if the form of the duration dependence is misspecified. Since in general the choice of this specification is not based on strong prior information but instead chosen because of its analytical tractability, empirical analyses based on such assumptions may be hazardous.

In this paper, we present a method for the nonparametric estimation of all determinants of the unemployment duration distribution. This method is designed to be applicable to discrete-time time-series data on gross outflows from different unemployment duration classes. Gross (or aggregate, or macro) data have the advantage that they provide the exact values of the exit probabilities (or exit rates) out of the different duration classes considered (averaged over unobserved heterogeneity).

The model and the estimation method proposed here explicitly take into account that individual exit rates are affected by macro effects like business cycle effects and seasonal effects. This is another advantage over the usual approach in micro econometric studies on unemployment durations. As a by-product, the estimation method presented here can be used to obtain estimates of business cycle effects and other calendar time effects on unemployment durations. (This however will not be our primary concern here. Also, we will not focus on the composition and changes of the aggregate unemployment rate.)

Section 2 presents the model and the estimation method. Basically, the model is a MPH model in which calendar time replaces the role of the observed explanatory (x) variables. The estimation method generalizes the method proposed in Van Ours (1992). It enables one to estimate the quantities of interest from ratios of observed hazards without the need to parameterize the determinants of the hazards. In Section 2 we also develop specification tests to test for the MPH specification, and we extend the model to allow for seasonal effects on the inflow into unemployment. (It should be noted from the outset that, in principle, the method proposed can also be applied for the analysis of other duration variables).

We apply the estimation method to CPS unemployment duration data from the US Bureau of Labor Statistics. Section 3 describes the data in some detail. Section 4 contains the results. In some respects, unemployment dynamics in the US differs a lot between individuals with different sex and race characteristics. It would be too restrictive to assume that the duration dependence and calendar time dependence patterns are the same for all four groups that can be distinguished. Since the data are disaggregated over these groups, the empirical analysis is therefore carried out separately for all groups. It turns out that the duration dependence patterns and the distributions of unobserved heterogeneity differ between groups with different sex/race characteristics. Section 5 concludes.

2. The model and the estimation method

2.1. *Model assumptions*

In this subsection we present the unemployment duration model and the underlying assumptions. In Subsections 2.2 and 2.3 we then examine what can be inferred about the model from macro data on exit rates out of unemployment. An estimation strategy is proposed that enables us to estimate the quantities of interest. Recall that our inferences are purely nonparametric. That is, we do not parameterize the model, and, strictly speaking, we estimate (summary measures of) functions rather than parameters.

We use two measures of time, each with a different origin. The variable t denotes the duration of unemployment, as measured from the moment the individual becomes unemployed. The variable τ denotes calendar time, which has its origin somewhere in the past. For simplicity we take t and τ to have the same measurement scale (apart from the difference in origin). Both t and τ are discrete variables. As an example, consider an individual who is unemployed for t periods at calendar time τ. If he fails to leave unemployment in period t, he will be unemployed for t+1 periods at calendar time τ+1.

For a good understanding of the model and the estimation method, it is useful to have an idea of the type of data for which this is all designed to be applicable to. Ideally, gross data give the total numbers of individuals in the labor market who are unemployed for t periods of time (t=0,1,2,...) at calendar times τ, τ+1, τ+2, etc.. By comparing the number of individuals who are unemployed for t periods of time at τ to the number unemployed for t+1 periods at τ+1, we observe the exit rate out of unemployment at calendar time τ for duration t. In other words, we observe the conditional probability that an individual leaves unemployment when being unemployed for t periods, when calendar time equals τ at the moment of exit, for different values of t and τ.

In the model, t is endogenous, whereas τ is an explanatory variable, in the sense that the exit rate out of unemployment for individuals with duration t may vary over calendar time. Thus, calendar time is assumed to capture macro effects (including business cycle and seasonal effects) on individual exit rates out of unemployment.

The model aims at explaining variations in unemployment duration distributions in terms of observed and unobserved individual characteristics, calendar time, and the duration dependence pattern. Usually, gross data do not contain information on individual characteristics that could be used as explanatory variables. At best, gross figures are collected separately for a few different groups of individuals. Let x denote the variables used to distinguish such groups. We will estimate the model separately for each group, i.e. for each possible value of x. In the sequel of this section, therefore, we suppress in word and notation the conditioning on the prevailing value of x.

We assume that all variation in the exit rates out of unemployment can be explained by the prevailing unemployment duration t and calendar time τ and by unobserved heterogeneity across individuals. We denote the unobserved heterogeneity variable by v. Consider an individual with unobserved characteristics v who is unemployed for t periods when calendar time equals τ. We denote the conditional probability that this individual leaves unemployment after t periods of unemployment by $\theta(t|\tau,v)$. By definition, this is the exit rate out of unemployment (or hazard rate) at t conditional on τ and v. The unemployment duration density conditional on calendar time and conditional on v can be constructed from these exit rates. For example, the probability that unemployment duration equals t, when calendar time was τ-t at the moment of inflow into unemployment, conditional on v, equals

$$\theta(t|\tau,v) \cdot \prod_{i=1}^{t} (1 - \theta(t-i|\tau-i,v)) \tag{2.1}$$

for all $t\in\{0,1,..\}$. We use the convention that the product term is one if t=0.

We make the following assumptions.

Assumptions

1. MPH: $\theta(t|\tau,v)$ has a mixed proportional hazard specification, i.e. there are functions ψ_1 and ψ_2 such that

$$\theta(t|\tau,v) = \psi_1(t) \cdot \psi_2(\tau) \cdot v \tag{2.2}$$

with ψ_1 and ψ_2 positive and uniformly bounded from above. Further, the distribution of v is such that, for every t and τ, $Pr(0<\theta(t|\tau,v)<1) = 1$.

2. Independence of v and τ: v does not depend on the moment of inflow into unemployment and does not change during unemployment.

3. Variation over calendar time: the function ψ_2 is not constant.

The functions ψ_1 and ψ_2 represent the duration dependence and the time dependence of the exit rate out of unemployment. The distribution of v represents the distribution of the unobserved heterogeneity.

As we will see, Assumptions 1–3 basically ensure nonparametric identifiability of the model. In particular, they ensure that duration dependence and unobserved heterogeneity can be distinguished empirically. Assumption 1 is similar to the standard MPH assumption in reduced-form duration models for micro duration data (for an extensive survey of reduced-form duration models for micro data, see Lancaster (1990)). In models for micro duration data, dependence on calendar time is usually ignored, and the role of τ in the model above is replaced by the role of observed explanatory variables x. Elbers & Ridder (1982) prove that the latter type of models are nonparametrically identified if assumptions similar to above are satisfied. Whenever calendar time is included as a regressor in reduced-form duration models for micro data, it is usually included as a multiplicative term in the hazard rate (see e.g. Imbens (1991)). In Subsection 2.4 we will show that even when we relax the MPH Assumption a bit, we can still get some informative results.

Note that one important difference between the present model and these reduced-form models is that here we have discrete time, whereas in micro studies time is usually treated as continuous. Because of this, we had to introduce the last line of Assumption 1. Note that it implies that the support of v is bounded. This in turn implies that all moments of v exist.

Assumption 2 rules out that there are cohort effects in the distribution of the unobserved heterogeneity term. However, as will be shown in Subsection 2.4, this assumption can be relaxed somewhat without much loss. Assumption 3 is similar to the assumption in Elbers & Ridder (1982) that there is dispersion of observed explanatory variables. Without such an assumption, duration dependence cannot be distinguished empirically from unobserved heterogeneity.

2.2. *Observed exit rates*

As mentioned above, the data provide observations on the conditional probabilities that individuals leave unemployment when being unemployed for t periods, when calendar time equals τ at the moment of exit, for different values of t and τ. These probabilities are unconditional on the unobserved heterogeneity term v, and will be denoted by $\theta(t|\tau)$. The situation is similar to reduced-form analyses of micro duration data in which the model expresses the exit rates conditional on observed and unobserved explanatory variables but the data only provide information on exit rates conditional on the observed variables.

To express the observed exit rates $\theta(t|\tau)$ in terms of the exit rates $\theta(t|\tau,v)$, we have to integrate v out of the latter. Let $\boldsymbol{t}$ denote the random unemployment duration, and t its realization. We have that

$$\theta(t|\tau) \equiv \frac{Pr(\boldsymbol{t}=t|\tau)}{Pr(\boldsymbol{t}\geq t|\tau)} \equiv \frac{\mathrm{E}_v(Pr(\boldsymbol{t}=t|\tau,v))}{\mathrm{E}_v(Pr(\boldsymbol{t}\geq t|\tau,v))} \tag{2.3}$$

in which $Pr(\boldsymbol{t}=t|\tau,v)$ and $Pr(\boldsymbol{t}\geq t|\tau,v)$ can be expressed in terms of $\theta(t|\tau,v)$ (note that equation (2.1) gives $Pr(\boldsymbol{t}=t|\tau,v)$). By doing this, and by substituting equation (2.2), we get

$$\theta(t|\tau) = \frac{\psi_1(t).\psi_2(\tau).\ \mathrm{E}_v\left[v.\prod_{i=1}^{t} [1-\psi_1(t-i).\psi_2(\tau-i).v] \right]}{\mathrm{E}_v\left[\prod_{i=1}^{t} [1-\psi_1(t-i).\psi_2(\tau-i).v] \right]} \tag{2.4}$$

Thus, $\theta(t|\tau)$ can be expressed in terms of the "structural functions" ψ_1, ψ_2 and the distribution function $G(v)$ of v. In fact, we can be more specific on the way $G(v)$ enters such expressions. By expanding the products in the r.h.s. of (2.4) it follows that $\theta(t|\tau)$ depends on $G(v)$ only by way of the first t+1 moments of v. Denote $\mathrm{E}_v(v^i)$ by μ_i. We have the following result: $\theta(t|\tau)$

depends on $\{\psi_1(i),\ \psi_2(\tau\text{-}t\text{+}i),\ \mu_{i+1},$ with $i = 0,1,..,t\}$. We will call the elements of the latter set the "parameters", even though they really are values of functions on $\mathbb{N}$ and summary statistics of the underlying heterogeneity distribution, respectively.

If we observe $\theta(t|\tau)$ for a large number of values of τ and t, then the number of observations exceeds the number of unknown parameters. Suppose we observe $\theta(t|\tau)$ for $\tau\in\{T+1,T+2,...,T+n_\tau\}$ and for $t\in\{0,1,..,n_t-1\}$. We then have $n_t.n_\tau$ observations. The number of parameters in the expressions for the observed exit rates equals $3n_t+n_\tau-1$ (namely, n_t moments of v, n_t terms $\psi_1(0)..\psi_1(n_t-1)$, n_τ terms $\psi_2(T+1)...\psi_2(T+n_\tau)$, and n_t-1 terms $\psi_2(T-n_t+2)...\psi_2(T)$). From equation (2.2) it follows that two parameters can be normalized arbitrarily. As a result, the number of observations minus the number of remaining parameters equals $(n_t-1).(n_\tau-3)$. This is positive if $n_t>1$ and $n_\tau>3$.

Consequently, at first sight it seems that all parameters can be estimated from a sufficiently large sample. (For the moment we are silent on exactly how the parameters should be estimated. The idea is that each observed exit rate $\theta(t|\tau)$ should be as close as possible to the model expression corresponding to it.) However, it is clear that the sample must be quite large to have some freedom of inference. For example, if $n_\tau=20$ and $n_t=4$ then the number of observations equals 80 while the number of estimable parameters equals 29. Usually, in econometrics, the number of parameters is much smaller relative to the number of observations. Moreover, if we have a sample in which n_t is relatively small (which is typically the case, see Section 3) then, for each $\psi_2(\tau)$ parameter, the number of observations that contain information on that parameter is extremely small, so the estimate of it would be unreliable. However, note that we are primarily interested in estimating the duration dependence and unobserved heterogeneity parameters. For this, the calendar time dependence parameters are nuisance parameters. Thus, it would be nice if the parameters of interest could be estimated without the need to have an extremely large sample.

It turns out that the ideas in Van Ours (1992) can be used to achieve this aim. Basically, these ideas amount to substituting values of past observed exit rates into the expressions (3.4) for $\theta(t|\tau)$, and examining ratios of the resulting expressions for different t.

Consider expression (2.4). We denote the expectation in the numerator of the r.h.s. by $a(t,\tau)$, and the denominator of the r.h.s. by $b(t,\tau)$. Consequently, we have that

$$\theta(t|\tau) = \psi_1(t).\psi_2(\tau).a(t,\tau) \ / \ b(t,\tau) \tag{2.5}$$

Note that $b(t,\tau)$ is nothing but $Pr(t \geq t|\tau)$. We can therefore rewrite it as

$$b(t,\tau) = \prod_{i=1}^{t} (1 - \theta(t-i|\tau-i)) \tag{2.6}$$

so $b(t,\tau)$ can be expressed completely in terms of observed past exit rates. Now consider $a(t,\tau)$. This term can be expressed completely in terms of observed past exit rates and moments of v by way of recursion. It depends on ψ_1 and ψ_2 by way of the products $\psi_1(t-i).\psi_2(\tau-i)$. From (2.5), we have that $\psi_1(t-i).\psi_2(\tau-i)$ equals $\theta(t-i|\tau-i).b(t-i,\tau-i)/a(t-i,\tau-i)$. Now suppose that $a(t-i,\tau-i)$ has been expressed in terms of past observed exit rates. Then $\psi_1(t-i).\psi_2(\tau-i)$ can be expressed that way as well. Consequently, the same is true for $a(t,\tau)$. The recursion starts with $a(0,\tau-t)$, which equals μ_1.

As an example, consider $a(1,\tau)$. From (2.4), it equals $\mu_1-\psi_1(0).\psi_2(\tau-1).\mu_2$. From (2.5), we have that $\psi_1(0).\psi_2(\tau-1)$ equals $\theta(0|\tau-1).b(0,\tau-1)/a(0|\tau-1)$. There holds that $b(0|\tau-1)$ equals 1, and that $a(0|\tau-1)$ equals μ_1. Consequently, $a(1,\tau)$ equals $\mu_1 - \theta(0|\tau-1).\mu_2/\mu_1$. For $t \geq 2$, the resulting expressions for $a(t|\tau)$ become quite lengthy. However, using the algorithm presented here, it is easy to calculate $a(t|\tau)$ numerically once its determinants are quantified.

In the general case, we have that $a(t,\tau)$ can be expressed completely in terms of observed past exit rates $\theta(0|\tau-t),..,\theta(t-1|\tau-1)$ and the moments $\mu_1,..,\mu_{t+1}$. In fact, it can be shown that the result can be written as μ_1 times an expression depending on $\theta(0|\tau-t),..,\theta(t-1|\tau-1)$ and the ratios μ_2/μ_1^2, $\mu_3/\mu_1^3,...,\mu_{t+1}/\mu_1^{t+1}$. Now let us return to $\theta(t|\tau)$. From the results just derived, and by (2.5), we have:

$$\theta(t|\tau) = \psi_1(t).\psi_2(\tau).\mu_1 \ . \left\{ \begin{array}{c} \text{expression depending on } \theta(i-1|\tau-t+i-1) \text{ and} \\ \mu_{i+1}/\mu_1^{i+1}, \text{ with } i=1,2,..,t \end{array} \right\} \tag{2.7}$$

for $t \geq 1$. (If $t=0$ then the expression between the accolades is one.) If there is no unobserved heterogeneity (i.e. if v equals some number μ_1 with probability one), then, from (2.4), $\theta(t|\tau) = \psi_1(t).\psi_2(\tau).\mu_1$. Consequently, in that case, the expression between accolades in (2.7) equals one. If there is unobserved heterogeneity, then in general this expression is smaller than one.

In the next subsection we will examine ratios of observed exit rates written as in (2.7), for different t. It turns out that such ratios can be used to estimate the parameters of interest.

2.3. *Ratios of observed exit rates*

Consider the ratio $\theta(t|\tau)/\theta(t\text{-}1|\tau)$. From equation (2.7) it follows that this ratio can be rewritten as

$$\frac{\theta(t|\tau)}{\theta(t-1|\tau)} = \frac{\psi_1(t)}{\psi_1(t-1)} \cdot \Big\{ \text{expression depending on } \theta(i\text{-}1|\tau\text{-}t\text{+}i\text{-}1) \text{ and } \mu_{i+1}/\mu_1^{i+1}$$

$$\text{with } i=1,2,..,t, \text{ and, if } t\geq 2, \text{ on } \theta(i\text{-}1|\tau\text{-}t\text{+}i) \text{ with } i=1,2,..,t\text{-}1 \Big\} \tag{2.8}$$

Denote μ_i/μ_1^i by γ_i ($i\geq 2$), and $\psi_1(t)/\psi_1(t\text{-}1)$ by η_t ($t\geq 1$). The parameters η_t represent the duration dependence of the exit rate as a function of t, whereas the parameters γ_i represent the normalized moments of the distribution of unobserved heterogeneity. Equation (2.8) implies that $\theta(t|\tau)/\theta(t\text{-}1|\tau)$ depends on the parameters η_t and $\gamma_2,..,\gamma_{t+1}$, but not on the parameters of ψ_2. It may be instructive to consider some special cases of (2.8) explicitly. If $t=1$ we get

$$\frac{\theta(1|\tau)}{\theta(0|\tau)} = \eta_1 \cdot \frac{1 - \gamma_2.\theta(0|\tau\text{-}1)}{1 - \theta(0|\tau\text{-}1)} \tag{2.9}$$

whereas if $t=2$ we get

$$\frac{\theta(2|\tau)}{\theta(1|\tau)} = \eta_2 \cdot \frac{1 - \theta(0|\tau\text{-}1)}{(1\text{-}\theta(1|\tau\text{-}1)).(1\text{-}\theta(0|\tau\text{-}2))} \cdot$$

$$\cdot \frac{1 - \gamma_2.\theta(0|\tau\text{-}2) - \theta(1|\tau\text{-}1).(1\text{-}\theta(0|\tau\text{-}2)).\dfrac{\gamma_2 - \gamma_3.\theta(0|\tau\text{-}2)}{1 - \gamma_2.\theta(0|\tau\text{-}2)}}{1 - \gamma_2.\theta(0|\tau\text{-}1)} \tag{2.10}$$

Such ratios of observed exit rates can be used to estimate the parameters of interest. In doing so, we deal with the problems encountered in the previous subsection. First of all, recall that (2.8) does not depend on the nuisance parameters of ψ_2 anymore. As a result of this, the number of parameters is now much smaller relative to the number of observations. Moreover, for each parameter η_t and γ_i, the number of observations that contain information on it is relatively large. This can be clarified by examining ratios $\theta(t|\tau)/\theta(t\text{-}1|\tau)$ recursively, starting with $t=1$. Suppose that, like in the previous subsection, we have $n_t.n_\tau$ observed exit rates. Let $n_\tau\geq n_t\geq 2$. From (2.9) it is clear that from the data on $\theta(1|\tau)/\theta(0|\tau)$ we can estimate η_1 and γ_2. This amounts to estimating 2 parameters from n_τ-1 ratios of observed exit rates. (Note that we now have to throw away observed ratios for which the expressions corresponding to (3.8) depend on exit rates for the periods before the sample period. Also note that γ_2 is identified because $\theta(0|\tau-1)$ varies with τ by virtue of Assumption 3.) Given the estimate of γ_2, we can estimate η_2 and γ_3 from the data on $\theta(2|\tau)/\theta(1|\tau)$. This amounts to estimating 2 parameters from n_τ-2 ratios of observed exit rates; etc. Note that the number of parameters cannot be reduced further by normalizations since the η_t and γ_i are defined as ratios of the original parameters. Also note that an estimation method based on recursive examination of ratios of observed exit rates is not efficient, since γ_i enters the expression of $\theta(t|\tau)/\theta(t-1|\tau)$ for every $t\in\{i-1,i,..\}$.

So, for typical values of n_τ, the parameters of interest are estimable reliably by using (2.8). The estimates of $\eta_1,\eta_2,...$ show the evolution of the duration dependence of the exit rate $\theta(t|\tau,v)$. The estimates of $\gamma_2,\gamma_3,...$ give information on the distribution of unobserved heterogeneity. We will return below to the issue of recovering $G(v)$ from $\gamma_2,\gamma_3,...$

If one is interested in the parameters of ψ_2, then an analogous estimation method can be used. The ratios $\theta(t|\tau)/\theta(t|\tau-1)$ only depend on the parameters $\psi_2(\tau)/\psi_2(\tau-1)$ and (if $t\geq1$) on $\gamma_2,..,\gamma_{t+1}$, in a way similar to (2.8). Thus, by using observations on such ratios for fixed τ and for different t, we can estimate these parameters. However, recall that as t increases the expressions for $\theta(t|\tau)$ become increasingly cumbersome. For t=0 we get that $\theta(0|\tau)/\theta(0|\tau-1)$ = $\psi_2(\tau)/\psi_2(\tau-1)$, so these ratios of observed exit rates provide direct estimates of the calendar-time dependence of $\theta(t|\tau,v)$ (note however that such estimates are each based on one observation only).

Let us return to the ratio $\theta(t|\tau)/\theta(t-1|\tau)$. If there is no unobserved heterogeneity, then $\theta(t|\tau)/\theta(t-1|\tau) = \eta_t$, so this ratio does not depend on τ. (This can be checked in (2.9) and (2.10) by noting that in that case $\mu_i=\mu_1^i$ for every $i\geq1$, so γ_i=1.) If there is unobserved heterogeneity, then in general these ratios do depend on τ. For example, there holds that $\theta(1|\tau)/\theta(0|\tau)$ depends on $\theta(0|\tau-1)$ if and only if $\gamma_2\neq1$, which in turn holds if and only if there is unobserved heterogeneity (note that always $\gamma_2\geq1$). The exit rate $\theta(0|\tau-1)$ varies with τ by virtue of Assumption 3, so, in sum, $\theta(1|\tau)/\theta(0|\tau)$ varies with τ if and only if there is unobserved heterogeneity.

The ratio $\theta(t|\tau)/\theta(t-1|\tau)$ can be thought of as the discrete-time equivalent of the derivative of log $\theta(t|\tau)$ w.r.t. t. The results in this subsection show that we are able to identify the parameters associated with the distribution of unobserved heterogeneity from the cross effects of t and τ in log $\theta(t|\tau)$. Thus, identification of this distribution is achieved by exploiting the fact that exit rates vary over calendar time. (There is a strong analogy with MPH models for micro duration data in which the role of τ is replaced by observed regressors x, see Van den Berg (1992).) Clearly, the MPH assumption is crucial for this. It is therefore important to test that assumption. The next subsection deals with this.

2.4. *Specification tests and model generalizations*

The model specification can be tested in a number of ways. Consider the estimates of $\gamma_2,..,\gamma_{n_t}$. Let us normalize by taking $\mu_1=1$, so γ_i equals $E(v^i)$. If the model is correct, then $\gamma_2,..,\gamma_{n_t}$ are mutually consistent as moments of a distribution with positive support (i.e. support in $<0,\infty>$) and mean one. This can be tested for. Suppose $n_t=3$. If $\gamma_2<1$ or $\gamma_3<\gamma_2^2$ then there is no distribution with positive support that is able to generate such moments (see Shohat & Tamarkin (1970); e.g. $\gamma_2<1$ would imply $Var(v)<0$). If $\gamma_2\geq 1$ and $\gamma_3\geq\gamma_2^2$ then, in contrast, there are such distributions, except for the cases $\{\gamma_2=1, \gamma_3>1\}$ and $\{\gamma_2>1, \gamma_3=1\}$. So, a relatively simple procedure would be to test H_0: $\{\gamma_2\geq 1, \gamma_3\geq\gamma_2^2\}$ versus its opposite. If γ_4 is available as well, then an additional necessary condition for the specification to be correct is that $(\gamma_4-\gamma_2^2).(\gamma_2-1) - (\gamma_3-\gamma_2)^2 \geq 0$ (see Shohat & Tamarkin (1970)).

If these tests do not result in rejections, then one can usually find a discrete distribution that is able to generate the γ_i estimates (see Shohat & Tamarkin (1970) and Lindsay (1989)). Lindsay (1989) provides formulas for recovering the underlying discrete distribution from the moments. It should be noted that in general there will also be non-discrete distributions that are able to generate a given finite set of moments. Consequently, if the estimated γ_i are moments of some distribution, then in general there will be more than one distribution function $G(v)$ consistent with them.

The moment tests proposed above are informative on the validity of Assumption 1. Suppose that in reality $\theta(t|\tau,v)$ is not multiplicative in t, τ, and v, but instead contains interaction terms. Then, in particular cases, this shows up in the γ_i estimates being inconsistent with the moment restrictions above. For example, suppose that the duration dependence pattern for individuals with large v differs from that for individuals with small v (this is observationally equivalent to a model in which the individual v changes as a function of duration), in the following way: $\theta(t|\tau,v) = \psi_1(t,v).\psi_2(\tau)$, with

$\psi_1(0,v)=v$ and $\psi_1(1,v)=1/v$. It can be shown that then the estimate of γ_2 asymptotically is smaller than one. Also, the tests may detect misspecification of the unit of time period. If in reality the model is correct for weekly periods but it is assumed to be correct for monthly periods, then this may turn up in the γ_i estimates.

The model can also be tested by estimating it for subsamples distinguished by their range of values of τ, and comparing the results. Such a procedure may be interpreted more positively as making the results less sensitive to structural changes.

It should be noted that not every misspecification of the model results in inconsistent estimates. For example, if the individual level of v is allowed to change as a function of duration in the following way: $v(t) = v(0).\psi_3(\tau)$, with $v(0)$ being a random drawing from $G(v)$, then the model is equivalent to the model of Subsection 2.1. A more interesting generalization that we can deal with concerns incorporating seasonal effects. The remainder of this subsection will be devoted to this.

Analogous to De Toldi, Gouriéroux & Monfort (1992) and Imbens & Lynch (1992), one may distinguish two types of seasonal effects on the exit rate. First, there may be an effect that affects every individual in a similar way. For example, there may be less activity on the labor market during the holiday season. Secondly, there may be an effect that only affects the individuals in the inflow into unemployment. For example, the success of individuals in the inflow at the end of the schooling season may on average be worse than that at other times of the year. If the unit of time period is small enough, then the first effect will be captured by the $\psi_2(\tau)$ terms in the model. To incorporate the second effect, which may be labeled the cohort effect, we allow for dependence of $G(v)$ on the moment of inflow, so we relax Assumption 2.

For the moment, suppose there are two seasons, labeled by indices A and B. We assume that the season affects a scale parameter of the distribution of unobserved heterogeneity,

$$G_B(v) = G_A(\omega.v) \tag{2.11}$$

with $\omega>1$, so A is the "good" season and B is the "bad" season. (It can be shown that this is observationally equivalent to assuming that $\theta(t|\tau,v)$ contains a fourth multiplicative term depending on the season prevailing at the moment of inflow.) As a result, $\mu_{i,A} = \omega^i.\mu_{i,B}$ and $\mu_{i,A}/\mu_{1,A}^i = \mu_{i,B}/\mu_{1,B}^i$. The latter ratio is denoted by γ_i.

In this model, all moments entering the expression for $\theta(t|\tau)$ that corresponds to (2.7) are moments of the heterogeneity distribution for the season at $\tau-t$. Consequently, if $\tau-t$ is a good season, then the expression for $\theta(t|\tau)$ that corresponds to (2.7) is a multiplicative factor ω larger than if $\tau-t$ is a bad season. Apart from this, there are no differences between the expressions for $\theta(t|\tau)$ for different seasons at the moment of inflow. This implies that the expressions for the ratios of observed exit rates differ in a very simple way from the corresponding expressions in Subsection 2.3. If $\tau-t$ is a bad season but $\tau-t+1$ is a good season, then $\theta(t|\tau)/\theta(t-1|\tau)$ is equal to a factor $1/\omega$ times the corresponding expression in Subsection 2.3. If $\tau-t$ is a good season but $\tau-t+1$ is a bad season, then $\theta(t|\tau)/\theta(t-1|\tau)$ is equal to a factor ω times the corresponding expression in Subsection 2.3. If $\tau-t$ and $\tau-t+1$ are seasons of the same type, then $\theta(t|\tau)/\theta(t-1|\tau)$ equals the corresponding expression in Subsection 2.3.

Thus, the analysis can easily be extended to allow for seasonal (or cohort) effects, which can be estimated along with the other parameters. Note that the method described above can be generalized to more than two seasons. In fact, the duration of a season may be taken to equal the unit of time. For example, if monthly data are observed, then we may distinguish 12 seasons corresponding to the months of the year. Let $s \in \{1,2,..,12\}$ denote the number of the month and let G_s denote the distribution function of v in the inflow into unemployment at month s. Analogous to (2.11), we postulate that

$$G_1(v) = G_{12}(\omega_1.v)$$

$$G_s(v) = G_{s-1}(\omega_s.v) \qquad s \in \{2,3,..,11\} \qquad (2.12)$$

$$G_{12}(v) = G_{11}(v/(\omega_1.\omega_2...\omega_{11}))$$

Thus, we have 11 additional parameters. Again, these appear as multiplicative factors in the expressions for the ratios of hazard rates.

Alternatively, one might want to model a calendar time trend in $G(v)$ by assuming that $G(v)$ at τ equals $G(\omega.v)$ at $\tau-1$, for every τ. However, it can be shown that this would make the η_t parameters unidentified.

3. The dataset

In our empirical analysis we use unpublished CPS data from the US Department of Labor which give monthly information on unemployment by weekly duration classes. Analyzing unemployment durations by using CPS data has been popular for some time. Many studies use information on distributions of in-progress spells of unemployment to calculate durations of completed spells of unemployment, relying on steady state assumptions (see e.g. Butler & McDonald (1986)). Sider (1985) and Baker (1992) relax the steady-state assumption. In our framework, cohorts are distinguished by the value of τ at the moment of inflow into unemployment, so no steady-state assumption is needed.

The data do not enable us to make a distinction between employment as destination and departure from the labor force as alternative destination. But, as Abowd & Zellner (1985) show, the share in the outflow from unemployment of workers becoming employed is larger than that of workers leaving the labor force.

We use information for the period 1967-1991. As, for example, Sider (1985) and Baker (1992) point out, there are several problems connected to the use of these data. First of all, the way in which the data are collected implies that we do not have actual cohorts. However, we may consider the data as synthetic cohorts. Second, we need data in which the frequency at which the data are collected equals the sizes of the unemployment duration classes. This implies that we have to aggregate the weekly duration classes into monthly duration classes. Finally, the data are influenced by phenomena like digit preferences and the tendency of respondents to report 'weeks of unemployment' as whole months.

Because of this we made the same corrections as in Baker (1992). Baker reallocated 30 percent of the respondents at 4, 8, 12, 16 and 26 weeks, 40 percent of those at 52 weeks, and 50 percent of those at 78 and 99 weeks, in each month of the sample to adjacent later weeks. Since the information on exit rates becomes more unreliable at longer durations we only used information on exit rates for the first four months of unemployment.

In the analysis we use time series of monthly exit rates out of unemployment for four groups of workers: white males, white females, black males and black females.

The CPS data used in Baker (1992) are more disaggregated than those used in the present paper, in the sense that the vector of observed individual characteristics x is much larger than here. Baker (1992) investigates whether business cycles affect the aggregate mean unemployment duration mainly by changing the distribution of x in the inflow into unemployment, or mainly by changing the exit rates for all unemployed individuals simultaneously. In terms of our model, this amounts to distinguishing whether τ affects the mean unemployment duration because $G(v)$ varies with the moment of inflow τ, or because ψ_2 varies with τ. Baker (1992) finds no evidence for the former phenomenon. This result is confirmed by Imbens & Lynch (1992), who use micro data. This supports our assumption that $G(v)$ does not depend on τ, at least to the extent in which τ represents the business cycle.

- - - - - - - *Table* 1 *about here* - - - - -

Table 1 shows yearly averages for 1970, 1980 and 1990 of the distribution of elapsed unemployment durations by duration class for all four groups. From this table some large differences over the duration classes appear. For the duration class less than 1 month the main difference is between male and female workers. In 1980 for example on average about 44% of the male unemployed workers was unemployed for less than 1 month, while for female unemployed workers this was about 52%.

- - - - - *Figure* 1 *about here* - - - - -

The developments of the yearly averages of the monthly exit rates are shown in Figure 1. Apart from cyclical fluctuations, the exit rate for the first month declines over the seventies, is stable over the eighties and declines again in the beginning of the nineties. The exit rates for the second, third and fourth month of unemployment are lower and show more fluctuations. For white male workers there is a definite sequence in the exit rates out of unemployment from high to low. For the other groups of workers, in particular the groups of black workers, the exit rate for a higher duration class is sometimes larger than those for a lower duration class. The latter phenomena may reflect behavior, but they may also reflect inaccuracies due to the small numbers of unemployed workers in the higher duration classes (see also Appendix 1).

Figure 1 shows that the exit rates out of unemployment are quite high. At the end of the sixties about 90% of the unemployed workers left unemployment within the first quarter. In the eighties this was about 75–85%. It is clear that there are large cyclical fluctuations in the exit rates for the first quarter. Furthermore, it appears that the exit rate for females is higher than for males and the exit rate for white workers is higher than for black workers. In 1990 the average exit rate in the first quarter of unemployment was 83% for white male workers, 87% for white female workers, 78% for black male workers and 84% for black female workers.

It is clear from this discussion that the four groups that are distinguished have quite different unemployment dynamics characteristics. Also, for each group there have been major changes in gross outflows during the period that the data span. The latter can be interpreted as justifying Assumption 3.

The model of Section 2 predicts that there is unobserved heterogeneity in the unemployment duration distribution if ratios of observed exit rates for different duration classes change over calendar time. This prediction can be used to perform a simple eyeball test. From Figure 1 it is clear that such ratios do change over calendar time. Thus, we anticipate the presence of significant unobserved heterogeneity.

4. The results

4.1. *Parameter estimates*

In the empirical analysis we use the information about the first four monthly exit rates out of unemployment. Following Section 2 we specify 3 linear equations, as follows: log $\theta(t|\tau)/\theta(t-1|\tau)$ equals the log of the corresponding expression on the r.h.s. of (2.8), plus an error term (so each $t\in\{1,2,3\}$ defines one equation). The error terms represent specification errors that are identically distributed over equations and over observations. We assume that the errors in a given equation are independent across the observations. On the other hand, we allow the errors in different equations to be contemporaneously related. So, at a given point of calendar time, the specification errors for different ratios of exit rates may be related. Note that we do not make a parametric assumption on the distribution of the error terms.

In the data we found a number of inconsistent (<0, or very small) monthly exit rates. In some cases this led to very large ratios $\theta(t|\tau)/\theta(t-1|\tau)$. We therefore skipped those observations from the dataset for which $\theta(t|\tau)$ was smaller than 0.05. This restriction is arbitrary, but the use of similar restrictions with different boundaries did not lead to substantially different results.

The 3 equations contain 3 heterogeneity parameters $(\gamma_2,\gamma_3,\gamma_4)$ and 3 duration dependence parameters (η_1,η_2,η_3). Furthermore we allowed for seasonal effects as specified in (2.12), introducing 11 additional parameters $\omega_1,...\omega_{11}$. We estimated the parameters using Seemingly Unrelated Nonlinear Regression. The estimation period was 1967.04-1991.12. The estimation results are shown in Table 2.

- - - - - *Table 2 about here* - - - -

The estimation results indicate that for white workers there is negative duration dependence of the exit rate out of unemployment, in particular after the first month of unemployment. For white male workers, the exit rate in the third month is 86% of the exit rate in the second month and the exit rate in the fourth month is 80% of the exit rate in the third month. This may be due to a stigma effect of not being short-term unemployed.

For both male and female black workers η_1 is significantly larger than one, indicating that there is significant positive duration dependence in the second month of unemployment in comparison to the first. However, for black males the sign of the duration dependence changes as the spell proceeds. The less negative duration dependence for blacks (relative to whites) during the first few months can be "explained" in a number of ways (relatively strong anticipation of unemployment benefits exhaustion; relative importance of particular recall options; relatively large non-pecuniary utility of being short-term unemployed; increase in transitions from unemployment to nonparticipation; etc.). However, in the absence of additional information it is hard to assess the power of such possible explanations. In the micro-econometric literature on unemployment durations it is always assumed that the duration dependence parameters do not depend on individual characteristics like race and gender.

The γ_i estimates show that there is significant unobserved heterogeneity, for all groups. The moment-inequality specification tests proposed in Subsection 2.4 give similar results for each of the four subgroups in the data. In particular, neither of the three inequalities is rejected, for any subgroup, using conventional levels of significance. This supports our model specification. All γ_i estimates are significantly larger than one. Further, $\gamma_3-\gamma_2^2$ is significantly larger than zero whereas $(\gamma_4-\gamma_2^2).(\gamma_2-1) - (\gamma_3-\gamma_2)^2$ does not differ significantly from zero, for each subgroup. Using results in Shohat & Tamarkin (1970), this implies that, for each subgroup, $G(v)$ can be approximated well by a discrete distribution with two positive point of support. In the next subsection we will examine the implications of the γ_i estimates for the exit rates more closely.

From Table 2 it also follows that there are seasonal effects in the heterogeneity distribution. The average quality of the newly unemployed workers increases substantially from October to March of each year to decrease again in subsequent months. This may be because in the fall a large proportion of the inflow into unemployment consists of schoolleavers who have no work experience and who may all compete for only a fraction of the set of jobs available.

It may be interesting to compare our empirical results on the presence of duration dependence and unobserved heterogeneity to those in the literature on the parametric analysis of unemployment durations. Butler & McDonald (1986) estimate these phenomena in a parametric setting using CPS data. They take a Weibull specification for $\psi_1(t)$ and assume that $G(v)$ is a Generalized Gamma distribution, and they find evidence for the presence of unobserved heterogeneity and positive duration dependence (so $\psi_1(t)$ is increasing). However, their model does not account for dependence of the unemployment duration hazard on individual characteristics x or calendar time τ. Consequently, the model is nonparametrically unidentified, and the results on duration dependence and unobserved heterogeneity are determined by the assumed parametric functional forms for $G(v)$ and $\psi_1(t)$.

A number of studies based on US unemployment duration data for male individuals estimate duration dependence and unobserved heterogeneity in a parametric setting, allowing for dependence of the hazard on observed explanatory variables x. The results in Flinn & Heckman (1983) are not statistically significant. Heckman & Singer (1984) find evidence for the presence of unobserved heterogeneity. They take a Weibull specification for $\psi_1(t)$ and find that the results on the sign of the duration dependence are very sensitive to the assumed family of distributions for $G(v)$.

Meyer (1990) uses a flexible functional form for $\psi_1(t)$ and assumes $G(v)$ belongs to the Gamma family. He finds evidence for unobserved heterogeneity. Further, in general $\psi_1(t)$ does not display strong duration dependence during the first 3 months of unemployment. The hazard does display spikes near points of time at which benefits entitlement ends, but these points of time are well beyond the three-month period we examine in our analysis. Thus, Meyer (1990)'s results are not inconsistent with ours.

Appendix 2 shows the estimation results with respect to the heterogeneity and duration dependence parameters for two subperiods: 1967.04–1979.12 and

1980.01–1991.12. The estimates are similar to those in Table 2, indicating that the estimation results are quite robust. We also estimated the model obtained by adding specification errors to the original equations (2.8) (rather than adding them after taking logs of the l.h.s. and r.h.s.). Again the results are similar to those in Table 2. The main difference is that for all groups the duration dependence is slightly less negative than in Table 2.

4.2. *Implications*

Using the information from Table 2 we can study the evolution of the average exit rate over the duration of unemployment for the different groups of workers in case of a stationary labor market ($\psi_2(\tau)$ constant). In that case the evolution of the average exit rate only depends on the heterogeneity distribution $G(v)$ and on the actual duration dependence as summarized in $\psi_1(t)$ (see equation (2.4)). Figure 2 shows this in detail.

– – – – – – *Figure 2 about here* – – – – – – –

Figure 2a shows how unobserved heterogeneity influences the average exit rate. The exit rates are plotted as fractions of the exit rates in the first month. Duration dependence is assumed to be absent. There is an obvious decline in average exit rate, which is largest for black females and smallest for white males. Due to heterogeneity the average normalized exit rate in the fourth month is 59% for black females and 70% for white male workers. Figure 2b shows the influence of actual duration dependence, by assuming there is no heterogeneity. These results have been discussed above.

Figure 2c shows the combined effect of duration dependence and heterogeneity. For white workers, duration dependence and heterogeneity work in the same direction on the average (or observed) exit rate. As a result, there is a substantial decline in exit rate over the duration of unemployment. For white males, duration dependence is the dominant factor, whereas for white females unobserved heterogeneity dominates.

For black workers, duration dependence and heterogeneity generally work in opposite directions. However, the latter effect always dominates. As a result, Figure 2c shows a substantially smaller decline for black workers.

5. Conclusion

In this paper we show both theoretically and empirically that it is possible to distinguish unobserved heterogeneity from duration dependence in unemployment durations by using aggregate time series on exit rates out of unemployment.

We analyze US unemployment data for the period 1967–1991 distinguishing four groups of workers: white males, white females, black males, black females. We find that unobserved heterogeneity is relevant for all four groups, causing the average exit rate out of unemployment to decline over the duration of unemployment. Furthermore, actual negative duration dependence, i.e. a decline of the exit rate for a given individual, appeared to be the largest for white male workers. The effect for white female workers is smaller, but significant. For black workers we do not find significant negative duration dependence throughout the duration of unemployment. From this we conclude that in the US labor market stigma effects related to unemployment durations are dominant for white workers, but not for black workers. Except for white males, though, the effect of unobserved heterogeneity dominates the duration dependence effect. Finally, there is a significant effect of the season at the moment of inflow into unemployment on the exit rate out of unemployment.

Several topics for future research emerge. First, it seems worthwhile to combine the aggregate data with micro data containing information on explanatory variables x. This might make it possible to estimate the quantities of interest under weaker assumptions.

Another topic for further research would be to improve the foundation of the stochastic specification of the equations of the empirical model. It is plausible that the aggregate observations on numbers of unemployed per duration class contain measurement errors (in particular for the groups of male and female blacks and for high-duration classes). However, it can be shown that incorporating this would lead to a model (*i*) with a complicated error covariance structure and (*ii*) that cannot be estimated with the method of this paper. Consequently, it seems that any analysis based on such a model would have to be parametric.

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Appendix 1. Crossing of lines

We occasionally observe that an exit rate of a certain duration class exceeds the exit rate of the "previous" duration class: $\theta(t|\tau) > \theta(t-1|\tau)$. This is not incompatible to our model. Consider for example $t=1$; then $\theta(1|\tau)>\theta(0|\tau)$ if and only if

$$\eta_1.[1-\gamma_2.\theta(0|\tau-1)]/[1-\theta(0|\tau-1)] > 1 \qquad \text{(A2.1)}$$

which we may rewrite as:

$$\theta(0|\tau-1)].(1-\eta_1.\gamma_2) > 1-\eta_1 \qquad \text{(A2.2)}$$

Now, we may distinguish three cases:

1. $\eta_1.\gamma_2=1$, then there is no solution for $\theta(0|\tau-1)$, (A2.1) holds if $\eta_1>1$
2. $\eta_1.\gamma_2<1$, then $\theta(0|\tau-1) > (1-\eta_1)/(1-\eta_1.\gamma_2)$, (A2.1) never holds.
3. $\eta_1.\gamma_2>1$, then $\theta(0|\tau-1) < (1-\eta_1)/(1-\eta_1.\gamma_2)$, which is only possible if $\eta_1>1$.

In conclusion: (A2.1) only holds if $\eta_1>1$. This result can be extended to show that the model is able to generate crossing time series for exit rates.

Appendix 2. Estimation results of heterogeneity and duration dependence parameters for subperiods[1]

1967.04 – 1979.12

	white male	*white female*	*black male*	*black female*
γ_2	1.124 (0.020)	1.093 (0.021)	1.208 (0.025)	1.177 (0.022)
γ_3	1.388 (0.061)	1.267 (0.069)	1.659 (0.080)	1.514 (0.075)
γ_4	1.862 (0.143)	1.524 (0.177)	2.434 (0.199)	2.039 (0.194)
η_1	0.953 (0.032)	0.899 (0.037)	1.056 (0.060)	1.026 (0.054)
η_2	0.784 (0.039)	0.851 (0.040)	1.076 (0.080)	1.011 (0.081)
η_3	0.913 (0.047)	0.899 (0.060)	0.979 (0.075)	1.134 (0.112)

1980.01 – 1991.12

	white male	*white female*	*black male*	*black female*
γ_2	1.144 (0.044)	1.102 (0.035)	1.243 (0.043)	1.194 (0.033)
γ_3	1.421 (0.163)	1.234 (0.132)	1.688 (0.168)	1.526 (0.121)
γ_4	1.947 (0.450)	1.337 (0.357)	2.331 (0.498)	1.953 (0.336)
η_1	0.970 (0.048)	0.951 (0.049)	1.153 (0.062)	1.127 (0.054)
η_2	0.928 (0.045)	1.048 (0.042)	1.120 (0.067)	1.181 (0.069)
η_3	0.793 (0.043)	0.914 (0.059)	0.973 (0.081)	0.893 (0.085)

[1] In the estimation we allowed for seasonal effects, which are not presented here.

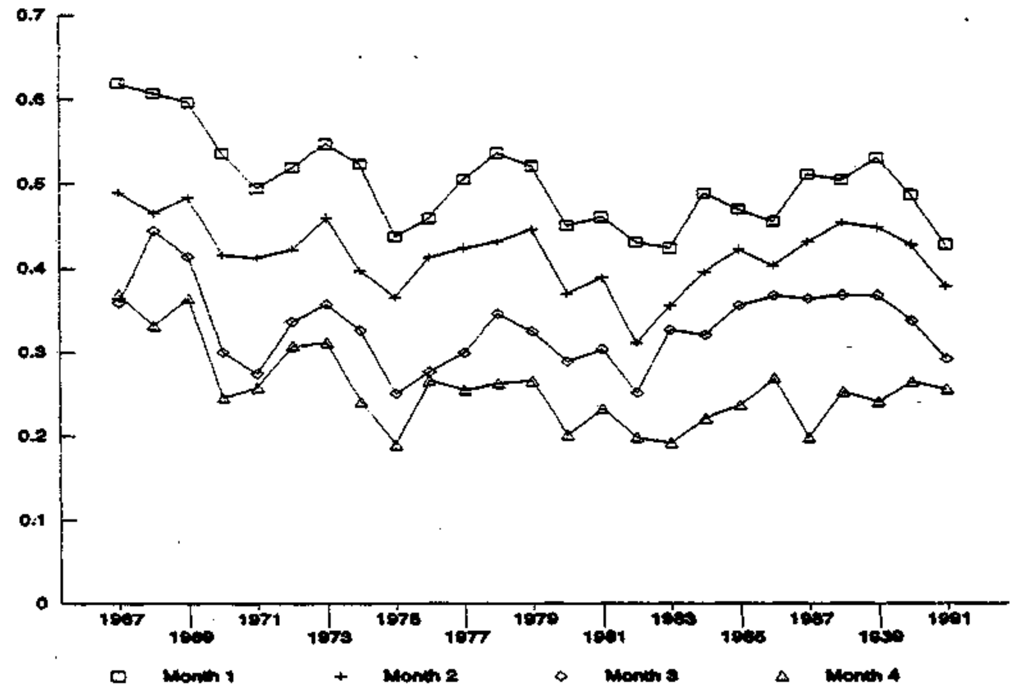
Table 1 Elapsed unemployment durations by duration class: (yearly average in %)

	white male workers			*white female workers*		
	1970	1980	1990	1970	1980	1990
Month 1	45.5	36.1	39.8	52.7	45.3	47.5
Month 2	20.3	19.5	20.0	19.2	20.5	20.7
Month 3	11.3	12.0	11.1	10.2	10.8	10.9
Quarter 2	15.7	19.9	16.8	13.0	15.1	13.7
Quarter 3	3.7	5.7	4.1	2.5	4.3	3.0
Quarter 4+	3.5	6.8	8.1	2.4	4.0	4.1
Total	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**

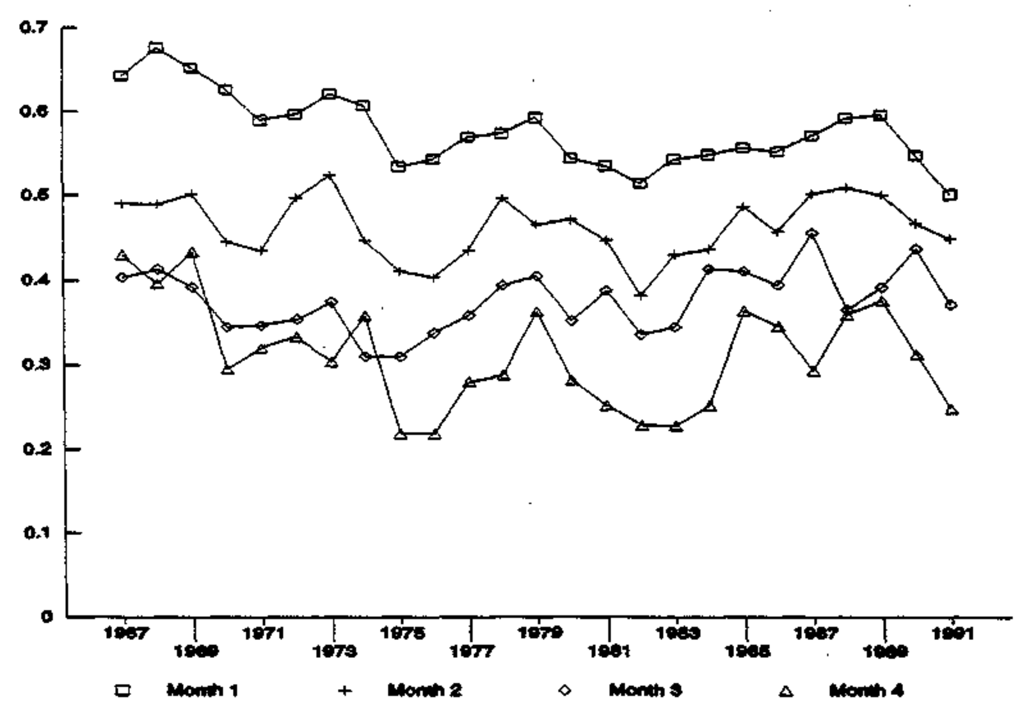
	black male workers			*black female workers*		
	1970	1980	1990	1970	1980	1990
Month 1	43.1	33.2	34.9	50.2	40.5	43.3
Month 2	21.2	18.8	21.2	20.3	20.3	22.0
Month 3	12.4	12.3	12.7	9.8	11.5	11.0
Quarter 2	16.0	19.7	16.4	13.0	16.0	14.1
Quarter 3	3.9	6.2	4.0	3.3	4.9	3.3
Quarter 4+	3.4	9.8	10.8	3.4	6.8	6.3
Total	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**

Figure 1 Monthly exit rates from unemployment

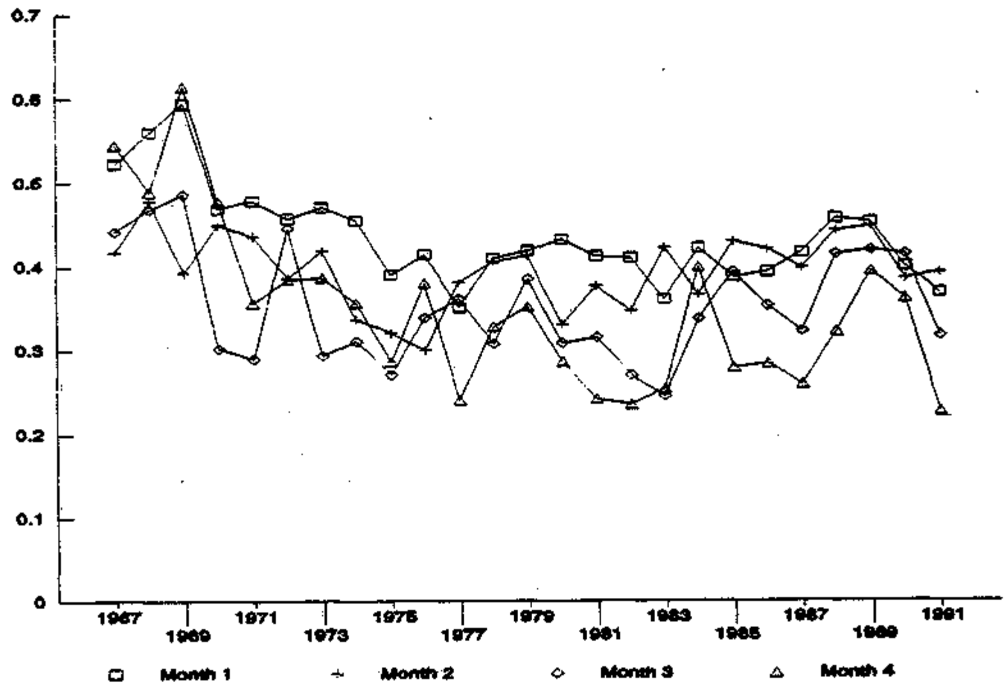
White male workers



White female workers



Black male workers



Black female workers

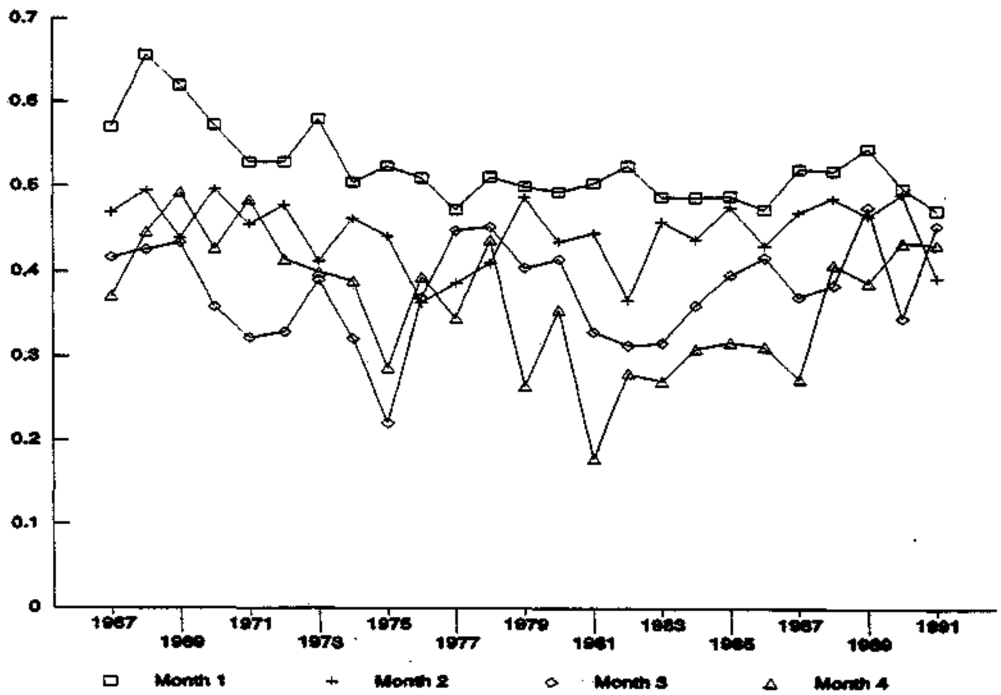


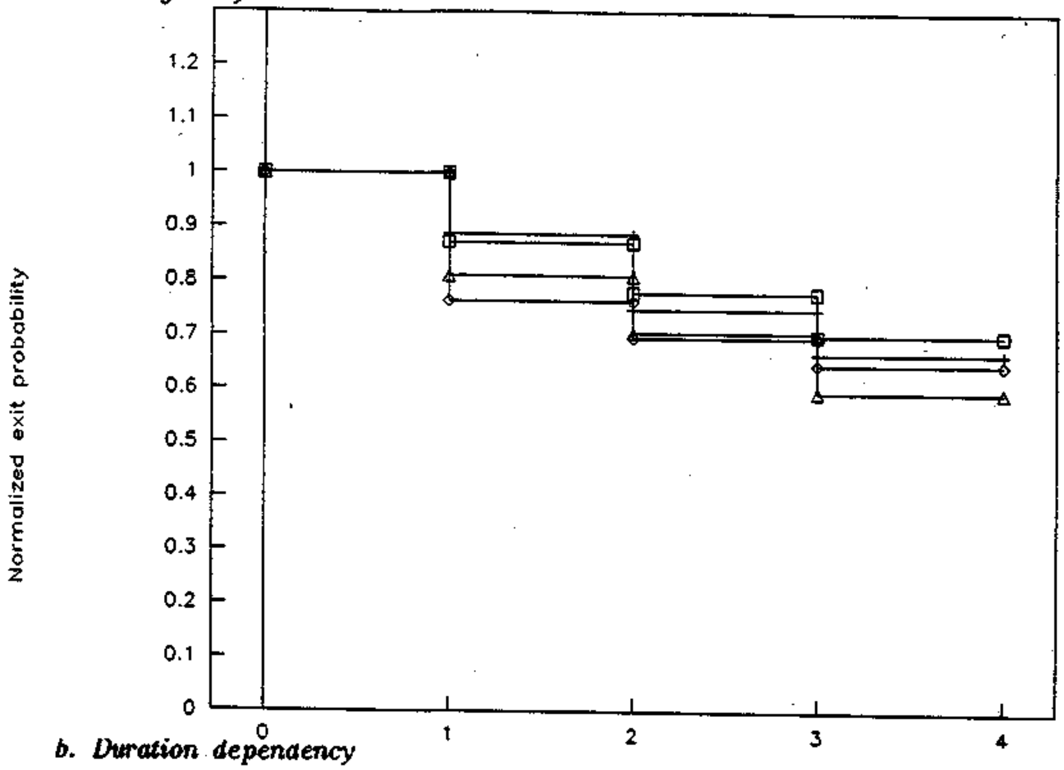
Table 2 Estimation results 1967.04-1991.12

	white male	*white female*	*black male*	*black female*
γ_2	1.128 (0.015)	1.113 (0.014)	1.237 (0.016)	1.190 (0.015)
γ_3	1.385 (0.049)	1.308 (0.048)	1.744 (0.051)	1.540 (0.052)
γ_4	1.810 (0.117)	1.558 (0.121)	2.608 (0.128)	2.060 (0.134)
η_1	0.957 (0.021)	0.951 (0.024)	1.125 (0.039)	1.085 (0.033)
η_2	0.855 (0.026)	0.958 (0.027)	1.062 (0.049)	1.090 (0.053)
η_3	0.803 (0.029)	0.873 (0.040)	0.924 (0.050)	1.009 (0.069)
ω_1	0.906 (0.026)	0.859 (0.021)	0.899 (0.066)	1.071 (0.052)
ω_2	0.865 (0.025)	0.874 (0.021)	0.855 (0.058)	0.821 (0.041)
ω_3	1.065 (0.028)	1.040 (0.024)	0.967 (0.062)	1.036 (0.055)
ω_4	1.065 (0.028)	1.031 (0.024)	1.072 (0.066)	1.009 (0.052)
ω_5	1.052 (0.028)	1.028 (0.024)	1.169 (0.073)	1.095 (0.055)
ω_6	1.027 (0.027)	1.043 (0.024)	0.912 (0.056)	0.945 (0.045)
ω_7	1.074 (0.028)	0.991 (0.023)	1.034 (0.061)	0.946 (0.045)
ω_8	1.074 (0.028)	1.063 (0.024)	1.168 (0.066)	1.134 (0.051)
ω_9	1.058 (0.028)	1.046 (0.024)	1.100 (0.064)	0.979 (0.044)
ω_{10}	0.950 (0.025)	1.029 (0.024)	0.853 (0.056)	0.993 (0.046)
ω_{11}	0.913 (0.025)	0.952 (0.022)	0.925 (0.067)	0.940 (0.043)

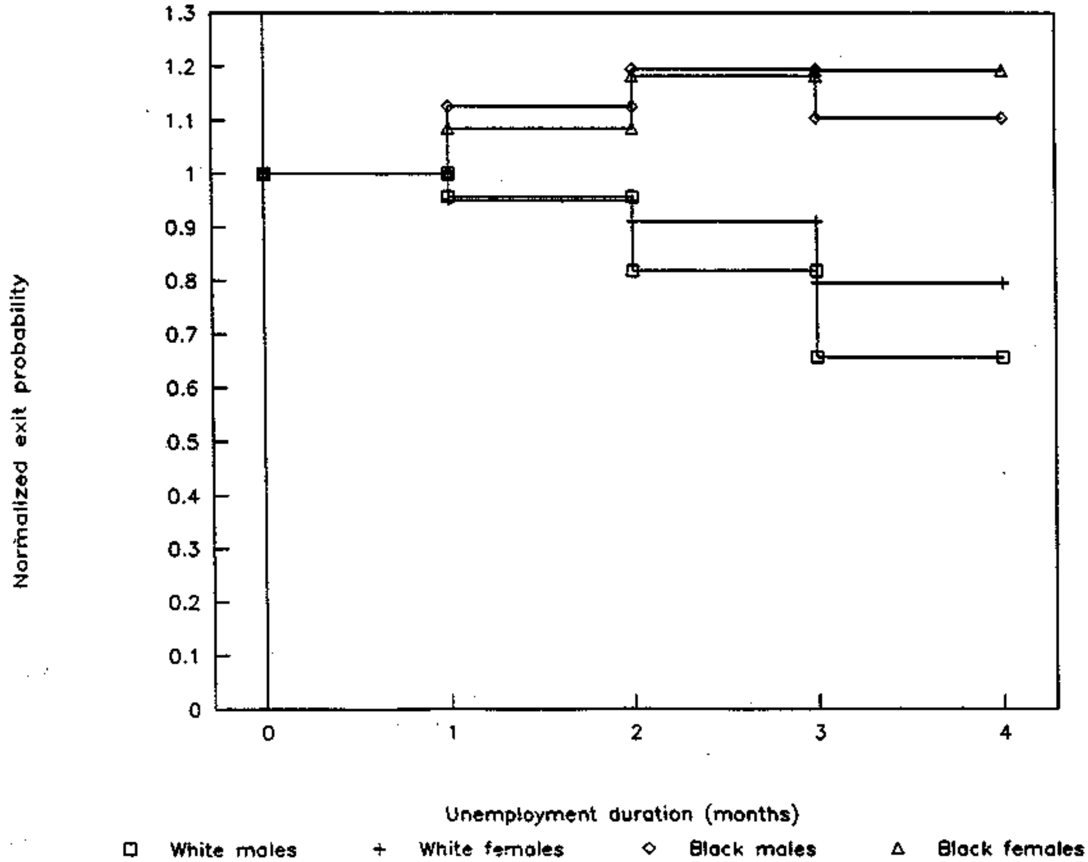
Standard errors between parentheses

Figure 2 Exit rate out of unemployment over the duration of unemployment in a stationary labor market (normalized)

a Heterogeneity



b. Duration dependency



EQ.3.2963M

c. Combined effect