Faculteit der Economische Wetenschappen en Econometrie

Serie Research Memoranda

Space-Time Dynamics, Spatial Competition and the Theory of Chaos

P. Nijkamp
A. Reggiani

Research Memorandum 1991-94
december 1991

vrije Universiteit *amsterdam*

Abstract

In economics and regional science we observe recently a growing interest in the use of non-linear dynamic models. In particular non-linear deterministic models appear quite appealing, since they may exhibit both oscillating and chaotic behaviour and hence may provide an endogenous explanation of the periodicity and/or irregularity observed in the dynamics of spatial economic systems.

In this paper the aim is to analyse consequences of spatial dynamic competition in regional systems including the possibility of chaotic behaviour. Non-linear deterministic models often used in ecology and biology will be investigated in this framework. Although these models by definition do not display oscillating behaviour, it will be shown that the particular case of a 'chaotic' evolution of May-type in one region will lead to an oscillating divergent behaviour in the entire system, if the intrinsic growth rates of the competing regions exceed a critical value (beyond which instability begins).

The analysis will be illustrated by means of simulation experiments for both the cases of two and three competing regions.

1. Introduction

Recently increasing attention is being paid in economics and regional science to non-linear dynamic problems such as susceptibility of systems to sudden changes, adjustment processes, uncertainty and perturbations at both micro and macro levels (see also Domanski, 1990). In the context of modelling this also meant an increasing use of (continuous and discrete) dynamic disequilibrium models able to capture both regular and irregular movements of phenomena (see among others Beckmann and Puu, 1985). In particular, economic theory and modelling has adopted various concepts and research strategies from the natural sciences, for example from biology, ecology and classical physics. Consequently, we may classify the various non-linear dynamic models developed in the past decade and which are capable of exhibiting bifurcations, oscillations and also self-organising structures, according to the following prototypes:

a) models based on logistic laws of biological populations (see, e.g., many analyses based on Harris and Wilson's (1978) models);
b) models based on the dissipative structures emerging from physics and chemistry (see, e.g., Allen, 1982);
c) models derived from the interaction laws in synergetics (see e.g., Weidlich and Haag, 1983);
d) models based on ecological/biological processes of a prey/predator type (see, e.g., Dendrinos and Mullally, 1981);
e) models derived from ecological theories of innovation diffusion (see, e.g., Blommestein and Nijkamp, 1987 and Sonis, 1987).

Moreover, it has recently been demonstrated (see Nijkamp and Reggiani, 1990a and Reggiani, 1990) that the family of spatial interaction models (emerging from physics and/or statistical mechanics) constitute a <u>unified framework</u> for all above mentioned models as well as for the class of discrete choice models emerging from economic theory. Since spatial interaction models can be reconducted, in their dynamic form, to the logistic structure (see Nijkamp and Reggiani, 1990b), it is interesting to underline the relevance of the role played by the <u>logistic law</u> in all these models, also because an important feature of the logistic growth is its capability, in discrete time, of generating aperiodic and chaotic behaviour (see May, 1976).

Chaos theory has recently become very popular in scientific modelling for its intrinsic characteristic of displaying deterministic, erratic behaviour, which is largely similar to irregular - endogenous - fluctuations observed in reality. In particular, 'chaotic' motions incorporate the feature that small uncertainties may grow exponentially (although all time paths are bound), leading to a broad spectrum of different trajectories in the long run, so that precise or plausible predictions are - under certain conditions - very unlikely.

Informative surveys of chaos theory and its relevance for the social sciences are quoted in Nijkamp and Reggiani (1990b, 1990c). Interesting applications of chaos theory can also be found in economics (see for a survey, Nijkamp and Reggiani, 1990b), while specific applications in regional sciences are, for example, the following:

- regional industrial evolution (White, 1985)
- urban macro dynamics (Dendrinos, 1984)
- spatial employment growth (Dendrinos, 1986)
- relative population dynamics (Dendrinos and Sonis, 1987)
- spatial competition and innovation diffusion (Nijkamp, 1990 and Sonis, 1986)
- migration systems (Reiner et al., 1986)
- urban evolution (Nijkamp and Reggiani, 1990c and Zhang, 1990)
- transport systems (Reggiani, 1990)

These applications of chaos stem directly from the prototype models described in a) - e); consequently, it is easy to notice once again that most of the above mentioned models can be reconducted to the discrete logistic growth of biological population.

The relevance of the logistic law of a May-type will in particular be shown in this paper, by analyzing the related impact in a spatial competition system. More specifically, we will first investigate the dynamics of a spatial competition model (emerging from ecology) for two regions by showing the absence of oscillating behaviour (see Section 2). Furthermore, we will study, in the above system, the particular case of a region following the logistic evolution of a

May-type. The theoretical analysis will reveal that the 'chaotic' regime in May's equation will only have a significant impact on the whole system (in terms of irregular behaviour), if the growth rate of the competing region will overcome a critical value (at which a Hopf bifurcation begins). Simulation experiments will confirm the above analysis, while they will also be used for the case of three competing regions (Section 3).

In synthesis this particular case of a competition model will show that:

1) stable behaviour emerges for low growth rates (despite the influence of a 'chaotic' regime);
2) only for high growth rates (exceeding a certain critical value) the impact of the chaotic region will be relevant by producing unexpected fluctuations in the entire system.

2. Dynamics of Spatial Competition

In recent years the number of studies devoted to mathematical models which seek to capture some of the essential dynamic features of regional and urban systems has drastically increased. In this context the potential of models derived from ecology and biology is increasingly recognized, starting from Samuelson (1971) who analyzed a prey-predator model for describing economic competition. We may also refer here to other applications of ecological-biological models, e.g. models formalyzing the introduction of new technologies (see, e.g., Camagni 1985, Nelson and Winter 1982, and Sonis 1986), the labour market evolution (Nijkamp and Reggiani 1991), or population dynamics (Dendrinos and Mullally 1981). A common characteristic of all these models is the use of continuous time (though sometimes not entirely consistent with the related simulative experiments) as well as the use of deterministic equations.

Stochasticity in prey-predator models is rarely analyzed (see, e.g. Campisi 1986); it was recently referred to as 'inner instability', in a prey-predator system of three equations (see Gilpin, 1979) and in a discrete prey-predator system (see Peitgen and Richter, 1986).

In the present section we will analyze a slightly different version of the prey-predator model, viz. a general spatial competition model, by showing the absence of oscillating behaviour (and hence the main difference with the standard prey-predator model) and consequently the absence of periodic cycles. This characteristic is important, since it hampers the emergence of chaotic behaviour. However in Section 3 we will show how a particular case of a spatial competition model will give rise to irregular, aperiodic cycles, probably leading to chaos.

As a starting point we will use here for the sake of simplicity, the prototype model of system competitions developed by Johansson and Nijkamp (1987) in their analysis of urban and regional development:

$$\dot{x}_i = \alpha_i \; x_i \; (N_i - x_i - \beta_{ij}x_j) - \gamma_i x_i \qquad (2.1)$$

where:

x_i = production (or income) of place i

α_i = entry (expansion), growth rate of i

γ_i = exit (depreciation) rate of i

N_i = carrying capacity of production level x_i

β_{ij} = competition coefficient describing the inhibiting effect of each centre j on its competitor i ($\beta_{ij} > 0$)

It should be noted that in the following pages we will always assume $\beta_{ij} > 0$, i.e., the case of the 'pure' competition model. If, for example, $\beta_{ij} < 0$, we would get the prey-predator model where the development of the one centre would increase the other one. We will not treat here this case, since it has been analysed several times in the literature as already noticed above.

We will in subsequent subsections investigate now the stability conditions related to system (2.1) (for example, in the case of two equations) both in continuous and discrete time.

2.1 Competitive interactions in continuous time

We will first analyse here system (2.1) in its two-dimensional case (i=1,2), where x_1 = x and x_2 = y:

$$\dot{x} = a\,x\,(N - x - by) - cx$$
$$\dot{y} = d\,y\,(K - y - ex) - fy \qquad (2.2)$$

where the various coefficients have the same meaning as their corresponding ones in (2.1).

Obviously system (2.2) can be written as follows:

$$\dot{x} = a\,x\,(m - x - by)$$
$$\dot{y} = d\,y\,(n - ex - y) \qquad (2.3)$$

where

$$m = N - (c/a)$$
$$n = K - (f/d) \qquad (2.4)$$

This system can also be written as:

$$\dot{x} = x\,(m^1 - ax - cy)$$
$$\dot{y} = y\,(n^1 - fx - dy) \qquad (2.5)$$

where of course:

$$m^1 = am \; ; \; c = ab$$
$$n^1 = dn \; ; \; f = ed \qquad (2.6)$$

System (2.5) has been studied by several authors starting from Volterra (1926), Lootka (1925) and Gause (1934).

In particular Maynard Smith (1974) analysed system (2.5) for biological processes, by showing -if a non-trivial equilibrium exists- two cases:

i) a stable equilibrium for $m^1/a < n^1/f$ and $n^1/d < m^1/c$ (see Fig. 1);

ii) an unstable equilibrium with both inequalities reversed (see Fig. 2) for which either species can win.

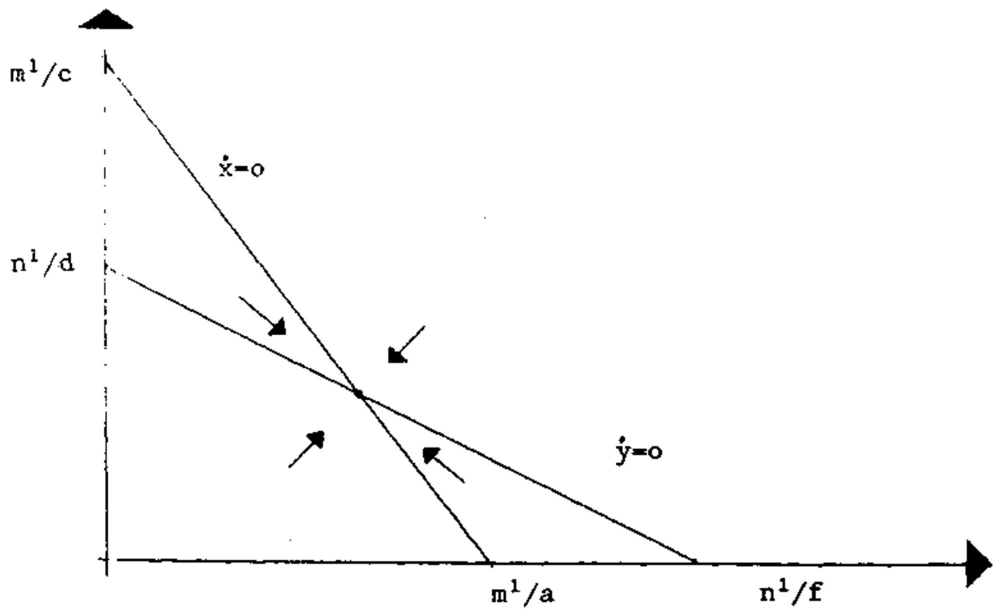

Fig. 1. Stable equilibrium in a competition between two regions x and y

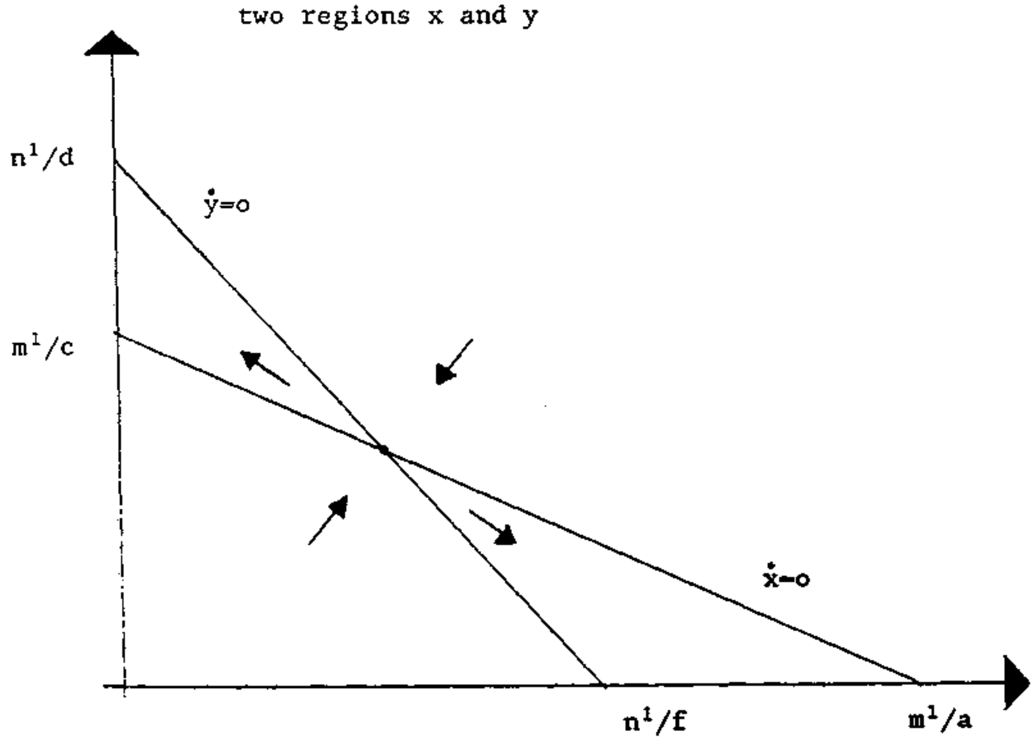

Fig. 2. Unstable equilibrium in a competition between two regions x and y

Consequently for the economic system (2.2) the conditions for stability related to (2.6) and (2.4) are $[N-(c/a)] < \{[K - (f/d)]/e\}$ and $[K - (f/d)] < \{[N-(c/a)]/b\}$, respectively.

It is important to underline that Maynard Smith (1974) showed that, both for system (2.5) and for the general case:

$$\begin{aligned} \dot{x} &= g\ x\ (x,\ y) \\ \dot{y} &= h\ y\ (x,\ y) \end{aligned} \qquad (2.7)$$

the non-trivial equilibrium if - it exists - is stable or unstable but <u>non-oscillatory.</u>

In particular the equilibrium is stable when:

$$\frac{\delta g}{\delta x} \cdot \frac{\delta h}{\delta y} > \frac{\delta g}{\delta y} \cdot \frac{\delta h}{\delta x} \qquad (2.8)$$

where the differentials $\left(\frac{\delta .}{\delta .}\right)$ are calculated at a non-trivial equilibrium point.

Expression (2.8) shows at the lefthand side the 'marginal' inhibiting effects of each place (region) on itself and at the right-hand side the effects of each place (region) on its competitor. In other words, following Gilpin and Justice (1972) it is clear that << a necessary and sufficient condition for the stability of a competitive equilibrium is that the product of the intraspecific growths regulation be greater than the product of the interspecific growth regulations >> (Maynard Smith 1974, p.64).

Conditions (2.8) stems directly from the formal expression of (2.5) and (2.7), since by definition an increase in one place reduces the 'velocity of increase' of the other one. This also means that if the two systems have identical needs with the same resources, the more efficient system will eliminate its competitor.

This result has in our analytical framework another important meaning. It hampers the emergence of chaotic behaviour in systems of type (2.5), since a pre-condition for chaos is the emergence of a chain of Hopf bifurcations (at least three) which are based on neutral stability' (or limit cycle or oscillatory behaviour) (see Schuster 1988).

2.2 Competitive interactions in discrete time

Let us now consider a general system describing competition between species with discrete generations:

$$\begin{aligned} x_{t+1} &= x_t \; p\,(x_t \, , \, y_t) \\ y_{t+1} &= y_t \; q\,(x_t \, , \, y_t) \end{aligned} \qquad (2.9)$$

Also for the general system of type (2.9) Maynard Smith (1974) showed that the equilibrium - if it exists - is either stable or unstable, but in either case non-oscillatory. Moreover he demonstrated the absence of oscillations in a long run. It should be noted however that the same result does not emerge in interactions of a prey-pedator type, where also divergent oscillations can emerge.

In the next section we will show how a very particular case of (2.7) (i.e., when the first place (region) has no competition effect on the second one), can indeed show, under certain conditions, the emergence of oscillating (likely chaotic) behaviour.

3. Impact of Chaos in Spatial Competition

In this section we will consider the particular case in which a region follows a 'chaotic' evolution of the well-known May type of model. In other words, we suppose that in the first equation of system (2.3) the competition coefficient b measuring the effect to which center y presses upon the resources used by x is equal to 0. This hypothesis may be plausible from a spatial economic viewpoint, as this may reflect hierarchy in spatial systems. This may imply a situation where higher-order places (or regions) have a decisive influence on lower-order places (or regions), without being influenced by means of feedback effects by lower-order places (or regions). Both Christaller-Lösch systems but also international trade block dominances may be described by such analytical systems.

We will analyse the impact of this 'chaotic' region in the case of both two competing regions and three competing regions.

3.1 The case of two competing regions

3.1.1 Analysis

Under the above mentioned hypothesis of a 'chaotic' region, the implications for the continuous system (2.3) are as follows:

$$\begin{aligned} \dot{x} &= a\, x\, (m - x) \\ \dot{y} &= dy\, (n - e\, x - y) \end{aligned} \qquad (3.1)$$

We will now analyse the discrete version of system (3.1), since this makes more sense from an economic viewpoint in terms of empirical applications (data are usually only available in discrete form), while also simulation experiments are usually carried out in discrete terms.

The discrete form of system (3.1) is therefore the following (see Annex A for the mathematical contents):

$$\begin{aligned} x_{t+1} &= \varphi\, x_t\, (1 - x_t) \\ y_{t+1} &= d\, y_t\, (n+1/d - e\, x_t - y_t) \end{aligned} \qquad (3.2)$$

where (see expression (A.4) in Annex A and equation (2.4):

$$\varphi = m\, a + 1 = a\, N - c + 1 \qquad (3.3)$$

It is interesting to note that the first equation in system (3.2) is exactly the equation thoroughly analysed by May (1976). In particular May showed the presence of a rich spectrum of irregular behaviour for the following range of parameter values:

$$3 < \varphi < 4 \qquad (3.4)$$

and the presence of 'chaotic' behaviour with cycles with every integer period, as well an uncountable number of aperiodic trajectories for:

$$3.824 < \varphi < 4 \qquad (3.5)$$

Further analysis of this equation with reference to its relationships with logit models, pertaining to the family of discrete choice models, can be found in Nijkamp and Reggiani (1990b).

Let us now investigate the equilibrium conditions related to the particular system (3.2). For the ease of calculus, we will rewrite system (3.2) as follows:

$$x_{t+1} = x_t\ (\varphi - \varphi\ x_t)$$
$$y_{t+1} = y_t\ (z - v\ x_t - dy_t) \qquad (3.6)$$

where:

$$z = dn + 1 - d\ K - f + 1$$
$$v = de \qquad (3.7)$$

An equilibrium analysis related to system (3.6) shows two fixed points: a trivial one P_1 (0,0) and a non-trivial one P_2 $\left(\frac{\varphi - 1}{\varphi} ; \frac{z\varphi - v\varphi + v - \varphi}{\varphi d}\right)$ (see Annex B). The stability analysis for P_1 shows unstable behaviour when $\varphi > 1$ and $z > 1$.

If we now examine the stability of point P_2, we get the critical value (see equation (B.9) in Annex B):

$$z^* = (v\ \varphi^2 + 2\ \varphi^2 - 3\ \varphi - 3\ v\ \varphi + 2\ v)\ /\ (\varphi^2 - 2\varphi) \qquad (3.8)$$

at which a Hopf bifurcation (i.e. a bifurcation of a fixed point into a closed orbit) occurs. This implies that for $z > z^*$ the fixed point P_2 becomes unstable, with the possibility of oscillations. This result is indeed interesting, since it shows that <u>if</u> the first equation in system (2.2) (and consequently in system (2.9)) is reduced to an equation of a May type (leading to chaos), we get in the whole system the possibility of <u>oscillating behaviour</u> based on Hopf bifurcations, and hence <u>unpredictable movements</u>. This also underlines the relevance of 'chaotic' evolution, since it can introduce oscillations in a system which in itself is not oscillatory (see Section 2).

The above result can also be illustrated graphically, by means of simulation experiments. In particular by considering certain values of φ from (3.4) and by fixing the parameter v we can investigate the dynamics of the system at hand according to the conditions of z^* that emerged in the stability analysis, at which a Hopf bifurcation occurs (see Table 1).

z^*	φ	v
3.37	3.1	0.69
3.12	3.6	0.69
3.04	3.9	0.69
2.90	3.99	0.54

Table 1. Values of some growth parameters leading to a Hopf bifurcation

It should be noted that the last values for φ which we have considered in our analysis (i.e. $\varphi = 3.9$; $\varphi = 3.99$) are the ones leading to chaos in a May equation.

3.1.2 Simulation experiments

For the first simulation (Fig.3) we have assumed the following parameter values:

$z = 4.1 > z^*$ $\varphi = 3.1$ $v=0.69$ $d = 1$

with the initial conditions:

$x = y = 0.1$

The results are contained in Fig.3.

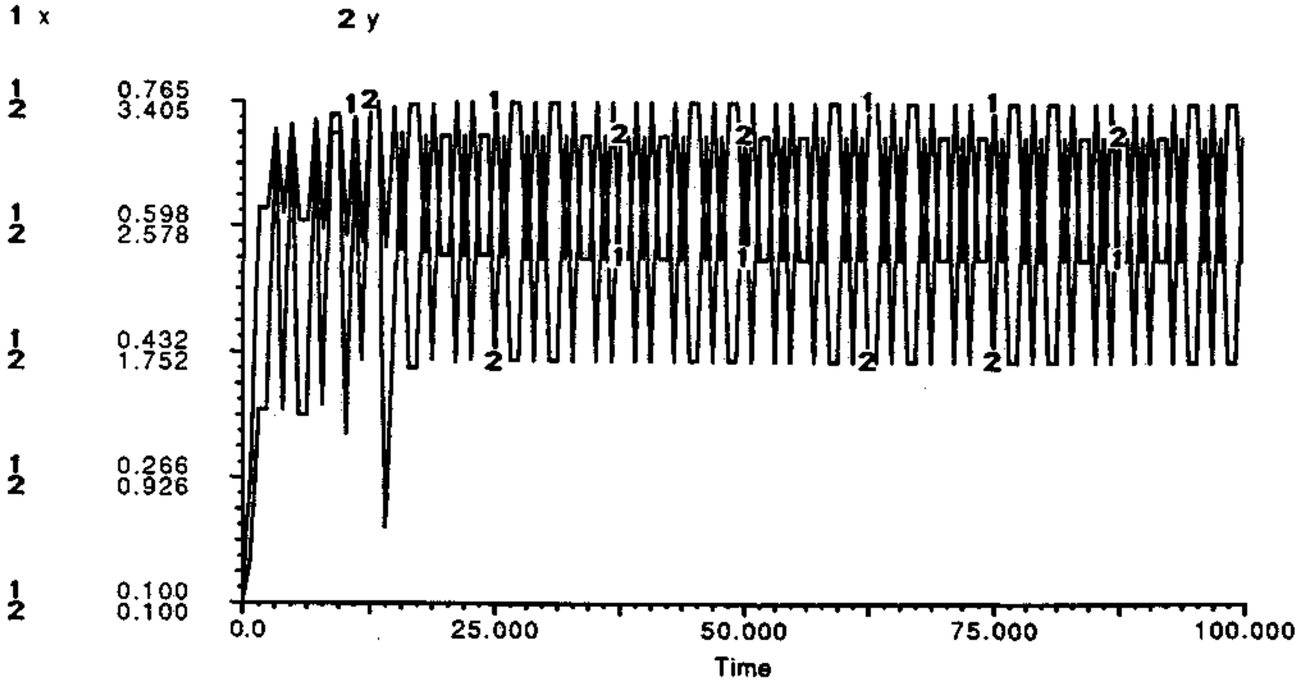

Fig. 3. Regular oscillatory behaviour for the two regions x and y

Fig. 3 shows a periodic behaviour. It is intersting to note that the value of the parameter d has no impact on the emergence of unstable behaviour (see equation (3.8)). It can be derived 'a posteriori' from equation (3.7).

In the following simulations we will consider values of φ that display in the related May equation both irregular behaviour (for example, for $\varphi=3.6$) and chaotic behaviour (for example, for $\varphi=3.9$). Then we will consider two cases:

$z < z^*$ and $z > z^*$

Therefore, for the second simulation (Fig. 4) we will assume:

$z = 1.4 < z^*$ $\quad \varphi = 3.6 \quad$ $v = 0.69$

while for the third simulation (Fig. 5) we will assume:

$z = 1.4 < z^*$ $\quad \varphi = 3.9 \quad$ $v = 0.69$

with the same initial conditions:

$x = y = 0.1$

Fig. 4 shows that by keeping the growth parameter z below the critical value z^* the related equation shows stability in the long run. The same happens even if we assume a 'chaotic' value of φ ($\varphi=3.9$) (see Fig. 5). The competing region y is more oscillatory in the short run; however, in the long run it reaches stability (by being eliminated by region x).

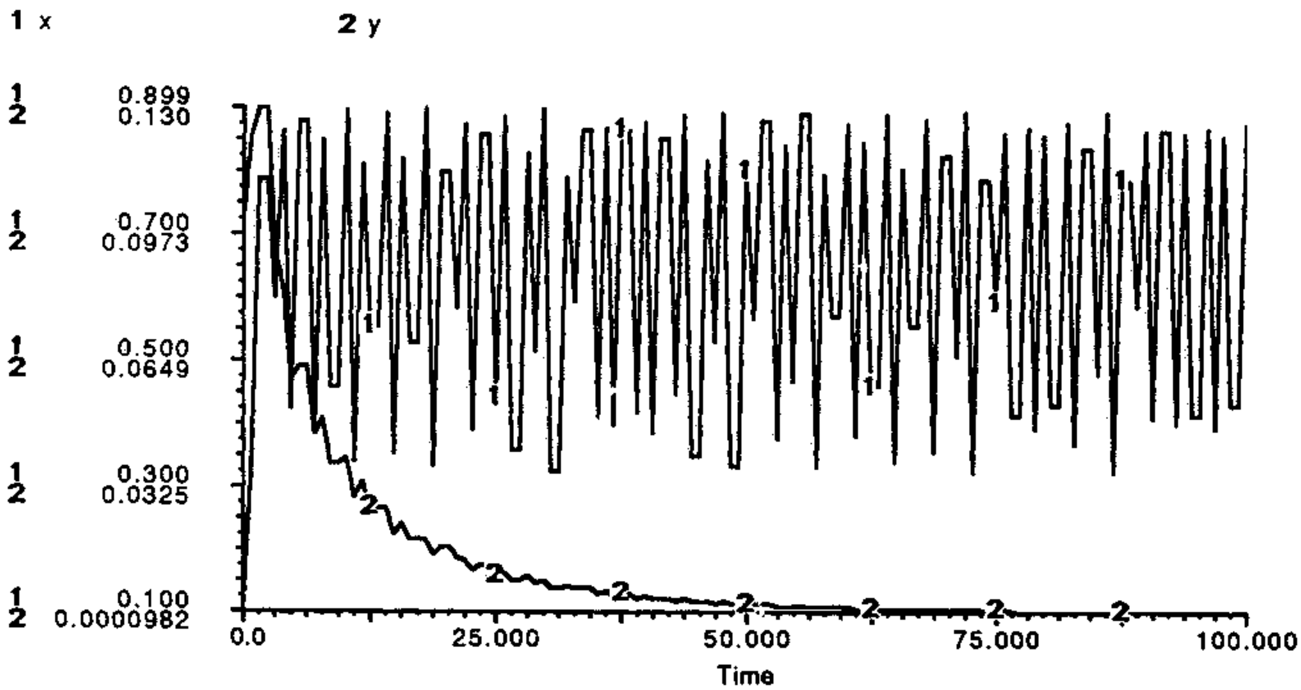

Fig. 4. Oscilatory behaviour for region x and stable behaviour (in the long run) for region y for $\varphi < z^*$ and $\varphi = 3.6$

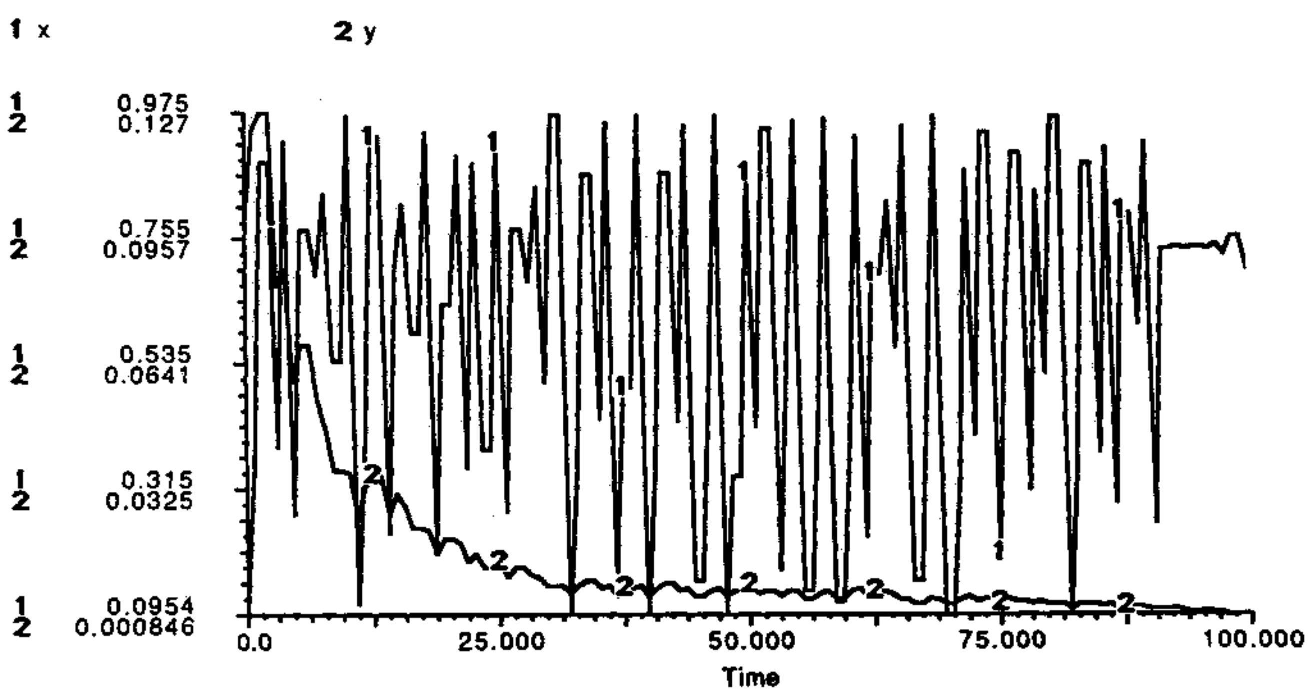

Fig. 5. Again oscillatory behaviour for region x and stable behaviour (in the long run) for region y for $\varphi < z^*$ and $\varphi = 3.9$ ('chaotic' value)

However, if we consider a value of the growth parameter z beyong the critical value z^* (by keeping the same values of φ and v considered in Fig.4 and Fig. 5), we can clearly observe a destabilization of region y.

In particular Fig. 6 shows the emerging irregular behaviour in the evolution of region y for the following parameter values:

$z = 3.5 > z^*$ $\varphi = 3.6$ $v = 0.69$

while Fig. 7 displays an even more irregular pattern, due to the increased value of φ ($\varphi = 3.9$, i.e. a 'chaotic' value) correlated with a value of $z = 3.2 > z^*$ (by keeping $v = 0.69$ and the same initial conditions $x = y = 0.1$).

3.1.3 Some remarks

In conclusion, the analysis conducted in the case of two competing regions, of which one is 'chaotic', shows the following results:

- the importance of the growth rates of the two competing regions, and hence
- the importance of the speed of change of these parameters in real systems
- the importance of a 'chaotic' evolution.

In particular we can underline the following findings:

a. stability occurs at low values of the growth rates;

b. the chaotic region becomes unstable according to its increasing growth rates. Hoever it does not completely destablize (in a long run sense) the competing region, unless the growth rate of the competing region exceeds the critical value z^* at which a Hopf bifurcation occurs.

This result also broadens the statement made by Maynard Smith (1974), defining the absence of oscillating behaviour in a two-dimensional spatial competitive system (see Section 2). In other words, we have shown in a spatial competition system of two regions the possiblity of oscillating behaviour based on Hopf bifurcations, when a region follows the particular evolution of a 'chaotic' type.

The next step will now be the analysis of the impact of a 'chaotic' region in a three - dimensional system, where the other two regions follow the usual spatial competition's low of type (2.9).

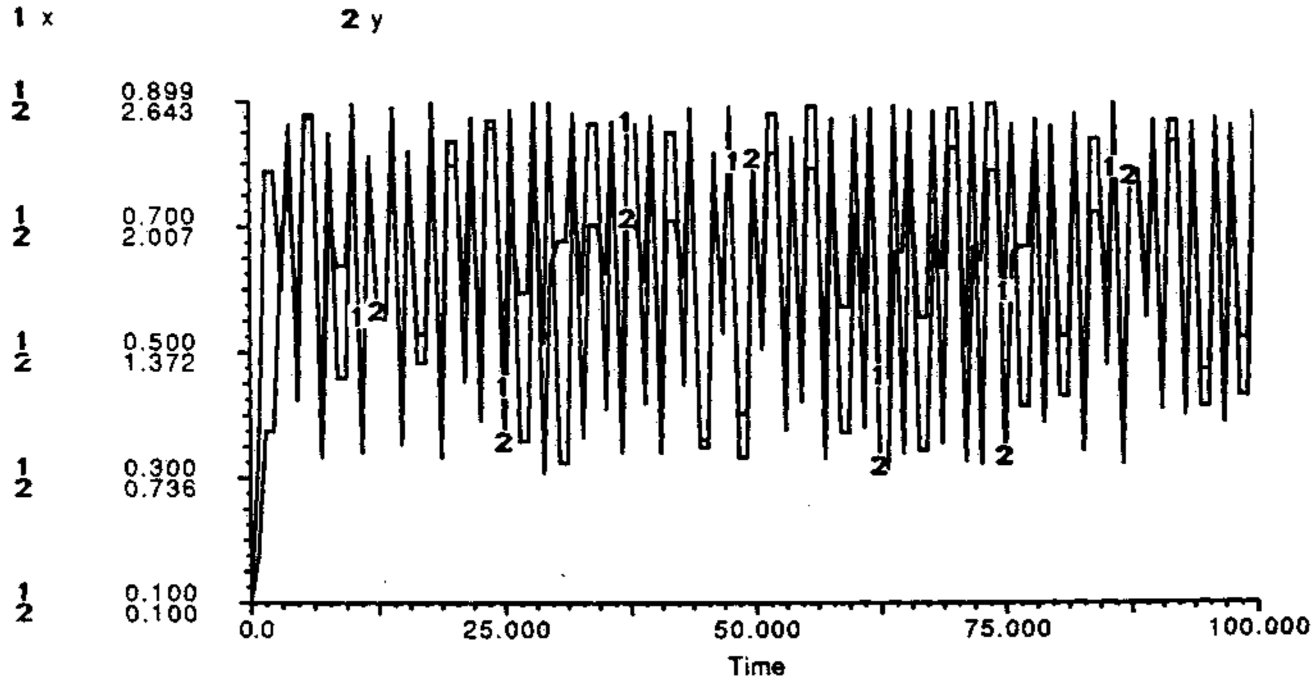

Fig. 6. Irregular behaviour for both the regions x and y for $\varphi >$ z^* and $\varphi = 3.6$

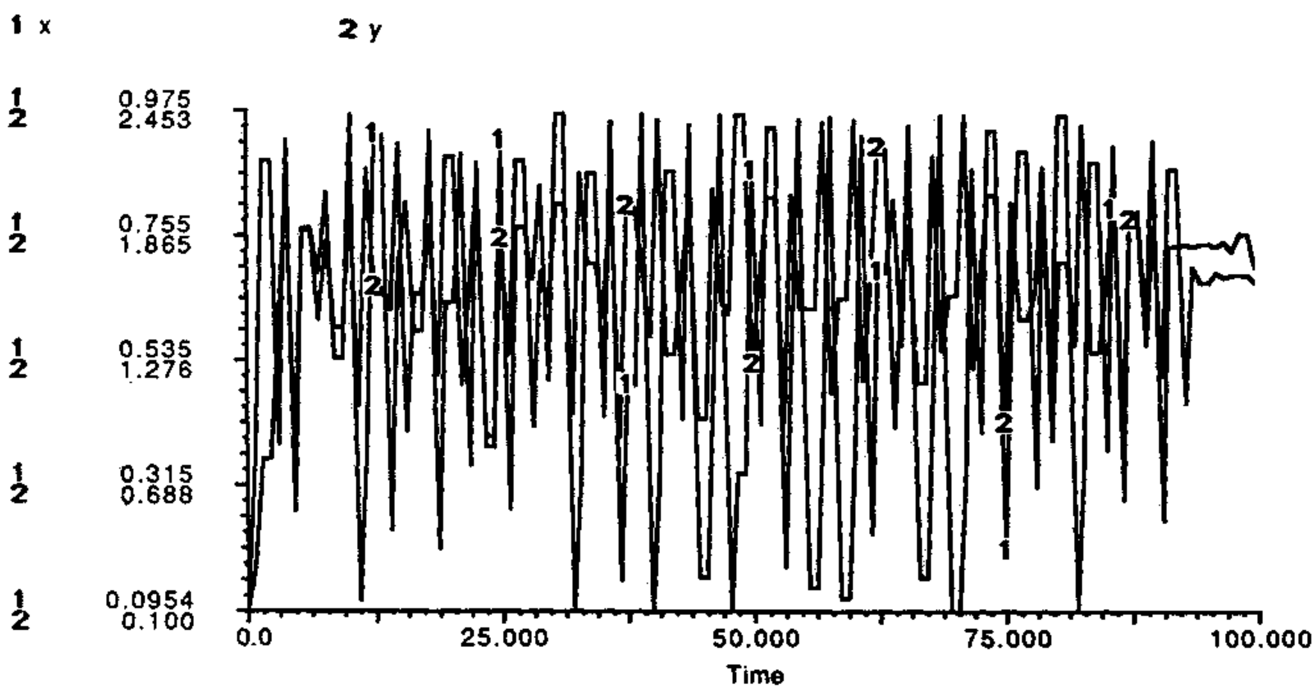

Fig. 7. Again irregular behaviour for both the regions x and y for $z > z^*$ and $\varphi = 3.9$ ('chaotic' value)

3.2 The case of three regions

3.2.1 Introduction

We will consider here the case of three regions in which one follows the May evolution already described in the first equation of system (3.6).

Consequently we will face the following system:

$$
\begin{aligned}
z_{t+1} &= z_t\ (\varphi - \varphi\, z_t) \\
x_{t+1} &= x_t\ (a' - b'x_t - c'y_t - d'z_t) \\
y_{t+1} &= y_t\ (e' - f'x_t - g'y_t - h'z_t)
\end{aligned}
\qquad (3.8)
$$

where a', b' c', d', e', f', g', h' and φ are positive parameters.

Since in this case the theoretical analysis concerning the stability of the three-dimensional system (3.8) is analitically difficult to manage (see Kloeden, 1981), we will illustrate now some simulation experiments displaying the irregular behaviour in the whole system owing to the influence of the 'chaotic' region z.

3.2.2 Simulation experiments

Firstly, we will examine a to-dimensional system in x and y; for example, we will assume the following initial system:

$$
\begin{aligned}
x_{t+1} &= x_t\ (3.3 - x_t - 0.5\, y_t) \\
y_{t+1} &= y_t\ (3.5 - 0.3\, x_t - y_t)
\end{aligned}
\qquad (3.9)
$$

As underlined in Section 2, system (3.9) can display either stable or unstable behaviour, but non-oscillatory. In this particular case, the stability conditions shown in Fig. 1 of Section 2, reveal a stable behaviour, since $3.3 < 3.5/0.3$ and $3.5 < 3.3/0.5$.

Let us now introduce a 'chaotic' equation in the variable z as illustrated in (3.8):

$$
\begin{aligned}
z_{t+1} &= z_t\ \ (\varphi - \varphi\, z_t) \\
x_{t+1} &= x_t\ (3.3 - x_t - 0.5y_t - 0.8z) \\
y_{t+1} &= y_t\ (3.5 - 0.3x_t - y_t - 0.5z)
\end{aligned}
\qquad (3.10)
$$

By considering now a low value of φ (φ=2.8) leading to stability in region z, we get for regions x and y the emergence of regular oscillatory behaviour (see Fig. 8). However, by increasing the values of φ toward the level of its chaotic values (φ=3.9), we clearly obtain a very irregular pattern for the whole system in x, y, z (see Fig. 9).

Obviously also in this three-dimensional case there will be a correlation between φ and the growth rates of x and y a'^*, e'^* at which instability begins (analogously to the two-dimension case), certainly for high values of the parameters a' and e' as in the example adopted in (3.10), according to the observations made in Section 3.1.2.

4. Conclusion

Given the recent, increasing interest in chaos theory for its capability in generating, by means of deterministic non-linear equations, endogenous fluctuations very much like as the ones observed in reality, this paper has paid attention to a particular form of chaotic behaviour, viz. the logistic discrete growth, since it constitutes the basis of many dynamic non-linear models adopted so far in economics and in regional science. In particular, our aim was to analyze the impact of a logistic growth in a more general system, such as the spatial competition model which in itself is non-oscillatory.

Our analysis, which has been carried out for the case of both two and three competing regions, shows on the one hand the relevance of a chaotic pattern, and on the other hand the relevance of the growth rates of competing regions.

In particular we can underline that chaotic behaviour in a region emerges only in a certain limited range of its growth rate and that it influences (in terms of irregularity) the competing region only if the related intrinsic growth rate (depending also on the inter-regional competition coefficient) exceeds a critical value.

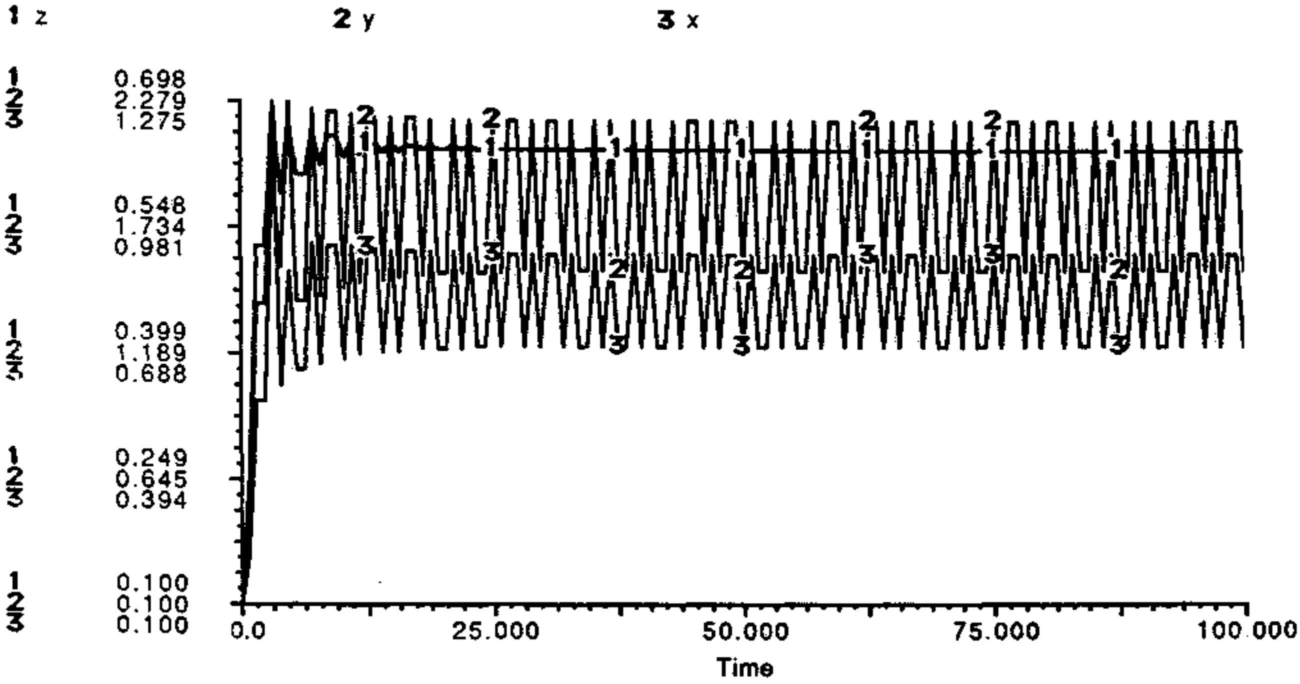

Fig. 8. Regular oscillatory behaviour for both the regions x and y, and stable behaviour for the region z for $\varphi = 2.8$

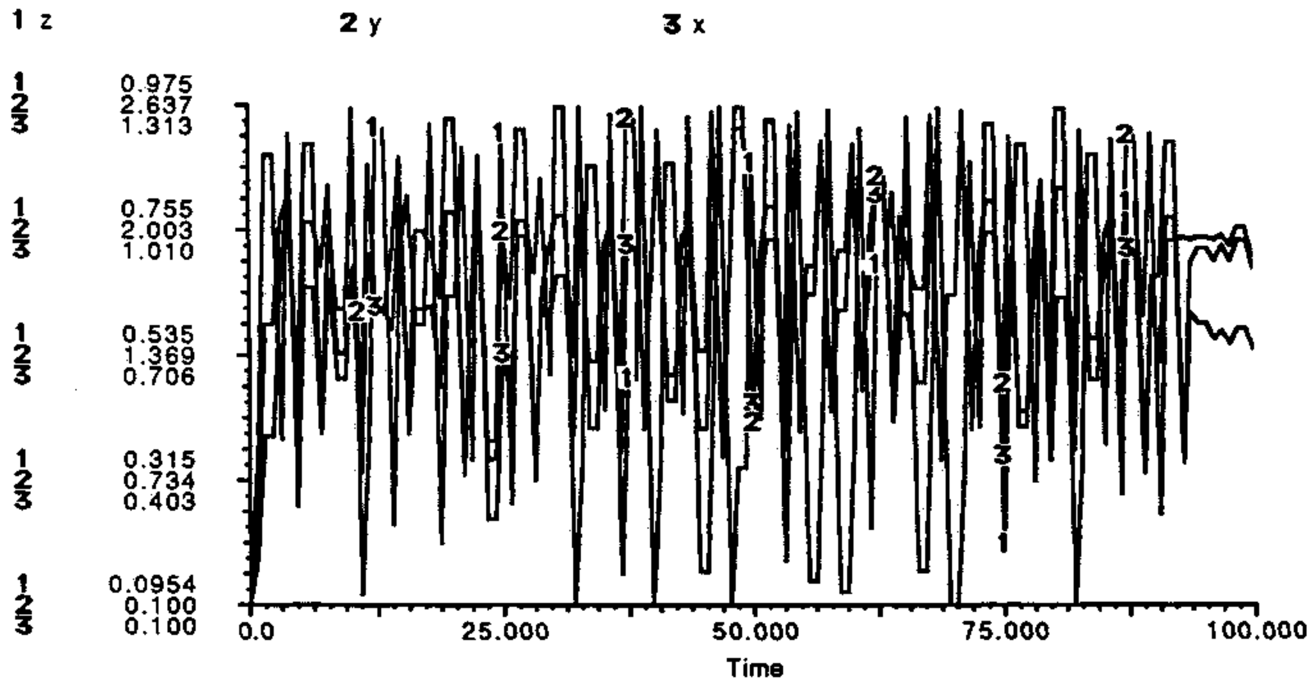

Fig. 9. Irregular behaviour in the whole system for $\varphi = 3.9$ ('chaotic' value)

This result provides fruitful insights into the actual mechanisms of competition and suggests the following future directions of research:

a. the need of a theory on interactive competition coefficients (actually non-existent); for example, study of decisive variables on which the competition coefficients may be dependent (e.g., density of population, accessibility, environmental-technological conditions, etc.)

b. the need of empirical research by gathering data on the speed and spread of change of competition coefficients as well as of the intrinsic growth rates.

c. the necessity of defining the 'real' domain of the relevant parameter space (e.g.,extensive stable domain or not) in order to test the dynamic (in) stability for both the predictable and unpredictable systems at hand.

Acknowledgement

The C.N.R. grants n. 90.02147.CT11 and n.88.03086.10 are gratefully acknowledged by the second author.

References

Allen, P.M., Self-Organization and Dissipative Structures, Self-Organization in the Urban System (C. Schieve and P. Allen, eds.), University of Texas Press, Austin, 1982, pp. 132-158.

Beckmann, M.J. and T. Puu, Spatial Economics: Density, Potential, and Flow, North-Holland, Amsterdam, 1985.

Blommestein, H. and P. Nijkamp, Adoption and Diffusion of Innovations and Evolution of Spatial Systems, Economic Evolution and Structural Adjustment, (D. Batten, J. Casti and B. Johansson, eds.), Springer Verlag, Berlin, 1987, pp. 368-380.

Camagni, R., Spatial Diffusion of Pervasive Process Innovation, Papers of the Regional Science Association, vol. 58, 1985, pp. 83-95.

Campisi, D., Lotka-Volterra Models with Random Fluctuations for the Analysis of Oscillations in Urban and Metropolitan Areas, Sistemi Urbani, 2/3, 1986, pp. 309-321.

Dendrinos, D.S., Turbulence and Fundamental Urban/Regional Dynamics, Paper presented at the American Association of Geographers, Washington D.C., April, 1984.

Dendrinos, D.S., On the Incongruous Spatial Employment Dynamics, Technological Change, Employment and Spatial Dynamics, (P. Nijkamp, ed.), Springer-Verlag, Berlin, 1986, ppl 321-339.

Dendrinos, D.S. and H. Mullally, Evolutionary Patterns of Urban Populations, Geographical Analysis, vol. 13, no. 4, 1981, pp. 328-344.

Dendrinos, D.S. and M. Sonis, The Onset of Turbulence in Discrete Relative Multiple Spatial Dynamics, Applied Mathematics and Computation, vol. 22, 1987, pp. 25-44.

Domanski, R., Self-Organization of Evolving Economic Space, Dynamics and Conflict in Regional Structural Change, (M. Chalterji and R.K. Kuenne, eds.), MacMillan, London, 1990, pp. 79-94.

Farmer, R.E.A., Deficit and Cycles, Journal of Economic Theory, vol. 40, 1986, pp. 77-88.

Gause, G.F., The Struggle for Existence, Williams and Wilkins, Baltimore, 1934.

Gilpin, M.E., Spiral Chaos in a Predator-Prey Model, American Nature, vol 113, 1979, pp. 306-308.

Gilpin, M.E. and K.E. Justice, Reinterpretation of the Invalidation of the Principle of Competitive Exclusion, Nature, 1972, vol. 236, pp. 273-301.

Harris, B. and A.G. Wilson, Equilibrium Values and Dynamics of Attractiveness Terms in Production-Constrained-Spatial-Interaction Models, Environment and Planning A, vol. 10, 1978, pp. 371-338.

Johansson, B. and P. Nijkamp, Analysis of Episodes in Urban Event Histories, Spatial Cycles, (Van den Berg, L., L.S. Burns and L.H. Klaassen, eds.), Gower, Aldershot, 1987, pp. 43-66.

Kloeder, P.E., Chaotic Difference Equations in R^n, Journal of Australian Mathematical Society, Series A, Vol. 31, 1981, pp. 217-225.

Lawrerier, H.A., Two-Dimensional Iterative Maps, Chaos, (A.V. Holden, ed.), Princeton University Press, Princeton, 1986, ppl 58-95.

Lotka, A.J., Elements of Physical Biology, Williams and Wilkins, Baltimore, 1925.

May, R., Simple Mathematical Models with Very Complicated Dynamics, Nature, vol. 261, 1976, pp. 459-467.

Maynard Smith, J., Models in Ecology, Cambridge University Press, London, 1974.

Nelson, R.R. and Winter S.G, An Evolutionary Theory of Economic Change, Harvard University Press, Cambridge, Mass., 1982.

Nijkamp, P., Regularity and Chaos in Economics: Are Economic Predictions Useless? Research Paper, Dept. of Economics, Free University, Amsterdam, 1990.

Nijkamp, P. and A. Reggiani, Spatial Interaction Models as a Unified Framework for Dynamic Transportation Models, Research Memorandum, 1990-82, Faculty of Economics, Free University, Amsterdam, 1990a.

Nijkamp, P. and A. Reggiani, Logit Models and Chaotic Behaviour: A New Perspective, Environment and Planning A, vol. 22, 1990b, pp. 1455-1467.

Nijkamp, P. and A. Reggiani, Theory of Chaos: Relevance for Analysing Spatial Processes, Spatial Choice and Processes (M. Fischer, Nijkamp, P. and Y. Papageorgiou eds.), North Holland, Amsterdam, 1990c, pp. 49-80.

Nijkamp, P. and A. Reggiani, A New Methodology for the Analysis of Dynamic Spatial Interaction Models, Sistemi Urbani, 1991 (forthcoming).

Peitgen, H.O. and P.H. Richter, The Beauty of Fractals, Springer Verlag, Berlin, 1986.

Reggiani, A., Spatial Interaction Models: New Directions, Ph.D. Dessertation, Dept. of Economics, Free University, Amsterdam, 1990.

Reichlin, P., Equilibrium Cycles and Stabilization Policies in an Overlapping Generations Model with Production, Journal of Economic Theory, vol. 40, 1985, pp. 89-102.

Ruelle, D. and Takens, F., On the Nature of Turbulence, Communications in Mathematical Physics, vol. 20, 1971, pp. 167-192.

Samuelson, P., Generalized Predator-Prey Oscillations in Ecological and Economic Equilibrium, Proceedings of the National Academy of Sciences, vol.58, 1971, pp. 980-983.

Schuster, H.G., Deterministic Chaos, VCH, Veinheim, 1988.

Sonis, M., A Unified Theory of Innovation Diffusion, Dynamic Choice of Alternatives, Ecological Dynamics and Urban/Regional Growth and Decline, Ricerche Economiche, vol. XL, n. 4, 1986, pp. 696-723.

Sonis, M., Log-Linear Relative Dynamics: Unification of Theories, Economic Evolution and Structural Adjustment, (D. Batten, J. Casti and B. Johansson, eds.), Springer Verlag, Berlin, 1987, pp. 166-184.

Volterra, V., Variazione e Fluttuazione del Numero di Individui in Specie Animali Conviventi, in: Mem. Accad. Nazionale Lincei, vol. 31, ser. 6, no. 1, 1926.

Weidlich, W. and G. Haag, Concepts and Models of a Quantitative Sociology, Springer, Berlin, 1983.

White, R.W., Transitions to Chaos with Increasing System Complexity: The Case of Regional Industrial Systems, Environment and Planning A, vol. 17, 1985, pp. 387-396.

Wilson, A.G., Catastrophe Theory and Bifurcation, Croom Helm, London, 1981.

Zhang Wei-Bin, Spatial Sinergetics, Springer, Berlin, 1990.

Annex A

Transformation of a continuous system into a discrete system.

In this annex we will show how the continuous system (3.2) can be approximated in a discrete form.

For the ease of reference we will write again the first equation of system (3.1):

$$\dot{x} = a\, x\, (m - x) \qquad (A.1)$$

We can now approximate equation (A.1) in discrete time by considering a unit time period as folows:

$$x_{t+1} - x_t = ax\, (m-x) \qquad (A.2)$$

which can also be written as:

$$x_{t+1} = \varphi\, x_t\, (1 - \frac{\varphi - 1}{\varphi m} x_t\,) \qquad (A.3)$$

where:

$$\varphi = am + 1 \qquad (A.4)$$

It is now clear that if we make the transformation (see also Nijkamp and Reggiani, 1990b and Wilson, 1981):

$$x^* = x_t\, (\varphi-1)/\varphi m \qquad (A.5)$$

equation (A.3) can be written in the canonical form:

$$x^*_{t+1} = \varphi\, (1 - x^*_t\,) \qquad (A.6)$$

For the sake of simplicity we have rewritten equation (A.6) in system (3.2) in x instead of x^*, since this is a translation of coordinates without any impact on the behaviour of the system. Next, we will consider the approximation in discrete terms of the second equation in (3.1) as follows:

$$y_{t+1} - y_t = d\, y_t\, (n - ex_t - y_t) \qquad (A.7)$$

or

$$y_{t+1} = d\, y_t\, (n + 1/d - ex_t - y_t) \qquad (A.8)$$

Consequently equations (A.6) and (A.8) make up in system (3.2) the difference version of system (3.1).

Annex B

Stability analysis for a particular competing system.

In this annex we will analyse the stability properties of system (3.5). For the ease of reference we will write again system (3.6):

$$x_{t+1} = x_t \ (\varphi - \varphi x_t)$$
$$y_{t+1} = y_t \ (z - v\ x_t - d\ y_t\ ) \qquad \text{(B.1)}$$

It is easy to find the fixed points for which $x_{t+1} = x_t$ and $y_{t+1} = y_t$. These are P_1 (0,0) and

$$P_2 \ \left(\frac{\varphi-1}{\varphi}\ ;\ \frac{z\varphi - v\varphi + v-\varphi}{\varphi d}\right)$$

The local behaviour of the map (B.1) at its fixed points P_1, P_2 is governed by its local linearization for which the Jacobian J:

$$J = \begin{bmatrix} \varphi - 2\varphi x^* & 0 \\ - v\ y^* & z-vx^* - 2dy^* \end{bmatrix} \qquad \text{(B.2)}$$

taken at the fixed point P (x^*, y^*) is the corresponding matrix.

Consequently we get:

$$J(P_1) = \begin{bmatrix} \varphi & 0 \\ 0 & z \end{bmatrix} \qquad \text{(B.3)}$$

In other words the multipliers of P_1 (i.e. the eigenvalues λ_1 and λ_2 of (B.3)) are:

$$\lambda_1 = \varphi \qquad \lambda_2 = z \tag{B.4}$$

It is clear that the trivial fixed point P_1 (0,0) shows a rich spectrum of stable/unstable behaviour according to the values of the growth rate φ and z of the region x and y respectively.

In particular we can underline the following results (for the basic notations related to the stability of discrete-time system see, e.g., Lawerier, 1986 and Lorenz, 1989):

a) if $\varphi < 1$ and $z < 1$, the fixed point P_1 is stable,

b) if $\varphi < 1$ and $z > 1$, the fixed point P_1 is a saddle;

c) if $\varphi > 1$ and $z > 1$, the fixed point P_1 is a repelling node

d) if $\varphi = 1/z$ a Hopf bifurcation occurs.

We will now analyse the Jacobian (B.2) taken at P_2:

$$J\ (P_2) = \begin{bmatrix} 2-\varphi & 0 \\ \frac{v}{\varphi d}\ (z\varphi - v\varphi + v - \varphi) & \frac{v\varphi - \varphi z - v + 2\varphi}{\varphi} \end{bmatrix} \tag{B.5}$$

Here we consider the trace Tr of (B.5) as well as its determinant Det:

$$T_r\ J\ (P_2) = J_{11} + J_{22} = \frac{v\varphi - \varphi z - v + 4\varphi - \varphi^2}{\varphi} \tag{B.6}$$

$$\text{Det } J\ (P_2) = J_{11} * J_{22} = \frac{(2-\varphi)\ (v\varphi - \varphi z - v + 2\varphi)}{\varphi} \tag{B.7}$$

It follows from Ruelle-Takens theorem (1971) that a <u>Hopf bifurcation occurs</u> at $z = z^*$ such that $\text{Det } J\ (P_2)|_{z=z^*} = 1$, i.e:

$$(2-\varphi)\ (v\varphi - \varphi z - v + 2\varphi) = \varphi \tag{B.8}$$

or

$$z^* = \frac{v\varphi^2 + 2\varphi^2 - 3\varphi - 3\ v\varphi + 2v}{\varphi^2 - 2\varphi} > 0 \tag{B.9}$$

Thus this implies that for $z > z^*$ the eigenvalues have a modulus larger then unity, i.e. the fixed point becomes unstable.

It should be noted that the Hopf bifurcation in discrete-time models has been rarely applied in economics, despite a few exceptions (see, e.g., Cugno and Montrucchio, 1984; Farmer, 1986; Lorenz, 1989 and Reichlin, 1985). Our interest in finding the parameter values z* leading to a Hopf bifurcation consists in the fact that a Hopf bifurcation is one of the possible routes to chaos (see Schuster, 1988).