SERIE RESEARCH MEMORANDA

TESTING FOR CO-INTEGRATION WITH
SPOT PRICES OF SOME RELATED
AGRICULTURAL COMMODITIES

by

Ben Vogelvang

Research Memorandum 1990-1

January 1990

VRIJE UNIVERSITEIT
Faculteit der Economische Wetenschappen en Econometrie
AMSTERDAM

TESTING FOR CO-INTEGRATION WITH SPOT PRICES OF SOME RELATED AGRICULTURAL COMMODITIES

by

Ben Vogelvang

January 1990

A B S T R A C T

In this paper the hypothesis is tested whether prices of related agricultural commodities are co-integrated, as a similar behaviour over time of their time series data is often observed. Various integration and co-integration tests are briefly surveyed and commented, after which they are applied on the time series of four related agricultural commodities and a world-inflation indicator. This research has also been done because we are interested to compare empirically the bivariate approach of Engle and Granger, and the multivariate tackling of Engle and Yoo, and Johansen. We conclude that co-integration relationships between these prices exist, and that the results obtained by using the methods of Johansen appeared to be the most informative about the existence these relationships.

Table of contents

1. INTRODUCTION

Introductory remarks

Analysis of co-integrated time series has become a rather popular subject in econometric research in recent years. The reason for this popularity is that it has attractive and interesting properties in applied work concerning the analysis of time-series. Many, more theoretical, studies in this area have been published in the econometric literature, starting with Granger (1981) who introduced the concept of co-integration and made the link with the error-correction models which have been introduced earlier by Davidson, Hendry, Srba and Yeo (1978). More recently, also various applied studies have been published concerning the testing of co-integrated variables of e.g. macro-economic and commodity models. References to a number of these theoretical and applied studies will be given in the sections 2 and 3 where some methodological aspects of testing for co-integration, and a number of applied studies will be discussed in a short survey.

The use of flexible dynamic short-run relationships between economic variables which have a constraint on a long-run equilibrium relationship is wide spread in econometric modelling. Many of these models have been specified and estimated as error-correction models. In these models, deviations of long-run equilibrium levels in the short run, are "corrected" in following periods. In more recent years the concept of co-integrated variables and the relationship of co-integration to error-correction models has been developed further. Co-integrated variables have an error-correction representation. Engle and Granger (1987) clearly formulate that problem and the solution for the bivariate case. The multivariate case has been studied by e.g. Engle and Yoo (1987) and Johansen (1988), (1989).

Although the concept of integrated and co-integrated time series can be studied in the recent econometric literature, a short summary of this item, concerning hypotheses and the belonging statistical tests will be given in the present and next section, as this paper is not intended for time-series specialists only. Therefore, this survey will be convenient for them who are not yet familiar with the many aspects of this subject, while it is superfluous for others.

Observed time series will mostly be non stationary and can be represented by an ARIMA model in that case. The non stationarity of the time series may be caused by a (stochastic) time trend which implies that the series has a unit root. Stationarity of the series may be obtained by first differencing, in which situation the original series is called to be integrated of order one. When various (economic) related variables are observed with non-stationary time series, which are all integrated of e.g. order one, it is still possible that a linear combination of these variables, in levels, is stationary. In that case the series are called to be co-integrated. Or, in the way Engle and Granger (1987) define co-integration in a general way: the components of a vector time series x_t are said to be co-integrated of order d, b, [notation: $x_t \sim CI(d,b)$], if all components of x_t are integrated of order d [notation: $x_t \sim I(d)$], and there exists a vector α ($\neq 0$) such that $z_t = \alpha' x_t \sim I(d-b)$, with $b > 0$. The vector α is called the co-integrating vector. Necessary for an equilibrium relationship is that $z_t \sim I(0)$, implying that the equilibrium error will not drift from zero and the co-integrated series can be modelled by an error-correction representation. When in practice a co-integrating vector has been estimated for the static interrelationship between the variables, one has to check whether that relationship can be identified to the economic theory, or has an economic interpretation. If this relationship concerns more than two variables, it is also possible that more than one co-integration relationship exists. In section 2 the procedure of testing and estimating for co-integration will be briefly discussed.

Motivation of the project

In the present paper, the hypothesis is examined that various related commodity prices deviate in the short run from long-run stochastic equilibrium levels. Commodity prices which have been formed on various related markets are influenced by common factors, which may cause similar price trends. This hypothesis stems from the observation that important events on one market do not influence the prices of only that particular commodity, but also prices of related commodities, while in the absence of such events prices develop also in a comparable way, (possible along their own time trends; see the remarks concerning trends in the next section) which may be caused by e.g. world inflation or common expectations about economic prospects in general. So, the basic idea is that some stochastic

equilibrium relationship exists between the prices of related commodities, i.e. the stochastic trends in the various prices, represented by their non-stationary parts, are linearly interrelated. The commodity prices which will be analysed in this manner are quarterly spot prices of coffee, cocoa, tea and sugar. Many similarities can be noticed between these commodities. An important fact is that they are all commodities which can be taken in store. Another important feature is that three of these commodities are involved in an international commodity agreement (with varying success). Tea is the only one that is traded unrestrictedly. This might be a potential reason for the absence of co-integration for all the prices. Coffee, cocoa (beside other uses) and tea are beverages which may be considered, to a certain extent, as substitutes for each other, while sugar is often used complementary with the others. Although futures prices are not involved in this analysis, it is well-known that some of them are often traded on the same futures exchange. These are potential reasons that co-integration of their prices may be expected. Secondly, one more reason for this research is that we are interested to see how the various tests differ or just correspond to each other in empirical research. The power of the various test statistics is not common knowledge as far as we know, although e.g. the remarks made by Schotman (1988) [see also section 4.2] indicate that the power seems to be extremely low in small samples. In this paper we will not investigate the power of these tests, but we are interested to observe possible differences or similarities in the empirical outcomes. For that reason we will compute many test statistics both of the bivariate and multivariate approach which are known from the literature, and compare the results.

Organization of the paper

This paper proceeds as follows. The statistical methodology which can be used for testing the hypothesis of integrated and co-integrated variables, is briefly summarized in section 2. Then the problem and belonging statistical tests have been clarified, so that a number of examples of results of applied work done by some other authors, will be given in section 3. Then in section 4, all the empirical results for the test statistics applied on the four agricultural commodity prices, which have been analysed in the present project, will be presented and discussed, the estimation results of VAR and error-correction models will be reported too, and lastly in section 5 some conclusions are formulated.

2. STATISTICAL METHODOLOGY

In this section a brief summary of commonly used statistical tests and their use in studying long-run properties of multivariate time series is given. Engle and Granger (1987) propose a two-step estimator for a co-integrated system and give a thoroughly survey of a number of tests for co-integration, and of the relationship between co-integration and a VAR or ECM representation of bivariate time series. The "Granger Representation Theorem" which has also been proved in Engle and Granger (1987), shows precisely that co-integrated series can be represented by an error-correction model, i.e. the relationship which was mentioned before:

$$A(L)\nabla x_t = -\gamma z_{t-1} + u_t.$$

With $A(L)$ being a $(n \times n)$ matrix with elements that are polynomials in the lag operator, x_t is a vector with n components, u_t is a stationary process and $\gamma \neq 0$ [γ is $(n \times r)$, $r \leq n-1$, where it is assumed that r linearly independent co-integrating vectors exist]. Co-integrated variables should not be modelled by a VAR model in first differences, as that model will be misspecified because the error-correction term has been omitted. When estimating in levels, it is possible that parameters near the unit circle are underestimated. The approach of Engle and Granger, will also be used in this paper.

A necessary condition for time series to be co-integrated is that each series is integrated of the same order. In particular the values $d = b = 1$ in the definition of Engle an Granger are considered in the applied literature, which will also be done in this paper. So, from now on two variables can have a long-run equilibrium relationship if they are both integrated of first order. In that case the series are stationary after first differencing. If the $I(1)$-variables are co-integrated, they will have a co-integration relation with an equilibrium error that is $I(0)$, which implies that the relationship is stationary. The hypothesis that a series is $I(1)$ can be determined by testing whether that series have a unit root. In most applied work, tests for co-integration have been done for bivariate systems. Therefore, the bivariate case will be explained first, after which the multivariate situation will be considered. For systems with more than two variables the procedure becomes somewhat more complicated.

Various tests can be used to test the hypothesis that a series has a unit root. Well-known popular tests are the (augmented) Dickey-Fuller tests, see Dickey and Fuller (1979) and the Phillips test, see Phillips (1987). Also the Durbin-Watson value of the series itself is sometimes computed, which is zero under the null hypothesis of a unit root. It is well-known that the difference between the Dickey-Fuller and the Phillips test concerns assumptions about other possible stochastic behaviour of the time series. The series is $I(1)$ under the H_0 and $I(0)$ under the alternative of Dickey-Fuller, while it may have any autoregressive or moving average representation under the null or alternative when using the Phillips test. The Phillips test is often recommended for that more realistic assumption. The null hypothesis that a series x_t is integrated of first order can be tested in the following manners, using the above mentioned tests.

a. Dickey-Fuller test (DF)

Run the regression $\nabla x_t = \phi x_{t-1} + \varepsilon_t$, and compute the t-statistic for ϕ. The time series has a unit root if ϕ differs not significantly from zero. Also a constant term and a linear trend term can be inserted. Tables of these test-statistics can be found in Fuller (1976) for equations without a constant term, with a constant term, and with a constant and linear trend term, see also Dickey and Fuller (1979). These test statistics are denoted by τ, τ_μ and τ_τ respectively. However, if a constant term is inserted, the null hypothesis is that the series has a unit root and drift, while it has even an explosive trend under the null if a trend term is also inserted in the regression equation! See also the remarks made in the next paragraph. The null is not rejected if the t-statistic is below its table value.

b. Augmented Dickey-Fuller test (ADF)

The ADF-test makes a correction for possible autoregressive behaviour of the process by adding lags of ∇x_t in the regression equation (cf. the Phillips test). So run the regression $\nabla x_t = \phi x_{t-1} + \sum_{i=1}^{p} \beta_i \nabla x_{t-i} + \varepsilon_t$, and test in the same way as before with the t-statistic of ϕ. To perform this test, it is again possible to include a constant and a trend term, and to use the tables in Fuller (1976). The ADF-test allows for more dynamics in the regression than the DF-test. The number of lags (p) is determined empirically, such that the disturbance term ε_t is random.

c. Phillips test

This is the Z_α test from page 287 in Phillips (1987), which test for a unit root, while both under H_0 and H_1 the series may have any AR(I)MA representation. The tables in Fuller (1976) are also applicable for this test statistic.

d. Durbin-Watson test (DW)

Run the regression $x_t = \alpha + \varepsilon_t$, and compute the DW-statistic which is zero under the null hypothesis of a unit root. Tables for this statistic can be found in Sargan and Bhargava (1983).

In the empirical econom(etr)ic literature one can observe that the ADF-test is a frequently used test for unit roots in applied work, and is often used with a constant term. However when using a constant term in the regression equation one has to realize that the hypothesis is tested that the series has a unit root and drift, which may give problems in the next stage: testing for co-integration. According to Engle and Granger (1987) (p.255): "any known deterministic components can be subtracted before the analysis is begun". The constant term represents the coefficient of a deterministic linear trend term when the equation is written in levels. Variables with different deterministic trends cannot be co-integrated in general. In case the unit-root test is performed as a first step to test for co-integration, deterministic components have to be removed before, or the assumption has to be made that the vector with constants is orthogonal with the co-integrating vector [see e.g. Engle and Yoo (1987)], which seems to be a rather ad-hoc or even implausible restriction in empirical work, at first sight. If it is not clear whether the series has a non-zero drift, the τ-statistic is the appropriate Dickey-Fuller test. The same holds for the Phillips test. Deterministic trends have only to be eliminated before applying the Phillips test, when it is obvious that they exist. For instance, Stock and Watson (1986) detrend the time series of real U.S. GNP by extracting a 1.5% annual trend growth before testing for a unit root. Alternatively for this situation one can also use the "common trends" representation, which has been introduced by Stock and Watson (1988). They show how a set of time series can be decomposed into stationary components, deterministic and stochastic trends.

If the null hypothesis of $I(1)$ variables cannot be rejected, co-integration tests can be performed for each pair of variables, in the bivariate case. The same test statistics as previously used, are applied on the residuals of the co-integrating regression. The co-integrating regression is just the linear static regression between the two variables. If the variables are co-integrated then o.l.s. estimates of the co-integrating vector α are consistent [as shown by Stock (1987)], and the static relationship can be interpreted as the long-run relationship. Otherwise, the estimated vector α has no meaning if the variables are not co-integrated, and the regression is called a "spurious regression", as explained by Granger and Newbold (1974). Notice that a difference with respect to the unit root tests, as used before, is that we check now whether the residuals do not have a unit root. Therefore the null hypothesis reads now: the variables are not co-integrated. This negative formulation is caused by the fact that the variables are not co-integrated if the equilibrium error is $I(1)$, which is the null hypothesis of the unit root tests. Engle and Granger (1987) give a survey of a number of co-integration tests. The Durbin-Watson test, which is often called the Co-integration-Regression-Durbin-Watson test (CRDW-test) in this situation, is together with DF an appropriate test for the first order system. But when testing for co-integration, Engle and Granger recommend the ADF-test as an appropriate test, because the critical values of the CRDW-test are very sensitive to the particular parameters within the null hypothesis, although the CRDW-test is useful for a quick check because of its simplicity. The ADF-test allows for more dynamics in the regression, though it is clear that the test will be over parameterized in the first order case and may be correctly specified in higher order cases [although this will never be the case if the series contains MA components]. For that reason the CRDW and the ADF-tests may give different results. It is also well-known that the tables of Fuller (1976) can not be used, because the co-integrating vector is unknown and has been estimated. So the critical values of Dickey-Fuller co-integration tests are different from those which have been used for testing for unit roots in the variables. Engle and Granger (1987) computed critical values for the bivariate case and a sample size of 100 by means of a Monte Carlo simulation. These critical values are given in their article.

When the null hypothesis of a first order integrated variable has not been rejected against an $I(0)$ variable, the same test statistics can be used to test for a unit root in the first differences, i.e. test the H_0: $x_t \sim I(2)$ against the alternative that $x_t \sim I(1)$ (at least when DF/ADF is used). Depending on the properties of the time series in question, the CRDW-test and the Phillips-test are applied on ∇x_t, while the DF and ADF-test regress $\nabla^2 x_t$ on ∇x_t and lags of $\nabla^2 x_t$. Lastly, if the assumption can be made that the stochastic equilibrium between the variables is in levels, all the tests are carried out for the observed values of the variables. However, in many cases, economic variables may be in equilibrium concerning their proportional rates of changes, which implies that the tests will be applied on the logarithms of the variables.

Theoretical studies concerning techniques for a multivariate tackling of testing for co-integration have been developed in the last years. Two approaches can be noticed in the publications resulting from these studies. The procedure of Engle and Yoo (1987) which is an extension of the bivariate procedures of Engle and Granger (1987), while a different approach has been developed by Johansen (1988), (1989), which concerns testing and estimation procedures within a maximum likelihood framework. Testing for co-integration in the multivariate case gives two more "problems". Firstly, one has to decide how to normalize the co-integration relationship, when it is relevant to express one variable as linear combination of the others, and secondly, more than one co-integration relationship among the variables may exist. Engle and Yoo (1987) propose the same two-step method for the multivariate case as in the bivariate situation. They consider only one co-integration regression among all the involved variables. Normalization can be a problem if not any natural normalization exists, but Engle and Yoo report that only little differences in the results from their experiments are caused by different normalizations. Although different test statistics can be found from the same data set, we expect the differences to be of minor importance as it concerns in fact only dividing by a constant (when no zero restrictions are included). So in the first step, the co-integration relation is estimated and in the second step the residuals are tested (in the same way as the Dickey-Fuller tests do) for (the absence of) a unit root. Engle and Yoo (1987) computed critical values for this t-test up to a maximum of five variables, also by means of a Monte Carlo simulation. As mentioned before,

Johansen (1988) has elaborated a quite different manner of hypotheses testing and estimating co-integration vectors, by using maximum likelihood techniques. He derives also a likelihood-ratio test, with the purpose to test for the number of co-integration vectors. This has been further developed in Johansen (1989) where also a constant term and seasonal dummies are included in the model. In section 4.4 for more details of that procedure will be given.

3. A BRIEF SURVEY OF THE RESULTS OF SOME EMPIRICAL STUDIES ON CO-INTEGRATED VARIABLES

Nowadays various publications can be encountered, where the hypothesis of co-integrated variables has been tested for variables of different types of models; like models for commodity markets, financial markets etc.. Some examples from that literature will be presented and commented in this section, with the intention to serve as illustration of the topics discussed in the previous section. In general it can be noticed that, although obvious deterministic components in the time series have to be removed before testing for co-integration can start as was argued in the previous section, in various of the empirical studies no attention has been paid to this phenomenon. Alternatively, if obvious trends are observed, the τ_μ-statistic can be computed, and later on the restriction can be imposed that the vector with constants is orthogonal with the co-integrating vector. But occasionally, without any explanation the τ_μ-statistic of the ADF-test is computed. Perhaps it is only lack of information, but it should have been useful knowledge to have some statistical information about possible trend behaviour of the series to justify the use of τ_μ when the ADF test is used, and about the consequences for the co-integration hypothesis.

First some applications to commodity markets will briefly be reviewed. For example, two papers dealing with possible co-integrated variables of commodity market models have been presented on the Conference on International Commodity Market Modelling in Washington D.C., 24-26 October 1988. In these papers various hypotheses have been tested, which make allowance for linkages among related commodity markets; see Durand and Blöndal (1988) and Wolak and Kolstad (1988). Durand and Blöndal examined

equilibrium relationships between the levels and changes of consumer and commodity prices (in pairs) to test the hypothesis that commodity prices are useful indicators of OECD price developments. They computed the DW and ADF(τ) test statistics to test the integration and co-integration hypothesis. Although they conclude that all the time series are $I(1)$, they found no clear evidence of any equilibrium relationship between these price levels. However, just a few relationships between changes in a number of commodity prices and consumer prices could be established, by using Granger-causality tests. In our opinion they take the significance levels rather (or too) large when testing for unit roots, which makes it at least doubtful whether the consumer price indices are $I(1)$. Wolak and Kolstad derived a model of homogeneous input demand under price uncertainty. They tested empirically [with the help of ADF(τ_μ)] the validity of the model for the imports of steam coal into Japan from five countries by modelling the five matched price series, which were found to be strong co-integrated, with the ECM specification.

A recently published study is the article of Ardeni (1989). He investigated the "law of one price", i.e. that each commodity has a single price throughout the world when it is defined in a common currency. This stems from the more general "purchasing power parity" (PPP) hypothesis, which states that the exchange rate between two currencies is proportional to the ratio of the price levels in the countries concerned. The PPP-hypothesis has been a research topic in various studies [see e.g. Schotman (1988)]. The reason to assume that the prices of agricultural commodities are perfectly arbitraged, is the presumption that commodities are traded in flexible-price markets. Ardeni gives a clear exposition of the problems related to co-integration and spurious regressions on the basis of some empirical studies, before presenting his empirical analysis of seven prices of commodities and exchange rates in four countries. He uses the ADF(τ_μ) test for unit roots. The τ_μ-statistic is motivated by the upward-trending behaviour of the time series. Almost all variables turned out to be non stationary in levels. When testing for co-integration it appeared that only three of the fifteen tested bivariate relationships may concern a co-integrating relationship. For this reason Ardeni concludes that in general the "law of one price" does not hold.

With respect to other markets it is interesting to consider the following two articles. Baillie and Selover (1987) applied the tests for co-integration in monetary models for exchange rates by using data from five countries. They analysed why a number of popular and simple models of exchange rate determination have broken down and failed in forecasting nominal exchange rates. They consider the PPP-hypothesis too. The authors show, by using ADF(τ) that the various variables of the model are integrated of different order, and the existence of a lack of co-integration between these variables. Therefore, a long-run relationship does not exist between these variables, and the authors conclude that the used monetary model is inappropriate. The PPP-hypothesis was also rejected for four of the five countries. MacDonald and Murphy (1989) examined [by using ADF(τ_μ)] the possible existence of a long-run relationship between inflation and interest rates in four countries. They did not found co-integration for the entire sample 1955-1986, although they found some evidence of co-integrated variables for a sub period with a regime of fixed exchange rates.

Lastly, the approach of Johansen to test for co-integration vectors has been applied by Johansen and Juselius (1988) (1989), and also by Kunst (1988). Kunst applies this concept to test for a number of co-integration relationships in a macro-economic system using Austrian data for six macro-economic variables, with the intention to test the validity of a neoclassical growth model. Johansen and Juselius illustrate the techniques of Johansen by testing for co-integration vectors for a demand for money model, by using Danish and Finnish data.

4. EMPIRICAL TEST RESULTS FOR FOUR AGRICULTURAL COMMODITY PRICES AND AN INFLATION INDICATOR

4.1 *Introduction*

The results of the research presented in the present paper concern the spot market prices of related agricultural commodity markets. It does not concern the "law of one price", as considered in the previous section because we are not looking at the price of one commodity in several countries. In many econometric research on commodity markets, the modelling

of a single commodity market is often encountered. However, if for instance an important reason for modelling a commodity market is to analyse the price formation of that particular commodity, one may wonder whether prices of related commodities are also important for the determination of the price level. In other words, circumstances on other, but related, markets influence the price too. This may concern price behaviour in the short run, or in the long run due to possible existing more or less (stochastic) long-run equilibrium levels between prices of the various commodities, represented by common non-stationary factors, as it has been explained in section 1. An example of this multivariate tackling of commodity markets is e.g. the study of Rausser and Walraven (1988). Rausser and Walraven analysed dynamic welfare effects of overreacting of futures market prices as a result of a monetary shock. They notice in general that: "studies of futures market efficiency which search for single series martingale or random walk processes cannot be expected to classify markets correctly. Linkages among markets forces inefficiencies in one market to be transmitted to related markets." They established a vector-ARMA model for an eight-market system (financial and commodity markets). Overshooting was found for all the futures markets, but for commodity markets to a much greater degree than the financial markets, however, the period length of the overreacting of agricultural commodity markets was much shorter.

The commodity prices which have been investigated in this paper are the spot-market prices for coffee, cocoa, tea and sugar[1]. It concerns monthly price indices from the period 1976 – 1986, which are plotted in Figure 1. The sample period is limited, as no monthly price indices have been published before 1976, only quarterly figures are available before that year. Looking at Figure 1, more or less similar developments in the commodity prices can be noticed, of course beside specific price movements of the particular commodities. For instance, the high prices for all the commodities in 1976 and 1977 have been caused by occurrences on the coffee market, while at the end of this sample period more individual price movements can be observed. Prices have their own short-term movements, but they have also the tendency to have similar stochastic trends. Therefore the

---

[1] World export price indexes of primary commodities and non-ferrous base metals, Monthly Bulletin of Statistics, U.N.

MONTHLY PRICE INDICES

1976 – 1986 (1975=100)

Legend:
- □ COFFEE
- ⊙ COCOA
- ▲ TEA
- + SUGAR
- × U.V.EXP
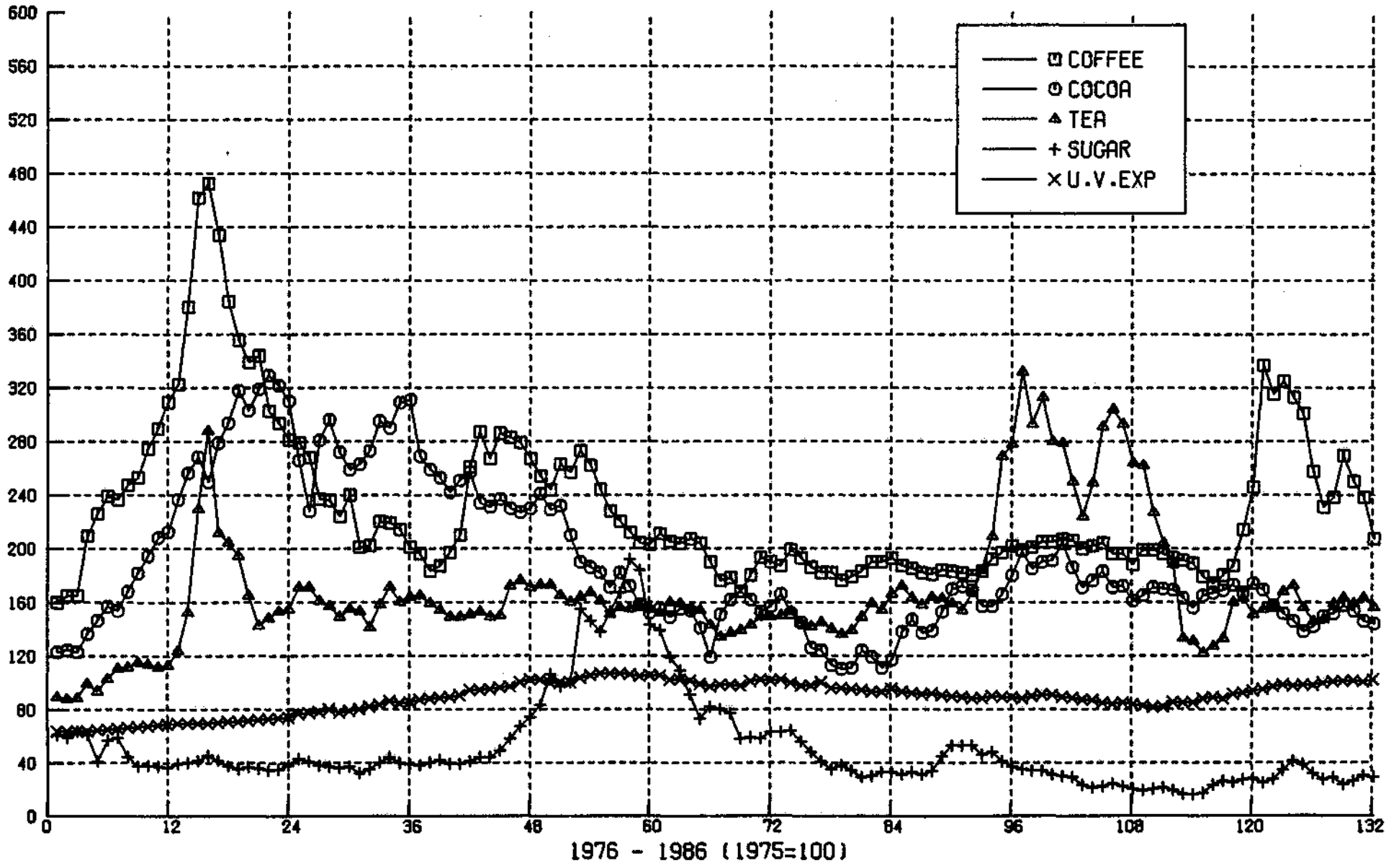
hypothesis will be tested that these stochastic trends are linearly interrelated. We did not test for seasonality in the price series, as it concerns prices of commodities which can be kept in store, and so they are assumed to be less (or not) sensitive to the influence of the harvest period on prices compared to commodities which cannot be stored. Maybe some attention to this subject will be paid in a following study. In the introduction it has been mentioned that commodity prices on various related markets may be influenced by common factors, and inflation is a possible common factor. The U.N. index of unit values of exports of manufactured goods from developed market economies is often used as world-price deflator in the empirical econometric literature. Therefore it can be tested whether the commodity prices are possibly also co-integrated with an indicator of world inflation. Because the above mentioned index is only available on a quarterly basis, we use the index of export unit values in industrial countries (abbreviated in this paper as: u.v.exp), which is monthly published in the International Financial Statistics of the I.M.F., which series has been plotted in Figure 1 too. In the next section, the test results of the bivariate analysis will be given, followed by the results of the multivariate approaches in section 4.3 and 4.4.

4.2 The bivariate results

The test statistics discussed in section 2 have been used to test the hypothesis that the spot prices of coffee, cocoa, tea and sugar are two by two co-integrated. It has already been mentioned in the introduction that these methods can be used to test for the presence of long-run equilibrium relationships between economic variables, stemming from economic theory or from observed behaviour of the particular variables. This latter reason is the basis for the research that is presented in this paper. As we will assume that prices may be in equilibrium in percentage changes, all tests have been performed for the logged variables. It is obvious from Figure 1 that the prices do not behave with a clear drift. Anyhow, only when prices are forced up by e.g. a permanent inflation, it should be possible to assume that prices will rise into heaven in the long run. We observe here that after a period with price increases, prices go down again and the other way round. They seem to move within a lower and upper bound. For this reason the

τ-statistics of the Dickey-Fuller tests will be computed. After the results for the entire sample period have been discussed, some attention will also be given to the effects of splitting up the sample in sub samples because of possible differences between the beginning and the end of the sample period, due to the price movements in the years 1976 and 1977.

First the results of the tests will be presented which have all been computed for the entire sample period. In Table 1 the outcomes of the DW-test, DF(τ), ADF(τ) and the Phillips Z_α test are reported for the variables in levels (logarithms), and in Table 2 for the variables in first differences. All statistics have been computed to check whether differences arise, originating from the characteristic properties of the various test statistics. The results of the ADF-test which are reported have been obtained by running regressions with various values for p. The choice of p, which appeared to be an appropriate value considering the absence of autocorrelation in the the residuals, is given in parentheses. Critical values at the 5% level are also given, where the critical value of .22 for the Durbin-Watson statistic has been roughly determined by extrapolating the values of Table 1 in Sargan and Bhargava (1983).

Table 1: *Unit-root test values, H_0 : prices are $I(1)$, variables in levels, monthly* 1976 – 1986

Commodity	DW	DF	ADF	Z_α
Coffee	.10	-.26	-.32 (1)	.03
Cocoa	.06	-.18	-.15 (2)	.02
Tea	.12	.29	.11 (1)	.07
Sugar	.06	-.67	-.68 (3)	-.28
U.v.exp	.02	2.45	.80 (12)	.37
Critical value 5%	.22	-1.95	-1.95	-7.9

Table 2: *Unit-root test values*, H_0 : *prices are* $I(1)$, *variables in first differences, monthly* 1976 - 1986

Commodity	DW	DF	ADF	Z_α
Coffee	1.34	-7.91	-4.93 (4)	-89.3
Cocoa	1.70	-9.80	-8.72 (1)	-101.2
Tea	1.43	-8.24	-6.74 (1)	-93.6
Sugar	1.65	-8.89	-5.55 (2)	-102.6
U.v.exp	1.75	-9.44	-3.64 (3)	-117.9
Critical value 5%	.22	-1.95	-1.95	-7.9

These results clearly show that the Durbin-Watson, the Dickey-Fuller and the Phillips test statistics do not reject the null hypothesis of first-order integrated price series, against the various alternative hypotheses. The value of 2.45 of the DF-test for u.v.exp is a striking value in Table 1. It has the wrong sign, and its magnitude indicates rather explosive than stationary or $I(1)$ behaviour. However, this result is quite similar to that of Durand and Blöndal (1988). They have negative not-significant (ADF) t-values for the commodity prices and positive t-values in the range [1.5, 3.3] for the consumer price indices of OECD countries. Their ADF tests had been computed with $p = 4$, while we needed $p = 12$ to eliminate first order autocorrelation. However, the Phillips test does not reject the null for u.v.exp, and as the first differenced series of u.v.exp does clearly not have a unit root, it seems plausible to conclude that the u.v.exp variable has a unit root and any ARIMA representation. The ADF value is lower for the same reason of more dynamics, so that both tests show the inappropriateness of the DF test. Although only the results for the logged variables have been tabulated, it can also be reported that the results for the levels, which have been computed too, do hardly differ from the results which have been presented for the logged variables.

Now it has been established that all the series are $I(1)$, co-integration tests for all pairs of variables have been computed. By using the augmented Dickey-Fuller test, the appropriate values of p has been given

again in parentheses. The value of $p = 12$ was used again, which was in some cases (coffee-sugar, cocoa-sugar, u.v.exp-sugar) necessary to eliminate first order autocorrelation, but the resulting test values do no differ very much compared with the results from regressions with fewer lags included. All the results are given in Table 3.

Table 3: *Co – integration test values between pairs of commodity prices*
H_0: *variables are non co – integrated* , *monthly* 1976 – 1986

	Cocoa	Tea	Sugar	U.v.exp	Test
Coffee	.12	.12	.06	.11	CRDW
	-2.04	-2.74	-1.18	-1.99	DF
	-2.10 (2)	-3.43*(2)	-1.55 (12)	-3.03 (4)	ADF
Cocoa		.12	.06	.07	CRDW
		-2.71	-1.17	-2.04	DF
		-3.41*(1)	-1.53 (12)	-2.08 (2)	ADF
Tea			.14	.13	CRDW
			-2.93	-2.49	DF
			-3.75*(1)	-3.26*(1)	ADF
Sugar				.06	CRDW
				-1.20	DF
				-1.34(12)	ADF

Critical values at the 5% level: CRDW .39, DF -3.37, ADF -3.17
a * indicates a significant value at the 5% level

According to the CRDW and the DF test statistic values the null hypothesis of non co-integration can not be rejected. However, the results of the augmented Dickey-Fuller test are different. Significant values are found for tea and coffee, tea and cocoa, tea and sugar and tea and u.v.exp. Referring to the remarks of Engle and Granger (1987), cited in section 2 of this paper, concerning the performances of the various test statistics for testing for co-integration, it can be remembered that the ADF-test was recommended as the most appropriated approach. Therefore we set more value on the outcomes of the ADF-test than to the other test statistics. One more reason is that the more dynamic specification was mostly needed to whiten the residuals. In fact it is useless to give all the results of the test statistics if it is obvious that more than first order dynamics is present in the series. Again it is worth mentioning that the tests have been applied on the observed prices too, and just as before, the results are rather

similar. Although the results do not show spectacular co-integrated variables, it seems acceptable to state that the implication of the results obtained so far, is that the conclusion may be drawn that the price of tea plays a particular role within this group of commodity prices and inflation index. These variables deviate in the short run from each other but have the tendency to return to an equilibrium relationship with the price of tea. It looks surprising that, in this analysis, it seems to be the price of tea which determines the long-run development and not e.g. the price of such an important commodity like coffee, which often seems to carry away other commodity prices with its occasionally heavy fluctuations. So this latter effect of the coffee price on the other commodity prices is apparently a short-run effect only.

We computed the same statistics for the first and the last half of the sample period, to check whether these results are rather stable or not. In Table 4 we report the unit root test results, and in Table 5 the results from the co-integration tests. The critical value of 0.4 for the DW-statistic has been determined in the same rough manner as the value of .22 for the entire sample has been determined before.

The unit root tests show that the prices still have a unit root and so are also non stationary in the two sub periods, although some small differences in the results from the two sub periods can be observed. The results of the co-integration tests are also not very different in the two sub periods. In the second half of the sample period only evidence of co-integrated variables is found with tea and u.v.exp, while that evidence has become weaker in the first half period. In the first sub period, the null hypothesis can only be rejected for the prices of tea and coffee, and of tea and sugar, although one has to realize that the critical values have been computed by Engle and Granger for a number of 100 observations, while our entire period consists of 132 observations, and so the results for the two sub periods are obtained only by using 61 observations. See e.g. Schotman (1988) for some remarks concerning the interpretation of test results with respect to unit roots and co-integration. The power of the unit-root and the co-integration tests appears to be extremely low in small samples. The non rejection of the null when testing for non co-integration indicates that the test cannot detect a possible long-run relationship,

Table 4: *Unit-root test values,* H_0: *prices are* $I(1)$, *variables in levels,*
first and second half of the period 1976 − 1986

Commodity	DW	DF	ADF	Z_α	period
Coffee	.10	-.33	-.31 (4)	.02	1st part, levels
	.16	.11	.01 (1)	.02	2nd part, levels
	1.31	-5.10	-3.10 (3)	-42.4	1st part, 1st dif
	1.40	-5.80	-4.26 (4)	-49.3	2nd part, 1st dif
Cocoa	.06	-.10	-.18 (3)	-.01	1st part, levels
	.17	.29	.30 (2)	-.01	2nd part, levels
	1.50	-6.19	-3.70 (2)	-48.0	1st part, 1st dif
	1.57	-7.87	-7.22 (1)	-48.4	2nd part, 1st dif
Tea	.18	.47	.42 (4)	.07	1st part, levels
	.11	.05	.01 (4)	.01	2nd part, levels
	1.40	-5.17	-4.34 (4)	-42.8	1st part, 1st dif
	1.47	-6.17	-3.85 (3)	-51.1	2nd part, 1st dif
Sugar	.06	1.11	.63 (3)	.04	1st part, levels
	.10	-1.32	-1.27 (2)	-.37	2nd part, levels
	1.83	-6.15	-3.15 (2)	-55.9	1st part, 1st dif
	1.43	-6.02	-5.47 (1)	-44.7	2nd part, 1st dif
U.v.exp	.06	3.37	-1.42 (12)	.43	1st part, levels
	.01	.37	.33 (12)	-.00	2nd part, levels
	1.61	-5.24	-2.80 (2)	-44.3	1st part, 1st dif
	1.91	-7.94	-3.93 (2)	-63.6	2nd part, 1st dif
Critical value 5%	.40	-1.95	-1.95	-7.8	

which should not be interpreted as a clear objection of co-integrated variables. Moreover, it seems that the more volatile price behaviour in the beginning of the sample period influences the outcomes of the test statistics, but the information contained in the entire sample strengthens the earlier formulated conclusions about the price of tea. So far, the cautious conclusion may be that within the group of coffee, cocoa, tea, sugar and the inflation indicator, the analysis indicates that the development of the price of tea seems to play a crucial role in a possible long-run equilibrium relationship among these commodity prices. Whether the fact that tea is the only commodity without an international commodity agreement plays a role in the obtained outcomes, is not clear. It is quite well possible that the agreements of the other commodities cause short-run deviations from an equilibrium relationship which exists in the long run.

Table 5: *Co-integration test values between pairs of commodity prices*
H_0: variables are non co-integrated, first and second half
of the period 1976-1986

	Cocoa	Tea	Sugar	U.v.exp	Test
	first half sample period				
Coffee	.10	.19	.08	.12	CRDW
	-1.09	-2.78	-1.18	-1.40	DF
	-1.14 (4)	-3.18*(3)	-2.26 (12)	-2.24 (4)	ADF
Cocoa		.30	.11	.07	CRDW
		-2.57	-1.91	-.65	DF
		-2.75 (4)	-1.59 (12)	-.85 (3)	ADF
Tea			.18	.22	CRDW
			-2.93	-2.49	DF
			-3.33*(4)	-2.95 (4)	ADF
Sugar				.12	CRDW
				-1.41	DF
				-1.91(12)	ADF
	second half sample period				
	Cocoa	Tea	Sugar	U.v.exp	Test
Coffee	.17	.10	.11	.18	CRDW
	-1.54	-1.32	-1.96	-1.85	DF
	-1.64 (2)	-1.91 (3)	-1.61 (12)	-2.35 (1)	ADF
Cocoa		.18	.11	.22	CRDW
		-1.82	-2.08	-1.97	DF
		-2.87 (2)	-2.41 (3)	-2.35 (1)	ADF
Tea			.13	.19	CRDW
			-1.34	-1.67$_*$	DF
			-1.83 (4)	-3.57(12)	ADF
Sugar				.16	CRDW
				-1.51	DF
				-1.51 (4)	ADF

Critical values at the 5% level: CRDW .39, DF -3.37, ADF -3.17
a * indicates a significant value

Therefore, it is interesting to proceed with the research by using the multivariate approaches. In the next sections, the results of the multivariate tackling of testing for co-integration will be reported. First the method of Engle and Yoo (1987) will be applied on the commodity prices in section 4.3, followed by Johansen's method in section 4.4.

4.3 *The multivariate approach of Engle and Yoo*

In section 2 it was already observed that one of the first "problems" in the application of the method of Engle and Yoo may be the question how to normalize the co-integrating equation, although the results may not differ very much in our opinion. Engle and Yoo report only small differences in their outcomes. We obtain also (unimportant) different results with various normalizations. This item was also commented by Ardeni (1989): "a co-integrating vector can exist in one case but not in the other". This is not true in our opinion. His ADF results show only small differences. As in our analysis of the commodity prices not an obvious normalization exists, we used, for reasons of completeness in this paper, each of the four prices once as a dependent variable, and estimated the co-integrating relationship both for the entire sample and the two sub sample periods. Let z_t be the vector with residuals after the co-integrating equation has been estimated. Just as before the residuals are tested for the absence of a unit root. Engle and Yoo tabulated critical values of the t-statistic of $\hat{\rho}$ in the following two regressions:

$$\nabla \hat{z}_t = \hat{\rho} z_{t-1}$$

$$\nabla \hat{z}_t = \hat{\rho} z_{t-1} + \sum_{i=1}^{4} \hat{\delta_i} \nabla z_{i-1}.$$

Our results are given in Table 6. The null hypothesis which is tested, is again that the variables are not co-integrated.

The t-values of the augmented regression results are probably more reliable than the non-augmented obtained values, considering the Durbin-Watson values of the residuals after the co-integrating relationships have been estimated. When the null hypothesis is tested using data both of the entire and the splitted sample periods, it is not rejected for all the equations. So the normalization question is (not surprisingly) not of any importance at the present stage of the research. Considering all the test statistics of Table 6, it cannot be concluded from these results that the four prices are co-integrated. Also can be seen that the obtained results differ only in a little way with respect to the used sample. The results

Table 6: *Co−integration test values*, H_0: *prices are not co−integrated,*
multivariate approach of Engle and Yoo

Dependent variable	t-value	t-value augmented	critical values at 5% level		period
Coffee	-2.21	-3.19	-4.58	-4.15	entire
	-2.15	-2.60	-4.76	-4.36	1st half
	-2.37	-3.92	-4.76	-4.36	2nd half
Cocoa	-2.41	-2.49	-4.58	-4.15	entire
	-1.72	-2.13	-4.76	-4.36	1st half
	-2.28	-3.05	-4.76	-4.36	2nd half
Tea	-2.53	-3.04	-4.58	-4.15	entire
	-3.08	-3.91	-4.76	-4.36	1st half
	-1.77	-2.75	-4.76	-4.36	2nd half
Sugar	-1.64	-1.74	-4.58	-4.15	entire
	-1.71	-2.49	-4.76	-4.36	1st half
	-1.96	-2.74	-4.76	-4.36	2nd half
U.v.exp	-2.11	-2.12	-4.58	-4.15	entire
	-2.05	-2.44	-4.76	-4.36	1st half
	-2.15	-3.24	-4.76	-4.36	2nd half

here are just opposed to the results obtained in the bivariate case, where
the evidence for possible co-integration became stronger if all the data
were used. It has already been emphasized in section 2 that one should not
conclude to easy that variables are co-integrated, as long-term equilibrium
relationships and spurious regressions may be difficult to distinguish if
the evidence from the test statistics is rather weak. Therefore we are
finally interested to see the results from applying the approach of
Johansen, to test first for the number of co-integrating relationships, as
it is quite well possible that co-integrating relationships exist among sub
sets of this set of variables.

4.4 The multivariate approach of Johansen

This section describes shortly the theoretical approach of Johansen
(1988) and follows the way like it has been applied by Johansen and Juselius
(1988) to the problem of testing for and estimating of co-integration
vectors in a multivariate context, together with our application to the

commodity prices. The procedure consists of the following stages. The first step is to test for the number of co-integrating vectors in a multivariate system. This is done with a likelihood-ratio test, that has an asymptotic distribution which can be approximated by a χ^2 distribution. Johansen computed fractiles of the distribution by simulation, up to five co-integrating relationships. In following steps the co-integrating space is estimated, which dimension equals the number of co-integrating vectors, after which hypotheses about possible restrictions on the co-integration vectors are tested concerning economic interpretations of the results.

The mentioned authors consider first the unrestricted vector-autoregressive (VAR) process, of p variables and possibly a constant term and seasonal dummies, which is integrated of order 1:

$$X_t = \Pi_1 X_{t-1} + \ldots + \Pi_k X_{t-k} + \varepsilon_t, \tag{4.1}$$

with ε_t being a sequence of i.i.d. p-dimensional Gaussian random vectors, distributed as $N(0,\Lambda)$. This specification can be rewritten as:

$$\nabla X_t = \Gamma_1 \nabla X_{t-1} + \ldots + \Gamma_{k-1} \nabla X_{t-k+1} - \Pi X_{t-k} + \varepsilon_t, \tag{4.2}$$

where $\Gamma_i = -I + \Pi_1 + \ldots + \Pi_i$,
and $\Pi = I - \Pi_1 - \ldots - \Pi_i$.

Then the research concentrates on the matrix Π, because of the long-run information that can be obtained from knowledge of this matrix, which is often called the impact matrix. The following situations may occur concerning the matrix Π:

$\text{rg}(\Pi) = p$, The matrix has full rank, implying a stationary process X_t;

$\text{rg}(\Pi) = 0$, The matrix Π is zero, implying an integrated vector process X_t;

$\text{rg}(\Pi) = r$, $0 < r < p$, implying the existence of $p{\times}r$ matrices α and β of rank r, giving a non-linear constraint on the coefficients $\Pi_1, \ldots, \Pi_k$: $\Pi = \alpha\beta'$, with β being the matrix with co-integrating vectors; $\beta'X_t$ is stationary.

With reference to Johansen we memorize that the parameters α and β cannot be estimated, as they are not uniquely determined, but the space spanned by β can be estimated. First two matrices of residuals are computed, originating from the regression of ∇X_t on $\nabla X_{t-1}, \ldots, \nabla X_{t-k+1}$, and X_{t-k} on

the same set of regressors ∇X_{t-1}, ..., ∇X_{t-k+1}. Denote these residuals by R_{0t} and R_{kt}, then the moment matrices S_{00}, S_{kk} and S_{k0} are computed. Johansen proposes the next procedure to test for the number of co-integration relationships. Solve the equation

$$|\lambda S_{kk} - S_{k0}S_{00}^{-1}S_{0k}| = 0,$$

giving the p eigenvalues λ_i and determine the corresponding eigenvectors. Let E be the matrix with eigenvectors, then E is normalized such that $E'S_{kk}E = I$. The number of co-integrating vectors r is determined by means of the likelihood-ratio test statistic (4.3) for H_0: there are at most r co-integration vectors.

$$-2\ln(Q) = -T \sum_{i=r+1}^{p} \ln(1 - \hat{\lambda}_i), \qquad (4.3)$$

where $\hat{\lambda}_{r+1}$, ..., $\hat{\lambda}_p$ are the $p - r$ smallest eigenvalues.

Before going on with the theoretical description, we give now some empirical results with the commodity prices. First the order of the vector autocorrelation k of equation (4.1) has to be determined. Because it has not been rejected in section 4.2 that the variables of (4.1), and of course of the identical equation (4.2) too, contain a unit root, we looked only at the first 12 autocorrelations of the residuals, in stead of computing likelihood ratio tests or Box Pierce portmanteau lack-of-fit tests to get an impression of the absence of autocorrelation. The value $k = 3$ turned out to be not inappropriate according this visual inspection. This result, and how to test it, has to be elaborated more exactly in future research. The autocorrelations are given below, where the approximated confidence interval is ± 0.18.

Coffee:

.04 .01 -.09 .06 -.04 -.03 -.05 -.02 .06 .11 -.01 -.01

Cocoa:

-.02 -.01 -.10 .12 -.02 .09 -.20 .02 -.18 .03 -.11 -.04

Tea:

.03 -.02 .03 .04 -.09 -.12 .07 -.03 .01 .08 .07 .17

Sugar:

.02 .04 .02 .07 .11 .06 -.01 .04 .03 -.06 -.11 -.14

U.v.exp.

.02 .01 -.02 -.05 -.13 .05 -.00 -.10 -.02 -.04 -.09 -.02

So our unrestricted model is:

$$\nabla X_t = \Gamma_1 \nabla X_{t-1} + \Gamma_2 \nabla X_{t-2} - \Pi X_{t-3} + \varepsilon_t,$$

with $X_t' = (p^{\text{coffee}} \; p^{\text{cocoa}} \; p^{\text{tea}} \; p^{\text{sugar}} \; p^{\text{u.v.exp}})'$.

The unrestricted estimates, with standard errors given in parentheses, of the constant term c, $-\Pi$, Γ_1 and Γ_2 are:

$$\hat{c} = \begin{bmatrix} 1.05 \\ (.35) \\ 1.66 \\ (.34) \\ .22 \\ (.46) \\ -1.81 \\ (.62) \\ .03 \\ (.08) \end{bmatrix}, \qquad -\hat{\Pi} = \begin{bmatrix} -.12 & .03 & -.05 & -.00 & -.06 \\ (.03) & (.03) & (.03) & (.01) & (.06) \\ -.00 & -.12 & .02 & .01 & -.27 \\ (.03) & (.03) & (.03) & (.01) & (.06) \\ -.01 & .05 & -.13 & -.02 & .07 \\ (.04) & (.04) & (.04) & (.02) & (.08) \\ .03 & .16 & -.13 & -.07 & .39 \\ (.06) & (.06) & (.05) & (.02) & (.11) \\ -.00 & .02 & -.02 & -.01 & -.00 \\ (.01) & (.01) & (.01) & (.00) & (.01) \end{bmatrix},$$

$$\hat{\Gamma}_1$$

$$\begin{bmatrix} .10 & .18 & -.00 & .03 & .27 \\ (.10) & (.09) & (.07) & (.05) & (.41) \\ -.12 & .00 & -.03 & .06 & .04 \\ (.09) & (.09) & (.07) & (.05) & (.40) \\ .16 & .20 & .14 & -.00 & -.45 \\ (.13) & (.12) & (.10) & (.07) & (.55) \\ -.20 & .33 & -.06 & -.10 & .13 \\ (.17) & (.16) & (.13) & (.09) & (.74) \\ -.01 & -.02 & -.01 & -.00 & -.05 \\ (.02) & (.02) & (.02) & (.01) & (.09) \end{bmatrix},$$

$$\hat{\Gamma}_2$$

$$\begin{bmatrix} .04 & .06 & .01 & -.06 & .03 \\ (.09) & (.09) & (.07) & (.05) & (.40) \\ -.06 & -.31 & .11 & .04 & .66 \\ (.09) & (.09) & (.07) & (.05) & (.39) \\ -.06 & -.02 & -.02 & -.08 & -.34 \\ (.13) & (.12) & (.10) & (.06) & (.54) \\ .49 & .17 & -.08 & -.27 & .96 \\ (.17) & (.16) & (.13) & (.09) & (.72) \\ .03 & -.00 & -.02 & .00 & -.10 \\ (.02) & (.02) & (.02) & (.01) & (.09) \end{bmatrix}.$$

Then the necessary regressions have been run to determine the matrices with residuals R_{0t} and R_{kt}, and we compute the moment matrices S_{00}, S_{kk} and S_{k0}. These matrices are defined as $S_{ij} = T^{-1}\Sigma_{t=1}^{T} R_{it}R_{jt}'$ with $i,j \in \{0,k\}$, and the resulting matrices are given below.

$S_{00} \times 100$

$$\begin{bmatrix} .43 \\ .03 & .44 \\ .16 & .05 & .72 \\ .00 & -.13 & .20 & 1.35 \\ .01 & .01 & .01 & .02 & .02 \end{bmatrix}$$

$S_{kk} \times 100$

$$\begin{bmatrix} 4.11 \\ 2.50 & 5.91 \\ .45 & 1.55 & 5.46 \\ .66 & -.03 & -4.01 & 25.83 \\ -.85 & -1.24 & .33 & 2.05 & 1.57 \end{bmatrix}$$

$S_{k0} \times 100$

$$\begin{bmatrix} -.41 & -.14 & -.31 & -.01 & -.04 \\ -.05 & -.33 & -.21 & -.32 & -.24 \\ -.06 & -.02 & -.57 & .27 & -.02 \\ .08 & .32 & -.05 & -.50 & .20 \\ .04 & .10 & -.05 & -.09 & -.05 \end{bmatrix}$$

With these matrices we compute the eigenvalues and eigenvectors of the generalized real symmetric eigenvalue problem $|\lambda S_{kk} - S_{k0}S_{00}^{-1}S_{0k}| = 0$ by using the routine GVCSP from the IMSL library. This routine normalizes the eigenvectors such that a modified ∞-norm of each vector is one, which means that its largest component equals one. Denote this matrix with eigenvectors as W. Then we transform the matrix W with the inverse R^{-1}, where R is obtained by the Choleski factorization $W'S_{kk}W = R'R$. The resulting eigenvalues and eigenvectors E ($E = WR^{-1}$) are given in Table 7.

Table 7: *Eigenvalues* $\hat{\lambda}_i$ *and eigenvectors* $\hat{e}_i$ *of* $|\lambda S_{kk} - S_{k0}S_{00}^{-1}S_{0k}| = 0$

eigenvalues				
.2920	.2024	.1237	.0421	.0258

eigenvectors				
.8774	1.8677	4.5824	2.9403	.7326
4.4242	-1.9001	-1.0214	-2.0501	-1.1761
-2.4166	3.6917	.3891	-2.7541	-.8051
-1.0662	.6478	-.2767	.0831	-2.0718
8.5857	2.7219	-2.7318	3.6375	3.2064

The results of Johansen's likelihood ratio test statistic (4.3), to test for the number of co-integrating vectors, are given in Table 8, together with the fractiles at the .10, .05 and .025% significance level.

Table 8: *Results of the likelihood ratio test statistic* (4.3)
H_0: *There exist at most r co-integrating vectors*

null hypothesis	$-2\ln(Q)$	fractiles		
		.90	.95	.975
$r \leq 4$	3.38	2.9	4.2	5.3
$r \leq 3$	8.93	10.3	12.0	13.9
$r \leq 2$	25.97	21.2	23.8	26.1
$r \leq 1$	55.14	35.6	38.6	41.2
$r = 0$	99.68	53.6	57.2	60.3

The result of the LR-test (4.3) is that the null hypothesis of at most three or four co-integrating vectors cannot be rejected at the 5% (three vectors even at the 10%) level. So the matrix β which columns span the co-integration space, consists of the three eigenvectors belonging to the three largest eigenvalues (the first three columns of Table 7). Then, with reference to Johansen, we computed the matrix α and the restricted impact matrix $-\Pi_r$ as: $\alpha = -S_{0k}\beta$ and $\Pi_r = \alpha\beta'$. The results are (cf. the unrestricted estimates of Π):

$$
\hat\alpha = \begin{bmatrix} .0059 & .0177 & .0176 \\ .0273 & .0113 & -.0079 \\ -.0078 & .0202 & .0048 \\ -.0389 & .0042 & .0042 \\ -.0027 & .0051 & -.0021 \end{bmatrix}, \quad -\hat\Pi_r = \begin{bmatrix} -.12 & .03 & -.06 & -.00 & -.05 \\ -.01 & -.11 & .03 & .02 & -.29 \\ -.05 & .08 & -.10 & -.02 & .03 \\ .01 & .18 & -.11 & -.04 & .33 \\ .00 & .02 & -.02 & -.01 & .00 \end{bmatrix}.
$$

The next step concerns the economic interpretation of the matrices α and β, which is not straightforward in this multivariate analysis. If $r = 1$, it is probably possible to normalize with respect to one of the variables, and to test for any exclusion restriction. But with $r > 1$ we cannot arbitrarily normalize the co-integration equations. In fact, only the hypotheses which co-integrating vectors exist can be tested. With the matrix β, we have computed a basis of the co-integration space. Therefore hypotheses can be tested, stemming from economic knowledge, which we have of the problem that we are investigating, concerning restrictions on the co-integrating vectors: H_0: $\beta = H\phi$, with H a known ($p \times s$) matrix of constants and ϕ a ($s \times r$) matrix of unknown parameters, $(r \leq s \leq p)$. Or in the way Kunst (1988) formulates the problem: one is interested in testing whether the co-integrating vectors which make up the columns of the ($p \times r$) matrix β are included in the space generated by the columns of a ($p \times s$) matrix H. According to Johansen: if $s = p$, then no restrictions are placed upon the choice of the co-integration vectors, and if $s = r$, then the co-integration space is fully specified. These restrictions are imposed on all the co-integration vectors, as otherwise no meaningful conclusions can be drawn. Johansen proves that the next procedure can be used to test the hypothesis H_0. First solve the equation:

$$
|\lambda H'S_{kk}H - H'S_{k0}S_{00}^{-1}S_{0k}H| = 0,
$$

which give the s eigenvalues λ_i^*. Let E^* be the matrix with eigenvectors, then E^* is normalized such that $E^{*'}H'S_{kk}HE^* = I$. Further the null is tested with the likelihood-ratio test:

$$-2\ln(Q) = T \sum_{i=1}^{r} \ln[(1 - \lambda_i^*)/(1 - \hat{\lambda}_i)], \qquad (4.4)$$

with λ_i^* and $\hat{\lambda}_i$ the r largest eigenvalues. This test statistic is asymptotically distributed as $\chi^2(r(p-s))$. Johansen and Juselius (1988) find some indications concerning the formulation of H_0 from the eigenvectors in β, where e.g. opposite signs can indicate that the difference of two variables has to enter a co-integration relationship or one large coefficient may indicate that the corresponding variable is stationary. All the variables in our research turned out to be $I(1)$, therefore we will search only for relationships among various commodity prices. This implies for the matrix H that its columns consist of vectors with one -1 and further figures which equal zero or plus one, so that one price may be proportional to a linear combination of other prices. Constancy (or stationarity) of differences in the logs of the variables implies similar percentage changes of the variables. As we earlier found bivariate co-integration among the price of tea and the other prices we started with a (5×3) matrix H to test for various bivariate relationships. This resulted mostly in $\chi^2(6)$ values in the range [40. 50.], which clearly reject the null hypotheses. The same occurred when more than two variables were involved in a possible relationship. The results improve substantially when proportionality to four relationships are specified ($s = 4$, $r = 3$). It is well-known that linear combinations of co-integrating vectors are also co-integrating vectors. So we have looked for linear combinations of the vectors of β which indicate that some prices may be related, by more or less equal magnitude in absolute value of its components which are also clearly larger than the others. Then we assume that they are proportional to the vectors which are given in the matrices H. In this way, the best results have been obtained with the two following matrices:

$$H_1 = \begin{bmatrix} 0 & -1 & 1 & 1 \\ 1 & 0 & -1 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \qquad H_2 = \begin{bmatrix} 0 & -1 & 1 & 1 \\ 1 & 0 & -1 & -1 \\ -1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix},$$

with the vector of variables: $X_t' = (p^{coffee} \quad p^{cocoa} \quad p^{tea} \quad p^{sugar} \quad p^{u.v.exp})'$. In hypothesis H_1, four relationships have been specified with one commodity price depending on two other prices, while in hypothesis H_2 the last column specifies a relationship among all the prices. The three largest eigenvalues obtained by solving $|\lambda H'S_{kk}H - H'S_{k0}S_{00}^{-1}S_{0k}H| = 0$ and the values of the LR-test (4.4) are:

H_1: .2914 .2020 .1145, $-2\ln(Q) = 1.51$ $[\chi^2(3)_{.05} = 7.81]$

H_2: .2919 .2012 .1163, $-2\ln(Q) = 1.30$ [id   ]

Both the null hypotheses are clearly not rejected. As examples of the sensitiveness of the test results for the specification of the restrictions, the results are also given for the matrix H_3 which equals the first three columns of H_1 or H_2, and the matrix H_4 which equals H_2 without the third column (so $s = 3$):

H_3: .2451 .1789 .1058, $-2\ln(Q) = 14.61$ $[\chi^2(6)_{.05} = 12.59]$.

H_4: .2405 .1644 .0446, $-2\ln(Q) = 26.21$ [id   ]

As the χ^2-value of H_2 is the lowest value, we give the other results which belong to H_2. One more interesting property of the matrix H_2 is its "consistency" with the results found in the bivariate analysis of section 4.2. There we found co-integration of the price of tea with the other prices. In the matrix H_2 it is also the price of tea that is always co-integrated with the other prices. So it is logical to consider the obtained result with the matrix H_2 as a clear extension of the earlier obtained indications. The procedures of Johansen appear to be more informative when more than 2 variables are tested for possible co-integrating relationships.

The matrix ϕ^* consists of the eigenvectors belonging to the r largest eigenvalues λ_i^*. Then the matrix β^* is computed as $H_2\phi^*$, the matrix α^* equals just as before $-S_{0k}\beta^*$, and lastly the matrix $\Pi^* = \alpha^*\beta^{*\prime}$. The computed matrices are given below.

$$\phi^* = \begin{bmatrix} 7.4255 & 1.2183 & 1.4683 \\ 2.1034 & 1.4114 & -2.0670 \\ 4.0209 & 2.6841 & 3.7550 \\ -1.0436 & .5727 & -.5554 \end{bmatrix}, \qquad \beta^* = \begin{bmatrix} .8739 & 1.8454 & 5.2666 \\ 4.4482 & -2.0384 & -1.7313 \\ -2.3449 & 3.4499 & -.3354 \\ -1.0436 & .5727 & -.5554 \\ 8.4853 & 3.2024 & -1.1540 \end{bmatrix},$$

$$\alpha^* = \begin{bmatrix} .0061 & .0168 & .0177 \\ .0274 & .0112 & -.0085 \\ -.0075 & .0191 & .0020 \\ -.0386 & .0032 & .0010 \\ -.0027 & .0052 & -.0016 \end{bmatrix}, \qquad -\Pi^* = \begin{bmatrix} -.13 & .04 & -.04 & .01 & -.09 \\ -.00 & -.11 & .02 & .02 & -.28 \\ -.04 & .08 & -.08 & -.02 & .00 \\ .02 & .18 & -.10 & -.04 & .32 \\ .00 & .02 & -.03 & -.01 & .00 \end{bmatrix}.$$

It is a striking feature how close these estimates are to the earlier restricted and unrestricted estimates, which indicate that the model is not really forced by the restrictions. The detected co-integration relationships do not look unrealisticly. If one of the prices changes, other prices may change directly or indirectly too. Because it concerns long-run relationships, it is of course not ruled out that individual short-run behaviour occurs without a reaction of other prices. So it is quite well conceivable that the dramatical coffee price increases in 1976-'77 (see Figure 1) did have also its influence on other prices, while less heavy price movements of e.g. tea or coffee in the eighties was of no any influence on the price formation of the other agricultural commodities. We will conclude this report by a last section where some conclusions are formulated.

5. CONCLUSIONS

In this research report, we applied the various developed test procedures to test for co-integration with prices of related agricultural commodities. Two matters of interest were raised in this paper:

1. Are related commodity prices in any way co-integrated, and
2. How do the various existing test statistics behave in this empirical research concerning the detection of possible co-integration.

These two questions can be commented together. After it had been established that all the variables are integrated of first order, the results showed that in the bivariate analysis the prices appeared to be co-integrated in pairs with the price of tea. This had been concluded by using the well-known augmented Dickey-Fuller test, to test for (the absence of) unit roots in the residuals of the static regression equation of pairs of variables. The DF- and CRDW-test seem to be less conclusive indeed as the data on the variables show mostly more than first order dynamics. Then in the next step, we wished to investigate whether a possible multivariate coherent interrelationship between these prices exist, by using the test of Engle and Yoo. This test did not detect a multivariate long-run relationship among all the prices. Quite different outcomes were obtained when the procedures of Johansen were applied. The likelihood-ratio test of Johansen indicated the existence of three or possible four co-integration relationships. These relationships could be linked to various inter-relationships between different sub groups of the prices and one relationship among all the prices, by using a second LR-test. Relationships which seem all to be realistic. The fact that tea is the only commodity without an international agreement may be of importance for this result, as effective agreements of the other commodities might disturb the relationship in the short run. This result appeared to be rather stable, as deviations from the detected coherence result in rejection of the hypothesis that the prices are interrelated. It has also been interesting to see how the results of the maximum likelihood approach extended the conclusions which had been obtained before by using the bi-variate techniques, while the test of Engle and Yoo did not indicate a possible co-integration relationship among all the variables. The reason for this last phenomenon, opposed to the result

from Johansen's method, is also an interesting item for further research. The last column of the matrix H_2 is in fact the relationship which was tested using the test of Engle and Yoo. To conclude this paper, it seems proper to infer from this research that the procedures of Johansen (and Juselius) are very useful in empirical work, and secondly that it may be of importance to consider occurrences on related commodity markets when only one commodity market is modelled.

REFERENCES

Ardeni, P.G. (1989): 'Does the law of one price really hold for commodity prices?', *American Journal of Agricultural Economics*, 71, 661-669.

Baillie, R.T, and D.D. Selover (1987): 'Cointegration and models of exchange rate determination', *International Journal of Forecasting*, 3, 43-51.

Davidson, J.E., D.F. Hendry, F. Srba, and S. Yeo (1978): 'Econometric modelling of aggregate time series relationships between consumer's expenditure and income in the U.K.', *Economic Journal*, 88, 661-692.

Dickey, D.A., and W.A. Fuller (1979): 'Distribution of the estimators for autoregressive time series with a unit root', *Journal of the American Statistical Association*, 74, 427-431.

Durand, M., and S. Blöndal (1988): 'Are commodity prices leading indicators of OECD prices?', *Conference paper on the Conference on International Commodity Market Modelling*, Washington, 20p.

Engle, R.F., and C.W.J. Granger (1987): 'Co-Integration and Error Correction: Representation, Estimation and Testing', *Econometrica*, 55, 251-276.

Engle, R.F., and B.F. Yoo (1987): 'Forecasting and testing in co-integrated systems', *Journal of Econometrics*, 35, 143-159.

Fuller, W.A. (1976): 'Introduction to statistical time series', New York, Wiley.

Granger, C.W.J. and P. Newbold (1974): 'Spurious regressions in Econometrics', *Journal of Econometrics*, 26, 1045-1066.

Granger, C.W.J. (1981): 'Some properties of time series data and their use in econometric model specification', *Journal of Econometrics*, 35, 143-159.

Johansen, S. (1988): 'Statistical analysis of cointegration vectors', *Journal of Economic Dynamics and Control*, 12, 231-254.

Johansen, S. (1989): 'Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models', Preprint University of Copenhagen.

Johansen, S., and K. Juselius (1988): 'Hypothesis testing for co-integration vectors - with an application to the demand for money in Denmark and Finland', Preprint University of Copenhagen.

Johansen, S., and K. Juselius (1989): 'The full information maximum likelihood procedure for inference on cointegration – with applications', Preprint University of Copenhagen.

Kunst, R. (1988): 'Cointegration in a macro-economic system", *Conference paper on ESEM*, Bologna, 31p.

MacDonald, R., and P.D. Murphy (1989): 'Testing for the long run relationship between nominal interest rates and inflation using co-integration techniques', *Applied Economics*, 21, 439-447.

Phillips, P.C.B. (1987): 'Time series with a unit root', *Econometrica*, 55, 277-301.

Rausser, G.C., and N. Walraven (1988): 'Dynamic welfare analysis and overshooting', *Working paper no.* 491, Dept. of Agric. and resource economics, University of California, 25p.

Sargan, J.D. and A. Bhargava (1983): 'Testing residuals from least squares regression for being generated by the Gaussian random walk', *Econometrica*, 51, 153-174.

Schotman, P. (1988): 'Testing the long-run implications of the parity conditions: co-integration, exogeneity or spurious regressions?', Erasmus University Rottterdam, report 8837/a.

Stock, J.H. (1987): 'Asymptotic properties of least squares estimators of co-integrating vectors', *Econometrica*,55,1035-1056.

Stock, J.H. and M.W. Watson (1986): 'Does GNP have a unit root?', *Economics Letters*, 22, 147-151.

Stock, J.H. and M.W. Watson (1988): 'Testing for common trends', *Journal of the American Statistical Association*, 83, 1097-1107.

Wolak, F.A., and C.D. Kolstad (1988): 'A model of homogenous input demand under price uncertainty, *Conference paper on the Conference on International Commodity Market Modelling*, Washington, 33p.