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**QUALITATIVE MULTICRITERIA METHODS FOR  
FUZZY EVALUATION PROBLEMS**

**An illustration of economic-ecological evaluation**

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## ABSTRACT

It is nowadays increasingly realized that environmental and resource problems generally have complicated economic and ecological dimensions. Therefore, there is a clear need for models offering a comprehensible and operational representation of a real-world environmental system. A great variety of quantitative (descriptive and forecasting) models has been developed for compound environmental systems, but there is still a need in a planning context for evaluation methods taking into account information of a "mixed" (qualitative and quantitative) type.

This paper presents a new discrete multicriteria decision aid (MCDA) model whose impact (or evaluation) matrix may include either crisp, stochastic or fuzzy measurements of the performance of an alternative  $a_n$  with respect to a criterion  $g_m$ . No traditional weighting of criteria is used in this method.

From an empirical point of view, this model is particularly suitable for economic-ecological modelling incorporating various degrees of precision of the variables measured. From a methodological point of view, two main issues will be faced here:

- the problem of equivalence of the used procedures in order to standardize the various evaluations (of a mixed type) of the performance of alternatives according to different criteria;
- the problem of comparison of fuzzy numbers typical of all fuzzy multicriteria methods.

An illustrative numerical example will be presented at the end of the paper.

**Keywords: fuzzy sets, multicriteria methods, mixed information, sustainable development**



## 1. Introduction

Environmental management is essentially conflict analysis characterized by socio-economic, environmental and political value judgements. Therefore, in planning for ecological "sustainability" it is very difficult to arrive at straightforward and unambiguous solutions [5]. This implies that such a multi-related planning process will always be characterized by the search for acceptable compromise solutions, an activity which requires an adequate evaluation methodology. Multiple criteria evaluation techniques aim at providing such a set of tools [1, 33].

From an analytical point of view, a central characteristic of sustainable development is *economic-ecological integration*.

Nowadays, it is increasingly realized that environmental and resource decisions generally have economic and ecological consequences. This implies that such problems are characterized inter alia by social, psychological, physicochemical and geological aspects. Models aiming at structuring these cross-boundary problems of an economic and environmental nature are usually called "*economic-environmental*" or "*economic-ecological models*". Since the complexity of this type of problems is high, there is a clear need for models offering a comprehensible and operational representation of a real-world environmental system. The strong quantitative tradition in economics has enabled researchers to include environmental elements - measured in a cardinal metric - fairly easily in conventional models focusing on the interface of economics and the environment. Nevertheless, in integrating economic and environmental models, some difficult methodological problems have to be faced, such as differences in time scales (in contrast to ecology, economics is mainly analyzing short-term and medium-term effects), differences in spatial scales (the spatial scale of many ecological variables is sometimes fairly low, whereas the scale of many economic variables is usually rather high) and *differences in measurement levels of relevant variables* [9, 13].

However, qualitative aspects are harder to deal with in traditional models and therefore there is a clear need for methods that are able to take into account information of a "mixed" type (both qualitative and quantitative measurements). For the sake of simplicity, we will refer here to *qualitative information* as information measured on a nominal or ordinal scale, and to *quantitative information* as information measured on an interval or ratio scale.

In multicriteria evaluation theory, a clear distinction is made between quantitative and qualitative methods. Essentially, there are two approaches for dealing with qualitative information: a direct and an indirect one [18]. In the *direct approach*, qualitative information is used in the evaluation method without a transformation into quantitative units; in the *indirect approach*, qualitative information is first transformed into cardinal information, while later on one of the existing quantitative multicriteria methods is used. Such a cardinalization is especially attractive in the case of available information of a "mixed type". In this case, the application of a direct method would imply that only the qualitative part of the whole available (quantitative and qualitative) information is used, which would give rise to an inefficient use of information. In the indirect approach, this loss of information is avoided; the question is of course, whether there is an adequate basis for the application of a certain cardinalization scheme.

Another problem related to the available information is the uncertainty contained in this information. Ideally, the information should be precise, certain, exhaustive and unequivocal. But in reality, it is often necessary to use information which does not have these characteristics and therefore to face uncertainty of a stochastic and/or fuzzy nature. In fact, if the available information is insufficient or delayed, it is impossible to establish exactly the future state of the problem faced, so that then a case of *stochastic uncertainty* is emerging.

*Fuzzy uncertainty* does not concern the occurrence of an event, but the event itself, in the sense that it cannot be described unambiguously. This situation is very common in human systems. Spatial systems in particular, are complex systems characterized by subjectivity, incompleteness and imprecision [15].

Therefore, the combination of different levels of measurement with different types of uncertainty has to be considered as an important research issue in multicriteria evaluation.

The model developed in the present paper, is a discrete multicriteria method whose impact (or evaluation) matrix may include either crisp, stochastic or fuzzy measurements of the performance of an alternative  $a_n$  with respect to a judgement criterion  $g_m$ . No traditional weighting of criteria is used in this method. Throughout this paper the assumption that a higher value of a criterion is preferred to a lower one (the higher, the better) is made.

From an empirical point of view, this model is particularly suitable for economic-ecological modelling incorporating various degrees of precision of

the variables taken into consideration. From a methodological point of view, two main issues will be faced here:

- the problem of equivalence of the used procedures in order to standardize the various evaluations (of a mixed type) of the performance of alternatives according to different criteria (e.g. the EVAMIX method [18]);
- the problem of comparison of fuzzy numbers typical of all fuzzy multicriteria methods.

## 2. Definition of a Fuzzy Region of Satisfactory Alternatives

Given a "consistent family" of mixed evaluation criteria  $G=\{g_m\}$ ,  $m=1,2,\dots,M$ , and a finite set  $A=\{a_n\}$ ,  $n=1, 2,\dots, N$  of potential alternatives (actions)<sup>1</sup>, a set of satisfactory alternatives can be obtained by defining a fuzzy interval of feasible and acceptable values for each criterion.

From an operational point of view, in public decision making a single point-value solution (e.g. weights) tends to lead to deadlocks in the evolution of the decision process because it imposes too rigid conditions for a compromise. When a higher degree of flexibility is to be achieved, the use of a fuzzy region of satisfactory solutions could in principle offer more room for mutual consensus. A natural and flexible way of defining such a region is by means of linguistic propositions.

In traditional mathematics, variables are assumed to be precise, but when we are dealing with our daily language, imprecision usually prevails. Intrinsically, daily languages cannot be precisely characterized at either a syntactic or semantic level. Therefore, a word in our daily languages can technically be regarded as a fuzzy set. In order to allow a formal analysis, a mathematical translation of such linguistic propositions is needed. This can be done by means of possibility theory [11, 15].

If  $R(x)$  is a fuzzy restriction (a fuzzy restriction is a fuzzy relation which acts as a flexible constraint on the values that may be assigned to a variable), then the effect of  $F$  (a linguistic value) on  $X$  (base variable) can be expressed as

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<sup>1</sup> In the terminology introduced by Vansnick [30], the decision model considered can be defined "*model A.A.E.*" (Alternatives, Attributes, Evaluators), where qualitative attributes are considered. In particular, in the present paper, the evaluations associated with each alternative can be real numbers, random variables with continuous and integrable density functions or fuzzy numbers (also with continuous and integrable membership functions).

$$R(x) = F \quad (1)$$

Where  $X$  is a variable in  $U$  (Universe of discourse),  $F$  is a fuzzy set in  $U$  and  $R(x)$  is a fuzzy restriction imposed by  $F$ . Therefore, this fuzzy restriction may be expressed in a linguistic proposition:

$$P : X \text{ is } F \quad (2)$$

which generates a possibility distribution

$$\pi_X = F \quad (3)$$

Associated with the possibility distribution there is a possibility distribution function such that

$$\text{Poss}(X=a) = \pi_X(a) = \mu_F(a) \quad (4)$$

Therefore, we can conclude that fuzzy restrictions, possibility distributions and fuzzy subsets are closely related. The set of linguistic propositions can be transformed into a set of possibility distribution functions. Such distributions in turn, impose a fuzzy restriction on the values that each single criterion  $g_m$  ( $m=1, 2, \dots, M$ ) may assume. Then a compatibility (membership) degree  $\mu_{F_m}(a_n)$  of each action to the linguistic restriction imposed on each criterion can be obtained. Such a computation is easy by means of standard fuzzy set rules for crisp evaluations. However, in the case of fuzzy or stochastic evaluations, the use of only intersections may lead to some inconsistencies. An example is given in Figure 1. Clearly, the evaluation does not satisfy the restriction completely, although the compatibility (membership) degree assumes the value 1.

In order to identify an index that can express the membership degree appropriately, the following properties may be imposed:

- (1) it should fall between 0 and 1;
- (2) it should assume the value 0 if the intersection is empty;
- (3) it should assume the value 1 if the evaluation completely satisfies the restriction indicated by the decision-maker.

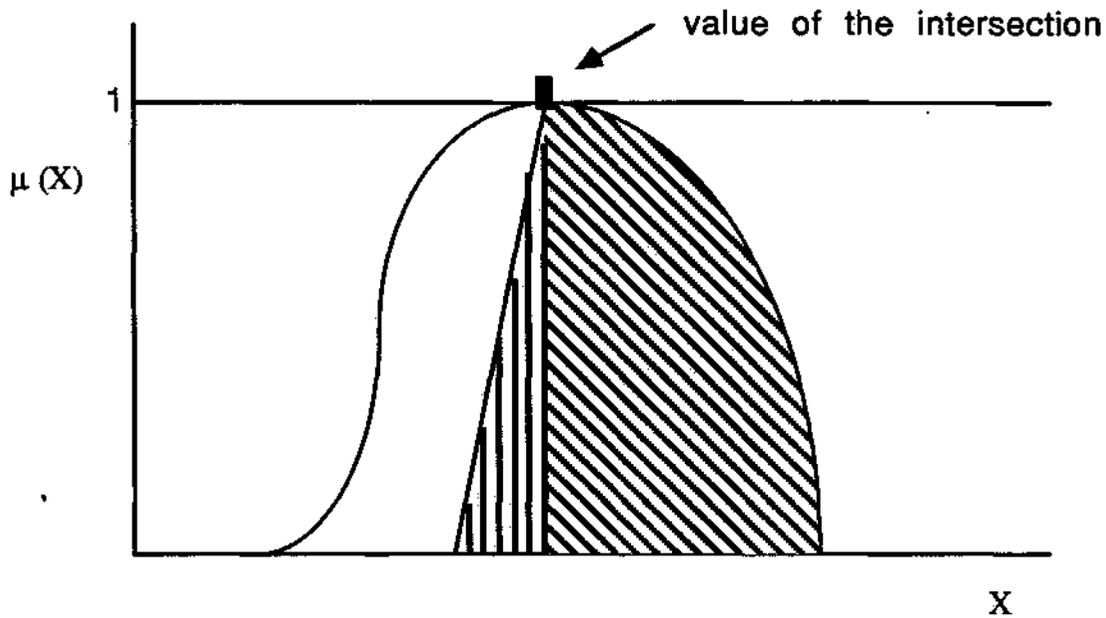


Figure 1. Example of a possible inconsistency

Therefore, we propose the following index: the membership degree  $\mu_{F_m}(a_n)$  of each action to the linguistic restriction imposed on each criterion is computed as the ratio between the area of the intersection and the area bounded by the function representing the fuzzy number (or a density function). Note that this index can be interpreted as the "degree of coincidence"<sup>2</sup> of a fuzzy restriction with respect to a fuzzy (or stochastic) evaluation.

Given the family of  $M$  linguistic restrictions  $F_1, F_2, \dots, F_M$ , the region of satisfactory solutions ( $\Delta$ ) is given by

$$\Delta = \mu_{F_1 \wedge F_2 \wedge \dots \wedge F_M}(a_n) \quad (5)$$

On the base of such information, the analyst can provide a first ranking of alternatives ranging from the most compatible to the least compatible one.

<sup>2</sup> The *degree of coincidence* of a fuzzy set  $A$  with respect to a fuzzy set  $B$  ( $w(A,B)$ ) is defined by Li and Liu [16] as follows:

$$w(A, B) = \frac{\int_U \min \{ \mu_A(x), \mu_B(x) \} dx}{\int_U \mu_B(x) dx}$$

where  $U$  is the Universe of discourse in which the two fuzzy sets are defined.

Possibly even an "optimal" solution  $a^*$  can be isolated by means of the simple rule

$$a^* = \max \mu_{\Delta}(a_n) \quad (6)$$

Of course, " $\wedge$ " may be replaced by any other operator (e.g., a t-norm). However, in the implementation of our procedure, in order to aggregate the various membership degrees  $\mu_{F_m}(a_n)$  and to find the aggregate membership degree  $\mu_{\Delta}(a_n)$  of the alternative  $a_n$  to the fuzzy region  $\Delta$ , the following operators are used:

- (1) the sum operator (completely compensatory),
- (2) the product operator (completely non compensatory),
- (3) the Zimmermann-Zysno  $\gamma$ -operator (partially compensatory):

$$\left( \prod_{m=1}^M \mu_m \right)^{(1-\gamma)} \left( 1 - \prod_{m=1}^M (1 - \mu_m) \right)^{\gamma} \quad (7)$$

As shown by the authors, this is a convex combination of the product operator and the algebraic sum, which are respectively known as the algebraic representation of the intersection and the union. In this operator,  $\mu_m$  is the normalized degree of membership and  $\gamma$  is a parameter indicating the degree of compensation [36, 37].

When an alternative  $a_n$  has one or more  $\mu_{F_m}(a_n)=0$ ,  $\mu_{\Delta}(a_n)$  will be equal to zero, if one uses the product operator or the  $\gamma$ -operator (and hence the action will be eliminated from the set  $\Delta$ ). On the other hand, if one uses the sum operator,  $\mu_{\Delta}(a_n)$  will be greater than zero and then the action will belong to the set  $\Delta$ . Note that this aggregation is of a "complete" type [23,24] since these operators can be considered as utility functions.

Since in the above aggregation procedure, some analytical information is lost, an index of the "diversity" of the single  $\mu_{F_m}(a_n)$  can be useful. A measure of this "incertitude" of evaluation is provided by the entropy concept [10, 29]. The entropy of a fuzzy set  $A$  on a Universe  $X$ , with membership function  $\mu_A(x)$ , is given by

$$\frac{1}{N} \sum_{x \in X} \ln(x) \quad (8)$$

where  $N$  is the number of elements in  $X$  ( $X$  being finite) and  $\ln(x)$  is the incertitude of the evaluation along scale  $x$  given by

$$\ln(x) = -[\mu_A(x)\log_2\mu_A(x) + (1-\mu_A(x))\log_2(1-\mu_A(x))] \quad (9)$$

The entropy of a set  $A$  is 1 if for every  $x$ ,  $\mu_A(x)=0.5$  and is 0 if for every  $x$ ,  $\mu_A(x)=1$  or 0.

Thus it is possible to construct an impact matrix which provides four different kinds of information:

- (1) the evaluation (crisp, fuzzy or stochastic) of the performance of each alternative  $a_n$  according to each criterion  $g_m$ ;
- (2) the membership degree  $\mu_{F_m}(a_n)$  of each alternative  $a_n$  to the linguistic restriction imposed on each criterion  $g_m$ ;
- (3) the aggregate membership degree  $\mu_{\Delta}(a_n)$  of alternative  $a_n$  to the set  $\Delta$  of feasible and satisfactory actions.
- (4) the "degree of incertitude" inherent in such  $\mu_{\Delta}(a_n)$ .

This information may be used to eliminate possible actions which do not satisfy the decision maker. Other actions may also be eliminated, if the decision maker defines a minimum level of satisfaction  $\alpha$ ,  $0 < \alpha < 1$ , such as:

$$\begin{aligned} \mu_{\Delta}(a_n) < \alpha &\Rightarrow a_n \notin \Delta \\ \mu_{\Delta}(a_n) \geq \alpha &\Rightarrow a_n \in \Delta. \end{aligned}$$

### 3. Comparison of Fuzzy Sets

How to compare fuzzy sets is a key issue for decision models in a fuzzy environment. In the past years various attempts to develop fuzzy multicriteria methods have been undertaken. A survey can be found in [37]; recent approaches to the problem of ranking fuzzy numbers can be found in [14, 27]. In general, fuzzy approaches to multicriteria evaluation present the following limitations:

- most of them are limited to the use of triangular fuzzy numbers;
- the shape of the membership function is not taken into consideration or only a part of it is used (leading to a loss of information);
- a general problem is the one of the "sensitivity" (degree of discrimination) of the solutions.

Some authors claim that a low degree of discrimination is a negative feature; in contrast, others believe that in a fuzzy context, any attempt to reach a high degree of precision on the results is somewhat artificial. The latter viewpoint seems to be plausible. A major reason is that in many decision problems (including environmental issues) the marginal benefit of an extra unit in precision may be rather unimportant in a decision situation where it is often the wish of a decision-maker not to be confronted with single unambiguous and (sometimes) imposed solutions, but rather with a spectrum of open feasible solutions each having its own merits.

In general, a *semantic distance*  $S_d$  between two fuzzy sets, A and B, mirrors a possibility degree of equality between two fuzzy sets or a similarity degree between them. The larger the distance the smaller the possibility degree of equality. The most common distance is the so-called Hamming distance. For the continuous case, another possible approach is the computation of some moments of the membership distributions of the fuzzy sets, after which the similarity can be evaluated by means of traditional distances such as the Euclidean distance, the Bhattacharya distance, the Mahalanobis distance and so on [6, 11, 36]. Of course, in this case two problems have to be faced, viz. the correct selection of moments and the correct selection of the distance function.

Here we will illustrate a new distance metric that is useful in the case of continuous membership functions allowing also a definite integration. It has to be noted that we take into consideration the case of standard L-R fuzzy numbers. In general, a fuzzy number is a normalized and bounded convex fuzzy set in the real space. A special type of fuzzy number is the L-R fuzzy number; it is defined as follows:

$$\mu_A(x) = \begin{cases} (F_L(x-m)/\alpha), & \text{if } -\infty < x < m, \alpha > 0 \\ | & \\ 1, & \text{if } x=m \\ | & \\ (F_R(x-m)/\delta), & \text{if } m < x < +\infty, \delta > 0 \end{cases} \quad (10)$$

where  $m$ ,  $\alpha$ ,  $\delta$ , are the mean value, the left-hand and the right-hand variation, respectively.  $F_L(x)$  is a monotonically increasing membership function and  $F_R(x)$ , not necessarily symmetric to  $F_L(x)$ , is a monotonically decreasing function.

In order to compute such a distance, it is necessary that the area bounded by the membership function must be equal to 1. Generally, it is possible to change membership functions proportionally by multiplying them by a constant  $c \in \mathbb{R}^+$ , with  $c \leq 1$  for normal fuzzy sets and  $c \leq 1/m_A$  for subnormal fuzzy sets ( $m_A = \max_{x \in X} \mu_A(x)$ ) [20].

If  $\mu_{A_1}(x)$  and  $\mu_{A_2}(x)$  are two membership functions, we can write

$$f(x) = c_1 \mu_{A_1}(x) \quad \text{and} \quad g(y) = c_2 \mu_{A_2}(x) \quad (11)$$

where  $f(x)$  and  $g(y)$  are two functions obtained by rescaling the ordinates of  $\mu_{A_1}(x)$  and  $\mu_{A_2}(x)$  through  $c_1$  and  $c_2$ , such that

$$\int f(x) dx = \int g(y) dy = 1 \quad (12)$$

The distance between all points of the membership functions is computed as follows:

$$\text{if } f(x) : X = [x_L, x_U] \quad \text{and} \quad g(y) : Y = [y_L, y_U] \quad (13)$$

(where of course sets  $X$  and  $Y$  can be non-bounded from one or either sides), then

$$S_d(f(x), g(y)) = \int \int |x-y| f(x) g(y) dy dx \quad (14)$$

It is easy to show that this distance satisfies the properties of non-negativity and symmetry; the proof of the triangle inequality and a Monte Carlo type numerical procedure for the computation of such a distance can be found in [17]. It has to be noted that without the absolute value the equation (14) becomes a function of the sign thus allowing the computation of the possibility degree of a fuzzy set to be greater than another one (preference index).

As a special case, we consider first the case where the intersection of two membership functions is empty.

If  $x > y \quad \forall x \in X$  and  $\forall y \in Y$ , it follows that a continuous function in two variables is defined over a rectangle. Therefore, the double integral can be calculated as iterated single integrals:

$$\int \int |x-y| f(x) g(y) dy dx = \quad (15)$$

$$= \int \int (x-y) f(x) g(y) dy dx = \int \int [x f(x) g(y) - y f(x) g(y)] dy dx = \quad (16)$$

$$= \int x f(x) dx - \int f(x) E(y) dx = E(x) - E(y) = \quad (17)$$

$$= |E(x) - E(y)| \quad (18)$$

where  $E(x)$  and  $E(y)$  are the expected values of the two membership functions<sup>3</sup>; the latter result is true, since  $x > y$ .

Therefore, when the intersection is empty, this indicator is equal to the difference between their expected values. When the intersection between two fuzzy sets is not empty (see Figure 2), such an indicator is different from the difference between the expected values. This is the case of a double integral over a general region; since this is not vertically simple nor horizontally simple, it is not possible the computation by means of iterated integration, but it is necessary to take the limit of the Riemann sum. This problem can be easily overcome by means of numerical analysis.

This property of being dependent on the intersection is quite interesting since the overlapping area between two fuzzy sets is a key issue for determining how difficult their comparison is. In general, whatever method is chosen, the ranking can be questioned whenever a significant overlap exists.

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<sup>3</sup> The expected value of a fuzzy set A is equal to:

$$E(\mu_A(x)) = \frac{\int_{-\infty}^{\infty} x \mu_A(x) dx}{\int_{-\infty}^{\infty} \mu_A(x) dx}$$

provided that the integral converges absolutely; that is  $\int_{-\infty}^{\infty} |x \mu_A(x)| dx < \infty$   
 Otherwise, A has no finite expected value.

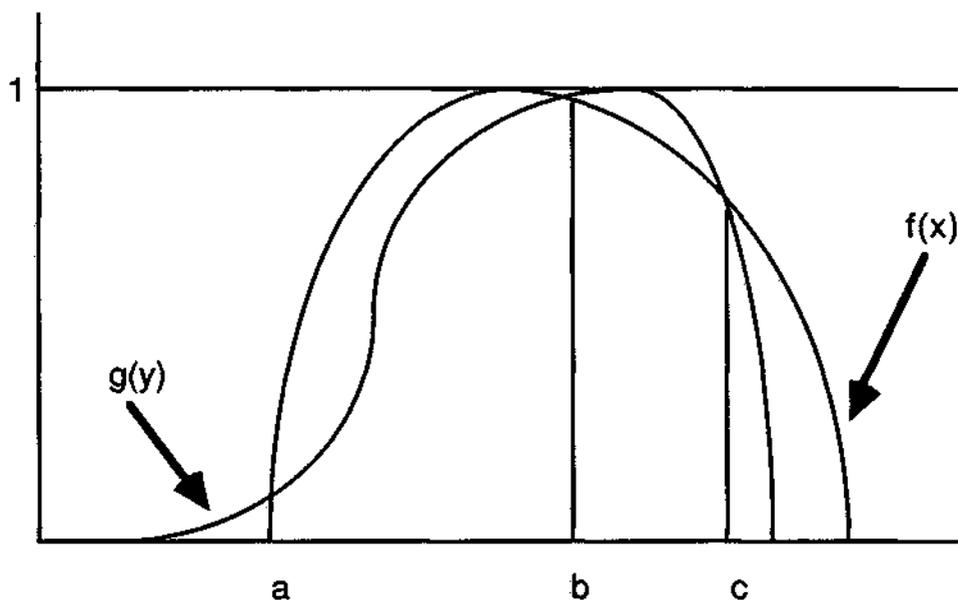


Figure 2. Intersection between two fuzzy sets

From a theoretical point of view, the following conclusions can be drawn:

- 1) the absolute value metric (simple difference) is a particular case of this type of distance (preference index);
- 2) the expected value is obtained as a representation of fuzzy sets only when their intersection is empty;
- 3) when the intersection between two fuzzy sets is not empty, their distance (difference) is different from the difference between their expected values;
- 4) in the case of fuzzy information being represented by L-R fuzzy numbers, when their intersection is empty, their distance (difference) is equal to the crisp numbers they represent only when they are symmetric; otherwise, their expected values are obtained;
- 5) by applying this preference index, the problem of the use of only one side of the membership functions, common to most of the traditional fuzzy multicriteria methods, is overcome.

It is interesting to note that also the *stochastic information* represented by means of density functions can be taken into account by means of this distance function (preference index). Of course in this case the condition

$$\int f(x) dx = 1 \quad (19)$$

is always true.

#### 4. Pairwise Comparison of Alternatives

Evaluation requires normally a judgement of the relative performance of distinct alternatives based on dominance relationships. Since the aggregation procedure used for the computation of  $\mu_{\Delta}(a_n)$  is of a complete type, it is useful to obtain further more "local" information by means of a *partial* aggregation procedure. This can easily be done by means of the notion of a fuzzy relation. Six different fuzzy relations are considered:

- 1) much greater than ( $\gg$ )
- 2) greater than ( $>$ )
- 3) approximately equal to ( $\cong$ )
- 4) very equal to ( $=$ )
- 5) less than ( $<$ )
- 6) much less than ( $\ll$ )

Analytically, membership functions pertaining to these fuzzy relations can be formulated in the equations below.

##### a) crisp evaluations

- 1) greater than

$$\mu_{>}(x,y) = \begin{cases} 0 & \text{if } x-y \leq 0 \\ \frac{1}{[1+c(x-y)^2]^{-1}} & \text{if } x-y > 0 \end{cases} \quad (c \in \mathbb{R}^+) \quad (20)$$

- 2) much greater than

$$\mu_{\gg}(x,y) = \begin{cases} 0 & \text{if } x-y \leq 0 \\ \frac{1}{[1+c(x-y)^2]^{-2}} & \text{if } x-y > 0 \end{cases} \quad (c \in \mathbb{R}^+) \quad (21)$$

3) approximately equal to

$$\mu_{\approx}(x,y) = e^{-c(|x-y|)} \quad (c \in \mathbb{R}^+) \quad (22)$$

4) very equal to

$$\mu_{\approx\approx}(x,y) = e^{-c(x-y)^2} \quad (c \in \mathbb{R}^+) \quad (23)$$

5) less than

$$\mu_{<}(x,y) = \begin{cases} [1+c(y-x)^2]^{-1} & \text{if } x-y < 0 \\ 0 & \text{if } x-y \geq 0 \end{cases} \quad (c \in \mathbb{R}^+) \quad (24)$$

6) much less than

$$\mu_{<<}(x,y) = \begin{cases} [1+c(y-x)^2]^{-2} & \text{if } x-y < 0 \\ 0 & \text{if } x-y \geq 0 \end{cases} \quad (c \in \mathbb{R}^+) \quad (25)$$

## b) fuzzy and stochastic evaluations

1) greater than

$$\mu_{>}(x,y) = \begin{cases} 0 & \text{if } \int \int (x-y)f(x)g(y)dydx \leq 0 \\ [1+c\left(\int \int (x-y)f(x)g(y)dydx\right)^{-2}]^{-1} & \text{if } \int \int (x-y)f(x)g(y)dydx > 0 \end{cases} \quad (c \in \mathbb{R}^+) \quad (26)$$

2) much greater than

$$\mu_{>>}(x,y) = \begin{cases} 0 & \text{if } \int \int (x-y)f(x)g(y)dydx \leq 0 \\ [1+c\left(\int \int (x-y)f(x)g(y)dydx\right)^{-2}]^{-2} & \text{if } \int \int (x-y)f(x)g(y)dydx > 0 \end{cases} \quad (c \in \mathbb{R}^+) \quad (27)$$

3) approximately equal to

$$\mu_{\approx}(x,y) = e^{-c \left( \int \int |x-y| f(x)g(y) dy dx \right)} \quad (c \in \mathbb{R}^+) \quad (28)$$

4) very equal to

$$\mu_{\ll}(x,y) = e^{-c \left( \int \int |x-y| f(x)g(y) dy dx \right)^2} \quad (c \in \mathbb{R}^+) \quad (29)$$

5) less than

$$\mu_{<}(x,y) = \begin{cases} \left[ 1 + c \left( \int \int (y-x) f(x)g(y) dx dy \right)^{-2} \right]^{-1} & \text{if } \int \int (x-y) f(x)g(y) dy dx < 0 \\ 1 & \\ 0 & \text{if } \int \int (x-y) f(x)g(y) dy dx \geq 0 \end{cases} \quad (c \in \mathbb{R}^+) \quad (30)$$

6) much less than

$$\mu_{\ll\ll}(x,y) = \begin{cases} \left[ 1 + c \left( \int \int (y-x) f(x)g(y) dy dx \right)^{-2} \right]^{-2} & \text{if } \int \int (x-y) f(x)g(y) dy dx < 0 \\ 1 & \\ 0 & \text{if } \int \int (x-y) f(x)g(y) dy dx \geq 0 \end{cases} \quad (c \in \mathbb{R}^+) \quad (31)$$

The use of such relations is inspired by the same philosophy as the definition of a "pseudo-criterion" [23], but here according to fuzzy principles, no precise boundary is established, thus allowing the contribution of the evaluation according to each single criterion to different preference modelling situations. Furthermore, the decision-maker is not asked to evaluate thresholds, which is always a difficult and perhaps arbitrary process, although it should be admitted that the choice of the membership functions is also somewhat arbitrary.

Given such information on the pairwise performance of the alternatives according to each single criterion, it is necessary to aggregate these evaluations in order to take into account all criteria simultaneously. The simplest way is the following [28]:

$$\frac{1}{M} \sum_{m=1}^M \mu_{\star m}(x,y) \quad (32)$$

where  $\mu_{*m}(x,y)$  indicates the evaluation of a given fuzzy relation for each pair of actions according to the  $m$ -th criterion. A disadvantage of this approach is that the diversity among the assessments of single fuzzy relations is not considered (since the preference intensities compensate completely one another). Thus, we propose the use of the following equation:

$$\frac{\sum_{m=1}^M \max(\mu_{*m}(x,y) - \alpha, 0)}{\sum_{m=1}^M |\mu_{*m}(x,y) - \alpha|} \quad (33)$$

where  $\alpha$  is a "minimum requirement" imposed on each fuzzy relation (of course a sensitivity analysis can easily be performed).

In order to have information on the diversity among the assessments of the single fuzzy relations, also in this case the entropy concept is useful; here, it is (see equation (9))

$$\mu_A(x) = \begin{cases} 0 & \text{if } \mu_{*m}(x,y) - \alpha \leq 0 \\ \mu_{*m}(x,y) & \text{if } \mu_{*m}(x,y) - \alpha > 0 \end{cases} \quad (34)$$

Thus a "fuzzy preference relation" is obtained:

$$\begin{cases} \mu_{\gg}(a,b) & H_L(\gg) \\ \mu_{>}(a,b) & H_L(>) \\ \mu_{\equiv}(a,b) & H_L(\equiv) \\ \mu_{=}(a,b) & H_L(=) \\ \mu_{<}(a,b) & H_L(<) \\ \mu_{\ll}(a,b) & H_L(\ll) \end{cases}$$

where  $\mu_{*}(a,b)$  is the overall evaluation of a given fuzzy relation for each pair of actions and,  $H_L(*)$  is the associated entropy level.

However it has to be noted that this is not a standard fuzzy preference relation, since it is not assumed to be reciprocal [28].

How to use such information in order to evaluate the different alternatives will be the topic of the next section.

## 5. Evaluation of the Alternatives

The information provided by a "fuzzy preference relation" can be used in different ways, e.g., the degree of truth ( $\tau$ ) of statements as: "according to most of the criteria

- **a is better than b,**
- **a and b are indifferent,**
- **a is worse than b"**

can be computed as follows.

The proportional linguistic quantifier "most" can be defined by the membership function

$$\mu_{\text{most}}(\omega) = \begin{cases} 1 & \text{if } \omega \geq 0.8 \\ 3.33\omega - 1.66 & \text{if } 0.5 < \omega < 0.8 \\ 0 & \text{if } \omega \leq 0.5 \end{cases} \quad (35)$$

It is evident that  $\forall \omega \in [0, 1]$  if  $\omega' > \omega'' \Rightarrow \mu_{\text{most}}(\omega') \geq \mu_{\text{most}}(\omega'')$ , thus it is a nondecreasing fuzzy quantifier [13].

The value  $\omega$  is a function of the aggregate membership degree and its entropy level, then after the transformation  $C(*) = 1 - H_L(*)^4$ , it is possible to define:

$$\omega(\text{a is better than b}) = \frac{\mu_{>>}(a,b) \wedge C(>>) + \mu_{>}(a,b) \wedge C(>)}{C(>>) + C(>)} \quad (36)$$

$$\omega(\text{a is worse than b}) = \frac{\mu_{<<}(a,b) \wedge C(<<) + \mu_{<}(a,b) \wedge C(<)}{C(<<) + C(<)} \quad (37)$$

$$\omega(\text{a and b are indifferent}) = \frac{\mu_{=} (a,b) \wedge C(=) + \mu_{\equiv} (a,b) \wedge C(\equiv)}{C(=) + C(\equiv)} \quad (38)$$

(where " $\wedge$ " may be replaced by any other operator).

Thus concerning each pair of actions a global linguistic evaluation characterized by its degree of truth, is obtained.

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<sup>4</sup> This transformation is necessary in order to apply some computational rules of approximate reasoning as defined by Zadeh in [34].

Such pairwise evaluations can be used directly by the decision-maker(s) in order to isolate a set of satisfactory solutions, or -if in a given decisional environment there is a need to perform further elaborations in order to get a ranking of the alternatives (in a complete or partial preorder)- the basic idea of positive (leaving) and negative (entering) flows of the PROMETHEE methods [8] can be adapted to the peculiarities of our procedure.

For each action we define:

$$\phi^+(a) = \frac{\sum_{n=1}^{N-1} \delta_n}{\sum_{n=1}^{N-1} C_n(>>) + \sum_{n=1}^{N-1} C_n(>)} \quad (39)$$

$$\text{where } \delta_n = \mu_{>>}(a,x) \wedge C(>>) + \mu_{>}(a,x) \wedge C(>) \quad (40)$$

and

$$\phi^-(a) = \frac{\sum_{n=1}^{N-1} \psi_n}{\sum_{n=1}^{N-1} C_n(<<) + \sum_{n=1}^{N-1} C_n(<)} \quad (41)$$

$$\text{where } \psi_n = \mu_{<<}(a,x) \wedge C(<<) + \mu_{<}(a,x) \wedge C(<) \quad (42)$$

However, it has to be noted that all the results obtained can provide to policy-makers "justifiable" or "defensible" decisions but in real world environmental decision making, it is necessary to interact with many actors (often each single actor is represented by complex organizations as town councils, trade unions, different associations and so on) each of them having different goals and values. Since, generally real-world problems are not direct win-lose situations, but a certain degree of compromise is needed, a procedure aimed at supporting environmental policy-makers must consider this problem of different (and often conflictual) evaluations [1, 9]. This should of course be the aim of further research.

## 6. An Illustrative Example

Suppose that there are 3 possibilities for improving the transportation system in a region, viz. highway construction, a road/bus system and a new train (railroad) system. Each of these 3 alternatives will be judged on the basis of 5 criteria, viz. costs, travel time, capacity, NO<sub>x</sub> emissions and landscape impacts. Some of these impacts are quantitative, but others are qualitative in nature. The impact (or evaluation) matrix related to the above problem is supposed to be the following:

Criteria	Units	Highway (a <sub>1</sub> )	Road/bus (a <sub>2</sub> )	Train (a <sub>3</sub> )
Costs	min gld	200 (.1)	250 (.1)	400 (.6)
Travel Time	linguistic	excellent (.1)	good (.85)	moderate (.6)
Capacity	mln km/year	20 (.5)	30 (.8)	40 (1)
NO <sub>x</sub> Emissions	ton/year	1000 (.3)	750 (.6)	100 (1)
Landscape	linguistic	bad (.2)	bad (.2)	moderate (.6)

(the values in brackets are the  $\mu_{F_m}(a_n)$ )

Table 1. Evaluation matrix of a transportation problem

For the  $\mu_{\Delta}(a_n)$  (see eq. (5)) the following results are obtained:

	Sum Operator	Product Operator	Entropy
Highway	0.60	0.49	0.52
Road/bus	0.69	0.60	0.60
Train	0.76	0.73	0.58

For the cost criterion, the following pairwise fuzzy relations are obtained (based on equations from (20) to (31)):

COSTS		
$\mu_{>>}(a_1, a_2)=0.23$	$\mu_{>>}(a_1, a_3)=0.87$	$\mu_{>>}(a_2, a_3)=0.80$
$\mu_{>}(a_1, a_2)=0.60$	$\mu_{>}(a_1, a_3)=0.96$	$\mu_{>}(a_2, a_3)=0.93$
$\mu_{\leq}(a_1, a_2)=0.17$	$\mu_{\leq}(a_1, a_3)\equiv 0$	$\mu_{\leq}(a_2, a_3)\equiv 0$
$\mu_{=}(a_1, a_2)\equiv 0$	$\mu_{=}(a_1, a_3)\equiv 0$	$\mu_{=}(a_2, a_3)\equiv 0$

The full set of outcomes is given in Appendix 1.

Then the aggregate values (according to all criteria) for the pair of actions  $a_1$  and  $a_2$  are (see equations (33) and (34)):

(	$\mu_{>}(a_1, a_2)=0.40$	$H_L(>)=0.38$
	$\mu_{>>}(a_1, a_2)=0.18$	$H_L(>>)=0.20$
	$\mu_{\equiv}(a_1, a_2)=0.49$	$H_L(\equiv)=0$
	$\mu_{=}(a_1, a_2)=0.36$	$H_L(=)=0$
	$\mu_{<}(a_1, a_2)=0.55$	$H_L(<)=0.17$
(	$\mu_{<<}(a_1, a_2)=0.50$	$H_L(<<)=0.28$

The outcomes for all pairs are given in Appendix 1.

It has to be noted that the entropy levels of these evaluations are much lower than the initial ones.

Then for each pair of actions, the following degrees of truth of a linguistic evaluation are obtained (see equations from (35) to (38)):

$a_1$ is better than $a_2$	$\tau=0$
$a_1$ and $a_2$ are indifferent	$\tau=0$
$a_1$ is worse than $a_2$	$\tau=0.57$
$a_1$ is better than $a_3$	$\tau=0.67$
$a_1$ and $a_3$ are indifferent	$\tau=0$
$a_1$ is worse than $a_3$	$\tau=1$
$a_2$ is better than $a_3$	$\tau=0.53$
$a_2$ and $a_3$ are indifferent	$\tau=0$
$a_2$ is worse than $a_3$	$\tau=1$

These results are mainly due to four factors:

- number of criteria in favour of an action;
- degree of compensation allowed in the aggregation process (in this application we have used a minimum requirement for each criterion, equal to  $\alpha=0.30$ );
- definition of the membership function of the linguistic operators;
- aggregation operator chosen for the approximate reasoning operations (in this application we have used the "min" operator which is known as

a representation of the logic "and", and therefore it is completely non-interactive (since a high value cannot compensate<sup>5</sup> a low one)).

By computing the  $\phi^+$  and  $\phi^-$  indices (see equations from (39) to (42)), the following results are obtained:

$$\phi^+(a_1)=0.564$$

$$\phi^-(a_1)=0.817$$

$$\phi^+(a_2)=0.643$$

$$\phi^-(a_2)=0.680$$

$$\phi^+(a_3)=0.980$$

$$\phi^-(a_3)=0.656$$

Then according to  $\phi^+$  the following ranking is obtained

$$a_3 \rightarrow a_2 \rightarrow a_1$$

and according to  $\phi^-$  the following ranking is obtained

$$a_3 \rightarrow a_2 \rightarrow a_1$$

Therefore, the intersection gives as a final result the complete preorder

$$a_3 \rightarrow a_2 \rightarrow a_1$$

This ranking is a function of all actions taken into consideration; on the contrary, the pairwise linguistic evaluations give information only on each single pair of actions. Thus both together can help the decision-maker(s) to reach a final decision.

In conclusion, we may say that since in a fuzzy environment a high precision of the results is illusory, the above procedure which aims at supplying the decision-maker(s) with as much information as possible and at making the entropy levels connected with these evaluations as small as possible, is a meaningful undertaking.

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<sup>5</sup> By compensation in the context of aggregation operators for fuzzy sets is meant the following: "Given that the degree of membership to the aggregated fuzzy set is  $\mu_{\text{Agg}}(x_k) = f(\mu_A(x_k), \mu_B(x_k)) = k$ ,  $f$  is compensatory if  $\mu_{\text{Agg}}(x_k) = k$  is obtainable for different  $\mu_A(x_k)$  by a change in  $\mu_B(x_k)$  [36 p. 36]".

## 7. Conclusion

The model developed in the present paper, is a discrete multicriteria method whose impact (or evaluation) matrix may include either crisp, stochastic or fuzzy evaluations of the performance of an alternative  $a_n$  with respect to a criterion  $g_m$ . No traditional weighting of criteria is used. Often in public decision making a single point-value solution (e.g. weights) tends to lead to deadlocks in a decision process because it imposes too rigid conditions for a compromise. A natural and flexible way of defining a region allowing more room for mutual consensus is by means of linguistic propositions. This can be done by means of possibility theory. From a methodological point of view, two main issues are then faced:

- the problem of equivalence of the procedures used in order to standardize the various evaluations (of a mixed type) of the performance of alternatives according to different criteria;
- the problem of comparison of fuzzy numbers typical of all fuzzy multicriteria methods.

Since in a fuzzy context, any attempt to reach a high degree of precision on the results tends to be somewhat artificial, a pairwise linguistic evaluation of alternatives is used. This is done by means of the notions of fuzzy relations and linguistic quantifiers. In the aggregation process, particular attention is paid to the problem of the diversity of the single evaluations, while the entropy concept is used as a measure of the associated "fuzziness". Such linguistic evaluations can be used in different ways according to the decision environment at hand.

In our approach a weighting of criteria is not assumed and no consideration is given to the "minority principle" (like the discordance index in the ELECTRE methods). Since in environmental and resource management and policy aiming at an ecologically sustainable development many conflicting issues and interests emerge, particular attention has to be given to the problem of different values and goals of different groups in society. This implies that such a procedure must be integrated with conflict minimization methods which allow policy-makers to seek for "defensible" decisions that could reduce the degree of conflict (in order to reach a certain degree of consensus) or that could have a higher probability of being accepted by certain groups of decision makers. This problem forms also an important item on a future research agenda of fuzzy multicriteria analysis.

## APPENDIX 1. Results of the Illustrative Example

The following pairwise fuzzy relations are obtained:

### COSTS

$\mu_{\gg}(a_1, a_2)=0.23$	$\mu_{\gg}(a_1, a_3)=0.87$	$\mu_{\gg}(a_2, a_3)=0.80$
$\mu_{>}(a_1, a_2)=0.60$	$\mu_{>}(a_1, a_3)=0.96$	$\mu_{>}(a_2, a_3)=0.93$
$\mu_{\equiv}(a_1, a_2)=0.17$	$\mu_{\equiv}(a_1, a_3)\equiv 0$	$\mu_{\equiv}(a_2, a_3)\equiv 0$
$\mu_{=}(a_1, a_2)\equiv 0$	$\mu_{=}(a_1, a_3)\equiv 0$	$\mu_{=}(a_2, a_3)\equiv 0$

### CAPACITY

$\mu_{\equiv}(a_1, a_2)=0.09$	$\mu_{\equiv}(a_1, a_3)\equiv 0$	$\mu_{\equiv}(a_2, a_3)=0.09$
$\mu_{=}(a_1, a_2)\equiv 0$	$\mu_{=}(a_1, a_3)\equiv 0$	$\mu_{=}(a_2, a_3)\equiv 0$
$\mu_{<}(a_1, a_2)=0.80$	$\mu_{<}(a_1, a_3)=0.94$	$\mu_{<}(a_2, a_3)=0.8$
$\mu_{\ll}(a_1, a_2)=0.62$	$\mu_{\ll}(a_1, a_3)=0.87$	$\mu_{\ll}(a_2, a_3)=0.62$

### NO<sub>x</sub>

$\mu_{\equiv}(a_1, a_2)\equiv 0$	$\mu_{\equiv}(a_1, a_3)\equiv 0$	$\mu_{\equiv}(a_2, a_3)\equiv 0$
$\mu_{=}(a_1, a_2)\equiv 0$	$\mu_{=}(a_1, a_3)\equiv 0$	$\mu_{=}(a_2, a_3)\equiv 0$
$\mu_{<}(a_1, a_2)=0.94$	$\mu_{<}(a_1, a_3)\equiv 1$	$\mu_{<}(a_2, a_3)\equiv 1$
$\mu_{\ll}(a_1, a_2)=0.90$	$\mu_{\ll}(a_1, a_3)\equiv 1$	$\mu_{\ll}(a_2, a_3)=0.98$

### TRAVEL TIME

$\mu_{\gg}(a_1, a_2)=0.52$	$\mu_{\gg}(a_1, a_3)=0.79$	$\mu_{\gg}(a_2, a_3)=0.33$
$\mu_{>}(a_1, a_2)=0.60$	$\mu_{>}(a_1, a_3)=0.86$	$\mu_{>}(a_2, a_3)=0.50$
$\mu_{\equiv}(a_1, a_2)=0.23$	$\mu_{\equiv}(a_1, a_3)\equiv 0$	$\mu_{\equiv}(a_2, a_3)=0.34$
$\mu_{=}(a_1, a_2)\equiv 0$	$\mu_{=}(a_1, a_3)\equiv 0$	$\mu_{=}(a_2, a_3)\equiv 0$

### LANDSCAPE

$\mu_{\equiv}(a_1, a_2)=1$	$\mu_{\equiv}(a_1, a_3)=0.34$	$\mu_{\equiv}(a_2, a_3)=0.34$
$\mu_{=}(a_1, a_2)=1$	$\mu_{=}(a_1, a_3)\equiv 0$	$\mu_{=}(a_2, a_3)\equiv 0$
$\mu_{<}(a_1, a_2)=0$	$\mu_{<}(a_1, a_3)=0.5$	$\mu_{<}(a_2, a_3)=0.5$
$\mu_{\ll}(a_1, a_2)=0$	$\mu_{\ll}(a_1, a_3)=0.33$	$\mu_{\ll}(a_2, a_3)=0.33$

On the basis of these for each pair of actions the following aggregate values are obtained:

(	$\mu_{>}(a_1, a_2)=0.40$	$H_L(>)=0.38$
	$\mu_{>>}(a_1, a_2)=0.18$	$H_L(>>)=0.20$
	$\mu_{\equiv}(a_1, a_2)=0.49$	$H_L(\equiv)=0$
	$\mu_{=}(a_1, a_2)=0.36$	$H_L(=)=0$
	$\mu_{<}(a_1, a_2)=0.55$	$H_L(<)=0.17$
(	$\mu_{<<}(a_1, a_2)=0.50$	$H_L(<<)=0.28$

(	$\mu_{>}(a_1, a_3)=0.57$	$H_L(>)=0.16$
	$\mu_{>>}(a_1, a_3)=0.54$	$H_L(>>)=0.25$
	$\mu_{\equiv}(a_1, a_3)=0.03$	$H_L(\equiv)=0.18$
	$\mu_{=}(a_1, a_3)=0$	$H_L(=)=0$
	$\mu_{<}(a_1, a_3)=0.71$	$H_L(<)=0.26$
(	$\mu_{<<}(a_1, a_3)=0.68$	$H_L(<<)=0.29$

(	$\mu_{>}(a_2, a_3)=0.47$	$H_L(>)=0.27$
	$\mu_{>>}(a_2, a_3)=0.37$	$H_L(>>)=0.32$
	$\mu_{\equiv}(a_2, a_3)=0.08$	$H_L(\equiv)=0.36$
	$\mu_{=}(a_2, a_3)=0$	$H_L(=)=0$
	$\mu_{<}(a_2, a_3)=0.70$	$H_L(<)=0.34$
(	$\mu_{<<}(a_2, a_3)=0.63$	$H_L(<<)=0.40$

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