91-8

Faculteit der Economische Wetenschappen en Econometrie

Serie Research Memoranda

A Product Form Extension for Discrete-Time Communication Protocols

Nico M. van Dijk

Research Memorandum 1991 - 8
January 1991

vrije Universiteit amsterdam

A PRODUCT FORM EXTENSION FOR DISCRETE-TIME COMMUNICATION PROTOCOLS

Nico M. van Dijk†
Bond University, Queensland, Australia

Abstract

A discrete-time communication framework is studied under an approximate so-called "stop protocol" related to the lost-calls-held protocol. An explicit insensitive product form expression as recently obtained under a less realistic recirculated protocol is shown to remain valid. This extension seems of both practical and theoretical interest. Applications include:

- Circuit switching structures
- MAN-allocation schemes
- CSMA and BTMA
- Rude-CSMA

Keywords

Random Access Protocols, Discrete-time, Stop Protocol, Recirculate Protocol, Product Form, Insensitivity.

†**Visiting Address:** School of Information and Computing Sciences, Bond University, Gold Coast, Queensland, 4229, Australia
Permanent Address: Department of Econometrics, Free University, The Netherlands

Contents

1 Introduction 3

2 Model, Protocol and Blocking Condition 4

3 Product form result 8

4 Applications 14

4.1 CSMA-protocols 14

4.2 Circuit switching 15

4.3 Interconnected Metropolitan Area Networks (MAN's) 15

4.4 Rude CSMA 16

5 References 18

1 Introduction

Integrated digitized transmissions of multi-type messages such as voice-data and video is the trend for current and future technological developments in the explosively growing field of telecommunications. Continuous-time modeling approaches such a most notably queueing analysis have proven most useful in classical traffic or communications engineering. These approaches generally assume that only one event, a message arrival or completion, can occur at a time. Present-day digitized transmissions or time-slotted devices such as a "router" in an "interconnected metropolitan area network", in contrast, typically involve multiple events at the same time and thus require a discrete-time rather than continuous-time modeling.

Several discrete-time analogues of classical continuous-time product form results for queueing networks have been reported over the last couple of years (cf. [3], [7], [9], [14], [15], [17], [18]). But blocking or interference phenomena as arising in communication random access protocols like CSMA, BTMA or circuit switching are herein not dealt with. Recently, in [22] such features have been studied in a discrete-time setting. An insensitive product form result was obtained under the assumption of a recirculating blocking protocol. Under this protocol both transmission requests and completed messages are lost and require a complete rescheduling or retransmission when blocking arises. In practical communications systems such as a circuit switched network, however, the so-called "communication protocol" seems much more natural. Under this protocol, which is also related to the lost-calls-held protocol and which will be referred to as "stop" protocol hereafter, the scheduling or transmission of a message is simply "stopped" or "interrupted" in blocked solutions. For example one may think of blocking due to

- limited capacities of common resources, such as channels, being exceeded, or
- end-to-end connections being interrupted when some priority message is sent along one of the interconnecting links, or
- a carrier or bus for transportation of packets being temporarily out of order or reassigned another packet.

As opposed to the recirculating protocol, the proposed stop protocol proposed seems more practical for the following reasons:

- It only interrupts an ongoing scheduling during the time that the system configuration is blocked rather than that it requires a complete new rescheduling.

- It almost completely decouples the scheduling and transmission mechanism so that ongoing transmissions are hardly affected by access blocking, as natural in practice.

- It can also model a state dependent delay factor of the provided service rate capacities or transmission scheduling times. This may occur for example in gracefully degradable computer communication systems with failures.

In a continuous-time framework equivalence of both protocols is intuitively obvious when all times involved are exponential. For the non-exponential case, this is not generally true but shown to remain valid in [2] for continuous-time communication systems that exhibit a product form. Herein the blocking was assumed to be non-randomized. For discrete-time systems this is no longer obvious at all as multiple transitions are involved at the same time.

This paper will study a discrete-time communication framework under a "stop" protocol. It will be shown that the insensitive product form result for the recirculate protocol remains valid. An analogue of the continuous-time equivalence result is thus obtained. This discrete-time extension is non-trivial as:

- Multiple occurrences can take place at the same time.

- The state of the system and thus also the blocking interferences change during the scheduling periods as opposed to the recirculate protocol under which blocking arises only at completion instants. This latter distinction is less crucial in continuous-time as changes then occur by one source at a time.

- Essentially different terms than under the recirculate protocol are involved (See remark 3.3).

- The stop protocol (See [21]) has not yet been investigated for randomized access schemes such as most notably Rude-CSMA (See section 4).

As all examples from [20] with $D(\cdot|\cdot) = 1$ and [22] fit in the framework, we will restrict the illustration and refer to these references for further examples.

The organisation is as follows. First in section 2 the model and stop protocol are discussed in detail. Next, in section 3 the insensitive discrete-time product form result is derived. Some illustration is provided in section 4 while a brief evaluation completes the paper.

2 Model, Protocol and Blocking Condition

We adopt the abstract framework given in [22] which, as per this reference and the examples in section 4 allows concrete applications such as circuit switching, CSMA and MAN blocking structures at the same time.

In this abstract setting, for convenience we use the terminology of "idle services" and "busy services" of a source.

Here in terms of a communication network, a "busy service" will actually correspond to the transmission of a message of that source, while an "idle service" actually corresponds to the scheduling period before that source requests or can start a next message transmission.

State description

Consider a set of N transmitters, such as satellites, terminals or in/output devices, which will be called sources hereafter. Each source is alternatively in an "idle" (non-transmitting or scheduling) and "busy" (transmitting) mode as according to the protocol described below. A state $H - \{h_1, \ldots, h_n)$ represents that sources $h_1, \ldots, h_n$ are busy. Write $\bar{H} = \{h | h \notin H\}, H + h = H \cup \{h\}, H - h = H/\{h\}, H + G = H \cup G$ and $H - G = H/G$ and denote by $\emptyset$ the state in which none of the sources is busy.

Idle-busy mechanism

The time is slotted in fixed intervals of length Δ. Sources can change their status only at the end of a time slot as follows.

When source h starts a "busy service", during which it is called busy, it requires k units of "busy service" with probability $p_h(k)$. In one time-slot a busy source h will receive one unit of busy service with probability ν_h. Conversely, when a source h starts an "idle service", during which it is called idle, it requires t units of idle service with probability $q_h(t)$. In contrast, though, in one time-slot it will request rather than receive one unit of idle service with probability γ_h. In other words that is, each request for a unit of "idle service" requires a geometric number of time-slots with parameter γ_h. When in a time-slot a unit of idle service is requested this unit will actually be provided and worked off in the state-dependent manner as described below.

Discrete-time stop protocol

Interferences of sources such as due to commonly used channels, message collisions at common recipients or message priorities may generally be involved. This interference will be modeled by a blocking function $A(\cdot|\cdot)$ as follows.

When at the beginning of a time-slot the system is in state H, while the group of idle sources that all request a unit of idle service during that time-slot is given by $Gc\bar{H}$, which occurs with probability

$$\prod_{h \epsilon G} \gamma_h[1-\gamma_h],$$

with probability

$$A(G|H)$$

all these sources are provided and worked off one unit of idle service. Conversely, with probability

$$1 - A(G|H)$$

none of these sources receives a unit of idle service, and thus the total group of requesting idle sources G is blocked or stopped to be serviced. In addition, in the latter case, any busy source which during this time-slot receives its last unit of "busy service" has to restart a new "busy survive". In particular, when $A(G|H) = 0$ idle servicing is completely stopped (interrupted).

Discussion of protocol

Under the recirculate protocol as described in [22], both idle and busy servicing is affected and requires a new idle or busy service to be restarted if blocking arises. In communications terminology that is, a total transmission request has to be rescheduled and a total message is to be retransmitted. Under the above protocol, in contrast, only one unit of "idle service" has to be rescheduled or is lost when the system is in a blocked configuration.

Most essentially, though, the "busy servicing" is not affected, up to the event discussed below. Message transmissions are thus continued, so as to resolve blocking conflicts more or less as quickly as possible. An almost complete decoupling of access blocking and ongoing message transmissions, as most natural, is thus achieved.

Only for technical complications in the proof, it is still assumed that a "busy service" or message transmission is to be repeated when during the time-slot in which the last service unit is received the idle sources would place requests that would lead to a conflict.

This modeling assumption seems justifiable as a reasonable approximation since:

- The actual initiation and completion of transmissions may require one and the same device (for example, think of an Ethernet-type network) so that a collision will occur, by which the message is lost. As the system may detect the collision but not the individual sources causing it, a retransmission of the message seems practical.
- The probability of the conflicting event to occur will generally be very small, as it requires during one time-slot, say of duration Δ, simultaneous multiple

service requests from idle sources as well as simultaneously, exactly the last busy service unit to be provided. For example, for each individual source this probability may already be or order Δ and thus for multiple requests of order Δ^2 or less. (Also see the asymptotic result in remark 3.4). On top of this, the multiple changes should lead to a conflict, which itself, in a well-designed realistic system, should not occur frequently.

Further, note that "busy services" (transmissions) can always successfully be completed during at least time-slots in which "idle services" (transmission requests) do not take place, the probability of which will always be positive as we assumed $\gamma_h < 1$.

Blocking condition 3.1

With C the set of possible admissible states of busy configurations, the following condition on the blocking function $A(\cdot|\cdot)$ is imposed. For any $H + G + h \in C$:

$$A(G + h|H) = A(h|H)A(G|H + h) \tag{2.1}$$

Remark 3.2 This condition can be shown to be directly related to the condition of reversibility for Markov chain models (see [11]), more precisely, to the existence of a function $P(\cdot)$ such that for any $H, H + G \in C$:

$$P(H + G) = P(H)A(G|H) \tag{2.2}$$

The condition (2.1) or (2.2) in turn can be checked as per the Kolmogorov criterion for reversibility (see [11]). Here, this would come down to the conditions: $A(h|H) > 0$ if $H, H + h \epsilon C$ and for any $H = \{h_1, \ldots, h_n\} \epsilon C$ and any permutation $(i_1, \ldots, i_n) \epsilon (1, \ldots, n)$:

$$P(H) = \prod_{k=1}^{n} A(h_{i_k}|h_{i_1}, \ldots, h_{i_{k-1}}) \tag{2.3}$$

Special application

The conditions (2.1), (2.2) and (2.3) are directly verified with

$$\begin{aligned} P(H) &= 1 \quad (H \epsilon C) \\ A(h|H) &= 1 \quad (H, H + h \epsilon C) \end{aligned} \tag{2.4}$$

provided C is such that

$$H \in C \Rightarrow H - h \in C \qquad \text{(For all } H \epsilon C \text{ and } h \epsilon H) \tag{2.5}$$

In correspondence with literature, (cf. [10], [12]) this type of blocking is called "coordinate-convex". As illustrated in [20] and section 4 the coordinate convex blocking allows a large class of present-day communication structures such as most notably:

- Circuit switching networks with limited common trunk groups
- CSMA or BTMA broadcasting interferences, and
- MAN channel allocation schemes.

We refer to this reference for detailed descriptions and limit the illustration to some basic examples in section 4.

3 Product Form Result

We will adopt the notation from [22] and follow its approach as closely as possible so as to highlight the contrast as much as possible. The presentation though will be kept selfcontained. Let

$$[(S_1, R_1), (S_2, R_2)]$$

with

$$(S_1, R_1) = ((g_1, s_1), \ldots, (g_m, s_m))$$
$$(S_2, R_2) = ((h_1, t_1), \ldots, (h_n, t_n))$$

be a generic notation for the state in which sources $S_1 = \{g_1, \ldots, g_m\}$ are currently idle with for source g_i still s_i units of idle service required up to completion of its current "idle service", $i = 1, \ldots, m$ and in which sources $S_2 = \{h_1, \ldots, h_n\}$ are currently busy with for source h_j still t_j units of "busy service" required up to completion, $j = 1, \ldots, n$. The state $[S_1, S_2]$ is the obvious notation for merely idle and busy source specification. Here one may note that $S_2 = H$ as per the notation of section 2. Further, we use the notation

$$[(S_1, R_1); (S_2, R_2)] - [(\alpha_1, R_1), (\alpha_2, R_2)] + [(\beta_1, R_1'); (\beta_2, R_2')]$$

to denote that subgroups $(\alpha_1, R_1) \subset (S_1, R_1)$ and $(\alpha_2, R_2) \subset (S_2, R_2)$ are replaced by new groups (β_1, R_1') and (β_2, R_2') respectively. Particularly, we write $R_j' = 1$ if all components of R_j' are equal to 1 and symbolize by $R_j' = R_j + 1$ that all components of R_j are increased by 1. Again, a notation $[S_1, S_2] - [\alpha_1, \alpha_2] + [\beta_1, \beta_2]$ is the obvious restriction to merely source specification. (One may note here that in this case necessarily either: $\beta_1 = \alpha_2$ and $\beta_2 = \alpha_1$, or: $[\beta_1, \beta_2] = [\alpha_1, \alpha_2]$). Let

(3.1)
$$r_h(s) = \gamma_h^{-1} \sum_{j=s}^{\infty} p_h(j)$$
$$v_h(t) = \nu_h^{-1} \sum_{j=t}^{\infty} q_h(j)$$

$$
(3.2) \qquad \begin{aligned} \sigma_h &= \gamma_h^{-1} \textstyle\sum_s s p_h(s) = \gamma_h^{-1} \sum_s r_h(s) \\ \tau_h &= \nu_h^{-1} \textstyle\sum_t t q_h(t) = \nu_h^{-1} \sum_t v_h(t) \end{aligned}
$$

We are now able to present the main result. To this end, first observe that the underlying probability structure implies that the process which keeps track of the number of residual "idle and busy service" requirements of all sources at time points $0, \Delta, 2\Delta, \ldots$ constitutes a discrete-time Markov chain. (In fact, one can also think of these residual numbers as numbers of residual phases where each phase itself has a geometric distribution). Without loss of generality, assume that this Markov chain is irreducible at some set of states. (Clearly, this set is determined by the $A(\cdot|\cdot)$ access mechanism for busy source configurations). Two results will now be proven. The first is the technical one. The second is the practical consequence revealing insensitivity.

Result 1 (Detailed product form) With c a normalizing constant

$$
(3.3) \qquad \pi([(S_1, R_1), (S_2, R_2)]) = cP(S_2) \prod_{h \epsilon S_1} r_h(s_h) \prod_{h \epsilon S_2} v_h(t_h)
$$

Proof It suffices to verify the global balance equations:

$$
(3.4) \qquad \pi(i) \sum_{j = i} p(i,j) - \sum_{j=i} \pi(j) p(j,i)
$$

where i and j symbolize the different possible states and where $p(.,.)$ represents the corresponding one-step transition probabilities. To specify these equations in more detail, the following notation is used for any $[S_1, S_2]$, subset $\alpha \subset S_1$ and subset $\beta \subset S_2$;

$$
(3.5) \qquad \begin{aligned} L(\alpha|S_1) &= \textstyle\prod_{h \epsilon \alpha} \gamma_h \prod_{h \epsilon S_1 - \alpha} [1 - \gamma_h] \\ M(\beta|S_2) &= \textstyle\prod_{h \epsilon \beta} \nu_h \prod_{h \epsilon S_2 - \beta} [1 - \nu_h] \end{aligned}
$$

Consider a fixed state $[(S_1, R_1), (S_2, R_2)]$ as symbolized by i in (3.4). The left hand side (probability flux out of state $[(S_1, R_1), (S_2, R_2)]$) in (3.4) then becomes:

$$
(3.6) \qquad \sum_{G_1 \subset S_1, G_2 \subset S_2} \pi([(S_1, R_1), (S_2, R_2)]) L(G_1|S_1) M(G_2|S_2)
$$

The right hand side (probability flux into state $[(S_1, R_1), (S_2, R_2)]$) equals:

$$\sum_{\substack{G_1 \subset S_1 \\ G_2 \subset S_2}} \sum_{\substack{\alpha_1 \subset G_1, \beta_1 = G_1 - \alpha_1 \\ \alpha_2 \subset G_2, \beta_2 = G_2 - \alpha_2}} \Bigg\{ \pi\left([(S_1, R_1), (S_2, R_2)] - [(\alpha_1 + \beta_1, R_1), (\alpha_2 + \beta_2, R_2)]\right.$$

$$\left. + [(\alpha_2, 1), (\alpha_1, 1)] + [(\beta_1, R_1 + 1), (\beta_2, R_2 + 1)]\right) L(\alpha_2 + \beta_1 | S_1 - \alpha_1 + \alpha_2)$$

$$M(\alpha_1 + \beta_2 | S_2 - \alpha_2 + \alpha_1) A(\alpha_2 + \beta_1 | S_2 - \alpha_2 + \alpha_1) \prod_{h \epsilon \alpha_1} p_h(s_h) \prod_{h \epsilon \alpha_2} q_h(t_h)$$

$$+$$

$$\pi\left([(S_1, R_1), (S_2, R_2)] - [(\alpha_1 + \beta_1, R_1), (\alpha_2 + \beta_2, R_2)] + [(\alpha_1, R_1), (\alpha_2, 1)]\right.$$

$$\left. + [(\beta_1, R_1), (\beta_2, R_2 + 1)]\right) L(\alpha_1 + \beta_2 | S_1) M(\alpha_2 + \beta_2 | S_2)$$

$$[1 - A(\alpha_1 + \beta_1 | S_2)] \prod_{h \epsilon \alpha_2} q_h(t_h) \Bigg\}$$

(3.7)

By substituting (3.3) this can be rewritten as:

$$\sum_{\substack{G_1 \subset S_1 \\ G_2 \subset S_2}} \sum_{\substack{\alpha_1 \subset G_1, \beta_1 = G_1 - \alpha_1 \\ \alpha_2 \subset G_2, \beta_2 = G_2 - \alpha_2}} \Bigg\{ P(S_2 - \alpha_2 + \alpha_1) \Bigg[\prod_{h \epsilon \beta_1} r_h(s_h + 1) \prod_{h \epsilon \beta_2} v_h(t_h + 1) \prod_{h \epsilon \alpha_1} v_h(1)$$

$$\prod_{h \epsilon \alpha_2} r_h(1) \prod_{h \epsilon S_1 - \alpha_1 - \beta_1} r_h(s_h) \prod_{h \epsilon S_2 - \alpha_2 - \beta_2} v_h(t_h) \Bigg] L(\alpha_2 + \beta_1 | S_1 - \alpha_1 + \alpha_2)$$

$$M(\alpha_1 + \beta_2 | S_2 - \alpha_2 + \alpha_1) A(\alpha_2 + \beta_1 | S_2 - \alpha_2 + \alpha_1) \prod_{h \epsilon \alpha_1} p_h(s_h) \prod_{h \epsilon \alpha_2} q_h(t_h)$$

$$+$$

$$P(S_2) \Bigg[\prod_{h \epsilon \alpha_1 + \beta_1} r_h(s_h) \prod_{h \epsilon \beta_2} v_h(t_h + 1) \prod_{h \epsilon \alpha_2} v_h(1) \prod_{h \epsilon S_2 - \alpha_2 - \beta_2} v_h(t_h) \Bigg]$$

$$L(\alpha_1 + \beta_2 | S_1) M(\alpha_2 + \beta_2 | S_2) [1 - A(\alpha_1 + \beta_1 | S_2)] \prod_{h \epsilon \alpha_2} q_h(t_h) \Bigg\}$$

(3.8)

Now from (3.2) we conclude:

$$(3.9) \qquad P(S_2 - \alpha_2 + \alpha_1) = P(S_2) A(\alpha_1 | S_2 - \alpha_2) [A(\alpha_2 | S_2 - \alpha_2)]^{-1}$$

provided $A(\alpha_2 | S_2 - \alpha_2) > 0$. This, however, follows from the fact that (S_1, S_2) is assumed to be an admissible configuration (otherwise global balance doesn't need to be verified in this state), so that the configuration $(S_1 + \alpha_2, S_2 - \alpha_2)$ is also admissible and thus, as the blocking condition requires $P(S_2) = A(S_2|\ \emptyset) = A(S_2-\alpha_2|\ \emptyset)A(\alpha_2|S_2-\alpha_2) > 0$, that $A(\alpha_2|S_2-\alpha_2) > 0$. Further, by (3.1):

$$r_h(1) = \gamma_h^{-1}$$

$$v_h(1) = \nu_h^{-1}$$

while by (3.5):

$$L(\alpha_2+\beta_1|S_1-\alpha_1+\alpha_2) = L(\alpha_1+\beta_1|S_1)[\prod_{h\epsilon\alpha_2}\gamma_h][\prod_{h\epsilon\alpha_1}\gamma_h]^{-1}$$

$$M(\alpha_1+\beta_2|S_2-\alpha_2+\alpha_1) = M(\alpha_2+\beta_2|S_2)[\prod_{h\epsilon\alpha_2}\nu_h][\prod_{h\epsilon\alpha_2}\nu_h]^{-1}$$

By substituting these expressions in (3.8) and noting that $\alpha_1+\beta_1 = G_1, \alpha_2+\beta_2 = G_2$ we obtain:

$$\begin{aligned}(3.10)\qquad &\sum_{\substack{G_1\subset S_1\\ G_2\subset S_2}} P(S_2)L(G_1|S_1)M(G_2|S_2)\prod_{h\epsilon S_1-G_1} r_h(s_h)\prod_{h\epsilon S_2-G_2} v_h(t_h)\\
&\sum_{\substack{\alpha_1\subset G_1,\beta_1=G_1-\alpha_1\\ \alpha_2\subset G_2,\beta_2=G_2-\alpha_2}}\left\{A(\alpha_2+\beta_1|S_2-\alpha_2+\alpha_1)\left[\frac{A(\alpha_1|S_2-\alpha_2)}{A(\alpha_2|S_2-\alpha_2)}\right]\right.\\
&\prod_{h\epsilon\alpha_1}[p_h(s_h)\gamma_h^{-1}]\prod_{h\epsilon\alpha_2}[q_h(t_h)\nu_h^{-1}]\prod_{h\epsilon\beta_1} r_h(s_h+1)\prod_{h\epsilon\beta_2} v_h(t_h+1)\\
&+\\
&\left.[1-A(\alpha_1+\beta_1|S_2)]\prod_{h\epsilon\alpha_1+\beta_1} r_h(s_h)\prod_{h\epsilon\alpha_2} q_h(t_h)\nu_h^{-1}v_h(t_h+1)\right\}\end{aligned}$$

Now from (2.1) - (2.3) first conclude that for any G_1, G_2 and H:

$$A(G_1+G_2|H) = A(G_1|H)A(G_2|G+G_1)$$

so that

$$\begin{aligned}&A(\alpha_2+\beta_1|S_2-\alpha_2+\alpha_1)A(\alpha_1|S_2-\alpha_2)\\
&= A(\alpha_1+\alpha_2+\beta_1|S_1-\alpha_2) = A(\alpha_2|S_2-\alpha_2)A(\alpha_1+\beta_1|S_2)\end{aligned}$$

Hence

$$(3.11)\qquad A(\alpha_2+\beta_1|S_2-\alpha_2+\alpha_1)\frac{A(\alpha_1|S_2-\alpha_2)}{A(\alpha_2|S_2-\alpha_2)} = A(\alpha_1+\beta_1|S_2)$$

Further, as $A(\alpha_1 + \beta_1 | S_2) = A(G_1 | S_2)$ regardless of the separation α_1, β_1, we can thus rewrite (3.10) as

$$\sum_{\substack{S_1 \subset G_1 \\ S_2 \subset G_2}} P(S_2) L(G_1|S_1) M(G_2|S_2) \prod_{h \epsilon S_1 - G_1} r_h(s_h) \prod_{h \epsilon S_2 - G_2} v_h(t_h)$$

$$\left\{ A(G_1|S_2) \sum_{\alpha_1 \subset G_1,\, \beta_1 = G_1 - \alpha_1} \prod_{h \epsilon \alpha_1} p_h(s_h) \gamma_h^{-1} \prod_{h \epsilon \beta_1} r_h(s_h + 1) + \right. \tag{3.12}$$

$$\left. [1 - A(G_1|S_2)] \prod_{h \epsilon \alpha_1 + \beta_1} r_h(s_h) \right\}$$

$$\left\{ \sum_{\alpha_2 \subset G_2, \beta_2 = G_2 - \alpha_2} \prod_{h \epsilon \alpha_2} q_h(t_h) \nu_h^{-1} \prod_{h \epsilon \beta_2} v_h(t_h + 1) \right\}$$

As by (3.1)

$$[p_h(s_h)\gamma_h^{-1}] + [r_h(s_h + 1)] = r_h(s_h)$$
$$[q_h(t_h)\nu_h^{-1}] + [v_h(t_h + 1)] = v_h(t_h)$$

interchanging of summation and factorization in the terms between braces $\{.\}$ in (3.12) lead to

$$\sum_{\substack{S_1 \subset G_1 \\ S_2 \subset G_2}} P(S_2) L(G_1|S_1) M(G_2|S_2) \prod_{h \epsilon S_1 - G_1} r_h(s_h) \prod_{h \epsilon S_2 - G_2} v_h(t_h)$$

$$\{A(G_1|S_2) + [1 - A(G_1|S_2)]\} \prod_{h \epsilon \alpha_1 + \beta_1} r_h(s_h) \prod_{h \epsilon \alpha_2 + \beta_2} v_h(t_h) \tag{3.13}$$

$$=$$

$$\sum_{\substack{G_1 \subset S_1 \\ G_2 \subset S_2}} \pi\left([(S_1, R_1), (S_2, R_2)]\right) L(G_1|S_1) M(G_2|S_2)$$

by substituting $\alpha_1 + \beta_1 = G_1, \alpha_2 + \beta_2 = G_2$ and (3.3). This proves equality of (3.6) and (3.7).

As an immediate consequence the following more practical result is obtained. This shows that the steady state busy source distribution is determined by the "idle and busy services" (scheduling and transmission times) only through their means.

Result 3.2 (Insensitivity result) With $\bar{c}$ a normalising constant and (σ_h, τ_h) the mean idle and busy service requirement as per (3.2), we have

$$\pi(H) = \bar{c} P(H) \prod_{h \epsilon H} [\tau_h / \sigma_h]$$

Proof Immediately by summing (3.3) over all possible residual numbers s_h and t_h for all h, recalling (3.2) and substituting $\bar{c} = c[\sigma_1\sigma_2 \ldots \sigma_M]^{-1}$.

Remark 3.1 (Group balance). As in [22] one may note that the proof has actually been established by showing that for each group of sources (G_1, G_2) individually the probability flux out of a state (S_1, S_2) due to that group is equal to the probability flux into that state due to that same group. (Group balance). The notion of "group balance" was introduced in [18] and further used in [3] and [10] as a key-property for product forms results, extending standard notions as local or job-local balance, when multiple changes can take place at the same time. As another immediate consequence of result 3.1, when compared with result 2 from [22] for the recirculate protocol, and referring to this reference for the precise definition, we conclude:

Result 3.3 (Equivalence result) For the system under consideration, that is under the blocking condition 3.1, the "stop" and "recirculate" protocol have exactly the same steady state busy source distribution.

Remark 3.2 (Protocol equivalence) As in the continuous-time case in [21], this equivalence is of both practical and theoretical interest as equivalence of both protocols does not hold generally. It seems to reveal some intrinsic property or related factor of product forms results.

Remark 3.3 (Comparison of proof) As a technical comparison with the proof under the recirculate protocol as given in [22], for example the term $A(\alpha_2 + \beta_1|S_2 - \alpha_2 + \alpha_1)$ in (3.7) would be replaced by $A(\alpha_2|S_2 - \alpha_2)$, and thus not take into the blocking effect (interruption of idle service) on β_1 nor the group α_1 having become busy. A similar remark holds for the $[1 - A(\cdot|\cdot)]$ term in (3.7). Also the first and second part of (3.7) have to be combined but do no longer have, as under the recirculate protocol, the direct combinable term $\prod p_h(s_h)$. These differences essentially led to the technical complications from (3.10) onward.

Remark 3.4 (Asymptotic) By assuming that for certain parameters λ_h and μ_h, and with $0(\Delta)/\Delta \to 0$ for $\Delta \to 0$:

$$\begin{aligned} p_h &= [1 - \lambda_h\Delta] + 0(\Delta) \\ q_h &= [1 - \mu_h\Delta] + 0(\Delta) \end{aligned}$$

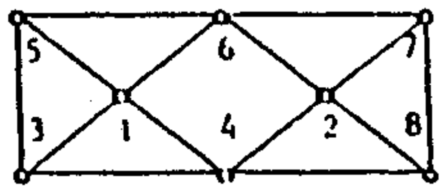
and letting $\Delta \to 0$, expression (3.3), taking (3.2) in to account, will consistently lead to the continuous-time product form result under the "stop" or "recirculate" protocol given in [21].

4 Applications

For the purpose of illustration and selfcontainedness, this section merely copies some examples presented earlier in [20] and [22]. Under a "stop" protocol and in a discrete-time setting no explicit expression for any of them seems to have been reported. In particular, the stop protocol has not been dealt with for randomized access blocking, that is with $A(\cdot|\cdot)$ taking values other than 0 or 1, as naturally arising in Rude-CSMA (example 4.4). The examples 4.1, 4.2 and 4.3 are all "coordinate convex" (see special application 3.1) so that $P(H) = 1$.

4.1 CSMA-protocols

[i] CSMA (cf. [2], [4], [5], [19]) – Let the sources correspond to transmitters that can be graphically represented such that adjacent sources (neighbours) cannot be busy (transmit) at the same time. In practice this is achieved by the so-called "Carrier Sense Multiple Access" (CSMA)-scheme in which a transmitter senses the state of its channels just prior to starting a transmission and where upon sensing a busy channel from a neighbour the transmission is aborted (inhibited). For example, in the figure below a transmission from source 1 prohibits any source 3...6 to start a transmission.



With $N(h)$ the set of neighbours of source h, the coordinate convexity condition (2.5) is guaranteed by

$$(4.1) \qquad C = \{H | h_2 \not\in N(h_1) \text{ for all } h_1, h_2 \in H\}$$

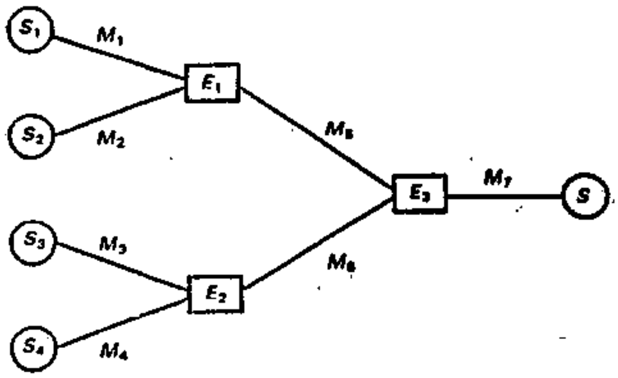
[ii] BTMA (cf. [19]) – In the above example the sources 1 and 2 which are outside hearing range can transmit at the same time. This will lead to a collision at nodes 4 and 6 which in turn will result in lost messages. This is known as the "hidden terminal problem". To eliminate this problem, the so-called "Busy Tone Multiple Access" (BTMA)-scheme has been introduced (cf. [19]). Under BTMA a node which senses a busy channel (in other words, which hears a transmitting neighbor) broadcasts a busy tone top all its neighbours to prevent idle neighbours from starting a transmission.

The set C from (4.1) now still applies (ie., is coordinate convex), provided we replace $N(h)$ by the set of all one and two-link neighbours (eg. $N(5) = \{2, \ldots, 7\}$).

4.2 Circuit switching

(cf. [6])

Consider a circuit switching network with 4 different types of sources with a fixed path along which a message from that source to the destination is to be transmitted. This transmission requires one trunk from each trunkgroup along this path. Interference thus arises with limited trunkgroups and messages using the same trunkgroups.



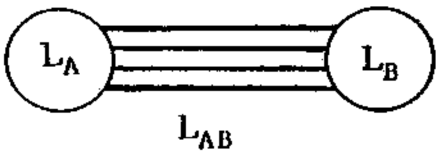
With M_i the number of trunks in trunkgroup i and n_i the number of busy sources of type i, the coordinate convexity condition (2.5) is satisfied by C the set of states H such that

$$(4.2) \qquad \begin{array}{ll} n_i \leq M_i & (i = 1, \ldots, 4) \\ n_1 + n_2 \leq M_5 & \\ n_3 + n_4 \leq M_6 & \\ n_1 + n_2 + n_3 + n_4 \leq M_7 & \end{array}$$

4.3 Interconnected Metropolitan Area Networks (MAN's)

(cf. [16])

Consider a communication system with two groups of subscribers, say a group A and B with M and N subscribers, such as representing two metropolitan or local are networks. Both within a group and in between the groups communication between subscribers might be possible. To this end, number all subscribers $1, \ldots, M + N$ and identify each possible connection from a source subscriber m to a destination subscriber n as a source (m, n). The description of section 2 then applies by stating that a connection is busy when a transmission along this connection takes place and idle otherwise.



For a given state H of busy connections let n_A, n_B and n_{AB} denote the number of busy connections within A, within B and in between A and B respectively. Assume finite numbers of L_A and L_B local circuits within A and B and L_{AB} circuits between A and B. Then the continuous-time model of [16] is extended to a coordinate convex discrete-time model by:

$$C = \{H | n_A \leq L_A, n_B \leq L_B, n_{AB} \leq L_{AB}\} \tag{4.3}$$

for the "dedicated allocation policy" with separate circuits for local and long-distance transmissions and by

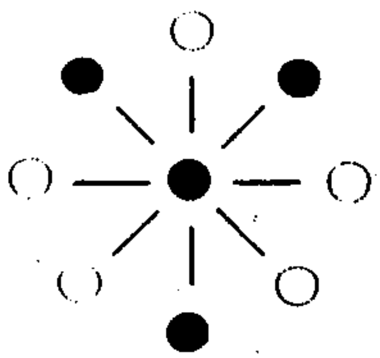
$$\begin{aligned} C = \{H | n_A \leq L_A + L_{AB}, n_B \leq L_B + L_{AB}, \\ n_{AB} \leq L_{AB} - (n_A - L_A)^+ - (n_B - L_B)^+\} \end{aligned} \tag{4.4}$$

where $(y)^+ = 0$ for $y \leq 0$ and $y^+ = y$ for $y > 0$, for the "shared allocation policy" in which the inter MAN circuits are shared among local and long-distance calls. As another shared allocation policy, each long-distance connection may require a local circuit within each local area, which is reflected by

$$C = \{H | n_A + n_{AB} \leq L_A, n_B + n_{AB} \leq L_B, n_{AB} \leq L_{AB}\} \tag{4.5}$$

4.4 Rude CSMA

(cf. [8], [13])



Another way to take into account the "hidden terminal problem" mentioned in 4.2 is introduced in [13] under the name of "rude-CSMA". In "Rude-CSMA" the access mechanism is randomized as according to

$$A(h|H) = x^{N_0^h(H)} y^{N_1^h(H)} \tag{4.6}$$

where $N_0^h(H)$ and $N_1^h(H)$ are the numbers of idle (not transmitting) and busy (transmitting) neighbours from h when the system is in state H and where x and y are given system parameters, with $0 \le x, y \le 1$. For instance $x = 1, y = 1$ corresponds to the ALOHA-protocol (no collisions), $x = 1, y = 0$ models the standard CSMA protocol as in example 4.1 (i) and other values of x and y may reflect for instance that sensing of channels is not always reliable. Condition (2.2) is verified with:

$$\begin{aligned} &P(H) = x^{-B_0(H)} y^{B_1(H)}, \quad \text{with} \\ (4.7) \qquad &B_0(H) : \text{number of idle pairs of neighbours in state } H \\ &B_1(H) : \text{number of busy pairs of neighbours in state } H \end{aligned}$$

Evaluation

In communication networks of today both the feature of source interferences and of digitization are most prevalent. Blocking or random access protocols as well as multiple changes or discrete-time slotting are thus to be taken into account. This paper aimed to relax a recently obtained exact result for such networks under a special recirculate protocol to a more natural blocking protocol. The proposed stop protocol provides a more natural interpretation and thus improvement over the recirculating protocol. It was shown that it preserved the exact appealing insensitive product form expression. This equivalence result is of interest in itself and seems promising for further extension.

Acknowledgement

The author would like to thank Les Berry, the Centre for Teletraffic Research at Bond University and Telecom Australia for arranging a short visit at this Centre during which this paper was written.

5 References

[1] Bharath-Kumar, K. (1980), "Discrete-time queueing systems and their networks", IEEE Trans. Commun. Com-28, 2, 260-263.

[2] Boorstyn, R.R., Kerschenbaum, A., Maglaris, B. and Sahin, V. (1987), "Throughput analysis in multihop CSMA packet radio networks", IEEE Trans. Commun., Com-35, 267-274.

[3] Boucherie, R. and Van Dijk, N.M. (1989), "Product forms for queueing networks with state dependent multiple job transitions", Research Report, Free University Amsterdam. To appear: Adv. Appl. Prob.

[4] Brazio, J. and Tobagi, F. (1985), "Conditions for product form solutions in multihop packet radio network models", SEL Technical Report 85-274, Computer Systems Laboratory, Stanford University, Stanford CA, April 1985.

[5] Brazio, J. (1987), "Capacity analysis of multihop packet radio networks under a general class of channel access protocols and capture models", Technical Report CSL-TR-87-318, Computer Systems Laboratory, Stanford University, Stanford, CA, March 1987.

[6] Burman, D.Y., Lehoczky, J.P. and Lim, Y. (1982), "Insensitivity of blocking probabilities in a circuit switched network", J. Appl. Prod. 21, 850-859.

[7] Daduna, H. and Schassberger, R. (1983), "Networks of queues in discrete time", Zeitschrift fur Operations Research, 27, 159-175.

[8] Felderman, R.E. (1987), "Extension to the rude-CSMA analysis", IEEE Trans. Commun., Com-35, 848-849.

[9] Henderson, W. and Taylor P.G. (1988), "Networks of queues with batch arrivals and batch services". To appear: Queueing Systems.

[10] Kaufman, J. (1981), "Blocking in a shared resource environment", IEEE Trans. Commun. Com-29, 1474-1481.

[11] Kelly, F.P. (1979), "Reversibility and stochastic networks", Wiley, New York.

[12] Lam, S.S. (1977), "Queueing networks with capacity constraints", IBM Journal Res. Devel. 21, 376-378.

[13] Nelson, R. and Kleinrock, L. (1985), "Rude-CSMA: A multihop channel access protocol", IEEE Trans. Commun., Com-33, 785-791.

[14] Pujolle, G., Claude, J.P. and Seret, D. (1985), "A discrete tandem queueing system with a product form solution", Proc. Intern. Seminar on Computer Networking and Performance Evaluation, North-Holland, Kyoto, 139-147.

[15] Pujolle, G. (1988), "Discrete-time queueing systems with a product form solution", MASI Research Report.

[16] Rubin, I. and Lee, J.K. (1988), "Performance analysis of interconnected metropolitan area circuit-switched networks", IEEE Trans. Commun., Com-36, 171-185.

[17] Schassberger, R. and Daduna, H. (1983), "A discrete-time technique for solving closed queueing models of computer systems", Technical Report Technische Universitat Berlin.

[18] Taylor, P.G., Henderson, W., Pearce, C.E.M., and Van Dijk, N.M. (1988), "Closed queueing networks with batch services". To appear: Queueing Systems.

[19] Tobagi, F.A. and Kleinrock, L. (1975), "Packet switching in radio channels: Part II: The hidden terminal problem in carrier sense multiple access and the busy tone solution", IEEE Trans. Commun., Com-23, 1417-1433.

[20] Van Dijk, N.M. (1988), "Product forms for random access schemes". To appear: Computer Networks and ISDN Systems.

[21] Van Dijk, N.M. (1989), "An equivalence of communication protocols for interconnection networks". To appear: Special Issue of "Computer Networks and ISDN-Systems" on ITC Specialist Seminar, Adelaide, September 1989.

[22] Van Dijk, N.M. (1990), "An insensitive product form for discrete-time communication networks". Performance '90 North-Holland, 77-89

[23] Walrand, J. (1983), "A discrete-time queueing network", J. Appl. Prob 20, 903-909.

1990-1	B. Vogelvang	Testing For Co-Integration with Spot Prices of Some Related Agricultural Commodities
1990-2	J.C.J.M. van den Bergh P. Nijkamp	Ecologically Sustainable Economic Development Concepts and Model Implications
1990-3	J.C.J.M. van den Bergh P. Nijkamp	Ecologically Sustainable Economic Development in a Regional System: A Case Study in Agri cultural Development Planning in the Netherlands
1990-4	C.Gorter P.Nijkamp P.Rietveld	Employers' Recruitment Behaviour and Re-Employment Probabilities of Unemployed
1990-5	K.Burger	Off-farm income and the farm-household the case of Kenyan smallholders
1990-6	H. Visser	Crowding out and the Government Budget
1990-7	P. Rietveld	Ordinal Data in Multicriteria Decision Making, a Stochastic Dominance Approach to Siting Nuclear Power Plants
1990-8	G. van der Laan P.H.M. Ruys D.J.J. Talman	Signaling devices for the supply of semi-public goods
1990-9	F.A.G. den Butter	Labour Productivity Slowdown and Technical Progress: An empirical analysis for The Netherlands
1990-10	R.W. van Zijp	Neo-Austrian Business Cycle Theory
1990-11	J.C. van Ours	Matching Unemployment and Vacancies: The Efficiency of the Dutch Labour Market
1990-12	B. Vogelvang	Hypotheses Testing Concerning Relationships between Spot Prices of Various Types of Coffee
1990-13	A.F. de Vos I.J. Steyn	Stochastic Nonlinearity: A Firm Basis for the Flexible Functional Form
1990-14	Y.H. van Emmerik D. de Jong W.W.A. Zuurmond D.N. Dukkers-van Emden	Opereren in overleg: geprotocolleerde samenwerking 1e-2e-lijn bij dagchirurgie
1990-15	T.J.J.B. Wolters	Mediation and Collective Bargaining: A Diagnostic Approach
1990-16	E.M.A. Scholten J. Koelewijn	Financieringsproblematiek van startende ondernemingen: een mogelijke verklaring op basis van empirisch onderzoek.
1990-17	E. Hüner H.P. Smit	Saturation and Model Specification of Passen ger car Ownership
1990-18	F.A.G. den Butter	Sociale zekerheid, de wig en economische groei
1990-19	F.A.G. den Butter R.F. v.d. Wijngaert	Who is Correcting the Error ? A Co-integration Approach for Wages, Wage Space and Labour Conflicts in the Netherlands
1990-20	J.P. de Groot R. Ruben	Sistemas de Producción y Transferencia de Teconología en la Economía Cafetalera de Centroamerica
1990-21	R. Ruben	Campesinado y Reforma Agraria en El Salvador
1990-22	J. van Ours G. Ridder	Vacancies and the Recruitment of New Employees
1990-23	A.F. de Vos J.J. de Vries	The Likelihood Function of a Generalized Gravity Model; Handling the Implicit Singularity of a Nonlineair Transformation
1990-24	D. van der Wal	Geloofwaardigheid en het Nederlandse wisselkoersbeleid
1990-25	R.J. Veldwijk M. Boogaard M.V. van Dijk E.R.K. Spoor	EDSOs, implosion and explosion: concepts to automate a part of application maintenance
1990-26	B. Hanzon	The area enclosed by the (oriented) Nyquist diagram and the Hilbert-Schmidt-Hankel norm of a linear system
1990-27	R.W. van Zijp	Why Lucas is not a Hayekian
1990-28	J. Rouwendal	On discrete choice under uncertainty: A generalization of the logit model and its application
1990-29	J. Rouwendal	On the equitable distribution of the housing stock
1990-30	J. Rouwendal	Stochastic market equilibria with efficient rationing (with an application to the Dutch housing market)
1990-31	J.A. Vijlbrief	The effects of unemployment insurance on the labour market
1990-32	J.G.W. Simons H.P. Wansink	Traffic Ban, a means to combat smog?
1990-33	J.C. van Ours T. Zoethout	De Interne Arbeidsmarkt van de Gemeente Amsterdam
1990-34	H.J. Bierens	A Note On the Limiting Distribution of Sample Autocorrelations in the Presence of a Unit Root
1990-35	T. Kuhlman	The Economic Integration of Refugees in Developing Countries: A Research Model.
1990-36	T. Kuhlman	Towards a Definition of Refugees.
1990-37	T. Kuhlman	Organized versus Spontaneous Settlement of Refugees in Africa.
1990-38	R. Zuidema	The Neo-Austrian View on Interest
1990-39	G.v.d.Laan	General Equilibrium in a Closed International Trade Model