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THE RISE OF NEW CLASSICAL ECONOMICS

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1. INTRODUCTION

From the 1970s onwards the New Classical research programme attracted much attention among economists. Its assumptions and theoretical conclusions evoked much controversy, which resulted in an extensive literature on the methodology of the NCE. The earliest analysis has been that of Maddock (1979) and (1984), who depicted the development of the New Classical research programme and provided a reconstruction along Lakatosian lines. He argued that the initial goal of the 'Rational-Expectations theorists' was to test the Natural Rate Hypothesis. However, in the mid 1970s the programme experienced a problem shift towards the theory of economic policy, and more in particular towards the neutrality proposition. A second approach in the discussion of the New Classical methodology is that of Klamer (1984), who concentrated on the style of argument (rhetorics) of the NCE. He also opined that the central claim of this programme is the neutrality proposition. Anticipated changes in the money supply will not affect real output and unemployment. It thus appears that in this regard he concurs with Maddock. Boland's (1986) critical analysis of the so-called disequilibrium foundation of equilibrium economics is the third discussion of the NCE, although its object is not restricted to this research programme. However, his criticisms of what he called Macroeconomics, Rational Expectations, Stochasticism, and Instrumentalism directly apply to the NCE. This also applies to his criticisms of the assumption of continuously clearing markets. A fourth analysis of the NCE was provided by Hoover (1988), who made a comparison between Friedman's and the New Classical methodology. He concentrated on the former's partial and the latter's general equilibrium analysis, and concluded that Friedman's economics is Marshallian in nature, whereas the NCE is a Walrasian type of monetarism. Hoover's analysis was criticized by Hirsch and De Marchi (1990). The latest contribution has been that of De Marchi (1990). He makes clear that Maddock's appraisal of Lucas's 'Expectations and the neutrality of money' (1972a) is incorrect, because it was written at a much earlier date than Maddock assumed. According to De Marchi (1990), it was written in 1970, which means that it should be interpreted as 'defining' the positive and negative heuristic of the

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NCE. That is, it set the modelling style.

Given the propensity of economists and economic methodologists to differ of opinion, one might expect that the various studies of the emergence and development of New Classicism would inevitably lead to differences concerning the central claims of this research programme. However, there has been a remarkable agreement on these claims, although emphasis differs. Given this large degree of agreement, the conclusion seems justified that there is a consensus as regards the solution of the problem of definition (or classification) of the NCE. The NCE can be defined as a scientific research programme which incorporates the following assumptions:

(1) the assumption of continuous market clearing;

(2) the Lucas supply function (including some version of the NRH);

(3) some version of the Rational Expectations Hypothesis; and

(4) some assumption about the information set of the individuals.[2]

This paper will analyze what versions of these assumptions were used by the early contributors to the programme. The analysis will be restricted to the work of Robert E. Lucas, Jr., Leonard A. Rapping, and Thomas J. Sargent.

The paper is organized as follows. Section 2 discusses the formalization of the Natural Rate Hypothesis (NRH) by Lucas and Rapping (1969a and 1969b). They adopted a partial-equilibrium model in which the concept of the 'representative individual' was incorporated. Such a framework is clearly incapable of analyzing economy-wide interdependencies. Phelps (1970) opined that partial-equilibrium analysis could not be used to account for the negative slope of the Phillips Curve. He introduced instead the so-called 'islands parable', which holds that an economy consists of markets which are imperfectly linked both physically and informationally. Moreover, Lucas and Rapping used the AEH, which is in danger of being inconsistent with the rationality postulate. This problem was solved by substituting the Rational Expectation Hypothesis (REH) for the AEH. The former will be discussed in section 3. The change from partial-equilibrium to general-equilibrium framework is addressed in section 4. Section 5 subsequently discusses an econometric issue which arose because of the substitution of the REH for the AEH. The REH implied that economic agents would include economic policy in their expectations. An anticipated change in policy then results in a change in expectations, and therefore in a change in behaviour. Lucas criticized 'orthodox' methods of policy evaluation because they neglected these behavioural changes. The exposition of the so-called 'Lucas critique' is followed in section 6 by a discussion of Lucas's attempt to test the NRH empirically. This

[2] Cf. also Barro (1981b, p. 41).

attempt evaded the issues concerning econometric policy evaluation which Lucas himself had raised. In contrast, Sargent's empirical tests, which were performed at about the same time as that of Lucas, did take the Lucas's critique into account. Before discussing these tests, a short detour will be made by outlining their 'prehistory'. This discussion of Sargent's early work will make clear that Friedman's NRH amounts to the same thing as Fisher's long-term framework of his solution to the Gibson paradox. Section 8 contains some conclusions.

2. THE FORMALIZATION OF THE NRH

The first step towards New Classical Macroeconomics was made by Lucas and Rapping (1969a).[3] They attempted to provide "... the rationalization in supply-and-demand terms of the observed correlation between unemployment rates and rates of inflation, or Phillips curve" (p. 20).[4] Friedman's and Schwartz' (1963 (1969)) 'tentative sketch' had already given this rationalization in verbal terms. Moreover, Phelps (1967) had shown that the Adaptive Expectations Hypothesis could be used as the transition mechanism from the short-term to the long-term Phillips Curve. However, he had not been able to include both types of curves in his mathematical model. Lucas and Rapping (1969a) tried to solve this problem by formalizing the NRU in such a way as to allow for short-term deviations from this rate. In their view, "... an adequate model must contain both a short run and a long run. There are, then, three features which we feel a model of the labor market (or, more broadly, the production-employment sector) should possess. First, it should incorporate the neoclassical feature that for fixed capital stock the aggregate supply schedule (relating the price of goods to real output) will become perfectly inelastic over a long period of stable aggregate demand. Second, the model should imply an elastic short-run aggregate supply function consistent with the observed fluctuations in real output and employment in the face of shifting aggregate demand. Finally, the transition from short-run to long-run market equilibrium should be described in full" (pp. 20 - 21). In other words, the vertical long-run Phillips Curve (i.e. the NRH) was accepted as the relevant framework in which to model the short-run Phillips Curve. The model which they used was a partial-equilibrium,

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[3] Page numbers refer to the 1981 reprint of Lucas and Rapping (1969a).

[4] They were clearly dissatisfied by "[r]ecent attempts to give a theoretical base to the Phillips curve [which] have been based largely on a view of the labor market as dominated by collective bargaining, where bargaining outcomes bear no explicit relation to supply-and-demand forces" (p. 20).

perfect-competition model of the labour market. Its most important aspect is the aggregate supply function, which has also become known as the *Lucas supply function*.[5] This function describes long-term as well as short-term output relations. As Lucas's and Rapping's labour demand function is a fairly straightforward application of the marginal productivity theory, the discussion here will concentrate on their supply function.[6]

Lucas and Rapping (1969a, p. 24) formulated their theory of the supply of labour in terms of the choice between goods and leisure, facing a single household (or individual) in a competitive market. The household's supply of labour was assumed to depend on current nominal wages, current prices, the present value of future real wages, the present value of future prices, the real rate of interest, and initial asset holdings (non-human wealth). By assuming homogeneity of degree zero in all arguments Lucas and Rapping subsequently deflated the nominal variables by current prices. The formulation of the household's supply function of labour reflects that it must not only choose between current goods and current leisure, but also between current labour supply and future labour supply. This means that there can be substitution between present labour supply and leisure, and between present and future labour supply.

An important step in Lucas's and Rapping's (1969a) analysis was to interpret the household as a *representative household*.[7] In this manner the household's supply of labour could be transformed into an aggregate supply of labour. But this procedure also entails an important narrowing of the problems which can be analyzed. By using the concept of the representative household Lucas and Rapping implicitly assumed that all individuals belong to the same 'class' and face similar optimization problems (which do not differ from each

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[5] Lucas (1981a, p. 5) himself gave credit to Rapping by stating that its proper name should have been the Lucas-Rapping supply function. In the literature on the New Classical Economics, however, it is called the Lucas supply curve. For several versions of this curve, see Minford and Peel (1981).

[6] Lucas's and Rapping's (1969a) labour demand function is based on an aggegate production function with constant elasticity of substitution, with constant returns to scale, with labour-augmenting technological change, and with a homogeneous real capital stock. Additionally, it is assumed that labour is a freely variable input, i.e., there are no adjustment costs in varying labour input.

[7] The concept of the 'representative entity' was first used by Marshall (1890 (1947)) who used the 'average firm', although not in a mathematical sense. In his view, "... a Representative firm is that particular average sort of firm, at which we need to look in order to see how far the economies, *internal and external*, of production on a large scale have extended generally in the industry and country in question" (pp. 317 - 18, italics in original). It is not a fictituous entity as "... we can see it fairly well by selecting, after a broad survey, a firm ... (or better still, more than one), that represents, to the best of our judgment, this particular average."

other in relevant aspects). The concept is a hypostatisation in the sense that it treats aggregates and index numbers as if they obey the principles of microeconomics. It is implicitly defined as the mathematical mean of the group (or subsystem) as a whole. In turn, this means that it does not allow for distributional effects. This emphasis on aggregates disregards their composition and its changes. The effects of redistributions between economic agents fall outside the scope of analysis. Furthermore, the problem of how coordination is brought about cannot be analyzed. Lucas and Rapping (1969a, p. 21) solved (or rather circumvented) this problem by assuming competitive labour markets in which a 'Walrasian auctioneer' ensures that equilibrium will prevail.

The Lucas supply function which was used by Lucas and Rapping (1969a) assumed that the aggregate labour supply depends on the current real wage rate (w_t), the anticipated real wage rate based on information available at time t (w_t^*), the nominal wage rate (r_t), and the deflated market value of initial non-human wealth (a_t).[8] In contrast to Friedman (1957), Lucas and Rapping (1969a) eliminated the latter (wealth) effect from their labour supply function. On empirical grounds they considered it to be small.[9] In other words, only changes in prices and wages are presumed to cause fluctuations in unemployment.

Following Friedman's (1957) Permanent Income Hypothesis (PIH), Lucas and Rapping (1969a, p. 26) interpreted the (current and expected) real wage rate as consisting of two components, namely a permanent and a transitory one. The representative household bases its labour-supply decision on some notion of its 'normal' real wage rate.[10]

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[8] Lucas and Rapping (1969a, p. 26) stated this labour supply function in mathematical form as:

$$\ln (N_t/M_t) = \beta_0 + \beta_1 \ln (w_t) - \beta_2 \ln (w_t^*) + \beta_3[r_t - \ln (P_t^*/P_t)] - \beta_4 \ln (a_t/M_t)$$

where N_t is the aggregate labour supply in total man-hours per year in period t, and M_t is an index of the number of households in the same period. The other symbols represent the variables as explicated in the text. As is argued below, Lucas and Rapping excluded the wealth component (i.e., they assumed that $\beta_4 = 0$).

[9] Lucas and Rapping (1969a, p. 50n12) admitted that the available evidence is not unanimous in this regard. When reflecting on his 1969a paper with Rapping, Lucas (1981a, p. 3) also gave a theoretical reason for excluding the asset effect. He argued that reduced employment caused by wealth increases is perceived as a positive phenomenon, whereas reduced employment in depressions are seen as negative phenomena. This means that a fall in the supply of labour which is induced by a rise in wealth cannot constitute *cyclical* unemployment.

[10] This means that the unemployed households are not presumed to know the prevailing real wage rate.

This rate is 'measured' by the wage-rate expectation of the representative household, w_t^*. Given the fact that this household is defined as the mathematical mean of the system as a whole, this means that the economy-wide permanent or 'normal' real wage rate is identical to the expected real wage rate. The transitory component indicates the degree in which the current real wage rate differs from the permanent rate ($w_t - w_t^*$). If the former is higher than the latter, then the supply of labour will be increased, because the households will intertemporally substitute their labour activities. Of course, the reverse also applies.

The transitory component in the real wage rate and in the general price level, and hence the cyclical changes in the supply of labour, can only be determined if the permanent component is known. The latter is reflected in the household's expectation, and hence an assumption about the household's expectations formation mechanism is required. Lucas and Rapping (1969a, pp. 27 - 28) adopted the Adaptive Expectations Hypothesis (AEH). It enabled them to reformulate their labour supply function into an unemployment-rate function, which expresses deviations from the permanent (or 'normal') rate of unemployment in terms of deviations from the permanent real wage rate and permanent price level.[11] This implies some sort of money illusion on the part of the economic agents. After all, a change in the general price level would lead to a change the unemployment rate. This property follows from the assumption that the agents form their expectations adaptively. Given the AEH, the NRH will only hold in the long run.

The analysis of Lucas and Rapping (1969a), and more in particular their version of the AEH and the consequent money illusion, is in danger of being inconsistent with the rationality postulate. In a second article, which was published in 1969, they acknowledged that their version of the AEH was based on "... an unreasonable stubbornness on the part of the households: if a sustained inflationary policy is pursued by the governments [i.e. if the rate of inflation continues to increase], households following the adaptive rule will continue *forever* to underpredict future prices" (p. 344, italics in original). They changed it by choosing a lag structure which fitted their data best. Thus, this new lag structure was *not* imposed on *a priori* grounds, but instead was inductively derived (cf. Maddock (1979, p. 198)), and could therefore be called 'ad hoc'.

Lucas and Rapping (1969a and 1969b) had thus formalized the idea of Friedman (1968) and Phelps (1967) that the negatively sloped Phillips Curve reflects short-term expectational errors. In the long run expectations will be correct, thus making the long-

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[11] Lucas and Rapping (1969a, p. 35) formalized this function as follows:

$$U_t = \beta_0 - \beta_1 \ln (w_t / w_{t-1}) - \beta_2 \ln (P_t / P_{t-1}) + \beta_3 U_{t-1} + u_t,$$

where β_1 and $\beta_2 > 0$, $0 < \beta_3 < 1$, and u_t is a random error term.

term Phillips Curve vertical. Furthermore, their empirical tests had indicated that "... statistical Phillips curves are highly unstable over time, and this instability is far too serious to be dismissed by a vacuous reference to structural change ..." (1969b, p. 349). They concluded that the curves "... are a weak foundation on which to base policy decisions" (p. 349), thus dismissing the Samuelson-Solow suggestion that the curves reflect exploitable trade-offs. This constituted a formidable attack on 'orthodox' stabilization policy. However, their results were not generally accepted.

Lucas's and Rapping's analysis was criticized on several accounts. Firstly, Rees (1970) criticized their assumption that labour markets continuously clear. He argued that economic agents will only withdraw their labour supply if they expect the current real wage rate to be temporarily below its 'normal' level. However, he did not ascribe such views to the unemployed during a prolonged depression, such as the Great Depression in the 1930s. He rejected the model's implication that "... unemployment arises from the recalcitrance of suppliers and not from deficiencies in aggregate demand" (p. 309).[12] In a reply Lucas and Rapping (1972, p. 60) argued that in their 1969b model real GNP *does* influence unemployment. They acknowledged that it does not enter in the unemployment rate function, but as this function is one of three structural equations in a three-equation system, it is also necessary to take the other equations into account. It then becomes clear that unemployment is a function of real wages, and that real wages are a function of real GNP per household.[13] In turn, they concluded, unemployment is also determined by real GNP. But when they concentrated on the effect of lagged unemployment on actual unemployment, it turned out that for the period 1930 - 1945 the persistence could not be fully explained by price expectations behaviour.[14] Thus Lucas's and Rapping's (1972) model was

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[12] Rees (1970, p. 309) also pointed out that Lucas's and Rapping's use of the term 'Phillips curve' is somewhat misleading. Phillips had assumed that the direction of causation runs from unemployment to changes in nominal wages. Lucas and Rapping followed Friedman's reinterpretation and reformulation by considering the relationship between unemployment and real wages, with causality running from the latter to the former.

[13] See Lucas and Rapping (1969b (1981)), equations (27) and (17) respectively.

[14] As Lucas and Rapping (1972, p. 62) acknowledged, "[b]y 1934 ... actual wages and prices had returned to their normal levels. .... Whatever the cause, the expectations model we used implies that in 1934 the unemployment rate should have been at its 1929 or 1930 level, as opposed to the observed 22 percent level. To make matters worse, our theory continues to 'miss' for the remainder of the depression years, accounting for the observed drift towards lower unemployment during this period but failing entirely to explain why this drift proceeded at so slow a pace."

confronted with an anomaly.[15] In his 1973 test Lucas would circumvent this anomaly by incorporating lagged income as an explanatory variable in the aggregate supply curve.[16]

The second problem which arose from Lucas and Rapping (1969b) concerns the framework in which they had conducted their analyses. As they themselves already acknowledged, their models were partial-equilibrium models, which means that the general interdependencies in the economy were disregarded. In his later work Lucas would solve this problem by adopting Phelps's (1970) 'islands parable'.[17]

Thirdly, Lucas and Rapping had used the AEH, which (even in its inductively improved version) remained unsatisfactory from a theoretical point of view because it is in danger of being inconsistent with the rationality postulate. Therefore, Lucas would substitute it by Muth's (1961) Rational Expectations Hypothesis (REH). However, the adoption of Phelps's parable would change the content of the REH. In time it would lead to Lucas's critique of standard methods of econometric policy evaluation.

3. THE RATIONAL EXPECTATIONS HYPOTHESIS

Presumably the best-known characteristic of New Classical analysis is the *Rational Expectations Hypothesis* (REH). Muth (1961, p. 315) had advanced the hypothesis that expectations are essentially the same as the predictions of the relevant economic theory. Or, as he formulated more exactly, "... expectations of firms (or, more generally, the subjective probability distributions of outcomes) tend to be distributed, for the same information set,

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[15] Favourably interpreted in a Lakatosian scheme, the reply by Lucas and Rapping is theoretically progressive as it contained an implicit recognition of Sargent's 'observational equivalence'. It implicitly predicted that the data cannot determine whether the appropriate model is an equilibrium or a disequilibrium model. However, if any *implicit* prediction is not recognized as a real (unexpected) prediction, then the reply is merely a defensive reply to Rees's criticism, and hence theoretically degenerative in a Lakatosian sense. But it should be recognized that Lakatos's framework does not capture an important element of economics as a competitive and hence communicative process, namely its rhetorics (including its attacks on other research programmes).

[16] The test itself will be discussed in more detail in section 6.

[17] Lucas (1981a, p. 6) opined that "[t]he best thing that happened to Rapping's and my [1969a] paper was that Edmund Phelps came across it and a number of related papers by others at a time when he himself was working on similar problems. ... Rapping and I had been thinking in sectoral terms typical of at least the more econometrically oriented macroeconomic tradition. We viewed ourselves as constructing a model of the 'wage-price sector', potentially suitable for combining with other models of other 'sectors' to provide a model of the entire economy. ... Phelps, as is evident from his introductory essay to the volume, was thinking in general-equilibrium terms ..."

about the prediction of the theory (or the 'objective' probability distributions of outcomes)" (p. 316). He argued that if economic theorists could predict better than the economic agents, they must have superior foreknowledge of the predicted outcome. But if they have, then the question arises why some entrepreneur does not study economics in order to sell his better predictions? After all, it would be rational for optimizing agents to use economic theory if this would lead to (higher) profits.[18] By linking economic theory and economic practice, Muth took a modest stand for economists with regard to the predictive powers of their theories. However, if a theorist wants to obtain definite outcomes of the assumed expectations formation mechanism (as Muth did), then he must specify a 'correct' model of the economy. Furthermore, he must make assumptions with regard to the content of the agents' information sets. Therefore, Muth (1961, p. 317) used ("[f]or purposes of analysis ...") a 'specific form' of the REH in a partial-equilibrium analysis, in which he assumed that the random disturbances are normally distributed. Individuals were also assumed to know these distributions, which means that on average their expectations are correct. That is, the expectations of the 'representative individual' are correct, and thus must be based on all relevant information. As will be shown in the next section, Lucas (1972a) transposed the REH to a general-equilibrium context, which has an important implication with regard to the individuals' information sets. This will be discussed in the next section, but first we must distinguish between types of REH.

The general formulation of the REH leaves room for several interpretations. A taxonomy may be formulated which discerns a strong and a weak form of the REH.[19] Both forms assume different information sets. The strong form holds that the representative individual knows at least (i) the 'true' structure of the model economy, (ii) the 'true' values of the parameters in that economy, and (iii) the past values of the relevant variables.[20] Stated differently, the strong form holds that individual expectations (taken separa-

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[18] In McCloskey's (1985, p. 88) words, "[t]he claim to know how economic actors predict ... runs up against the American Question ...: If you're so smart, why ain't you rich?"

[19] The strong form can be divided into three version, namely the descriptive, the micro-instrumentalist, and the macro-instrumentalist version. For an explication of the respective assumption about the agents' information sets, see Snippe (1986 - 87).

[20] Cf. Grossman (1980, p. 10). Runde and Torr (1985, p. 220) argued that the use of the concept of the 'representative individual' means that possible differences between individuals are disregarded. These differences may concern (1) their abilities to gather and process information, and/or (2) their forecast functions. Some individuals may not be able to gather and process information as efficiently as others, or (given their information) they may not be able to predict as accurately as others. Under these circumstances, there seems

tely) may be *in*correct, but that they are correct *in the aggregate*. The expectational errors are supposed to cancel out, presumably due to the 'Law of Large Numbers'.[21] As Muth (1961, p. 316 - 17) assumed that in the aggregate individuals do not make larger expectational errors than economic theory, his version of the REH may best be represented as a strong form. The weak form of the REH is merely a restatement of the rationality postulate. Individuals are assumed to optimize the information on which they base their decisions. Obviously, if information is not costless, the optimal information need not be sufficient to lead to Muth-rational ('correct') expectations. Moreover, the weak form may suffer from an indeterminacy, as it may not be possible to determine the optimal 'amount' of information. Information optimization presupposes expectations about its marginal returns and costs. However, expectations (being informed predictions) already presuppose information. In other words, expectations are needed in order to optimize information, while information is needed to form expectations. This may appear to be a problem of circularity, but it is not. It is a problem of infinite regress, because the information needed for expectations formation is of a different kind than the information for which the expectations are needed. Therefore optimizing information involves infinite regress, leaving the optimality of the information gathered by the individuals unexplained and hence undetermined. This can also be phrased in terms of the Shackle-Boulding paradox, which

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to be no reason to presume why these errors should cancel out. The New Classical models under consideration do not allow individuals to have such different abilities and/or functions, because their analysis exclusively deals with the 'representative individual'. Stated differently, New Classicals presume that individuals belong to the same class, in the sense that they do not differ from each other in any relevant aspect.

[21] Haltiwanger and Waldman (1989, p. 620) emphasize that a situation in which expectations are rational in the aggregate (because deviations from the average expectation cancel out) may differ from the situation in which all individuals hold rational expectations (i.e. in which there are no deviations from the mathematical mean). The difference between the respective rational expectations equilibria (REE, see below) depends on the nature of interaction between economic agents. They identify two types of interaction, namely 'congestion' and 'synergism'. The former is defined as the situation in which the larger the number of participants in a given activity, the lower is the incentive to participate (e.g. career choice). Synergism is defined as the opposite situation, in which the larger the number of participants, the higher the incentive to participate (e.g. the choice which currency to use in international trade). They concluded that "[o]nly under very special condition do standard [i.e. individual] rational expectations and aggregate rational expectations yield equivalent results. The difference between the two equilibria is larger when: (i) the divergence in expectations under aggregate rational expectations is increased; (ii) in a world which exhibits congestion, the severity of the congestion is decreased; (iii) in a world which exhibits synergism, the severity of the synergism is increased; and (iv) the activities exhibit synergism rather than congestion" (p. 631). New Classicals disregard these interactional effects by restricting their analyses to the representative individual.

holds that any planned (rational) knowledge acquisition process presupposes *un*deliberately acquired knowledge.[22] Hence the economics of information, as originated by Stigler (1961), fails to make clear how the marginal benefits of information can be equated with the marginal costs. In this sense the REH presupposes the existence of non-deliberate search procedures which it cannot explain. At the same time, and despite this drawback, it also appears to be the most sophisticated and least ad-hoc formalization of an expectations formation mechanism.

The content of rational expectations crucially depends on the model in which the REH is incorporated. The hypothesis cannot stand on its own, because otherwise it cannot be asserted which expectations are rational. It must therefore be analyzed in a model.[23] Muth's (1961) model had been a partial-equilibrium model, and so were those of Lucas and Rapping (1969a and 1969b). In two subsequent papers Lucas adopted a general-equilibrium framework, and more in particular, Phelps's 'islands parable', instead.[24]

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[22] Cf. Shackle (1952 (1955), pp. 17 - 18), Boulding (1966, p. 146), and Kirzner (1976 (1979), p. 142). The latter used the term 'Shackle-Boulding paradox'.

[23] This led Buiter (1980, p. 38) to conclude that "[t]he hypothesis appears to be in danger of being consistent with any conceivable body of empirical evidence, because the assumption of optimal use of the available information cannot be tested independently of an assumption about the available information set." Buiter's *irrefutability critique* (as Kamath (1989, p. 222) labeled it) amounts to the view that the REH is irrefutable, and hence non-scientific in a Popperian sense. Stated differently, the REH may be considered to be a part of the New Classical hard core, whereas the information set underlying the expectations belongs to the protective belt. It should be noted that Buiter's critique presumes that a single hypothesis can be tested. This runs counter to the Duhem-Quine thesis, which holds that only combinations of hypotheses can be tested, and that therefore crucial tests of isolated hypotheses cannot exist. This thesis seriously weakens Buiter's critique.

[24] It should be noted (as De Marchi (1990) did) that the articles referred to were already written in 1970, although they have been published as late as 1972. Presumably the first, entitled 'Expectations and the neutrality of money', was received by the *Journal of Economic Theory* on September 4, 1970, whereas the second, 'Econometric testing of the Natural Rate Hypothesis', was presented at a conference on October 30-31, 1970. Given the years of publication, we shall refer to them as Lucas (1972a) and (1972b) respectively. Although there is hardly any indication (except the dates mentioned) as to which paper was written first, the order given above will be maintained. The reason for this is a logical one. Lucas (1972a) tried to formalize Phelps's parable, using the REH, whereas in his (1972b) he addressed some (econometric) difficulties in testing models which include this hypothesis. As model specification preceeds empirical testing, Lucas presumably formulated the model before directing his attention towards the problems inherent in testing it.

4. FROM PARTIAL TO GENERAL EQUILIBRIUM

In his 'Expectations and the neutrality of money' (1972a, written in 1970) Lucas tried to give "... a simple example of an economy in which equilibrium prices and quantities exhibit what may be the central feature of the modern business cycle: a systematic relation between the rate of change in nominal prices and the level of real output" (1972a, p. 66).[25] In order to discuss this Phillips-Curve relationship he formulated a mathematical model of an economy in which individuals do not have 'money illusion'. The rationality postulate is adopted as an *a priori* principle. Moreover, prices are assumed to be perfectly flexible and market-clearing, and individuals form their expectations according to the REH. In this setting the fluctuations in real output (and employment) must be attributed to the fact that individuals have incomplete information. Furthermore, Lucas (1972a) assumed that individuals are price-taking agents, which means that a single agent cannot influence (or rather, has an infinitely small effect on) the price on his market. This assumption rules out any price-setting behaviour, which means that Lucas's model needed to specify a mechanism by means of which disequilibrium prices are changed into equilibrium (market-clearing) prices. He 'solved' this coordination problem by assuming that "[w]ithin each market, trading by auction occurs, with all trades transacted at a single, market clearing price" (p. 68). The 'Walrasian auctioneer' thus ensured that all prices are completely flexible, and that all markets clear continuously. The resulting general equilibrium does not imply that the actions are ex-post optimal in some absolute sense. That is, it does not need to be a *Natural Rate Equilibrium* (NRE) which is characterized by perfect knowledge and perfect foresight. The actions may well prove to be ex-post non-optimal, because in the process of acting individuals have obtained new (better) knowledge. Such a general equilibrium concept is called a *Rational Expectations Equilibrium* (REE).[26] It merely holds that, *given their information sets*, individuals form (ex-ante) optimal (and rational) expectations.

Given the REH, the content of the expectations is determined by the information which is contained in the individuals' information sets. Lucas (1972a, p. 68) postulated a two-period overlapping-generations model in which the individuals belonging to each new generation are stochastically distributed over two markets, with fraction $\Theta/2$ going to one

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[25] Page numbers refer to the 1981 reprint of Lucas (1972a).

[26] According to Radner (1989, p. 317), the REE is characterized by the fact that individuals learn from price changes. That is, in the REE individual decision-makers take account of 'the potential informational feedback of prices'.

and 1-(Θ/2) going to the other market. The allocation variable Θ is unknown, except indirectly via prices. After the distribution of agents over both markets, no communication is possible between the markets. There are three goods: labour, output, and money. Output is only produced by the young, and it cannot be stored. Neither output nor money can be inherited. In such a framework, exchange is only possible between the young and the old. The former sell their output against the money which the old have held over from the preceding period (p. 68). As in Phelps's parable, the agents must act on the basis of incomplete global and complete local information. Moreover, the former is lagged with one period. In short, information is heterogeneous across markets. As the agents are presumed to be identical (p. 68), information is homogeneous across agents on the same market. Money (m) is assumed to be the exogenous, primary variable. Its supply ($m_1 = m_0 x$, where m_1 applies to period 1 and m_0 to period 0) changes at random. Like the changes in Θ, the actual changes x are also indirectly known through changes in the general price level, although again this information will only become available in the next period. Both Θ and x are serially independent, which means that past deviations from their mean do not reveal any information as regards current and future deviations. Lucas (1972a, p. 73) assumed that the individuals know the 'true' probability distributions of changes in the money supply, and of changes in the allocation of individuals over both markets. The agents thus know the 'true' distributions of monetary as well as real disturbances.[27] Moreover, as the global information of the previous period is known, agents also know m_0 (presuming that they form expectations about m_1).[28] In conclusion, "... the state of the economy in any period is entirely described by three variables m, x, and Θ." That is, the price p can be expressed as a function of (m, x, Θ). The dynamic behaviour of the economy as a whole can then be obtained by describing the successive constellations of these three 'state variables'. The economic agents form their expectations on the known distributions of these variables. The state variables fluctuate at random, randomly disturbing p. The disturbances may have either a monetary or a real nature. The agents thus face a *signal extraction problem*: they must determine how much of a change in their local price must be attributed to real and how much to monetary disturbances. As global information is lagged

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[27] This is merely another way of stating that expectations are formed rationally in the sense of Muth (1961), as the latter's definition (which was given in section 3) reveals.

[28] Lucas (1972a, p. 73) stated that "... the *true* probability distribution of next period's price, $p' = p\ (m', x', \Theta') = p\ (mx, x', \Theta')$ is known, conditional on m, from the known distributions of x, x', and Θ'. Further information is also available to traders, however, since the current price, $p\ (m, x, \Theta)$ yields information on x", where the variables with the apostrophe denote variables in period 1, and those without apostrophe apply to period 0.

one period, the agents are bound to make expectational errors. They will mistake monetarily-induced price changes for changes which result from real disturbances. As Lucas (1972a, p. 78) stated, "... monetary changes have real consequences only because people cannot discriminate perfectly between real and monetary demand shifts." Economic agents then merely behave *as if* they suffer from money illusion.[29] However, Lucas (1972a, p. 79) continued by arguing that such expectational errors cannot be systematic, because rational individuals will form their expectations according to the REH. This means that the 'classical' *neutrality proposition* can be maintained in a manner which explicitly takes account of expectations.[30] *Anticipated* changes in the money supply will not induce changes in real variables, whereas *unanticipated* monetary changes will merely have transitory effects on these variables. This has important consequences for the effectiveness of monetary policy. As Lucas (1972a, pp. 78 - 79, italics in original) stated, "[s]ince [the agents'] ability to discriminate [between nominal and real disturbances] should not be altered by a proportional change in the *scale* of monetary policy, intuition suggests that such scale changes could have no real consequences." Hence, systematic monetary policy does not influence real economic activity. Monetary disturbances are merely capable of influencing real variables because the economic agents do not have sufficient information to solve the signal extraction problem adequately. In turn, this means that monetary policy can only be successful by 'fooling' people.

By combining Friedman's NRH, Phelps's 'island parable', and Muth's REH, Lucas (1972a) provided an example of an economy in which the existence of a Phillips Curve is consistent with the absence of money illusion. His model exhibited the four characteristics mentioned in the Introduction, which became integral parts of the modelling standard of New Classical Economics. As De Marchi (1990, p. 33) concluded, Lucas's

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[29] Lucas (1972a, p. 80) formulated this in terms of the following regression hypothesis of the Lucas supply function:

$$\ln Y_t = \beta_0 + \beta_1(\ln P_t - \ln P_{t-1}) + \epsilon_t$$

in which ϵ_t is a serially independent, zero-mean, random disturbance variable. Lucas argued that "[c]ertainly a positive estimate for β_1 would ... be interpreted as evidence for the existence of a 'trade-off' between inflation and real output." However, "... there is no such trade-off in the model under study, yet [the estimator of β_1] will turn out to be positive", due to the restricted information sets of the economic agents.

[30] This proposition holds that monetary disturbances merely lead to changes in nominal (and not in real) variables. In this sense money is presumed to be neutral. The proposition is often poetically rephrased as 'money is (merely) a veil' (over real economic activity).

(1972a) paper set the modelling style.[31] However, it appears that the content of the REH changed in the process of combining these propositions. Muth (1961) had used the hypothesis in a partial-equilibrium framework, which implies that in order to form (on average) optimal expectations, (the aggregate of) individuals must know the structure of their *local* market. This means that they must know how the other market participants will act. In a partial-equilibrium context, this implicit assumption may be a plausible one: 'hog farmers are likely to know the hog market' (and if they do not, they will go bankrupt). In Lucas's (1972a) general equilibrium framework an analogous assumption is presupposed. As was shown in section 3, Lucas (1972a) argued that the average of expectations will only be *in*correct if on aggregate the individuals do not anticipate a change in the supply of money. Otherwise their anticipation will be correct. Ex-post optimality is thus caused only by the incompleteness of information about the source of disturbance. Again, just as in Muth's analysis, this presupposes that the individuals know (in a probabilistic sense) what the state of the system will be. In Lucas's (1972a) general-equilibrium analysis this system is the economy as a whole. He thus implicitly assumed that on average the individuals know the structure of the economic system as a whole.[32] It turns out that Lucas's transposition of the REH from a partial-equilibrium to a general-equilibrium context has an important implication for the assumption about the individuals' information sets.

The substitution of the REH for the AEH not only changed the content of the individuals' information sets, it also invalidated a common econometric technique of evaluating economic policy.[33] This invalidation has been called the 'Lucas critique'.

5. THE LUCAS CRITIQUE

In 1970 Lucas presented a conference paper which was published in 1972 as 'Econometric

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[31] Hence, Lucas's paper can be seen as the beginning of the positive and negative heuristic of the New Classical research programme.

[32] It might be objected that Lucas (1972a) circumvented this strong assumption by presupposing (1) that all individuals which belong to the same generation are identical, and hence know each others' actions, and/or (2) that there is no communication between markets, in which case the behaviour of economic agents on other markets does not influence a particular agent's actions.

[33] Lucas (in Klamer (1985, p. 38)) remarked that in the 1960s he already knew the REH. However, he "... didn't understand then how fundamental a difference it made econometrically. I didn't realize that if you took it seriously you had to rethink the whole question of testing and estimation. I guess no one else did either, except for Muth." Sargent (in Klamer (1985, p. 61)) concurred with this view.

testing of the Natural Rate Hypothesis' (1972b). In this paper Lucas observed that the NRH is challenged by observations that a trade-off between changes in the inflation rate and changes in the level of real output exists. The question is whether the NRH can only be maintained on theoretical (*a priori*) grounds, or whether there is also some empirical justification? Can the NRH be formulated in a testable form, and if it can how should the test be conducted?

Lucas (1972b) claimed that the answer to the first part of this question is affirmative. He combined the NRH with the REH, so that there was no *a priori* reason to presume that the information set will be incomplete and hence that the expectations will be incorrect (p. 93).[34] This means that the mathematical mean of the expectational errors is zero. According to Lucas, if the REH is combined with the NRH, then "... rational expectations are *equivalent* to the existence of a natural output rate" (p. 96, italics in original). The next step in his paper was to add an aggregate demand schedule to the aggregate supply schedule. The former is interpreted as a policy rule, which is defined as "... a (possibly randomized) *rule* giving the current value of x_t as a function of the state of the system" (p. 96, italics in original)). Using a more concrete rule, Lucas derived the condition for market clearing by equating the aggregate demand schedule with that of aggregate supply. He assumed that the system is linear, which leads to the following solution for real output (p. 97):[35]

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[34] Page numbers refer to the 1981 reprint of Lucas (1972b).

[35] Lucas (1972b) derived this equation:

(*a*) $y_t = a\,(P_t - P_t^*)$

(*b*) $P_t^* = E(P_{t+1}|x_t, x_{t-1}, \eta_t)$

(*c*) $y_t + P_t = x_t$

(*d*) $x_t = \beta_1 x_{t-1} + \beta_2 x_{t-2} + \epsilon_t$

where the variables are those as given in the text. Lucas (1972b, p. 97) "conjectures" the solution of this system of equations as follows. He combined equations (a) and (c) in order to eliminate y_t. This yields:

(*e*) $(1 + a)P_t - aP_t^* = x_t$

Lucas subsequently assumed that the solutions are linear, hence:

(*f*) $P_t = \pi_1 x_t + \pi_2 x_{t-1} + \pi_3 \eta_t$

(*g*) $P_t^* = \pi_4 x_t + \pi_5 x_{t-1} + \pi_6 \eta_t$

Substituting these equations in (e) and (b), this yields six equations in π_1 to π_6, which can be solved in terms of β_1, β_2, and a. The resulting equation is (5.1).

$$(5.1) \qquad y_t = \frac{a\,(1 + a)\,(1 - \beta_1) - \beta_2 a^2}{(1 + a)\,[1 + a\,(1 - \beta_1)] - \beta_2 a^2}\, x_t - \\ - \frac{\beta_2 a^2}{(1 + a)\,[1 + a\,(1 - \beta_1)] - \beta_2 a^2}\, x_{t-1} - \left(\frac{1}{1 + a}\right) \eta_t$$

where y_t is the log of real GNP, x_t is the log of nominal GNP, the β's are the parameters in the policy rule, and ν_t is an independent random error term which is normally distributed with mean zero and variance σ^2. Given this version of the NRH, the question can now be addressed whether and how models of this type can be tested. Lucas (1972b, p. 99) opined that the common methods of testing would take the sum of the coefficients of x_t and x_{t-1} as a measure of the long-run effect of a once-and-for-all (permanent) demand shift. Suppose that this shift is caused by nominal factors, such as a change in the rate of inflation. The NRH then predicts that real GNP will not change, because economic agents do not suffer from money illusion. In terms of equation (5.1), this means that the sum of the coefficients of x_t and x_{t-1} must be zero. According to Lucas, this test has been the 'standard' test of the NRH. He argued that it would be an inappropriate one in case the NRH was combined with the REH, as is readily seen from equation (5.1). The coefficients of x_t and x_{t-1} contain the policy parameters β_1 and β_2, which means that a change in monetary policy will alter the coefficients. Presumably this will change their sum, which in that case does not remain zero. This means that the new policy "... cannot be evaluated by simply summing parameters implied by some previous, now irrelevant policy" (p. 99). Stated less formally, in a economic system in which economic agents form their expectations according to the REH, an *anticipated* change in monetary policy will immediately be incorporated in the (rational) expectations. This means that the agents will adapt their actions accordingly, thus changing the parameters of the behaviourial equations (and hence the structure of the economy). Any method of testing the joint NR/RE hypothesis which takes the parameters in the model as invariable is invalid because of the so-called 'cross-equational restrictions'. Lucas (1981a, p. 9) concluded that "[i]f the theory of 'Expectations and the Neutrality of Money' was the correct way to formulate the Friedman-Phelps natural-rate hypothesis, then it was evident that the econometric methods then being applied to test this hypothesis were entirely missing the point." This criticism of the econometric method under consideration has become known as the *Lucas critique*.[36] Lucas

[36] The Lucas critique can be interpreted as 'merely' an example of Goodhart's Law, which in its broadest sense holds that 'social laws' are subject to change if one attempts to exploit them. Lucas (1981a, p. 10) observed that Sargent (1971) had already achieved the

(1972b, pp. 99 - 100) subsequently outlined an alternative test procedure which estimated the parameters in equation (5.1) and then used the estimates for estimating a.[37]

Sargent (1971) also invalidated a common method of testing the NRH (or as he called it the 'accelerationist' thesis of Phelps and Friedman). Empirical tests of the NRH had assumed that economic agents form their price expectations adaptively. The price expectation then is a weighed average of past relative changes in the general price level.[38] Substitution of this price-expectation function into a Phillips-curve relationship yields the following equation (p. 33):[39]

$$(5.2) \qquad \frac{\Delta w_t}{w_{t-1}} = \alpha \sum_{i=0}^{m} v_i \frac{\Delta P_{t-i}}{P_{t-i-1}} + f(U_t, \ldots) + \varepsilon_t$$

where w_t is the nominal wage rate in period t, P_t is the general price level in t, U_t is the unemployment rate in that period, $f(U_t, \ldots)$ is the short-run Phillips curve with $\partial f/\partial U < 0$ and with the sequence of dots representing a list of other variables, and ε_t is an unobser-

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objective of making clear that 'orthodox' distributed-lag tests could not be used to test the NRH (the so-called 'Lucas critique'). He added that he himself "... did not know this at the time ...." This suggests that Lucas's 1972 paper was written at a later date than Sargent (1971). The problem, however, is that Lucas's paper was already presented at a conference in October 1970. This means that he could not have known Sargent's paper, unless the latter had been written at an earlier date. From a chronological point of view, the question then is whether Sargent wrote his 'A note on the "accelerationist" controversy' already in 1970 (or at an even earlier date)? It appears that Lucas and Sargent have discovered the 'Lucas critique' simultaneously, independently, and along different lines. In any case, its discovery had taken place well before Lucas's second paper on the issue, entitled 'Econometric policy evaluation: a critique' (1976), which is usually referred to as the paper in which he outlined his critique.

[37] Lucas's model contained four slope parameters, but the number of independent parameters is only three (namely a, β_1, and β_2). Lucas proposed first to estimate β_1 and β_2. He then assumed these estimates to be true values. This means that equation (5.1) is nonlinear in the still unknown parameter a. He concluded that "[a] standard Chi-square test, using a comparison of the sum of squared errors from Equation 19 [i.e. (5.1)] estimated with and without this restriction, can then be used to evaluate the hypothesis" (p. 99).

[38] Sargent (1971, p. 33) formulated the AEH as follows:

$$\pi_t = \sum_{i=0}^{m} v_i \frac{\Delta P_{t-i}}{P_{t-i-1}}$$

where the v_i's are positive parameters.

[39] Page numbers refer to the reprint of Sargent (1971) in Lucas and Sargent (1981).

vable random variable.[40] The first argument on the right-hand side of the equation is the public's expectation about inflation in t. The parameters v_i and α are subject to the following restrictions: $v_i \geq 0$ and $0 \leq \alpha \leq 1$. The NRH is supposed to be corroborated if α is close to unity, whereas it is considered to be discorroborated if it is closer to zero. Estimation of the $(m+2)$ parameters is possible only if some restriction is added.[41] Sargent observed that "[a]lmost always, the constraint that has been imposed is that the distributed lags in [the AEH] sum to unity" (p. 34). In his view, this constraint is usually justified by the argument that eventually economic agents will fully incorporate an unexpected change in the rate of inflation into their price-expectations. Under the AEH this will only hold if the new inflation rate remains unaltered for quite some time. Sargent claimed that in reality this is never the case.[42] Instead, it is "... most appropriate to ask what sort of expectations-generating scheme would be reasonable in the light of the actual behavior of the rate of inflation during the period being studied" (pp. 34 - 35). Rational economic agents will adopt an expectation formation mechanism which minimizes their expectational errors. The most reasonable restriction which can be imposed on the sum of the distributed lags is then the one which is compatible with the observed behaviour of the rate of inflation. Sargent (1971, pp. 35 - 36) subsequently showed that the appropriate restriction is that this sum is less than unity.[43] In turn, this means that the unity-restriction leads to overestimation of the v_i's and to underestimation of α. He concluded that "... as usually interpreted, those estimates tell us virtually nothing about the validity of the accelerationist thesis" (p. 37).

To summarize, Lucas (1972b) tried to answer two questions. The first question asked

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[40] As Maddock (1979, p. 158n4) observed, "[i]t is quite common in this [i.e. New Classical] literature to switch back and forth between income and unemployment by replacing log unemployment for log income minus log trend income."

[41] After all, there are $(m+2)$ parameters (v_0, ..., v_m, and α) and only $(m+1)$ degrees of freedom. This means that one restriction must be imposed on the v_i's in order to render the estimation of the $(m+2)$-th parameter (α) possible.

[42] It is interesting to note that Sargent here used an argument which appears to amount to the methodological position of descriptive realism, which can be described (not defined) as the methodological stance that the assumptions of a theory must describe reality as we observe it. But perhaps it is more appropriate to interpret his remarks in terms of the notion of 'plausibility', which he was to specify in his (1973b).

[43] He argued that if the unity-restriction would be appropriate, then the actual rate of inflation "... would display extremely strong serial correlation or 'drift'" (p. 36). As this is not the case, he concluded that the distributed lags sum to less than unity.

whether the combination of the NRH and REH (the NR/RE hypothesis) could be formulated in a testable form. The answer proved to be affirmative. The testable form turned out to pose a problem for orthodox policy-evaluation techniques. The Lucas critique showed that these techniques could not be used for rational-expectations models. Sargent (1971) also reached this conclusion, albeit along somewhat different lines. Both argued that changes in policy parameters influence behavioural parameters. Individuals will take changes in the policy rule into account by changing their actions. Lucas (1972b, p. 98) subsequently asked "[h]ow (if at all) can models of this [NRH/REH] class be tested?" He suggested a test procedure which would take the relationship between policy parameters and behaviourial parameters into account, but he did not carry out such a test. Instead, his 1973 attempt to test the NR/RE hypothesis evaded the problems implied by policy changes.

6. LUCAS'S PARTIAL TEST OF THE NRH

Lucas (1973) undertook an attempt to test his version of the NR/RE hypothesis. He divided aggregate supply into a normal (secular) and a cyclical component, y_{nt} and y_{ct} respectively. The former follows a trend which is identical for all markets z. The cyclical component, which consists of deviations from this trend, differs between markets. Contrary to Lucas's previous models, the equation describing this component contained deviations in lagged income as an explanatory variable. This inclusion was a reply to Rees's (1970) criticism that the model of Lucas and Rapping (1969b) could not account for persistence. Therefore, Lucas (1973) assumed that expectational errors have a drawn-out effect on aggregate output, which accounts for persistence in the economic time series under consideration.[44]

As he had done in his 1972-models, Lucas (1973) explained the Phillips curves in terms of expectational errors as regards the nature of changes in local prices. He assumed again that economic agents know all past deviations of aggregate supply from its trend value, and all past demand shifts (p. 134).[45] They can use this information to construct a

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[44] Lucas's (1973) aggregate supply curve was formalized as:

$$y_{ct}(z) = \gamma[P_t(z) - E(P_t|I_t(z))] + \lambda y_{ct-1}(z)$$

where $P_t(z)$ is the actual price in local market z at time t and $E(P_t|I_t(z))$ is the expected general price level on information available at t. The condition that $|\lambda| < 0$ reflects the persistence effect of past expectational errors.

[45] Page numbers refer to the 1981 reprint of Lucas (1973).

'prior' distribution of the general, economy-wide price level P_t, which was assumed to be normal, with mean $\bar{P}_t$ and variance σ^2. The deviations of the actual local price $P_t(z)$ from P_t are also normally distributed with zero mean and variance τ^2.[46] Rational economic agents will use all available information in their expectations formation process, hence price-expectations will depend on both $P_t(z)$ and P_t. Lucas added a function in which changes in aggregate demand (x_t) were due to either real (y_t) or nominal (P_t) factors. The process which describes the changes in x_t consists of two components, namely its mean δ and a random disturbance variable u_t. The test equation which Lucas subsequently derived is (p. 136):

$$(6.1) \qquad y_{ct} = -\pi\delta + \pi\Delta x_t + \lambda y_{ct-1}$$

where π is a measure of the response of real output to changes in aggregate demand. The first term of the right-hand side of this equation reflects the influence of the average change in x_t on deviations of aggregate supply from its trend. As the variable u_t is a random variable, this average change is also the *anticipated* change in aggregate demand. The second term on the right-hand side of (6.1) gives the *actual* change in aggregate demand, with the same parameter but with opposite sign. The equation thus implies that anticipated changes in x_t do not have any effect on the deviation of real output from its trend, whereas unanticipated changes in aggregate demand will affect the latter with magnitude π. Lucas (1973, p. 137) conjectured that economic agents will be more easily surprised if changes in the nominal component of x_t are less frequent. Economic policy can then more easily 'fool' people. This means that the 'tradeoff' between inflation and output is more favourable, and that the Phillips curve is flatter. He thus argued that the variance of x_t was inversely related to the slope parameter π.

Lucas's test over the period 1952 - 1967 concentrated on eighteen countries. Two types of nominal income behaviour could be distinguished: Argentina and Paraguay had experienced highly volatile and expansive policies, whereas the policies of the remaining sixteen countries had been relatively smooth and moderately expansive. It turned out that the variances of nominal GNP of the former were at least ten times those of the other countries, whereas their π's were smaller (by a factor of ten) than those of the latter. His conjecture thus was corroborated, although this result very heavily depended on the two extreme cases of Argentina and Paraguay.

Lucas's (1973) test can be criticized on two accounts. Firstly, as De Marchi (1990) pointed out, the test did not contain all the knowledge derived in Lucas's own

[46] The distributions σ^2 and τ^2 are assumed to be independent from each other.

process of discovery. In particular, Lucas did not consider his own critique on conventional econometric testing, although in his 1972b-article he had already proposed a solution to the ensuing problem of testing the NRH. This means that the test was only a 'partial' test of the knowledge which was available at the time. However, this is not to say that the Lucas's critique did not play any role in Lucas's (1973) test. In fact, the awareness of it made clear that the problems which were implicit in testing the joint NR/RE hypothesis had to be avoided. Lucas accomplished this by using international data while at the same time assuming changes in the policy rules to be absent. This enabled him to study the effect of different policy regimes without having to address the Lucas critique.

A second criticism which may be levelled against the test concerns the fact that Lucas included past deviations from the natural rate (of output) as an explanatory variable for the persistence in the temporary (i.e. cyclical) component of aggregate supply. Such an inclusion was not founded on economic theory and can be considered *ad-hoc*.

To conclude, Lucas's test had evaded his own critique on econometric policy evaluation by concentrating on inter-country differences in the slope of their Phillips curves and in the variability of nominal GNP. Furthermore, he used a model in which persistence was only explained by including lagged deviations of output from its trend. In contrast, Sargent's (1973a) and (1973b) tests took the Lucas critique (or rather Sargent's own version of it) into account. Furthermore, he showed that the rate of interest could be incorporated in the New Classical framework. Before discussing his tests, a short detour will be made in order to outline their 'prehistory'.

7. SARGENT'S TESTS OF THE NRH

7.1. Fisher's solution to the Gibson paradox

Sargent's work in the late 1960s and early 1970s was concerned with the relationship between the general price level and the (nominal and real) rate of interest. In his 'Commodity price expectations and the interest rate' (1969) he addressed what Keynes (1930 (1965), pp. 198 - 208) had called the 'Gibson paradox'.[47] 'Classical' theory implied that a rise (fall) in the interest rate would produce deflationary (inflationary) pressures because of the gap between desired savings and investment, and the ensuing fall (rise) in effective

[47] In the 1960s this paradox received wide attention. For a list of important articles and books, see Sargent (1972, p. 212n1).

demand. A.H. Gibson found that the data did not correspond to this pattern.[48] He showed that in reality the interest rate and the general price level tend to move together. Classical theory was therefore confronted with an anomaly. Presumably the best known solutions are those of Keynes (1930 (1965)) and Fisher (1930 (1961)).[49] Sargent (1969) concentrated on the latter's suggestion to distinguish between the nominal and the real interest rate. In particular, Fisher had stated that in equilibrium the former equals the latter *plus* the rate of inflation (p. 43).[50] In his view, causality ran from the rate of inflation to the rates of interest (pp. 36 - 37).[51] In a situation of perfect foresight, a change in the rate of inflation will be anticipated. In response, rational economic agents will alter their actions in such a way as to leave the real rate of interest unaffected. However, Fisher observed that economic agents do not have perfect foresight. The ensuing *unanticipated* change in the rate of inflation will not be translated immediately into a change in the nominal interest rate. There will be a transition period during which the real rate of interest will be 'distorted'. Fisher explained this incomplete adjustment by arguing that "... between price changes and interest rates a third factor intervenes. This is business, as exemplified or measured by the volume of trade. It is influenced by price change and influences in turn the rate of interest" (p. 429). Rising prices and lagging remunerations for the factors of production cause producers to earn larger profits. This will induce them to invest more, thus increasing the demand for credit. In turn, this leads to a rise in the

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[48] In particular, see Gibson (1923).

[49] Keynes (1930 (1965)) adopted the Wicksellian distinction between the 'natural' (r_n) and the market rate of interest (r_m). His explanation can be stated in six propositions: (1) r_m is very sticky in comparison to r_n. This means that it cannot maintain equilibrium between saving (S) and investment (I) (p. 203). (2) r_n exhibits long-term movements (extending over decades) because of the fact that the annual increase in the capital stock is relatively small with respect to this stock (p. 204). (3) If these movements are upward, then $r_m < r_n$ for quite some time. In this case $I > S$. If r_n is falling, then $I < S$ (p. 204). (4) If $I > S$, then the price level will fall, and vice versa (see Keynes's 'Fundamental Equations' (viii.) and (x.) (p. 138). This phenomenon "... is a slight, long-continued drag in a particular direction," and does *not* cause a credit cycle (p. 204). A fall (rise) in r_n thus has two effects. Firstly, it eventually induces a fall (rise) in r_m. Secondly, and more importantly, it will ensure that $I < (>) S$. In turn, this induces a fall (rise) in the general price level. The interest rates and this price level thus move together. Propositions (1) and (2) amount to what may be called proposition (2a), which holds that the market rate lags behind the natural rate. This view was already held by Wicksell, who considered it to be the cause of the business cycle. For a more detailed discussion of Keynes's and other solutions of the paradox, see Visser (1980, pp. 151 - 54).

[50] Page numbers refer to the 1961 reprint of Fisher (1930).

[51] As will be shown later, Fisher also allowed for the reverse causal relationship.

nominal rate of interest (1925, p. 180).[52] For reasons of convenience, this effect will be called *Fisher's indirect effect*.[53]

Fisher (1925) and (1930) had also calculated the length of this transition period. He said that this adjustment process could be described as a distributed lag function (1925, p. 184; 1930 (1961), p. 419). The effects of an unanticipated rise in the rate of inflation are presumed to be distributed over the future according to a probability distribution. He found that the lag weights of the distributed lag function declined slowly (1930 (1961), p. 427), and that the transition period was therefore rather long. He estimated that it could take ten to thirty years before the effects were fully incorporated in the nominal interest rate.[54]

7.2. The implausible length of the transition period

Sargent (1969) interpreted Fisher's distributed lag function as an expectations formation mechanism.[55] Given the calculated length of the transition period, this interpretation implies that it would take ten to thirty years before economic agents correctly anticipate the new rate of inflation and act accordingly. Sargent concurred with Cagan (1965, p. 257)

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[52] Fisher (1925, p. 180) called his article "... little more than an inductive verification of these established truths in their application to 'the business cycle' ..."

[53] This effect should not be confused with Wicksell's indirect mechanism, which holds that an expansion of the money supply will lead to an increase in the general price level, via the (market or nominal) rate of interest. This means that causality runs from the interest rate to the price level. Fisher's indirect effect relates the rate of inflation (and hence the change in the price level) to the nominal rate of interest via effective demand. Causality thus runs from (changes in) the price level to the interest rate. Fisher (1930 (1961), p. 443n21) acknowledged that he was not the first to discover the indirect effect. He stated that "Prof. Knut Wicksell was one of the first to recognize the influence of interest rates upon prices. See his book, *Geldzins und Güterpreise*; Prof. Alfred Marshall, Prof. Gustav Cassel, Rt. Hon. Reginald McKenna, Chairman of the Midland Bank of London, Mr. R.G. Hawtrey, of the Treasury of Great Britain, and many other well known economists, bankers, and business men have emphasized that business activity is influenced and may be largely controlled by manipulation of the discount rate" (p. 443n21).

[54] More in particular, "[f]or recent years in Britain, ... the effects of P' [i.e. of changes in the general price level] are distributed over 28 years" (p. 438).

[55] Sargent (1969, p. 132) formulated this mechanism as:

$$\pi_t = r_t - \rho_t = \sum_{i=0}^{\infty} w_i \frac{P_{t-i} - P_{t-i-1}}{P_{t-i-1}}$$

where $w_i \geq 0$ for all i.

that this is very implausible. He recalculated the length of the transition period, using an equation in which the nominal rate of interest r_t is explained in terms of (changes in) aggregate output x_t, relative changes in the real money supply m_t^*, the rate of inflation p_t, and a random disturbance term u_t. His estimates of the length of the lags did not differ much from those of Fisher, so that he could ascribe the implausibility of Fisher's lags to incorrect estimation. Instead, he explained the length of the lags in terms of what he called the extrapolative effect, which holds that an increase (decrease) in the general price level induces economic agents to expect prices to rise (fall) still further. This hampers the adjustment process, thus lengthening the transition period. Additionally he derived another result from his estimates. The parameters of the equation which was tested indicated that there was also a shorter-term regressive component in the process of expectations formation. This means that economic agents expected a rise (fall) in the rate of inflation to be followed by a fall (rise) in this rate. Sargent (1969, p. 138) argued that such expectations were rational because of "... the cyclical properties of price movements over the period under consideration."

It appears that Sargent was not satisfied with the results of his 1969-explanation of the length of the lags in Fisher's expectations formation mechanism. In his 'Anticipated inflation and the nominal rate of interest' (1972) he criticized the mathematical model which Fisher had used. He claimed that in his empirical work Fisher had implicitly assumed that the anticipated rate of inflation does *not* affect the real rate of interest, not even in the transition period (pp. 212 - 13).[56] This assumption can be formalized as:

$$(7.1) \quad r_t = \rho_t + \pi_t$$
$$(7.2) \quad \rho_t = \alpha + \epsilon_t$$

where r_t is the nominal rate of interest, ρ_t is the real rate of interest, π_t is the anticipated rate of inflation, α is a constant, and ϵ_t is a stochastic term which is uncorrelated with the nominal rate of interest. The second equation implies that a change in the anticipated rate

[56] It should be noted that Sargent presumably wrote his 1972-article in 1970. As he indicated in his (1987, p. 117n1), in 1970 and 1971 he attempted "... to formalize for students the relationships among the various hypotheses advanced in Milton Friedman's AEA presidential address (1968)." As I am concerned with the sequence in which the articles were written, and as there is no reason to the presume that this sequence differs from that in which they were published, I shall refer to the year in which they were published, although the articles must have been written one or two years earlier.

of inflation will affect only the nominal rate of interest, leaving the real rate unchanged.[57] Sargent doubted the plausibility of this relationship and tried to establish whether the equations given above "... can in general be taken to characterize correctly the relationship between anticipated inflation and the nominal rate of interest" (p. 213). In disequilibrium (or transition) periods the actual rate of inflation is not fully reflected in the nominal rate of interest. This implies that the real rate of interest must also have changed (systematically), and hence that it cannot be described by equation (7.2). Sargent concluded that the above equations should be replaced by one or more other equations in order to avoid the implausible explanation of the extremely long lags in terms of the process of expectations formation. The question then arises as to the correct way of modelling the relationship between the nominal rate of interest and the anticipated rate of inflation. This question was addressed in Sargent's 'Interest rates and prices in the long run' (1973a), in which he not only substituted the REH for the AEH, but also tried to show that Friedman's NRH and Fisher's solution to the Gibson Paradox are two sides of the same coin.[58]

7.3. Muth-rationality as a criterion for plausibility

Sargent (1973a) started his analysis by performing a test of the length of the transition period, based on the equations (7.1) and (7.2). He combined them with the AEH, which yielded equation (7.3):

$$(7.3) \qquad r_t = \alpha + \gamma \sum_{i=0}^{\infty} \lambda^i \frac{\Delta P_{t-i}}{P_{t-i-1}} + \varepsilon_t$$

where r_t is the nominal rate of interest, α is a constant, the sum is the adaptively formed expected rate of inflation, and ε_t is a stochastic term which is uncorrelated with the nominal rate of interest. Sargent's estimated this equation for the U.S. in the period 1870-

[57] It should be stressed that at least in his theoretical work Fisher already pointed out that the second equation does not hold in the short run, i.e., during the transition period (which can be very long, as was already shown). Hence it is misleading to label this equation "Fisher's formula" because this suggests that Fisher not only considered it to be valid in equilibrium, but also during the transition periods.

[58] Cf. Sargent (1987, p. 117n1).

1940.[59] The resulting estimates of the 'decay parameter' λ were close to unity. He concluded that "[t]hese estimates corroborate the main outlines of Fisher's findings" (p. 392). But these findings did not convince him that the lags were really very long. He maintained that "[w]hile Fisher's explanation of that [i.e. the Gibson] paradox formally 'works', the implied lags in forming expectations do seem extraordinarily long." This implies that plausibility enters as a criterion for the acceptance (or rejection) of empirical results and the underlying propositions. It is interesting to note that in his previous articles Sargent had not given any criterion for plausibility. This suggests that his rejection of Fisher's empirical estimates rested on *a priori* beliefs. However, in 1973 he acknowledged that "[t]o say that the estimates of the weights obtained by that [i.e. Fisher's adaptive-expectations formation] procedure are implausible apparently means that they do not resemble the weights that really characterize the process by which people seem to form expectations about future rates of inflation" (p. 392). Fisher's estimates can only be considered implausible if superior (extraneous) knowledge about the 'true' expectations formation mechanism is available. The problem is how to obtain such knowledge. According to Sargent (1973a), Muth (1961) had provided a source of information about the length of the lags by arguing that the forecasts of the economic agents will be identical (in a probabilistic sense) to those of statistical and economic theory. Sargent proposed to use these rational expectations as a "... yardstick against which we will judge the 'plausibility' of the expectations implied by Fisher" (p. 393). This means that he implicitly accepted the propositions that the REH adequately describes "the process by which people seem to form expectations about future rates of inflation."

Sargent restricted his attention to the class of autoregressive models which can be formulated as:[60]

$$(7.4) \qquad p_{t+1} = \sum_{i=0}^{\infty} v_i \, p_{t-i} + u_{t+1}$$

where p_t ($= \Delta P_t/P_{t-1}$) is the actual rate of inflation in period t, and u_t is an independently

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[59] The estimates are derived by a search procedure. As Sargent (1973a, p. 391n5) explained, "[o]ur procedure here was first to search over λ's ranging from .1 to .9 at steps of .1. Having found the value of λ, say λ_0, that, among these nine values of λ, delivered the smallest residual variance, we then searched again over $[\lambda_0 - .09, \lambda_0 + .09]$ at steps of .01 for the λ_0 associated with the minimum residual variance. This value was taken as our estimate of λ."

[60] The variables in the equations in the remainder of this section are expressed in deviations from their respective means.

and identically distributed random variable with mean zero and variance σ_u^2 (p. 393). The rate of inflation consist of two mutually uncorrelated components. The first (η_t) is deterministic, and is predictable "... with zero mean square error given a sufficient number of its own past values."[61] According to Sargent (1973a, p. 394), "[i]t is generally believed that economic time series exhibit no important deterministic (i.e. strictly periodic) components of variation, so that η_t is in effect zero." The other component (z_t) equals the sum of a number of independently and identically distributed random variables with zero mean and finite variance. In other words, it consists of 'white noise'. The rate of inflation p_t can then be formulated as follows (p. 394):

$$(7.5) \qquad p_t = z_t = \sum_{j=0}^{\infty} c_j u_{t-j}$$

This is the moving-average representation of p_t. By equating this representation with the autoregressive representation (7.4) Sargent derived a formulation of the optimal-forecast generating function. He assumed that this function is a low-order process which could be formulated as follows:

$$(7.6) \qquad p_t = a_1 p_{t-1} + a_2 p_{t-2} + b_1 u_{t-1} + b_2 u_{t-2} + u_t$$

where u_t is a white noise variable and a_1, a_2, b_1, and b_2 are parameters. By assuming $a_2 = b_2 = 0$, Sargent narrowed this function down to a first-order ARMA-process. In particular, he estimated function (7.7) (p. 398):

$$(7.7) \qquad \hat{\pi}_{t+j} = \frac{(a_1 + b_1)\, a_1^{j-1}}{1 + b_1 L}\, p_t$$

On the basis of this equation he derived a 99 percent confidence region for the estimates of a_1 and b_1 for the period 1870 - 1940. This region included the origin, which means that the hypothesis that the inflation rate behaves like white noise cannot be rejected.[62] More generally, if $a_1 = - b_1$, then the first-order ARMA-process follows a random walk, which means that the mean lag cannot be used to assess whether Fisher's estimates are implausi-

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[61] This condition as regards the number of past values indicates that Sargent presumed that the 'Law of Large Numbers' applies. Cf. section 3.

[62] In the origin $a_1 = b_1 = 0$. If we substitute these values in the first-order ARMA-process $p_t = a_1 p_{t-1} + b_1 u_{t-1} + u_t$, then $p_t = u_t$.

ble.[63] Sargent thus had to find another way of assessing Fisher's estimates, which should take the confidence regions into account. In order to achieve this he estimated the parameters of Fisher's equation jointly with the parameters of the ARMA-process in p_t (p. 400). The estimated equation was written as:

$$(7.8) \qquad r_{jt} = \alpha_j + \gamma_j \sum_{i=0}^{t-1} \lambda^i p_{t-i} + \lambda^t \eta_j + \epsilon_{jt} \qquad t = 1, \ldots, T$$

where j reflects the maturity of the bond. The parameters of (7.8) were jointly estimated with those of (7.9) for a given θ (which is a scalar that determines how much weight is placed on the observations on (7.9)):

$$(7.9) \qquad \theta\, p_{t+1} = \theta\, \beta_0 + \theta\, \gamma_1 \sum_{i=0}^{t-1} \lambda^i p_{t-i} + \theta\, \lambda^t\, \eta_j + \theta\, u_{t+1}$$

Sargent adopted a value of θ which made the residuals of both equations nearly homoscedastic (p. 401). The joint regressions led to estimates of λ and γ which were both lower than those implied by Fisher's analyses. In most cases they did not significantly differ from zero. This indicates that the past behaviour of p hardly influences its current value; the p-process can then be approximated by a random walk. Sargent (1973a, p. 402, italics in original) concluded that "[t]o work, Fisher's equation obviously requires both large values of λ *and* significant estimates for γ. Hence the introduction of the observations on the p-process [i.e. equation (7.9] has seriously weakened the ability of Fisher's equation to explain the 'Gibson pradox', implying that it is difficult *both* to accept Fisher's explanation of the Gibson paradox *and* to maintain that the extraordinary long lags in expectations estimated [by Fisher] are 'rational'." Sargent thus opined that Fisher's expectations formation mechanism does not yield plausible (in the sense of Muth-rational) expectations, and hence his explanation of the Gibson paradox cannot be considered plausible. This raised the problem of providing an explanation of the paradox which would be consistent with Muth's REH.

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[63] For $a_1 = -b_1$ the mean lags are given by $-b_1/(1+b_1)$, and thus depend only on b_1. The 99 percent confidence region mentioned above also contained parameter values which imply very long mean lags. Hence the length of the mean lags do not constitute reliable indicators of the plausibility of Fisher's estimates.

7.4. The inclusion of omitted variables

Sargent (1973a) suggested the following solution to the problem he himself had posed. He argued that Fisher had only taken into account a one-way causal relationship between the rate of inflation and the nominal rate of interest, with causality running from the former to the latter. In this sense the inflation rate appears to be exogenous. Sargent argued that this is unduly restrictive, and that it would be more appropriate to transform the interest rate into an endogenous variable. He suggested to test Fisher's model for feedback from the nominal rate of interest to the rate of inflation. This model can be represented as a special case of (7.10):

$$(7.10) \qquad \begin{vmatrix} 1-a(L) & -b(L) \\ -c(L) & 1-d(L) \end{vmatrix} \begin{vmatrix} r_t \\ p_t \end{vmatrix} = \begin{vmatrix} \epsilon_t \\ u_t \end{vmatrix}$$

where $a(L)$, $b(L)$, $c(L)$, and $d(L)$ are one-sided polynomials in the lag operator L, and where u_t and ϵ_t are mutually independent white noises (p. 405). If there is no feedback from r_t to p_t, then $c(L)$ must be zero. Referring to an early version of Sims (1972), Sargent argued that this model could be tested by analyzing equation (7.11):

$$(7.11) \qquad r_t = \sum_{j=-m_1}^{m_2} h_j \, p_{t-j} + v_t$$

where m_1 and m_2 are positive parameters, the h_j's are the estimated distributed lag parameters, and v_t is a statistical residual. Adopting the Granger-Sims notion of causality, the existence of a feedback from the nominal interest rate to the rate of inflation means that future values of the latter are correlated with current values of the former.[64] Hence Sargent tested the hypothesis that $h_j = 0$ for all $j < 0$. The test results suggested that "... an explanation of the interest-inflation relationship that does not permit feedback from interest to inflation is probably unduly restrictive" (p. 422).[65] It thus seemed that feedback should be allowed for. However, if in equation (7.10) $c(L)$ does not equal zero, then some interpretational problems emerge, because "... it will no longer be 'rational' to form

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[64] Granger-Sims 'causality' merely reflects correlation instead of (philosophical) causation. Hence it can give no more than an indication of causation.

[65] Sargent (1973a, pp. 425 - 26) acknowledged that Fisher had argued that causality need not always run from the rate of inflation to the interest rate, but that the reverse influence may also occur. In fact, Fisher (1930 (1961), p. 443n21) had argued that this influence was already discovered by many others.

expectations of inflation by looking at current and lagged rates of inflation alone, since current and past rates of interest are of some help in predicting subsequent rates of inflation" (p. 427). Therefore, Sargent proposed another interpretation of his test results. This interpretation holds that what appears to be feedback from interest to inflation is caused by some omitted variables which influence both r and p (p. 427). Sargent subsequently built a model in which the variable 'aggregate demand' (represented by 'changes in the money supply') was included, and in which both the interest rate and the rate of inflation were endogenous.

Sargent's (1973a) model described a closed economy with one good which was produced according to a linearly homogeneous production function in both labour and capital. The nominal rate of interest r and the rate of inflation p are mutually determined. Sargent generated artificial (annual) data which indicated that the nominal rate of interest r moved in the same direction as the rate of inflation p. That is, the generated data were characterized by the Gibson paradox. The values of the regression coefficients of r on future and past values of p were similar to the estimates which Sargent had derived from historical data.[66] However, the anticipated rate of inflation π was assumed to be constant (pp. 438 - 39). This means that the long mean lags which characterize the relationship between the nominal rate of interest and the rate of inflation cannot be explained in terms of long lags in the process of expectations adjustment. Additionally, the assumption makes clear that Sargent was more interested in the 'long-run' relationship between the nominal rate of interest and the rate of inflation than in the adjustment process.[67] His simulations indicated that the Gibson paradox need not be explained in terms of lags in the agents' expectations formation mechanism. He conjectured instead that "[t]he key reason that the Gibson paradox may infest the data generated by the model is the failure of wages and prices to adjust sufficiently quickly to keep output always at its full-employment level" (p. 442). In his view, "... it does not seem necessary to stress differences between nominal and real rates of return in order to explain the Gibson paradox" (p. 442). This conclusion implies that real rates of return (or interest) would behave similar to nominal rates. Stated

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[66] There was only one exception, namely the coefficient on the current value of p, which in the simulations proved to have become larger.

[67] When showing the way in which his model works, Sargent (1973a, p. 435) had noted that "[a]ssuming that the system is dynamically stable, the final resting place for all variables will be the same as if π had remained at its steady-state value throughout the adjustment process; but the path to steady-state equilibrium may be much different." The assumption that π remains unchanged thus means that the path to the final resting place is not considered to be relevant. In turn, this implies that Sargent was only interested in the 'final resting place' of his model.

in Lakatosian terms, Sargent's 1973a-article predicted a novel fact, and hence can be considered theoretically progressive. To determine whether the prediction was empirically progressive (i.e., whether 'reality' corroborated the prediction), Sargent regressed two measures of the real return on equities on wholesale commodity price inflation for the period 1871 - 1929 (pp. 444 - 45). His results corroborated his prediction that movements in the anticipated rate of inflation cannot explain the relationship between the nominal rate of interest and the actual rate of inflation (that is, the Gibson paradox). He concluded that the explanation of this paradox should focus on the relationship between movements in real rates of return and the price level. However, this relationship is somewhat problematical, because it relates a real variable with a nominal variable. This implies that rational economic agents suffer from money illusion. This is a similar problem to the one Lucas had faced concerning the Phillips curve. He had shown that this implication may only be apparent, by explaining the curve as a result of expectational errors. His interpretation of the insufficient wage and price adjustments as a consequence of incomplete knowledge and incomplete foresight also proved useful for Sargent in his attempt to explain the Gibson paradox. This paradox was also interpreted as a short-run phenomenon which arises because of the fact that economic agents make expectational errors.[68] In the long run, Sargent (1987, p. 117n1) argued, Friedman's NRH and Fisher's two-equations model are two sides of the same coin.

7.5. Sargent's (1973b) tests

It is interesting to note that Sargent (1969), (1972) and (1973a) did not refer to Lucas's work. This indicates that the latter's formalization of the NR/RE hypothesis did not exert any influence on Sargent's pre-1973 work.[69] Sargent's 'Rational expectations, the real rate

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[68] The substitution of the AEH by the REH played an important role in this regard. As Sargent (1987, p. 117n1) observed, "[w]hile working with adaptive expectations ... one obtains only weak or 'long-run' versions of Friedman's hypotheses. Switching to rational expectations (or 'perfect foresight') leads to much more precise and more immediate versions of Friedman's hypotheses. This led Neil Wallace and me to produce Sargent and Wallace (1975) and Sargent (1973b)."

[69] This also becomes clear from Klamer's (1985) respective discussions with Lucas, Rapping and Sargent. Lucas argued that Sargent and he "... didn't talk very much during the two years he [Sargent] was there" (p. 33). Rapping confirmed that Sargent "... did not pay much attention to what Bob [Lucas] and I were doing. He did not talk with Lucas much" (p. 225). Sargent himself stated when e met Lucas at Carnegie-Mellon, he didn't know what Lucas was up to, and he didn't completely understand his work. In fact, he "...

of interest, and the natural rate of unemployment' (1973b) connected Sargent's work to that of Lucas, as "[t]he argument in this paper is heavily dependent on the analysis of the natural rate hypothesis carried out by Lucas in a series of papers."[70] This suggests that as late as 1973 Lucas's work had a sudden and substantial impact on Sargent, presumably because the latter's work had reached a stage in which Lucas's contributions proved useful.

Sargent (1973b) started from his 1973a-conclusion that Fisher's 'long-run' framework and Friedman's NRH amounted to the same thing. However, the former's tests had been inappropriate because they implied that the lengths of the lags in the process of expectations adjustment were implausibly long. This raised the question of how the NRH could be appropriately tested. Sargent (1973b) attempted to outline the proper ways in which such a test could be executed. Additionally, he performed two of such tests. As the explanation of the inappropriateness of Fisher's tests was already discussed in Sargent (1973a), we shall concentrate on the ways in which he proposed to test the NRH.

Sargent (1973b) assumed the following equation:

$$(7.12) \qquad Un_t = \beta(p_t - Ep_t|\theta_{t-1}) + \sum_{i=1}^{q} \lambda_i Un_{t-i} + u_t, \qquad \beta < 0$$

where Un_t is the unemployment rate (which serves as a reverse index for the natural logarithm of real output y_t minus a constant k), p_t is the natural logarithm of the price level, Ep_t is the expected value of p_t formed at time t-1, θ_t is the information set at time t, λ is a parameter, q is the period of relevant lagged variables, and u_t is a normally distributed random disturbance term. The forecast of this term cannot be improved by including components of the information set which is available at the time. This means that u_t obeys $E(u_t|\theta_{t-1}, u_{t-1}, u_{t-2}, ...) = E(u_t|u_{t-1}, u_{t-2}, ...)$. Sargent (1973b, pp. 175 - 76) assumed that u_t can be described by the following process:

$$(7.13) \qquad u_t = \gamma u_{t-1} + \xi_{ut}, \qquad |\gamma| < 1$$

where ξ_{ut} is a normally distributed and serially uncorrelated random variable. Using (7.12) and (7.13) Sargent derived equation (7.14):

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learned from Lucas mostly by reading his stuff" (p. 60).

[70] Sargent (1973b (1981), p. 162) thereby referred to Lucas and Rapping (1969a), and Lucas (1972a), (1972b), (1973), and (1976). The page numbers of Sargent (1973b) refer to the 1981 reprint.

$$(7.14)\qquad Un_t = (\lambda_1 + \gamma)Un_{t-1} + \sum_{i=2}^{q}(\lambda_i - \gamma\lambda_{i-1})Un_{t-i} - \gamma\lambda_q Un_{t-q-1} + \\ + \beta(p_t - Ep_t|\theta_{t-1}) - \beta\gamma(p_{t-1} - Ep_{t-1}|\theta_{t-2}) + \xi_{tt}$$

The expectation of this equation conditional on all past rates of unemployment and all other information available at θ_{t-2} gave the equation from which Sargent derived his tests (p. 176):

$$(7.15)\qquad E(Un_{t-1}, \ldots, Un_{t-q-1}, \theta_{lt-2} = (\lambda_1 + \gamma\lambda_{t-1})Un_{t-1} - \gamma\lambda_q Un_{t-q-1} - \\ - \gamma\beta E[(p_{t-1} - Ep_{t-1}|\theta_{t-2})|Un_{t-1}]$$

where θ_{lt} is a subset of the information set θ_t. Based on this equation, Sargent proposed two tests. The first of these proposals concerned the case in which u_t is serially correlated, and follows the following *n*th order process:

$$(7.16)\qquad u_t = \sum_{i=1}^{n}\gamma_i u_{t-i} + \xi_{tt}$$

where ξ has the same properties as in equation (7.13). If the expectation of (7.15) is taken conditional on past values of the rate of unemployment and the subset of information θ_{lt-1}, then equation (7.16) indicates that the lagged error in the price expectations remains. Given the fact that u_t is lagged n periods, this means that the conditional expectation of Un_t can be written as:

$$(7.17)\qquad E(Un_t|Un_{t-1}, Un_{t-2}, \ldots, Un_{t-n-q}, \theta_{lt-n-1}) = \\ = E(Un_t|Un_{t-1}, Un_{t-2}, \ldots, Un_{t-n-q})$$

This equation implies that the forecast of Un_t based upon past observations of this unemployment rate cannot be improved by including components of the information subset θ_{lt-1}. These components will then have coefficients which do not differ significantly from zero when they are added to a regression of Un_t upon (enough) lagged values of itself. This proposition can be tested. Sargent (1973b, p. 176) added that "[t]he higher the order of serial correlation in the u's, the more periods components of θ_t must be lagged to warrant the implication that their coefficients are zero."

The second test proposal of Sargent (1973b) concerned the case in which u_t is *not* serially correlated, and hence in which the λ's are zero. This means that the economic agents cannot improve their forecasts by including more information. That is, the expected value of Un_t formed on any subset θ_{lt-1} of the full information set θ_{t-1} equals zero. Mathematically, this can be formulated as $E(Un_t|\theta_{lt-1}) = \beta(p_t - Ep_t|\theta_{tl-1}) + u_t = 0$. This

equation can then be tested empirically by regressing the rate of unemployment upon components of the information subset. Sargent in fact performed this test twice. He first regressed the rate of unemployment against its own lagged values and a subset θ_{lt-1} which consisted of the lagged price level (p_{t-1}, p_{t-2}, p_{t-3} and p_{t-4}) and the lagged nominal wage rate (w_{t-1}, w_{t-2}, w_{t-3} and w_{t-4}). Testing the null hypothesis that the coefficients on these lagged variables do not differ significantly from zero, he found that the regression corroborated this hypothesis at the 95% confidence level. However, his second regression which included a larger subset of θ_t led to a quite different conclusion.[71] Sargent again tested the null hypothesis that the coefficients of the components of the enlarged subset under consideration have zero coefficients, but this time he found that the null hypothesis must be rejected at the 99% confidence level. The natural rate hypothesis was thus discorroborated. However, Sargent cautioned not to reject the NRH too hastily.

Sargent (1973b) mentioned four circumstances in which a rejection of the NRH would be premature. Firstly, his test had assumed that there is no serial correlation in the u's. If in fact such serial correlation exists, then the test is biased towards rejection of the null hypothesis. Secondly, he argued that individuals may rationally form expectations on the basis of a smaller subset than the one used in the second test. He stated that in that case "... the essence of the natural rate hypothesis could stand unrefuted even though tests using large subsets θ_{lt-1} find systematic effects of θ_{lt-1} on Un_t." He thereby referred to the first test, which had considered a smaller subset and which did not allow for the rejection of the null hypothesis.[72] The third defense of the NRH against the failure of the test to corroborate this hypothesis held that the u's may have been correlated with components of θ_{lt-1}. Such a correlation means that the coefficients of the components will be biased upwards, and hence that the regression will be biased towards rejection of the null hypothesis. Finally, Sargent took a position which resembles 'sophisticated falsificationism'.

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[71] This subset included "... values of the logarithm of the money supply (currency plus demand deposits), seasonally adjusted (m), the federal and state and local government deficit on the national income accounts basis (Def); and the logs of the GNP deflator, seasonally adjusted (p), of the implicit deflator for personal consumption expenditures (pc), of the average hourly wage rate in manufacturing, seasonally adjusted (wr), of government purchases of goods and services (g), of total federal and state and local government employment, seasonally adjusted (ng), and of GNP (y). Each of these arguments is included lagged one, two, and three periods" (p. 178).

[72] This second defense may be criticized on the account that it is likely to be inconsistent with the rationality postulate. After all, the subset used in the second test contains variables such as the GNP deflator and GNP itself. Rational economic agents will presumably include these variables in their process of expectations formation because they will be relatively easy to gather and to process (i.e. without incurring large costs).

He argued that "... it has not been shown that an autoregression for unemployment yields ex ante predictions of unemployment inferior to those of a particular structural macroeconometric model that embodies a particular aggregate supply theory other than the natural rate hypothesis" (pp. 177 - 78). He concluded that there is no way of knowing whether such a better alternative theory exists until a so-called 'horse race' is held. The second test which Sargent (1973b) performed consisted of such a horse race.

Sargent's second test involved a comparison of the predictions of equation (7.12) with those of the following equation:

$$(7.18) \qquad Un_t = \sum_{i=1}^{q} \lambda_i Un_{t-i} + \beta(p_t - Ep_t|\theta_{t-1}) + \\ + \beta(1 - \alpha)(Ep_t|\theta_{t-1} - p_{t-1}) + u_t$$

This equation differs from (7.12) as it contains the third term on its right-hand side. This term represents the influence of the difference between the price forecast Ep_t based on information available at time t-1, minus the price at t-1. Equation (7.12) and hence the NRH implies that $\alpha = 1$. In contrast, equation (7.18) shows that "... if $\alpha < 1$ ($\alpha > 1$), then increases in the systematic part of the rate of inflation decrease (increase) the unemployment rate, contrary to the natural rate hypothesis" (p. 180). Sargent tested both equations for the United States over the period 1952:1 - 1970:4, using quarterly data.[73] The 'horse race' was thus held between the NRH, on the one hand, and, on the other, the hypothesis that the systematic part of the rate of inflation affects the rate of unemployment. The test itself pointed in the direction of rejection of the NRH. However, Sargent added that such a rejection would not be based on an 'unusuallly' high confidence level (p. 186).[74] He stated that the evidence "... would not be sufficiently compelling to persuade someone to abandon a strongly held belief in the natural rate hypothesis" (p. 187). Prior beliefs thus play a crucial role in the assessment whether the discorroboration of the null hypothesis should lead to its rejection.

8. CONCLUSIONS

New Classical Economics arose in the late 1960s and early 1970s from the attempt of

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[73] For a discussion of these problems and the respective ways in which they were solved, see Sargent (1973b, pp. 181 - 82).

[74] This raises the question why an *un*usual high level is required, and why a usually high level does not suffice?

economists such as Lucas, Rapping and Sargent to formalize Friedman's (1968) contention that the long-run Phillips curve would be vertical. By 1973 its main proponents were Robert E. Lucas, Jr., and Thomas J. Sargent, who had succeeded in the formalization. Lucas combined Friedman's NRH with Phelps's islands parable. His use of Muth's REH enabled him to circumvent the conclusion that rational economic agents can be constantly 'fooled'. Sargent, on the other hand, had concentrated on what Keynes had called the Gibson paradox. He rejected Fisher's analysis because of its implication as regards the lengths of the lags in the process of expectations adjustment, and he used Muth's REH as a criterion for plausibility. Like Lucas's explanation of the Phillips curve, Sargent's explanation of the Gibson paradox ran in terms of insufficiently quick price and wage adjustment, due to incomplete knowledge and incomplete foresight. It implicitly used the NRH as its framework.

The introduction of the REH led Lucas and Sargent to criticize standard econometric methods of policy evaluation. This 'Lucas critique' implied that past tests of the combination of the NRH and the REH were invalid. As a corollary, testing of the joint NR/RE hypothesis was yet to be undertaken. Lucas's 1973 test used international data in such a way as to evade the recommendations based on the critique. In the same year Sargent developed another way of testing the NR/RE hypothesis and carried out such a test. It indicated that forecasts of the rate of unemployment could be improved by taking a number of other variables into account. This meant that economic agents could improve their forecasts by using regressions such as carried out in Sargent's (1973b) test. However, Sargent did not reject the theory, even though it was discorroborated. He discerned three types of reasons why such a rejection would be inappropriately. The first reason implicitly built on the Duhem-Quine thesis which holds that a (social) scientist cannot test an isolated hypothesis.[75] Any discorroboration may then be caused by a 'false' supplementary hypothesis. That is, the initial conditions (assumptions) may not hold, or the ceteris-paribus clause may be violated. In particular, Sargent observed that the assumption about the absence of serial correlation in the disturbance term may not hold. Secondly, Sargent

[75] Duhem (1954 (1976), p. 8) had argued that "... the physicist [or, more generally, the scientist] can never subject an isolated hypothesis to experimental test, but only a whole group of hypotheses; when the experiment is in disagreement with his predictions, what he learns is that at least one of the hypotheses constituting this group is unacceptable and ought to be modified; but the experiment does not designate which one should be changed." Quine (1951 (1964)) provided an even stronger thesis. He extended Duhem's argument to include the 'laws of logic' and *all* laws of science. In fact, he claimed that "[t]he unit of empirical significance is the whole of science", and not merely a theory (p. 59).

'retreated' to the methodological position of sophisticated falsificationism. This position holds that a theory should not be rejected if no better alternative is available.[76] Sargent (1973b) set up a comparative test ('horse race') and its results indicated that there was no such alternative. The third reason involved prior beliefs. It was not used separately but rather in conjunction with the other two types of argument. Sargent's second test had provided "... some evidence for rejecting the natural rate hypothesis, although not at an unusually high confidence level." This indicates that prior beliefs play an important role in the New Classical assessment of whether a discorroborated theory (and in particular the NRH) should be rejected. In fact, they insulate the theory or hypothesis from rejection by demanding discorroboration at 'unusually high' confidence levels. It thus appears that there is still some validity in H.G. Johnson's (1971, p. 51) statement that "... the 'testing of hypotheses' is frequently merely a euphemism for obtaining plausible numbers to provide ceremonial adequacy for a theory chosen and defended on *a priori* grounds."

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[76] Naive falsificationism holds that any theory or hypothesis which has been discorroborated by the empirical data must be considered to be falsified, and hence must be rejected. This implies that naive falsificationism assumes that crucial tests are possible. In contrast, sophisticated falsificationism incorporates the Duhem-Quine thesis. As a corollary, it holds that a theory or hypothesis should be rejected only if there is a better alternative available.

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