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G. Munda
P. Nijkamp
P. Rietveld

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COMPARISON OF FUZZY SETS: A NEW SEMANTIC DISTANCE

G. Munda[1], P. Nijkamp[2] and P. Rietveld[2]

1) COMMISSION OF THE EUROPEAN COMMUNITIES
Joint Research Centre
Institute for Systems Engineering and Informatics
I-21020 ISPRA (VA), Italy

2) FREE UNIVERSITY
Department of Economics
De Boelelaan 1105
1081 HV Amsterdam
The Netherlands

ABSTRACT

The main features of complex impact systems are imprecision (different levels of measurement of the available information) and uncertainty (stochastic and/or fuzzy). This paper deals with the problem of the presence of such qualitative information. In particular, first the use of concepts of fuzzy set theory in order to represent qualitative information will be defended in the light of their mathematical and psychological properties. Then a new semantic distance and a generalization of the Minkowski p-metric, which is particularly useful for mixed information (crisp, fuzzy and stochastic), are presented. In the appendices, the proof of the fulfilment of the triangle inequality property (a necessary condition for a distance metric) and a numerical algorithm for the semantic distance are given.

Keywords: qualitative information, fuzzy sets, distance functions, physical planning

1. Introduction

In the modeling of a real world problem, the first phase is to structure this problem. This phase depends above all on the available information; in fact, the model must fit reality and not vice versa! Therefore, a flexible model able to take into account the possible different types of information is of a fundamental importance. As known from measurement theory [15], in structuring a problem, given a set A and some information about this set, there is a need to express this information by assigning to each element $a \in A$ a real number m(a). This real number is called the measure of a and the application m:A--->R is called a *scale of measurement*. The main scales of measurement are

- nominal scale
- ordinal scale
- interval scale
- ratio scale.

For simplicity, we will refer to *qualitative information* as information measured on a nominal or ordinal scale, and to *quantitative information* as information measured on an interval or ratio scale.

It has been argued that the presence of qualitative information in evaluation problems concerning socio-economic and physical planning is a rule, rather than an exception. Thus, there is a clear need for methods taking into account qualitative information.

Another problem related to the available information is the one of the uncertainty contained in this information. Ideally the information should be precise, certain, exhaustive and unequivocal. But in reality, it is often necessary to use information which has not those characteristics and therefore to face the uncertainty of a stochastic and/or fuzzy nature present in the data. In fact, if the available information is insufficient or delayed, it is impossible to establish exactly the future state of the problem faced, so that then a *stochastic uncertainty* is created. Another type of uncertainty derives from the ambiguity of this information, since in the majority of the particularly complex problems involving men, much of the information is expressed in linguistic terms, so that it is essential to come to grips with the *fuzziness* that is either intrinsic or informational typical of all natural languages. Therefore, the combination of the different levels of measurement with the different types of uncertainty have to be taken into consideration.

2. Representation of Qualitative Information by means of Fuzzy Sets

Human judgments, especially in linguistic form, appear to be plausible and natural representations of cognitive observations. Numerical verbalization seems to give rather precise statements which may appear imprecise to many observers, while linguistic verbalizations seem to preserve more information for these observers. We can explain this phenomenon by the notion of *cognitive distance*. A linguistic representation of an observation may require a less complicated transformation than a numerical representation, and therefore less distortion may be introduced in the former than in the latter [14]. We could say that the linguistic representation is cognitively closer to the mental description than the numerical representation.

In traditional mathematics, variables are assumed to be precise, but when we are dealing with our daily language, imprecision usually prevails. Intrinsically, daily languages cannot be precisely characterized on either the syntactic or semantic level. Therefore, a word in our daily languages can technically be regarded as a fuzzy subset.

Qualitative information can be represented by means of fuzzy sets in two different ways:

-using linguistic variables,
-using graphical procedures.

Both approaches will concisely be discussed.

2.1. Linguistic Variables

A linguistic variable is a fuzzy variable whose values are fuzzy subsets in a universe of discourse. The base variable of the linguistic variable is a precise variable which takes an individual value in its domain, i.e. the universe of discourse U. The domain of the linguistic variable is the collection of all possible linguistic values, fuzzy subsets defined in the same universe of discourse through the base variable. However, it has been noted [18] that in some cases, the fuzzy set which is assigned to the fuzzy restriction may not have a numerically-valued base variable. Therefore, in the qualitative information available for an evaluation or decision model, two different types of linguistic variables may be present:

(1) the meaning can be translated in a measure on an interval or ratio scale (quantitative base variable), e.g. age, distance, etc.;
(2) there is no meaning on an interval or ratio scale, and therefore

the base variable is also qualitative in nature, e.g. appearance, comfort, beauty, etc.

Type 1. If linguistic variables whose meaning can be translated in a measure on an interval or ratio scale are present in a decision model, generally it is because of a lack of information or of the right instrument of measurement. Therefore, we have a qualitative evaluation of a variable that in theory could be measured on an interval or ratio scale. So it is reasonable to suppose that it is possible to transform the qualitative information into a quantitative one with a certain degree of precision. The parameters, necessary in this case, may be easily established, because this is a case of the so-called "*informational fuzziness*" depending mostly on the subjective culture of the person in charge of the evaluation. For example, the proposition "that man is tall" may have different meanings for different people, but everybody can easily indicate the "tolerance interval" of his own evaluation.

Type 2. In the case of linguistic variables with no meaning on an interval or ratio scale, the qualitative information does not depend on lack of information, but on the nature of the information that is essentially fuzzy (*intrinsic fuzziness*). Therefore, whereas in the other cases the stochastic representation and the fuzzy one may be competitive, in this case the fuzzy representation is the only one possible. For example, if linguistic propositions (like "pretty girl", "beautiful flower" or "quality of life") clearly have no quantitative base variable, how can we represent them? It seems that there is a set of hidden and fuzzy standards in one's mind in a justification for this type of concepts, but they are more than a human being can rationally handle simultaneously [20].

A first approach to this problem may be to try *to decompose the concept* that one wants to represent into a series of quantitative measurable variables [20]. This approach presents two main problems, viz. the explicitation of the quantitative variables, and the aggregation procedure to be used.

A second approach is to define an *artificial quantitative base variable*, assuming that the real space is one-dimensional. The interval of the real space is chosen from [-1, 1], [0, 1], or [0, 10], etc., as desired and can be subdivided into a series of fuzzy sets representing linguistic values (e.g. very negative, moderately positive, very positive, etc.). Then, a link or mapping between quantitative (numerical) and qualitative (linguistic) values is established. This "*direct estimation*" approach has been criticized because of its lack of theoretical foundation. Recently, some psychologists [12, 13, 17] have developed a graded pair comparison procedure, which allows simultaneous testing of the necessary axioms, scaling of the responses in order to obtain memberships and tests of goodness of fit of the scale values. Subsequently, empirical experiments have demonstrated a high level of similarity between membership values determined through graded pair comparison and direct magnitude estimation! Thus it seems that a theoretical justification for the quantification of the vague meanings of inexact linguistic terms by means of direct estimation can be established.

A third interesting approach can be the notion of *type 2 fuzzy set*. A type 2 fuzzy set is a fuzzy set whose membership values are fuzzy sets on [0, 1]. This corresponds to the case that the decision-maker is not able (or not willing) to characterize the grade of membership by an exact number, but gives an evaluation such as "the grade of membership is high, medium", etc. It is always possible to define a fuzzy set of type n= 2, 3,....... if its membership function is a mapping from U to a set of fuzzy subsets of type n-1; therefore, it is possible that in order to reduce the fuzziness, many transformations are requested, thus diminishing drastically the computational efficiency of the algorithm.

Another possible approach has been developed above all in the field of psychological research [8], called the "*yes-no paradigm*". In this approach, an element x of the universe of discourse U, is presented to the subject and he has to decide whether the element is a member of A, A being a fuzzy subset of U. The fraction of positive responses across replications (within or across subjects) is considered a measure of $\mu_A(x)$. It has been noted that the main problem of this approach is that it confounds fuzziness with response variability and that it can be interpreted as an indication that words have various, but nevertheless precise meanings to different people and/or different times.

2.2. Interactive Graphical Procedures Approach

Hesketh and others [9] have noted that "by adapting traditional psychological measurement to deal with fuzzy concepts, new possibilities are open to both fields of enquiry". These authors have proposed a computerized graphic rating scale using linear membership functions and combining some rules of fuzzy set theory with others that are typical of Bayesian statistical theory in order to obtain as a compound index of the membership function the expected value of the re-scaled distributions. A traditional psychological procedure initially developed for the measurement of attitudes is the so-called "*semantic differential*". This approach requires an individual to describe a concept in terms of where it

falls between bi-polar objective descriptions. Thurstone [16], facing the problem of the meaning of attitudes, proposed a graphical representation of individual differences. Furthermore, he suggested that the range of opinions which a particular person is willing to endorse could also be represented graphically. "Using graphic representation, Thurstone demonstrated that an individual's opinion could be characterized in terms of three different measures, the range of opinions the individual is willing to endorse, their mean position on the scale, and the one opinion selected which best represented an attitude [9 p.429]".
Hesketh and others [9] in the fuzzy graphic rating scale, consider as a representation of a rater's perception, a point indicated by means of a mouse in a bi-polar scale and then the extension of the rating to the left or right (these extensions represent uncertainty inherent in this estimate). The main assumptions of this procedure are:

(1) the membership function takes the value 1 at the first point indicated by means of the mouse;
(2) the membership functions are linear;
(3) the union of two or more fuzzy ratings has a convex membership function;
(4) each fuzzy variable can be represented by means of its expected value, to be computed by rescaling its membership function.

In the light of these observations, different heuristic graphical procedures can be proposed in order to represent qualitative information as fuzzy sets.

How to compare fuzzy sets in decision models, is so far an unresolved problem. In the following section, a new semantic distance and a generalization of the Minkowski p-metric which is particularly useful for mixed information (crisp, fuzzy and stochastic) are presented.

3. A New Semantic Distance

In general, a *semantic distance* S_d between two fuzzy sets, A and B, mirrors a possibility degree of equality between two fuzzy sets or a similarity degree between them. The larger the distance the smaller the possibility degree of equality. The most common distance is the so-called Hamming distance.
For the discrete case it is:

$$S_d(A, B) = \sum_{i=1}^{n} | \mu_A(x_i) - \mu_B(x_i) | \qquad (1)$$

and for the continuous case:

$$S_d(A, B) = \int_a^b \mid \mu_A(x) - \mu_B(x) \mid dx \qquad (2)$$

For the continuous case, another possible approach is the computation of some features of the membership distributions of the fuzzy sets, after which the similarity can be evaluated by means of traditional distances such as the Euclidean distance, the Bhattacharya distance, the Mahalanobis distance and so on [4, 6]. Of course, in this case two problems have to be faced, viz. the correct selection of features and the correct selection of the distance function.

Now we will illustrate a new distance metric that is useful in case of continuous membership functions allowing also a definite integration. In order to compute such a distance, it is necessary that the area bounded by the membership function must be equal to 1. Generally, it is possible to change membership functions proportionally by multiplying them by a constant $c \in R^+$, with $c \leq 1$ for normal fuzzy sets and $c \leq 1/m_A$ for subnormal fuzzy sets ($m_A = \max_{x \in X} \mu_A(x)$) [7].
If $\mu_{A1}(x)$ and $\mu_{A2}(x)$ are two membership functions, we can write

$$f(x) = c_1 \mu_{A1}(x) \quad \text{and} \qquad (3)$$

$$g(y) = c_2 \mu_{A2}(x) \qquad (4)$$

where f(x) and g(y) are two functions obtained by rescaling the ordinates of $\mu_{A1}(x)$ and $\mu_{A2}(x)$ through c_1 and c_2, such that

$$\int_x f(x)\, dx = \int_y g(y)\, dy = 1 \qquad (5)$$

The distance between all points of the membership functions is computed as follows:

$$\text{if } f(x) : X = [x_L, x_U] \quad \text{and} \quad g(y) : Y = [x_{L'}, x_{U'}] \qquad (6)$$

(where of course sets X and Y can be non-bounded from one or either sides), then

$$S_d(f(x), g(y)) = \int_x \int_y |x-y| f(x) g(y) \, dy \, dx \qquad (7)$$

It is easy to show that this distance satisfies the properties of non-negativity and symmetry; the proof of the triangle inequality will be given in the Appendix.

As a special case, we consider first the case where the intersection of two membership functions is empty.

If $x>y$ $\forall x \in X$ and $\forall y \in Y$, it follows that a continuous function in two variables is defined over a rectangle. Therefore, the double integral can be calculated as iterated single integrals:

$$\int_x \int_y |x-y| f(x) g(y) \, dy \, dx = \qquad (8)$$

$$= \int_x \int_y (x-y) f(x) g(y) \, dy \, dx = \qquad (9)$$

$$= \int_x \int_y [x f(x) g(y) - y f(x) g(y)] \, dy \, dx = \qquad (10)$$

$$= \int_x x f(x) \, dx - \int_x f(x) E(y) \, dx = \qquad (11)$$

$$= E(x) - E(y) = \qquad (12)$$

$$= |E(x) - E(y)| \qquad (13)$$

where $E(x)$ and $E(y)$ are the expected values of the two membership functions; the latter result is true, since $x>y$.

Therefore, when the intersection is empty, their distance is equal to the distance between their expected values. When the intersection between two fuzzy sets is not empty (see Figure 1), their distance, is larger than the difference between the expected values since $|x-y|$ is always larger than $(x-y)$.

In the Appendix, a Monte Carlo type numerical procedure for the computation of such a distance is shown.

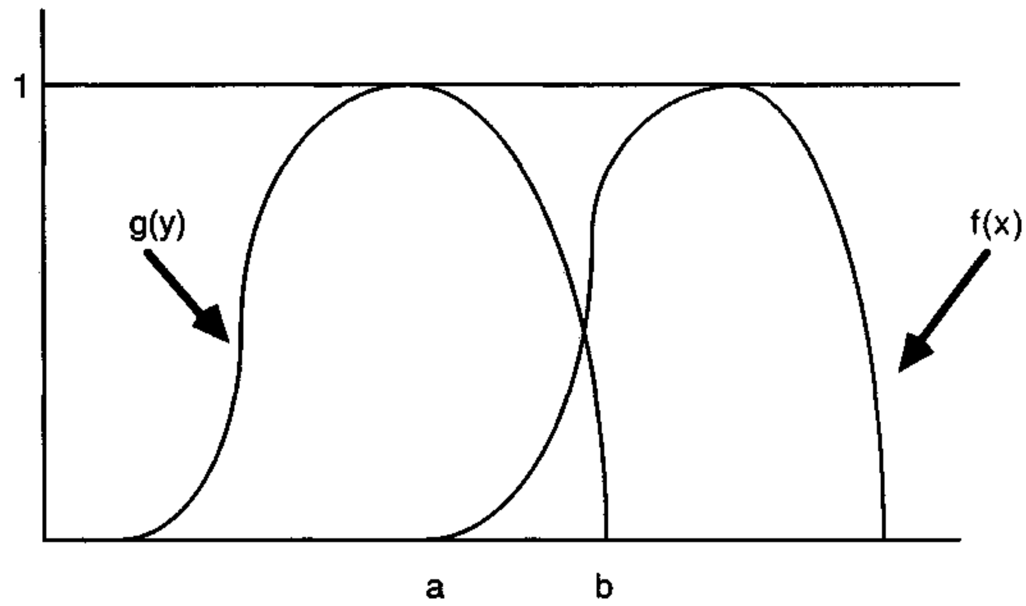

Figure 1. Intersection between two fuzzy sets

In order to illustrate the results of this distance function, the following numerical application can be useful. Let us take into consideration the following two fuzzy sets:

$$\mu_A(x)=(1+((x-25)/5)^2)^{-1} \qquad x\in[25, 100] \qquad (14)$$

and

$$\mu_B(x)=\begin{cases}0 & \text{if } x\in[0, 50]\\ (1+((x-50)/5)^{-2})^{-1} & \text{if } x\in(50, 100]\end{cases} \qquad (15)$$

In general, the expected value of a fuzzy set is equal to:

$$E(\mu_A(x))=\frac{\int_a^b x\mu_A(x)dx}{\int_a^b \mu_A(x)dx} \qquad (16)$$

where a and b are the lower and upper limits of the range of the variable x.

Computing the expected values of the two fuzzy sets taken into consideration here, the following results are obtained:

$$E(\mu_A(x))=255.785/7.521=34.009 \quad (17)$$

$$E(\mu_B(x))=3325/42.644=77.971 \quad (18)$$

Therefore by means of their expected values the distance between the two fuzzy sets is

$$E(\mu_B(x)) - E(\mu_A(x))=43.962 \quad (19)$$

Taking into consideration the semantic distance proposed here (computed by means of the numerical algorithm shown in the appendix with 1000 iterations), the following result is obtained:

$$S_d(\mu_A(x), \mu_B(x))= 49.642 \quad (20)$$

As one can see, since the intersection between the two fuzzy sets is not empty; the result obtained by our distance function is different from the simple difference between their expected values.

From a theoretical point of view, the following conclusions can be drawn:

1) the absolute value metric is a particular case of this type of distance;
2) the expected value is obtained as a representation of fuzzy sets only when their intersection is empty;
3) when the intersection between two fuzzy sets is not empty, their distance is greater than the difference between their expected values. It has to be noted that this is the case for fuzzy ordinal information; thus by means of our distance function, the two different cases can be distinguished;
4) in case of fuzzy information represented by L-R fuzzy numbers, when their intersection is empty, their distance is equal to the crisp numbers they represent only when they are symmetric; otherwise, their expected values are obtained;
5) by means of this semantic distance, the problem of the use of only one side of the membership functions, common to most of the traditional fuzzy multicriteria methods, is overcome.

4. A Generalization of the Minkowski p-metric

It is interesting to note that also the *stochastic information* represented by means of density functions can be taken into account by means of this distance function. Of course in this case the condition

$$\int_x f(x)\,dx = 1 \qquad (21)$$

is always true.

Generally, for crisp cardinal evaluations, the Minkowski p-metric is considered: given any two points $x,y \in R^N$, their distance is given by [11]:

$$||x-y|| = [\sum_{n=1}^{N} |x_n-y_n|^p]^{1/p} \qquad (22)$$

$p \in \{1, 2, \ldots\} \cup \{\infty\}$

It is then clear that we have:

for p=1 an absolute value metric (completely compensatory);
for p=2 a Euclidean metric (partially compensatory);
for $p \rightarrow \infty$ the Tchebycheff metric (completely non compensatory).

For the problem of mixed information, we propose the following generalization of the Minkowski metric: given R crisp cardinal outcomes and I ordinal, stochastic and fuzzy outcomes, the distance between two elements x and y is:

$$||x-y|| = [\sum_{r=1}^{R} |x_r-y_r|^p + \sum_{i=1}^{I} (\int_x \int_y |x-y| f_i(x)\, g_i(y)\, dy\, dx)^p]^{1/p} \qquad (23)$$

In our opinion, this distance has the great advantage of dealing simultaneously with different kinds of information, so it can be a very appropriate tool in order to increase the equivalence of the procedures used for the different types of available information.

5. Conclusions

The presence of qualitative information in impact and evaluation problems, for instance, in socio-economic and physical planning is a rule rather than an exception. In particular, the combination of the different levels of measurement with the different types of uncertainty have to be taken into consideration. Thus there is a clear need for methods taking into account qualitative information. In this paper, the use of some concepts of fuzzy set theory in order to represent qualitative information has been justified in the light of their mathematical and psychological properties; a new semantic distance for the comparison of fuzzy sets and a generalization of the Minkowski p-metric, particularly useful for mixed information (crisp, fuzzy and stochastic), have been presented. This distance has the great advantage of dealing simultaneously with different kinds of information, so that it can be a very appropriate tool in order to reduce the problem of the equivalence of the treatment of different types of information in qualitative decision models.

APPENDIX 1

Proof of the Property of Triangle Inequality

Let us assume 3 functions:
$f(x) : X \rightarrow R^+$, $g(y) : Y \rightarrow R^+$ and $h(z) : Z \rightarrow R^+$.
For the sake of generality, we assume that $X \cap Y \cap Z \neq \varnothing$.

We first prove that $\forall\, x \in X$, $\forall\, y \in Y$ and $\forall\, z \in Z$, the relationship

$|x-y| + |y-z| \geq |x-z|$ is always true.

The total number of possible cases is 3!

$x \geq y \geq z \rightarrow \quad (x-y) + (y-z) - (x-z)=0$

$x \geq z \geq y \rightarrow \quad (x-y) + (-y+z) - (x-z)= 2(z-y)\geq 0$

$y \geq x \geq z \rightarrow \quad (-x+y) + (y-z) - (x-z)= 2(y-x)\geq 0$

$y \geq z \geq x \rightarrow \quad (-x+y) + (y-z) - (-x+z)= 2(y-z)\geq 0$

$z \geq x \geq y \rightarrow \quad (x-y) + (-y+z) - (-x+z)= 2(x-y)\geq 0$

$z \geq y \geq x \rightarrow \quad (-x+y) + (-y+z) - (-x+z)=0$

therefore $|x-y| + |y-z| - |x-z| \geq 0 \;\; \forall\, x \in X, \forall\, y \in Y$ and $\forall\, z \in Z$.

Since $f(x)\geq 0$, $g(y)\geq 0$ and $h(z)\geq 0$ it follows:

$$\int\int\int[\,|x-y| + |y-z| - |x-z|\,]\, f(x)\, g(y)\, h(z)\, dz\, dy\, dx \;\geq 0$$

This integral can be decomposed as follows:

$$\int\int\int |x-y|\; f(x)\, g(y)\, h(z)\, dz\, dy\, dx \;\; +$$

$$\int\int\int |y-z|\; f(x)\, g(y)\, h(z)\, dz\, dy\, dx \;\; -$$

$$\int\int\int |x-z|]\; f(x)\, g(y)\, h(z)\, dz\, dy\, dx$$

since these triple integrals can be computed by means of iterated integrals and since, because of equation (5)

$$\int f(x)\,dx = \int g(y)\,dy = \int h(z)\,dz = 1,$$

it follows that the above sum of integrals is equal to:

$$\int\int |x-y|\; f(x)\, g(y)\, dy\, dx \;+$$

$$\int\int |y-z|\; g(y)\, h(z)\, dz\, dy \;-$$

$$\int\int |x-z|]\; f(x)\, h(z)\, dz\, dx$$

Therefore, we find the result:

$$d[f(x),\; g(y)] + d[g(y),\; h(z)] - d[f(x),\; h(z)] \ge 0$$

or:

$$d[f(x),\; g(y)] + d[g(y),\; h(z)] \ge d[f(x),\; h(z)] \qquad \text{Q.E.D.}$$

APPENDIX 2

A Numerical Algorithm for the Computation of the Semantic Distance

In this Appendix, in order to compute the semantic distance proposed here, a Monte Carlo type of numerical algorithm will be proposed.

Assumptions:

1) $f(x) : X = [x_L, x_U] \longrightarrow M$

$g(y) : Y = [x_{L'}, x_{U'}] \longrightarrow M$

where M is the membership space.

2) all $x \in X$ and all $y \in Y$ can be obtained by means of a random generator that supplies numbers $r \in [0, 1]$ and then

$x = rx_L + (1-r)x_U$ and

$y = rx_{L'} + (1-r)x_{U'}$

3) The probability to obtain a point P inside e.g. f(x), whose value on the x-axis is x_0, depends on the shape of the function; therefore, an auxiliary variable Z whose values $z \in [0, \max f(x)]$ are again obtained by a random generator is taken into consideration.

Procedure:

STEP 1 -draw a random number r_0;

STEP 2 -$x_0 = r_0 x_L + (1-r_0)x_U$;

STEP 3 -draw a random number z_0;

STEP 4 -if: $z_0 \leq f(x_0)$ then go to next step,

$z_0 > f(x_0)$ then return to step 1;

STEP 5 -draw a random number r_1;

STEP 6 -$y_1 = r_1 x_{L'} + (1-r_1)x_{U'}$;

STEP 7 -draw a random number z_1;

STEP 8 -if: $z_1 \leq g(y_1)$ then compute $|x_0-y_1|$;

$z_1 > g(y_1)$ then return to step 5;

This procedure must be repeated many times. If n values of $|x_0-y_1|$ with i= 1, 2,..........., n are obtained, then

$$\int_x \int_y |x-y| f(x)\, g(y)\, dy\, dx \cong \frac{\sum_{i=1}^{n} |x_i-y_i|}{n}$$

Therefore, the distance between two fuzzy sets is approximately equal to the arithmetic mean of all the points bounded by their respective membership functions obtained by drawing random numbers.

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