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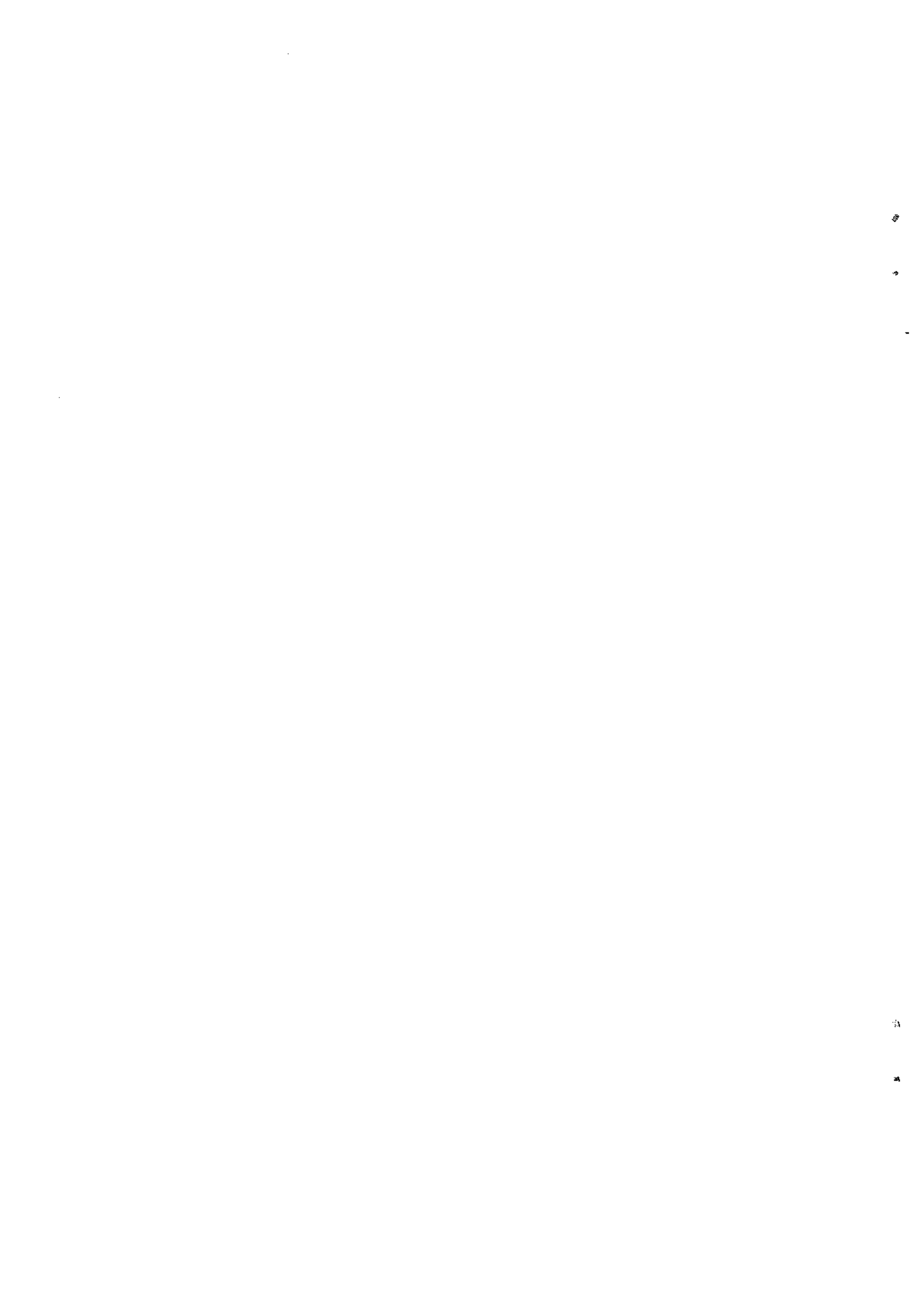
A MODEL FOR QUARTERLY UNEMPLOYMENT IN CANADA

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ABSTRACT

We propose a periodic cointegration model for the quarterly unemployment rate in Canada, i.e. a model in which long–run relations between unemployment and output and real wages are allowed to vary over the seasons. It emerges that in Canada this variation is related to the summer and winter season. Theoretical economic considerations result in a model that is able to explain the observed findings.

Keywords: Unemployment, Periodic models, Cointegration

JEL classification: C22, E24

AUTHORS' NOTE

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1. INTRODUCTION

The construction of dynamic econometric models for nonstationary seasonal time series has gained some attention recently. Examples of multivariate models that can handle complex seasonal fluctuations are the seasonal cointegration model proposed in Engle *et al.* (1991) and the periodic cointegration model as developed in Birchenhall *et al.* (1989) and Franses and Kloek (1991). Seasonal cointegration formalizes the notion that seasonal time series can have common nonstationary components. Periodic cointegration is an extension of the usual cointegration concept given in Engle and Granger (1987) by allowing that the parameters in the cointegration relations as well as the dynamic adjustment parameters vary over the seasons, see also Franses and Boswijk (1991). One of the typical characteristics of the periodic cointegration model is that it can generate asymmetric cyclical patterns for the variable to be explained. This is because shocks in the explanatory variables have varying long-run, as well as short-run, impacts on the time series process. For example, a positive shock in the first quarter can have another effect than a negative shock in the second quarter. Hence, cyclical behavior of a time series can be dependent on the seasons in which expansions and contractions occur. See Ghysels (1991) for empirical evidence of seasonal patterns in business cycle turning points.

A typical example of an economic variable that shows asymmetric cyclical behavior is the quarterly unemployment rate. Luukkonen and Teräsvirta (1991) document that asymmetries in unemployment can be found for several countries, and further that the evidence for Canada is rather strong. Given this, it seems worthwhile to investigate whether a periodic cointegration model can adequately explain the Canadian unemployment. In section 2 of this paper we discuss the model, a simple estimation method and the empirical results obtained. In section 3 we propose a tentative theoretical model that can

explain the observed outcome, which is that the cointegration relations vary over the winter and the summer seasons. In section 4 we give some concluding remarks.

2. A PERIODIC COINTEGRATION MODEL

Anticipating on the theoretical results in the next section, and also relying on empirical studies as Bierens and Broersma (1990), we consider an empirical dynamic model for unemployment rate ur_t, in which it is explained by the log of the industrial output index y_t, by the business loan interest rate ir_t, and by the log of the real wages w_t. This choice of variables reflects that our model should be consistent with economic theories. For example, the inclusion of ir_t is argued in Farmer (1985), and the neoclassical theory stresses the importance of the real wage rate. The time series of these variables, when decomposed as annual series per quarter, are depicted in the figures 1 through 4. All variables are seasonally unadjusted and obtained from the OECD tapes. The sample for estimation covers 1961.1–1987.4, although the forthcoming model will be estimated from 1964.1 onwards. The observations for 1988.1–1990.4 are also collected, but they will be used for the evaluation of the forecasting performance. To save space we do not report all computational details of the univariate and multivariate test procedures, but these can be obtained from the authors. All calculations are performed using PCGIVE version 6.1.

Univariate data analysis with the test procedure for seasonal unit roots proposed in Hylleberg et al. (1990) reveals that ir_t and w_t have a unit root at the zero frequency. This means that these variables require the use of the filter Δ_1, where $\Delta_k x_t$ is defined as $(1-B^k)x_t = x_t - x_{t-k}$. The unemployment and output variables seem to have the unit roots ± 1. Literally, this would mean

that one should apply the $(1-B^2)$ filter. However, from the simulations in Franses (1992) it appears that this phenomenon can occur when ur_t and y_t are periodically integrated. A simple example of a periodically integrated model is the first order autoregressive model $x_t = \phi_s x_{t-1} + \varepsilon_t$, $s = 1,2,3,4$, where it applies that $\prod_j^4 \phi_j = 1$, but not all $\phi_j = 1$, see Osborn (1988) for details.

In Franses and Kloek (1991) it is shown that univariate periodically integrated behavior can be explained by the presence of periodic cointegration relations. Given our choice of variables and the outcomes of the univariate test, a general representation of a periodic cointegration model can be

$$\phi_p(B)\Delta_4 ur_t = \sum_{s=1}^{4} \mu_s D_{st} + \sum_{s=1}^{4} \alpha_s D_{st}(ur + \beta_s y + \gamma_s w + \delta_s ir)_{t-k} + \psi_j(B)\Delta_4 y_t$$

(1)

$$+ \theta_q(B)\Delta_1 w_t + \eta_r(B)\Delta_1 ir_t + \varepsilon_t.$$

The D_{st} are seasonal dummies. The parameters with index s are allowed to vary over the seasons. The $\phi_p(B)$, $\psi_j(B)$, $\theta_q(B)$ and $\eta_r(B)$ are polynomials in B. For $\phi_p(B)$ the usual stationarity assumptions apply. The ε_t is an uncorrelated zero mean process with constant variance. The k in (1) is usually set equal to 4 or 8. The periodic cointegration relations are $(ur + \beta_s y + \gamma_s w + \delta_s ir)_t$ and the α_s are the periodically varying adjustment parameters.

A convenient procedure to check for the presence of periodic cointegration is proposed in Franses and Boswijk (1991). This method amounts to the application of four Wald tests for the joint significance of ur_{t-k}, y_{t-k}, w_{t-k} and ir_{t-k} in (1), which are given by

$$\text{Wald}_s = (n-(p+j+q+r)-20)*((RSS_r/RSS_u)-1)$$

(2)

where n is the number of observations used for estimation, and where RSS_r and

RSS_u refer to the residual sums of squares of the restricted and unrestricted model, respectively. Critical values are displayed in Boswijk (1991). In the case of no trend and three exogenous variables, the relevant critical values are 14.94 (10%) and 17.18 (5%). Rejection of the null hypothesis of these restrictions implies that there is some form of cointegration.

In the present application the following choices have been made for the empirical specification of (1). A dummy for the first quarter of 1975 is included to capture the observed structural change in the natural rate of unemployment, as noted in the OECD Economic Surveys for Canada (1991, p.69). The k is set equal to 4, and the $\phi_p(B)$ appears to be $(1-\phi_1 B-\phi_2 B^2-\phi_4 B^4 \Delta_1)$, $\psi_j(B) = \psi$, $\theta_q(B) = \theta B$, $\eta_j(B) = 0$. These decisions are based on several F test outcomes and diagnostic checks. The final model can not be rejected on the basis of LM tests for residual autocorrelation, heteroskedasticity, and the Jarque–Bera test for normality. The values of the $Wald_s$ tests in (2) are 12.471, 17.489, 7.519 and 8.560, respectively. Hence, it can be concluded that only for the second quarter there is some evidence of a stationary equilibrium relation.

Inspection of the parameter estimates establishes however the impression that the model in (1) is overspecified in the sense that parameters related to the first and the fourth quarter are almost equal, as well as those related to the second and the third quarter. This suggests that a restricted version of (1), i.e.

$$(1-\phi_1 B-\phi_2 B^2-\phi_4 B^4 \Delta_1)\Delta_4 ur_t = \sum_{q=1}^{2} \mu_q D_{qt} + \sum_{q=1}^{2} \alpha_q D_{qt}(ur + \beta_q y + \gamma_q w + \delta_q ir)_{t-4}$$

(3)

$$+ \psi\Delta_4 y_t + \theta\Delta_1 w_{t-1} + \varepsilon_t,$$

where the index q refers to winter (*win*) and summer (*sum*), may be worthwhile

to estimate. The corresponding $Wald_q$ tests as in (2) obtain values of 19.407 and 21.611, respectively. Hence, cointegration relations appear to present.

The model in (3) may be further simplified in case restrictions on the parameters are appropriate. In Franses and Boswijk (1991) it is shown that the restrictions can be checked via F tests that asymptotically follow standard F distributions. Given that the long-run parameters β_q, γ_q and δ_q can only be estimated as the product of α_q and some other parameters, it seems convenient to start a simplification strategy by testing whether $\alpha_q = \alpha$. If this hypothesis can not be rejected, one can pursue by testing for restrictions on β_q, γ_q and δ_q, conditionally upon α_q being equal over the seasons. If the hypothesis is rejected, one can apply the iterative procedure discussed in Franses and Boswijk (1991) for details.

For the estimated version of the model in (3) it is found that the F test statistic for the hypothesis $\alpha_{win} = \alpha_{sum}$ equals 0.290. Conditional upon this equality, we calculate the F statistics for the hypotheses $\mu_q = \mu$, $\beta_q = \beta$, $\gamma_q = \gamma$, and $\delta_q = \delta$. Their values are 6.295, 6.148, 14.86 and 0.732, respectively. Further, we checked for insignificant parameters, and we found that β_{sum} and γ_{sum} can be set equal to zero. The final simplified periodic cointegration model is (where heteroskedasticity–corrected standard errors are given in parentheses)

$$(1 - 0.850B + 0.171B^2 + 0.408B^4\Delta_1)\Delta_4 ur_t = -4.651\Delta_4 y_t + 7.530\Delta_1 w_{t-1}$$
$$(0.086)(0.078)(0.058)(1.017)(1.824)$$

$$+ 6.621D_{win} + 0.471D_{sum} + 1.018DUM751 \qquad (4)$$
$$(2.570)\phantom{D_{win}}(0.170)\phantom{D_{sum}}(0.125)$$

$$-0.112ur_{t-4} - 1.381D_{win}y_{t-4} + 3.445D_{win}w_{t-4} + 0.056ir_{t-4}$$
$$(0.021)\phantom{ur_{t-4}}(0.588)\phantom{D_{win}y_{t-4}}(0.940)\phantom{D_{win}w_{t-4}}(0.022)$$

This model is estimated for 96 observations, and the diagnostic test results for first and fourth order autocorrelation, for autoregressive conditional heteroskedasticity and for normality indicate that this model is adequate. Additionally, a Chow test statistic for parameter constancy over the period 1988.1–1990.4 obtains a value of 1.59.

The White (1980) test for functional form misspecification yields a value of 1.080. An F type LM test for bilinearity caused by $x_{t-1}\hat{\varepsilon}_{t-1}$, where x_t is $\Delta_4 ur_t$, results in a value of 8.420, see Saikkonen and Luukkonen (1988) for details of this test procedure. An LM test for bilinearity caused by variables like $x_{t-1}\hat{\varepsilon}_{t-1}$, $x_{t-4}\hat{\varepsilon}_{t-1}$, $x_{t-1}\hat{\varepsilon}_{t-4}$, and $x_{t-4}\hat{\varepsilon}_{t-4}$, results in 3.274. Hence, it seems that the nonlinear patterns in unemployment are not fully captured by the periodic cointegration model. However, an inspection of the residuals and the unemployment series suggests that this remaining nonlinearity is caused by the three observations in 1982.3, 1982.4 and 1983.1. When dummy variables for these observations are included, the above test statistics for model (4) obtain the values 0.320 and 0.838, respectively. Hence, the nonlinearity seems to be established by these three observations only. The parameter estimation results of (4) when these dummies are included are very similar to those reported in (4).

In summary, the unemployment rate in Canada can be modeled with periodic cointegration. This means that in terms of equilibria there is one equilibrium relation for the winter season, and another one for the summer season. The adjustment to disequilibrium errors however appears to be constant over the seasons.

3. ECONOMIC THEORY

This section proposes a simple but tentative economic model that can explain the empirical phenomena encountered. We base our output equation on the Lucas supply equation, see Lucas (1973). Instead of deviations from the natural rate of output caused by unanticipated price shocks, we assume the output level to deviate from its natural level due to unanticipated real wage shocks. Expected prices are underestimated when real wages exceed anticipated real wages, and output becomes lower than the natural output. We therefore assume a supply equation given by

$$y_t = y_n + \alpha(w_t - E_{t-1}w_t) + \varepsilon_{1t}, \ \alpha < 0 \tag{5}$$

where y_t is the log of output, y_n is the log of the natural level of output, and w_t is the real wage rate. The $E_{t-1}z_t$ denotes the expectation on z_t based on the information set I_{t-1}. In this section, the ε_{it} denotes an uncorrelated zero mean error term for $i = 1,2,..$

From this supply equation, which includes the supply of labor, we can easily derive an equation for the actual rate of unemployment, i.e.

$$ur_t = ur_n + \beta(w_t - E_{t-1}w_t) + \varepsilon_{2t}, \ \beta > 0 \tag{6}$$

where ur_n is the natural rate of unemployment.

To facilitate estimation of an empirical version of (6), some additional assumptions have to be made. Firstly, consider the relation

$$ur_n = \delta + \gamma y_n + \varepsilon_{3t} \tag{7}$$

where the natural rate of unemployment is related to the natural rate of output. Substituting (7) and (5) into (6) results in

$$ur_t = \delta + \gamma y_t + (\beta - \gamma\alpha)(w_t - E_{t-1}w_t) + \gamma\varepsilon_{1t} + \varepsilon_{2t} + \varepsilon_{3t} \tag{8}$$

Further, the formation of expectations has to be considered. For our example of quarterly data, we conjecture that a simple partial adjustment process is appropriate, or

$$E_{t-1}w_t = w_{t-1} + \varphi(w^*_{t-4} - w_{t-4}) + \varepsilon_{4t} \tag{9}$$

where w^*_t is the target real wage rate. This target wage can depend on the actual unemployment rate, such that the Phillips curve effect is represented. Further, it may depend on the actual output level, since an increase in the productivity is likely to lead to an increase in wages, and on the nominal interest rate, see Ashenfelter and Card (1982) and Farmer (1985), or

$$w^*_t = \alpha_1 ur_t + \alpha_2 y_t + \alpha_3 ir_t \tag{10}$$

This equation (10) does not include expectations of the variables, but includes the actual values of the explanatory variables. The reason for this is that it is often found that the wage bargaining process depends on the current economic situation instead of on the expected economic situation, see, e.g, Smith and Wilton (1978), Christofides, Swidinsky and Wilton (1980) and Lang (1991). Substituting (9) in (10) yields an operational form of the expected real wage process

$$E_{t-1}w_t = w_{t-1} + \xi\hat{w}_{t-4} + \kappa u_{t-4} + \lambda y_{t-4} + \mu ir_{t-4} + \varepsilon_{4t} \tag{11}$$

where the ξ through μ are functions of φ and the α_i in (10). Substituting this equation into (8), and rewriting this such that an error correcting equation emerges, gives

$$\Delta_4 ur_t = \delta + \gamma\Delta_4 y_t + (\beta-\gamma\alpha)\Delta_1 w_t \tag{12}$$

$$-(\kappa\beta-\kappa\gamma\alpha+1)[ur + (\frac{\gamma-\lambda\beta+\lambda\gamma\alpha}{\kappa\beta-\kappa\gamma\alpha+1})y - (\frac{\xi\beta-\xi\gamma\alpha}{\kappa\beta-\kappa\gamma\alpha+1})w - (\frac{\mu\beta-\mu\gamma\alpha}{\kappa\beta-\kappa\gamma\alpha+1})ir]_{t-4}$$

$$- \gamma\varepsilon_{1t} + \varepsilon_{2t} + \varepsilon_{3t} - (\beta-\gamma\alpha)\varepsilon_{4t}$$

which is similar to the model in (1).

This error correction model can have seasonal relations like those in (1) when additional assumptions on the parameters, or in fact relaxations, are made. One such relaxation is that the natural rates of unemployment and output are related in seasonally varying way, i.e. δ_s is more appropriate than δ. This may be caused by seasonal employment effects, i.e. there may be more employees available in the summer season and the adjustment costs of hiring or firing these employees may be less important then. A relaxation that can establish that cointegration relations as well as adjustment parameters are periodic is that producers and consumers have expectations that vary over the seasons, i.e. the φ in (9) becomes φ_s. Some empirical evidence of seasonally varying expectations is reported in Franses (1992b). Further, since the wage bargaining process depends on the current economic situation, the parameters in (10) may also be seasonally dependent. Finally, as observed in Smith and Wilton (1978), there are distinct seasonal bargaining patterns in the Canadian labour market. In Saikkonen and Teräsvirta (1985) a similar finding is reported for the Finnish bargaining process. These authors also fit a model like (4), but they do not test for periodic cointegration. Depending on the magnitudes of ξ_s, κ_s, λ_s and μ_s when they replace the parameters in (11), in practical occasions, one then may or may not be able to find these parameters to be varying over time. Seasonal heteroskedasticity can be established when α, β and/or γ vary per quarter. Summarizingly, there appear to be several economic motivations for the periodic cointegration model.

4. CONCLUSIONS

In this paper we propose a cointegration model for the quarterly unemployment rate in Canada, in which the parameters in the equilibrium relation vary over the winter and the summer season. An implication of this model is that shocks in the explanatory variables in the winter season have different long-run effects than those occurring in the summer season. Hence, this model can generate time series which show asymmetric cyclical patterns. The empirical results reported indicate that the periodic model does not remove all nonlinearity, although it turns out that this nonlinearity can be attributed to only a few observations. An economic theoretical model that can explain the observed empirical success of periodic cointegration is also discussed.

Given that quarterly unemployment series may also show nonlinear patterns in other countries, a natural topic for further research is to investigate whether more periodic cointegration relations can be found. Another issue is to evaluate the forecasting performance of the periodic model related to alternative models for nonstationary seasonal time series. Finally, since the assumption of periodically varying equilibrium relations implies that seasonal adjustment methods can destroy these relations, it may be worthwhile to study the effects of modelling and forecasting when the time series are seasonally corrected.
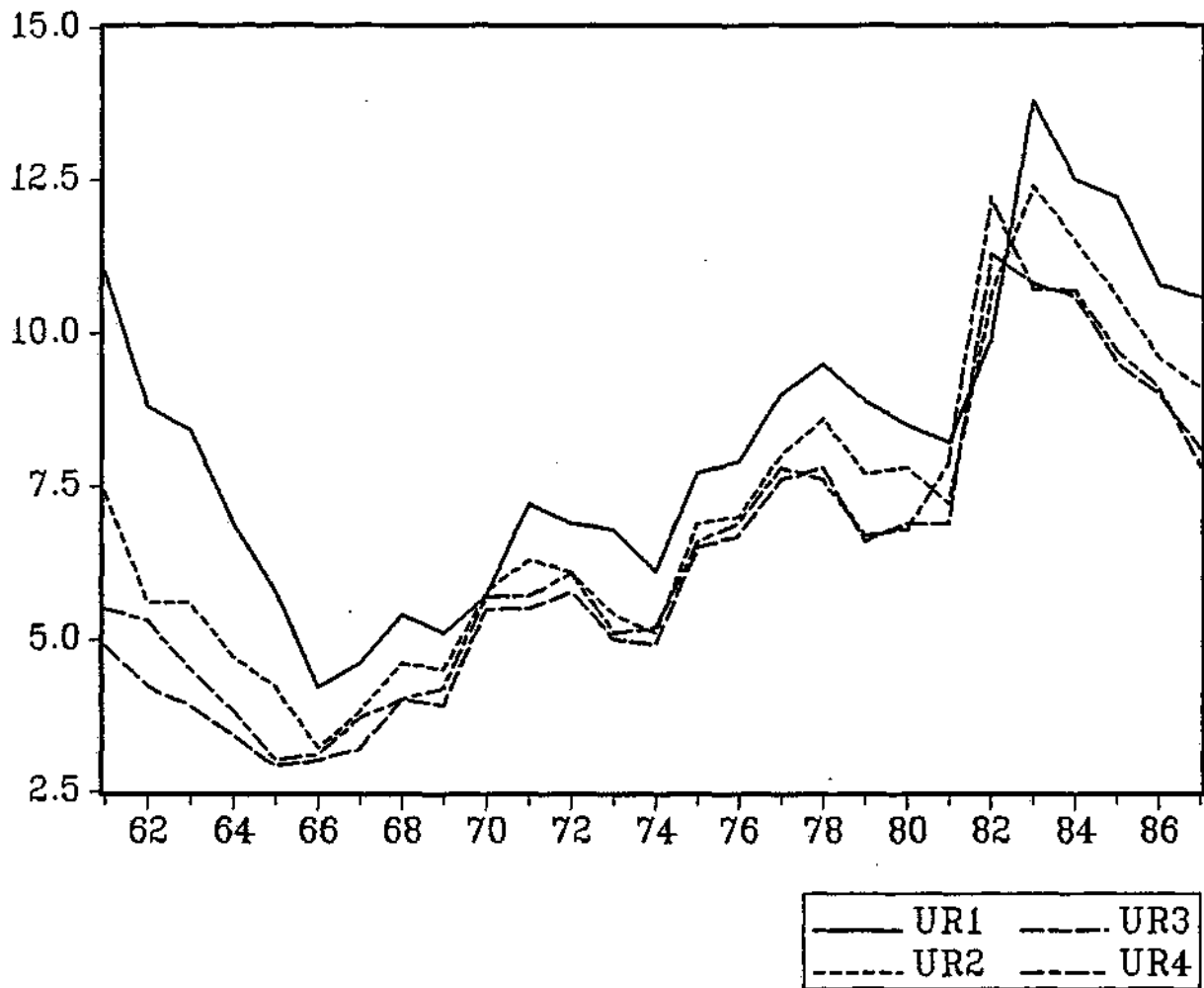
Figure 1. The Canadian unemployment rate per quarter, 1961.1–1987.4
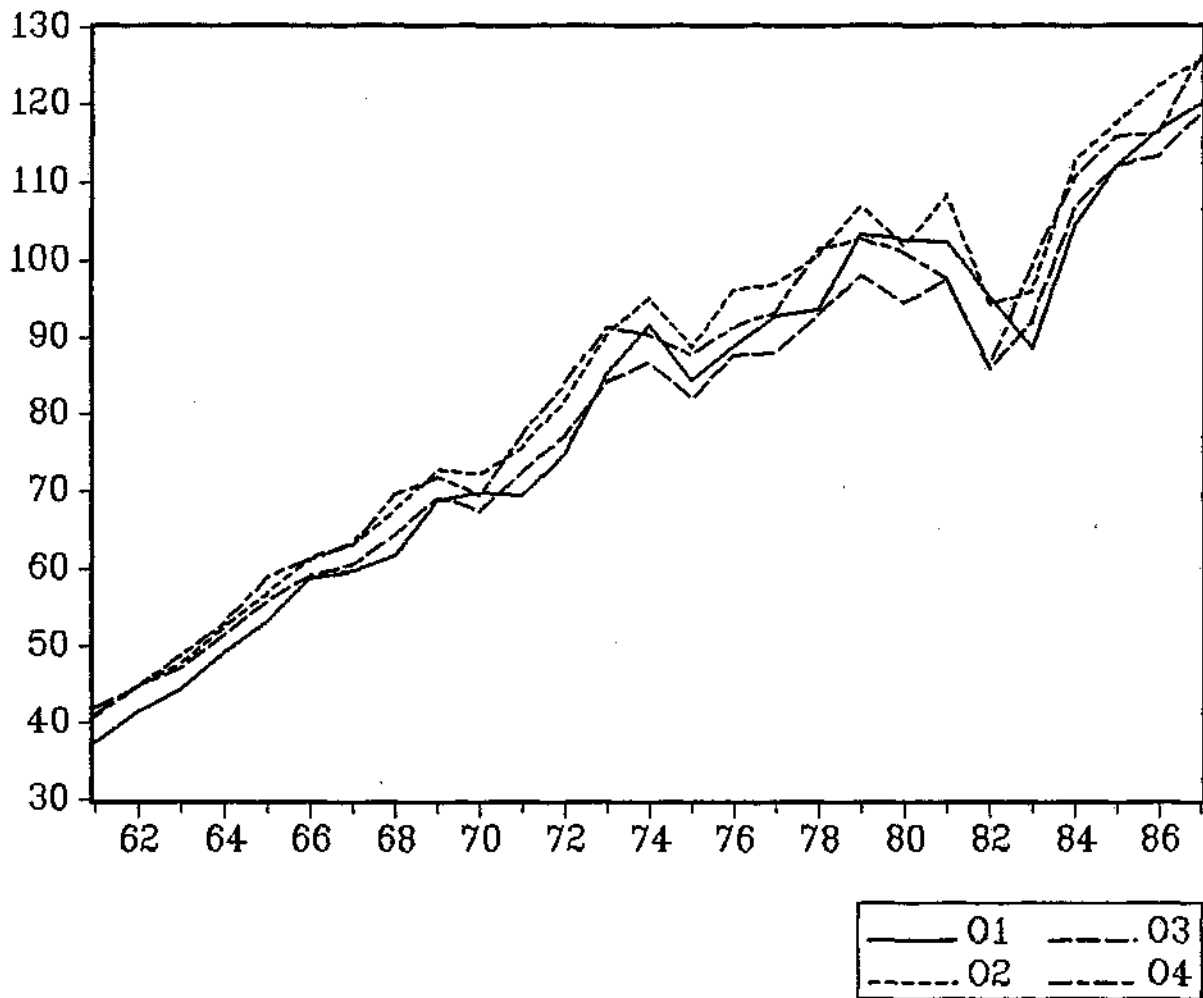
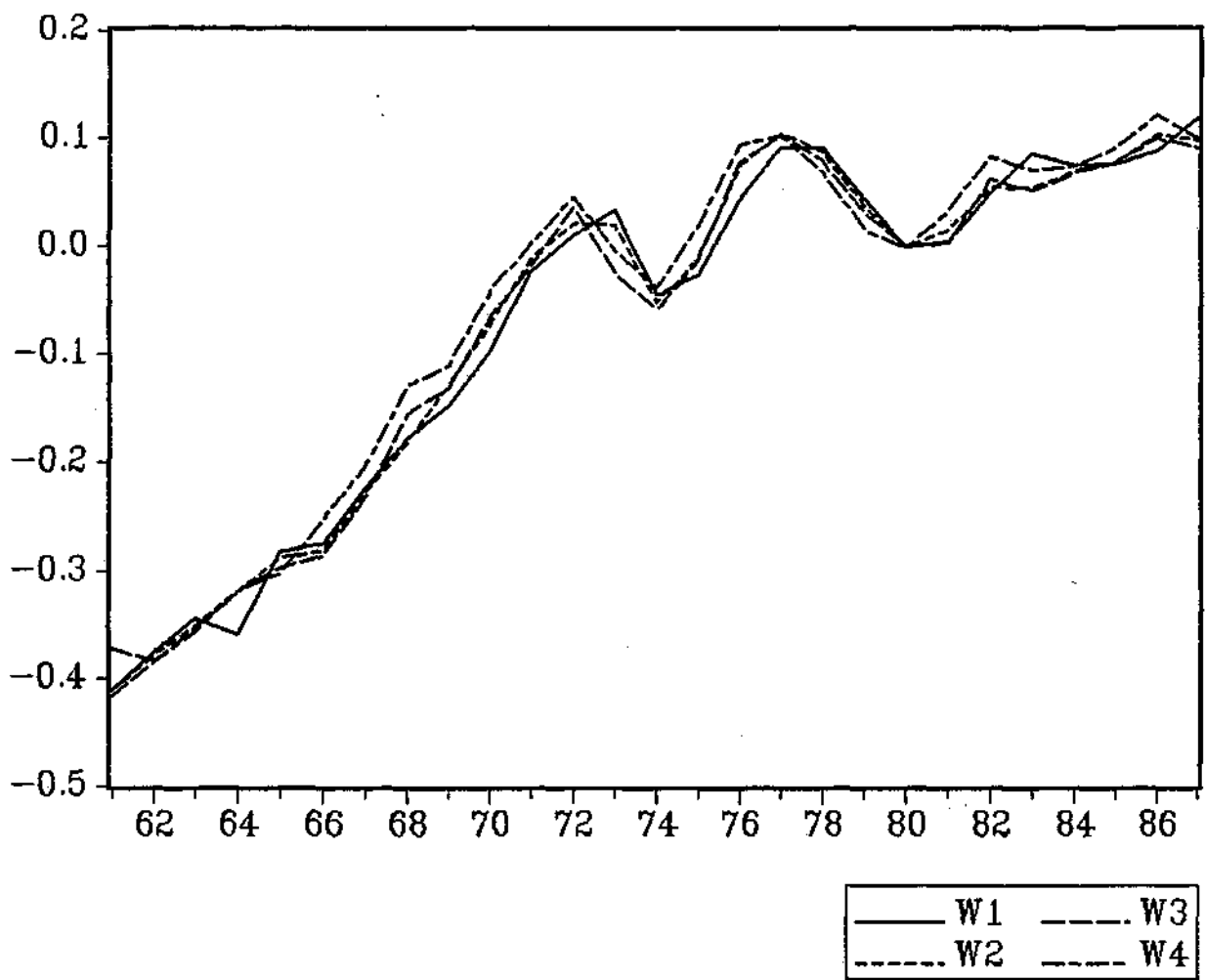Figure 2. The industrial output per quarter, 1961.1-1987.4

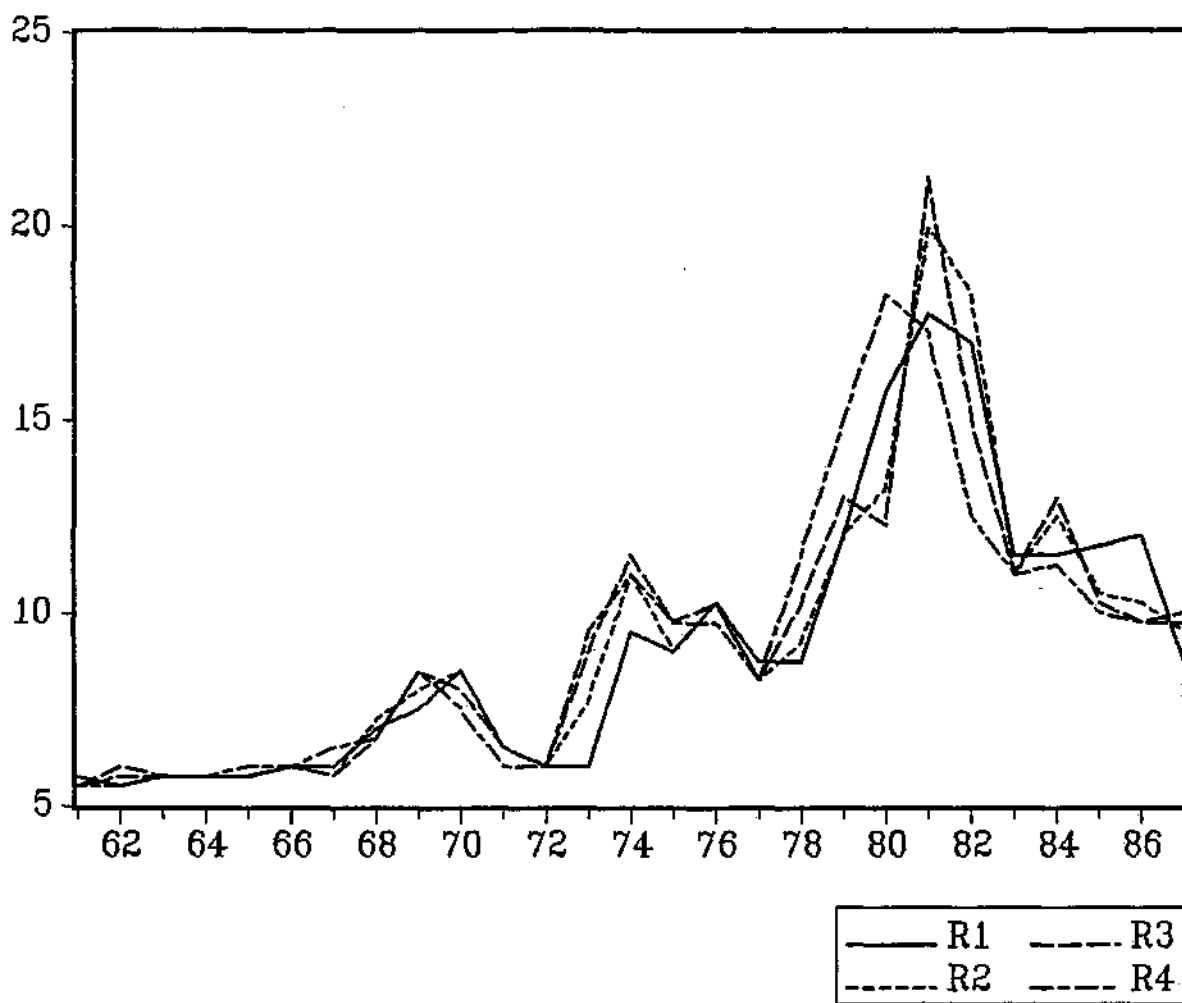Figure 3. Real wages per quarter, 1961.1-1987.4

Figure 4. Interest rate per quarter, 1961.1-1987.4

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