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ORDINAL DATA IN MULTICRITERIA DECISION MAKING, A STOCHASTIC DOMINANCE APPROACH TO SITING NUCLEAR POWER PLANTS

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Abstract

This paper addresses decision problems with ordinal data (weights, criterion scores). A random sampling approach is proposed to generate quantitative values which are consistent with the underlying ordinal information. An attractive feature of the approach is that it is applicable with mixed (quantitative/ordinal) data. Another feature is that the approach can be extended to rankings with degrees of difference.

The outcome of the approach is a distribution of performance scores. Stochastic dominance concepts are proposed to arrive at a final ranking of alternatives. An application of these procedures is given for a location study of nuclear power plants.

Keywords: multicriteria decision analysis, uncertainty, ordinal data, stochastic dominance.

1. Introduction

The complexity of many present day policy problems calls for a multi-dimensional analytical framework in order to capture a wide range of relevant aspects. Two problems can be mentioned in this respect. Firstly, the various aspects (criteria) to be taken into account are often difficult to compare: it is hard to arrive at quantitative figures to trade them off against each other. Secondly, for some relevant criteria it often proves to be difficult to obtain quantitative data on the impacts of policy alternatives.

As a result of these problems a need exists of multicriteria decision methods which can handle qualitative information on weights and criterion scores (cf Janssen et al., 1989, and Nijkamp et al., 1990). Among the methods which have been developed for this purpose are regime analysis (Hinloopen et al., 1983), QUALIFLEX (Paelinck, 1977) and a multidimensional scaling approach (Voogd, 1983). One of the problems with these methods is that they are not so easy to apply in the case of mixed data, i.e., when part of the criteria are quantitative and part are qualitative of nature. In some applications of these methods it even occurs that available quantitative data are 'downscaled' to ordinal data in order to be able to apply a qualitative multicriteria method (cf. Tweede Kamer, 1986). This is of course an unfortunate state of affairs which has lead to the development of special methods for mixed data. An example of this is the EVAMIX method (Voogd, 1983), which is based on concordance analysis.

In the present paper we will present a stochastic approach to ordinal data which is both applicable when all data are qualitative and when a mix of quantitative and qualitative data occurs. In this approach quantitative values for weights and/or criterion scores are generated which are consistent with underlying ordinal data. The approach is quite flexible since it can deal with various kinds of ranked data: usual ordinal data, ordinal data with degrees of difference, ties, and incomparable data. Thus, from the input side, the stochastic method bears a certain resemblance to Saaty's analytical hierarchy approach (Saaty, 1977). The stochastic approach entails the use of Monte Carlo procedures to generate the sets of possible outcomes for the alternatives. The concept of stochastic dominance is proposed to arrive at a final ranking of alternatives. The method is illustrated by means of an application to the location of nuclear power plants.

2. Ordinal data: a stochastic interpretation

As a starting point for the discussion of ordinal data in multicriteria decision making we take the case of ranked criteria. Suppose that criteria have been ranked in decreasing order of importance. Let λ_j denote the (unknown) quantitative value of the weight of criterion j $(j=1,..,J)$. Assume that the weights are non-negative and add up to 1. Then, the set of weights S which is consistent with the information on the ranking reads as follows:

$$S = [(\lambda_1, \ldots, \lambda_J) | 0 \leq \lambda_1 \leq \lambda_2 \ldots \leq \lambda_J \ ; \ \sum_j \lambda_j = 1] \qquad (1)$$

The set S is a convex polyhedron with J vertices. Thus, ordinal information on weights gives rise to a large set of possible quantitative values of the weights. The problem is how to make the set S tractable in the context of multicriteria analysis. In this paper we distinguish two approaches: the extreme value method and a stochastic approach.

The **Extreme value method** focusses on the vertices of the set S. For example, if there are three criteria, the weight combinations taken into consideration are: (0, 0, 1), (0, 1/2, 1/2), and (1/3, 1/3, 1/3), as also illustrated in Figure 1. Examples of this approach can be found in Paelinck (1977), Kmietovich and Pearman (1981) and Voogd (1983). As indicated in Janssen et al (1989), the extreme value method is easy to apply, but it has as a main disadvantage that the interior points of S are neglected.

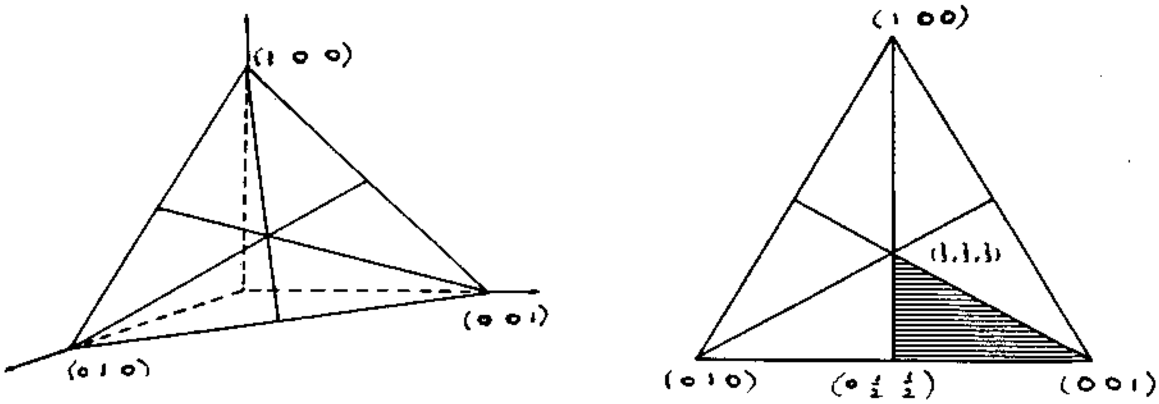


Fig. 1. Set of feasible weights in the case of 3 criteria.

In order to overcome this problem a <u>stochastic approach</u> is proposed. This is done by introducing the probability that a certain weights combination is the 'true' combination the decision maker has in mind. The probability distribution which is most easily to defend in the absence of further prior information is the uniform distribution: all elements in S are equally probable. This gives rise to the following distribution:

$$
\begin{aligned}
f(\lambda_1,\ldots,\lambda_{J-1}) = c \text{ if: } & 0 \le \lambda_1 \le 1/J \\
& \lambda_1 \le \lambda_2 \le 1/(J-1) - \lambda_1/(J-1) \\
& \\
& \lambda_{J-2} \le \lambda_{J-1} \le 1/2 - \lambda_1/2 - \ldots - \lambda_{J-2}/2 \qquad (2) \\
= 0 \text{ elsewhere} &
\end{aligned}
$$

where c can be shown to be equal to (J-1)!J! (Rietveld, 1989). Once the values of $\lambda_1,\ldots,\lambda_{J-1}$ are known, the value of λ_J can be found as: $1 - \lambda_1 - \ldots - \lambda_{J-1}$.

On the basis of this distribution one may proceed in two different directions. The first direction is that one focusses on the <u>expected values</u> of the weights, given distribution (2), as the best representation of the set S. As shown in Rietveld (1984, 1989) it is possible to derive the expected values (E) in an analytical way. The following results are found:

$$
\begin{aligned}
E(\lambda_1) &= 1/J^2 \\
E(\lambda_2) &= 1/J^2 + 1/[J(J-1)] \qquad (3) \\
&\vdots \\
E(\lambda_{J-1}) &= 1/J^2 + 1/[J(J-1)] + \ldots + 1/[J.2] \\
E(\lambda_J) &= 1/J^2 + 1/[J(J-1)] + \ldots + 1/[J.2] + 1/J.1
\end{aligned}
$$

In table 1, the outcomes of (3) are presented for some selected values of J. The table clearly reveals that this approach gives rise to a cardinalization which is different from the usual 'naive' approach to ordinal numbers. The naive approach - interpreting rank numbers as if they were cardinal - would, for example, in the case of J=3 amount to cardinal weights equal to 1/6, 2/6, 3/6, a result different from that in Table 1.

A disadvantage of the expected value method is that only one interior point of S is generated. If one wants to investigate more elements of S one may proceed in another direction: generation of a <u>random sample</u> of weight combinations on the basis of (2).

number of criteria	expected values $E(\lambda_1)$	$E(\lambda_2)$	$E(\lambda_3)$	$E(\lambda_4)$	$E(\lambda_5)$	$E(\lambda_6)$
2	.25	.75				
3	.11	.28	.61			
4	.06	.15	.27	.46		
5	.04	.09	.16	.26	.46	
6	.03	.06	.10	.16	.24	.41

Table 1 Expected values of ranked weights for various numbers of criteria.

It may be tempting to draw random combinations of weights by the following approach. Draw J numbers a_j $(j=1,\ldots,J)$ from a uniform distribution on the interval [0,1] and define λ_1 as the smallest value of a_j divided by $\Sigma\ a_j$. This does not lead to the uniform distribution defined in (2), however.

An operational approach to generate a random sample of weight combinations consistent with (2) is the following (see Rietveld, 1988). The approach consists of two steps. In the first step the marginal distribution of λ_1, the conditional distribution of λ_2 given λ_1, etc. are derived. In Appendix 1 it is shown how these distributions $f(\lambda_1)$, $f(\lambda_2|\lambda_1)$, $f(\lambda_3|\lambda_1,\lambda_2)$, etc. can be obtained in an analytical way on the basis of (2).

In the second step a random generator is used to draw subsequently a value of λ_1, based on $f(\lambda_1)$, a value of λ_2 based on $f(\lambda_2|\lambda_1)$, etc. In Appendix 1 it is indicated that no standard random generators exist to do this job because of the special forms the conditional distribution functions assume. It can be shown, however, that a standard random generator can be used after an appropriate transformation of the weights.

The above approach holds true for ordinal information on <u>weights</u>. In the case of ordinal information on <u>criterion scores</u> a similar approach can be followed. There is a difference, however, since weights are usually standardized such that they add up to 1, whereas for criterion scores other standardizations are used. A common way of standardizing criterion scores is to divide all elements by the highest attainable value. In this case, the highest standardized value is 1. Further, assume that alternatives are ranked in increasing order of attractiveness according a certain criterion

j, and that all criterion scores are non-negative. Then the set T_j of combinations of criterion scores which are consistent with the ordinal information reads:

$$T_j = \{(p_{j1},\dots,p_{jI}) \mid 0 \le p_{j1} \le p_{j2} \le \dots \le p_{jI} = 1\} \qquad (4)$$

where p_{ji} is the score of alternative i ($i=1,\dots I$) according to criterion j. For the convenience of notation we will drop the subscript j where possible. T is a convex polyhedral set. In the case of $I=3$, T assumes the form of a triangle with extreme points (0,0,1), (0,1,1) and (1,1,1). When we assume along the same lines as above that the p_i's are uniformly distributed on T, the following probability density function results:

$$g(p_1,\dots,p_{I-1}) = (I-1)! \text{ if } \begin{array}{l} 0 \le p_1 \le 1 \\ p_1 \le p_2 \le 1 \\ \vdots \\ p_{I-2} \le p_{I-1} \le 1 \end{array} \qquad (5)$$

$$= 0 \qquad \text{elsewhere}$$

The expected value approach can be shown to lead to the following results if applied to (5):

$$E(p_i) = i/I \qquad i=1,\dots,I.$$

Appendix I contains a description of the procedure for generating random combinations of criterion scores which are consistent with (5).

3. Rankings with degrees of difference

Consider ordinal information such as $x_1 \le x_2$ and $x_2 \le x_3$. Hitherto we have assumed that the degree of difference between x_1 and x_2 is equal to that between x_2 and x_3. In certain cases, decision-makers or analysts may be able to express their opinions in terms of rankings with varying degrees of difference. For example: x_1 is smaller than x_2, which in turn is considerably smaller than x_3. Information of this type is used in the analytical hierarchy process developed by Saaty (1977).

We will show that it is possible to develop the stochastic approach of section 2 in such a way that it can deal with rankings of weights or criterion scores with varying degrees of difference. For this purpose the following notation will be used:

$x \leq_m y$ x is smaller than y according to degree m

where m = 1,2,3,.... In our stochastic approach variations in the degree of difference are taken into account by introducing auxiliary variables. For example, when $x \leq_2 y$, an auxiliary variable b is added such that $x \leq b \leq y$. Similarly when $x \leq_3 y$, two auxiliary variables b and c are added such that $x \leq b \leq c \leq y$.

Consider the case of ranked information on criterion scores. The following notation will be used:

$$0 \leq_{m_1} p_1 \leq_{m_2} p_2 \ldots \leq_{m_I} p_I = 1 \tag{6}$$

where m_i is the degree of difference between p_{i-1} and p_i for $i=1,\ldots,I$. Further, let q_{ik} denote the k'th auxiliary variable between p_{i-1} and p_i $(k=1,\ldots,m_i-1)$. For notational convenience we set $q_{im_i}=p_i$. Then (6) is equivalent to:

$$0 \leq q_{11} \leq \ldots q_{1m_1} \leq q_{21} \leq \ldots \leq q_{2m_2} \leq \ldots\ldots \leq q_{Im_I} = 1 \tag{7}$$

For the stochastic aproach this means that instead of the original I variables now $\sum_i m_i$ variables have to be generated. The modified probability density function reads

$$g(q_{11},\ldots\ldots,q_{I,m_I-1}) = (\sum_i m_i - 1)! \tag{8}$$

for all q_{ik} satisfying (7).

Of the generated values only the $q_{im_i}(=p_i)$ are used in the further analysis. The structures of (8) and (5) are very similar; if $m_i = 1$ for all i, (8) and (5) coincide. A high degree of difference between subsequent criteria scores can be shown to lead to a small variance of the p_i. For expected values another result can be proved: if the degree of difference is the same for all subsequent criterion scores $(m_1 = m_2 = \ldots = m_I = N)$, the expected values of the criterion scores do not depend on N.

For weights, the introduction of varying degrees of differences leads to a more complex adjustment of the original formulations. Using the same notation as above, our point of departure is:

$$0 \le_{m_1} \lambda_1 \le_{m_2} \lambda_2 \le \ldots \le_{m_J} \lambda_J$$

$$\sum_j \lambda_j = 1 \qquad (9)$$

We introduce auxiliary variables μ_{jk} ($k=1,\ldots,m_j-1$) satisfying:

$$0 \le \mu_{11} \le \mu_{12} \ldots \le \mu_{1m_1} \le \mu_{21} \ldots \le \mu_{2m_2} \le \ldots \le \mu_{Jm_J}$$

$$\sum_j \mu_{jm_j} = 1 \qquad (10)$$

where λ_j is denoted as μ_{jm_j} for national convenience.

It can be shown that the probability density function f is equal to:

$$f(\mu_{11},\ldots,\mu_{J,m_J-1}) = (\sum_{j=1}^{J} m_j - 1)! \prod_{j=1}^{J-1} (J+1-j)^{m_j} \qquad (11)$$

for all μ_{jk} satisfying (10). Note that when $m_j = 1$ for all weights, (11) coincides with (2). In Appendix 1 it is shown how random samples of weights can be generated on the basis of (11).

4. Ties and incomplete rankings

Ties deserve special attention in ordinal data; the probability that ties occur is large in the case of a large number of observations. Consider a ranking such as $\lambda_1 \le (\lambda_2, \lambda_3) \le \lambda_4$. This may have different interpretations:

- λ_2 and λ_3 are exactly equal
- λ_2 and λ_3 are approximately equal
- λ_2 and λ_3 are incomparable: λ_2 may be both larger and smaller than λ_3, and the difference between the two is not necessarily small.

Each of these cases deserves its own treatment in the stochastic approach outlined above.

When observations are <u>exactly equal</u>, one only needs to draw one random value which is assigned to all observations concerned. An inspection of (8) and (11) reveals that this can be done in a consistent way by interpreting an exact equality as $<_0$ (i.e., $m_j = 0$ in such a case). Thus, there is no need to design special procedures to deal with exact equality: one can still use the formulas derived in Appendix 1[1].

In the case of incomparable observations (an incomplete ranking) one can still use the stochastic approach. Consider for example a cluster of incomparable observations consisting of λ_2,λ_3 and λ_4. Then random numbers $a \leq b \leq c$ are generated which are assigned to λ_2,λ_3 and λ_4 in a random way. Thus, in one case λ_2 may be assigned the largest value (c), and in another case the smallest value.

In the case of approximately equal observations, one may proceed as follows. In a first step, a value is generated for these observations as if they are exactly equal (along the lines sketched above). In the second step, the observations are assumed to be uniformly distributed in an appropriately defined interval around this value. Consider for example three clusters: $\{p_1\},\{p_2,p_3\}$ and $\{p_4,p_5\}$ with the following features: $p_1 \leq p_2 \approx p_3 \leq p_4 = p_5$, where $p_2 \approx p_3$ means that p_2 and p_3 are approximately equal. The standard stochastic approach leads to values a_1, a_2 and a_3 for the three clusters. Then in the last step, values for p_2 and p_3 are drawn from a uniform distribution on the interval $[\frac{1}{2}\ a_1 + \frac{1}{2}\ a_2,\ \frac{1}{2}\ a_2 + \frac{1}{2}\ a_3]$. This approach can also be followed for weights, but in that case an additional condition ($p_2+p_3 = 2a_2$) has to be imposed to ensure that the additivity constraint on the weights is satisfied.

We conclude that the stochastic approach outlined above is quite flexible. It can deal with all kinds of ordinal data on weights criterion scores:

- standard ordinal data
- ordinal data with degrees of difference
- ties of exactly equal observations
- ties of approximately equal observations
- incomparable observations.

The question remains how this stochastic approach can be used to generate a final ranking of alternatives. This will be the subject of the next section.

5. A stochastic dominance approach to multicriteria decision making.

The stochastic approach leads to a large set of cardinal values for weights and criterion scores which are consistent with ordinal data. Assuming that the uniformity assumption on which these values are based is appropriate, the stochastic approach gives an adequate representation of the distribution of the weights and criterion scores.

The cardinal values obtained in this way can be used as an input to various multicriteria decision methods such as ELECTRE (Roy, 1968) or utility based approaches. For each combination of values drawn another run has to be made of the multicriteria decision method. The question arises how the outcomes of all these runs can be summarized to arrive at a final ranking of alternatives. In the present paper we will show that stochastic dominance is a promising tool if a weighted summation utility structure is used.

Stochastic dominance is a concept to analyze decision making under risk (e.g., portfolio analysis as described by Whitmore and Findlay, 1978). Consider the distribution functions of the outcome x of two alternatives i and k: $F_i(x)$ and $F_k(x)$. If one wants to select the most attractive alternative, one may use the mean value as a criterion. If one is risk averse, the variance can be used as a second criterion, which leads to mean-variance analysis (Markowitz, 1987). However, as noted by Fishburn and Vickson (1978) mean-variance analysis has certain defects so that other approaches have to be considered.

Stochastic dominance is addressed to the following question. Let the mean value of x for alternative i be larger than for alternative k: $E(X,F_i) > E(,F_k)$. Does this result still hold true if a transformation is applied to x? For example: $v(x) = \ln x$. To answer this question, two classes of utility functions will be distinguished[2)] (V_1 and V_2):

- V_1 is the set of all utility functions v(x) which are continuous, bounded and increasing
- V_2 is the set of all utility functions v(x) which are continuous, bounded, increasing and concave.

Clearly, V_1 is a quite general set of utility functions. The concavity property added in V_2 means that V_2 is the set of risk averse utility functions.

We will use the symbols SD_1 for first degree stochastic dominance and SD_2 for second degree stochastic dominance. Then stochastic dominance of the first degree is defined as:

- $F_i\ SD_1\ F_k$ if and only if $E\ [v(x),F_i(x)] > E\ [v(x),F_k(x)]$

for all v in V_1

Similarly, stochastic dominance of the second degree is defined as:

- $F_i\ SD_2\ F_k$ if and only if $E\ [v(x), F_i(x)] > E\ [v(x), F_k(x)]$

  for all v in V_2

Thus, in the case of first degree stochastic dominance, the mean of distribution i is larger than that of distribution k for a quite general class of utility functions. In the case of second degree stochastic dominance the same result is obtained for the class of risk averse utility functions. Stochastic dominance can be shown to be transitive: F_i SD F_k and F_k SD F_ℓ imply F_i SD F_ℓ for both degree 1 and 2. Further, it can be shown that $F_i\ SD_1\ F_k$ implies $F_i\ SD_2\ F_k$.

Assume that x takes on values on the interval $I = [0,\infty]$. Then the following theorems can be proved (Fishburn and Vickson, 1978):

$$F_i\ SD_1\ F_k \text{ if and only if } F_k(x) \geq F_i(x) \text{ for all } x \text{ in } I$$

$$F_i\ SD_2\ F_k \text{ if and only if } \int_0^x F_k(y)d_y \geq \int_0^x F_i(y)d_y \text{ for all } x \text{ in } I.$$

These theorems are illustrated by means of Figure 2. In Fig. 2(a) the distribution functions do not cross: alternative i stochastically dominates alternative k to the first degree. Hence, stochastic dominance of the second degree also applies. In Fig. 2 (b), the distribution functions do cross. Hence, stochastic dominance of the first degree does not occur. In this case alternative i is stochastically dominant of the second degree with respect to alternative k since the size of area A is larger than of area B. Finally, in Fig. 2(c) stochastic dominance of neither degree 1 nor 2 occurs since the size of area A is smaller than of area B.

In most applications of stochastic dominance x refers to an uncertain monetary variable. In the present multicriteria decision context, x is defined as a weighted summation performance indicator:

$$x_i = \sum_j \lambda_j\ p_{ji} \tag{12}$$

where the stochastic background of x_i has been explained above. Thus by using a stochastic dominance approach one can address the question whether or not a ranking of alternatives on the basis of the mean value of x_i is sensitive to transformations of the functional form of x_i such as for example $x_i = (\sum_j \lambda_j\ p_{ji})^2$.

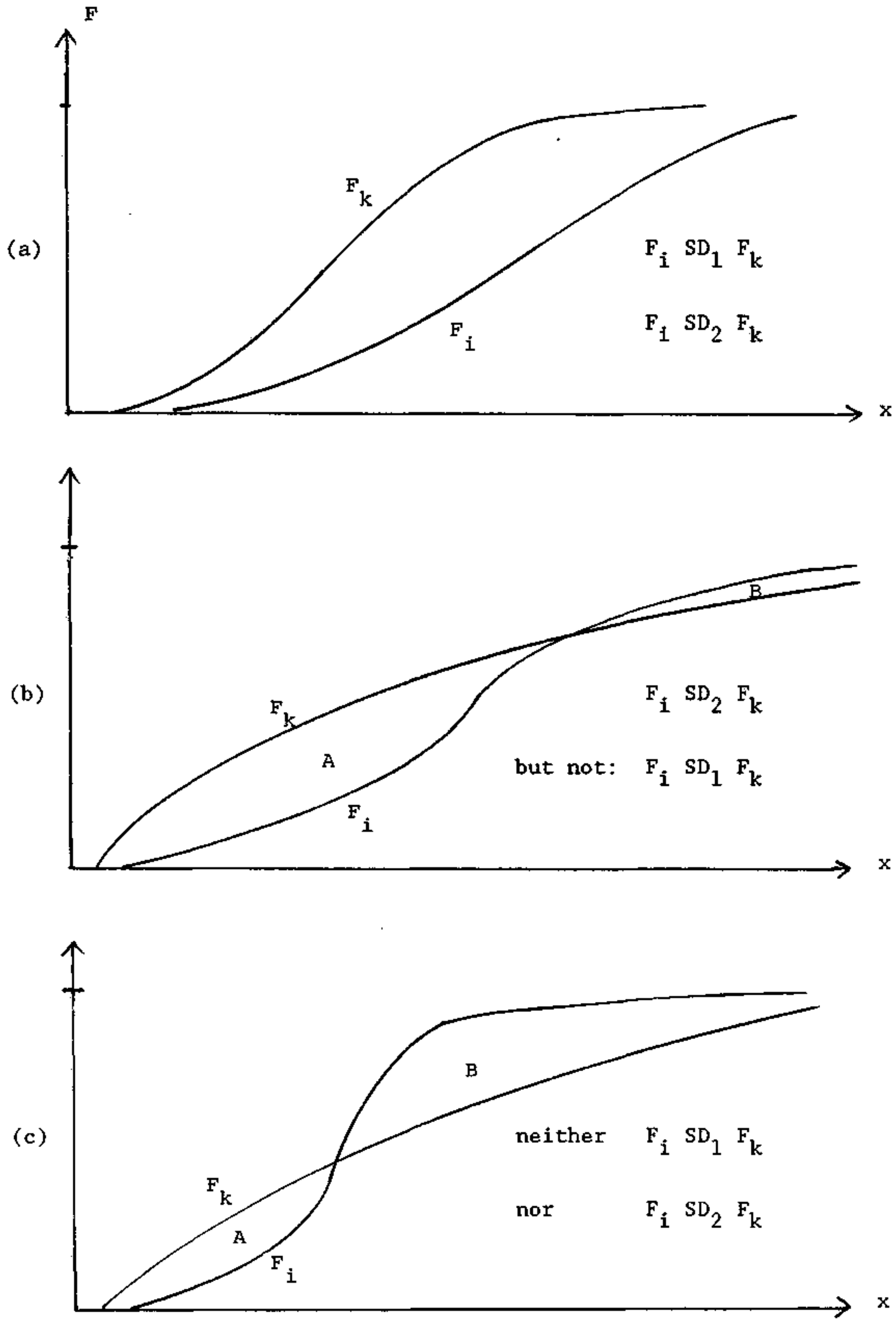
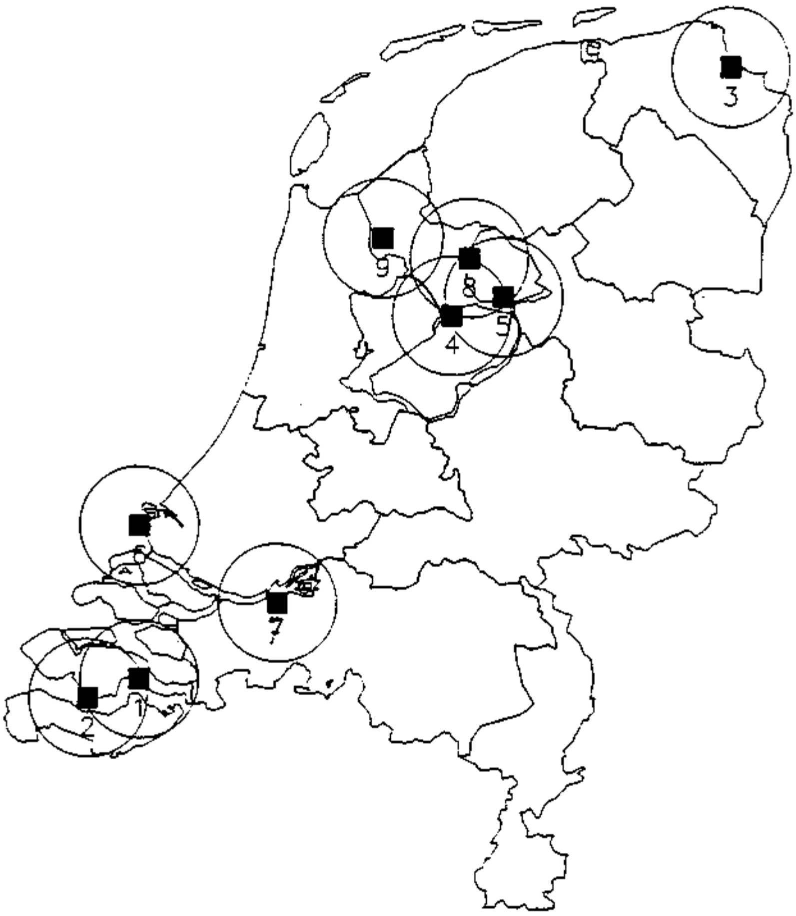


Fig. 2. Examples of stochastic dominance relationships.

6. Application: the siting of nuclear power plants in the Netherlands

Compared with other European countries, the share of nuclear power in total power production is very small in the Netherlands. After a long public debate in the beginning of the 1980's, the Dutch government expressed in 1985 the intention to build two new nuclear power plants with a capacity of 1000 MWe each. As a first step towards implementation a locational study was carried out. After some initial scoping, nine potential locations have been selected (see Figure 3).



0 potential site

0 20 km range around the potential site

1 Bath/Hoedekenskerke
2 Borssele
3 Eems
4 Flevo Noord
5 Ketelmeer
6 Maasvlakte
7 Moerdijk
8 West. NOP-dijk
9 Wieringermeer

Figure 3. Potential locations for nuclear power plants.

The Dutch national advisory council for physical planning RARO has carried out a multicriteria analysis of the site selection problem (Tweede Kamer, 1986). Fifteen criteria were formulated, only one of which measured in quantitative terms (population at risk); the remaining 14 criteria were measured in an ordinal way (see Table 2). The ordinal data are presented in increasing order, i.e., the higher a score the more favourable the performance of an alternative. Appendix II gives a more precise definition of the criteria. Table 2 shows that there are many ties and that in this case the degree of difference between the ordinal criterion scores is the same.

The RARO council has formulated the following ranking of weights to be attached to the evaluation criteria:

$$(\lambda_{10},\lambda_2) \leq (\lambda_8,\ \lambda_{11},\ \lambda_{12},\ \lambda_{13},\ \lambda_{14}) \leq (\lambda_6,\ \lambda_7,\ \lambda_9,\ \lambda_{15}) \leq (\lambda_3,\ \lambda_5) \leq \lambda_4 \leq \lambda_1$$

Thus, population at risk receives the highest weight, followed by industry at risk, which in its turn is followed by agriculture at risk and fresh water at risk (ex equo). Here again a considerable number of ties can be observed.

	Bath	Borssele	Eems	Flevo	Ketel	Maasvlak	Moerdijk	NO Polder	Wiering
Population	51	49	16	27	30	43	100	19	21
Evacuation	2	1	2	2	2	1	2	2	2
Agriculture at risk	2	2	2	2	2	3	1	2	2
Industry at risk	5	2	3	4	5	1	3	5	5
Fresh water at risk	2	2	2	1	1	2	1	1	1
Cool-water quantity	2	3	3	3	3	3	1	3	3
Cool-water quality	2	3	2	1	1	3	2	1	1
Air pollution	1	1	1	1	1	2	1	1	1
Thermal pollution	1	2	2	2	1	3	2	1	1
Indirect land use	3	2	2	3	4	3	1	4	4
Landscape	1	3	3	3	1	3	2	1	1
National environment	1	3	1	3	2	3	3	3	3
National grid	2	2	1	3	3	2	2	2	1
Infrastructure	1	2	2	1	1	2	2	1	1
Coal location	5	2	4	5	6	1	3	7	7

Table 2. Impact matrix.
(Source: Tweede Kamer 1986, pages 43-44)

We use the procedures described in sections 2-4 to transform the impact matrix in Table 2 into a score matrix P where p_{ji} denotes the cardinal score of alternative i on criterion j. Also a weight vector λ is calculated where λ_j denotes the weight of criterion j. Further a linear additive utility function is adopted to calculate the performance x_i of each alternative by means of (12).

By carrying out these calculations n times one finds empirically the cumulative distribution function (c.d.f) of x_i based on n drawings[3)].

As already mentioned in section 4, ties can be interpreted in various ways. First we discuss the results when all ties are interpreted as "exactly equal". Table 3 shows some descriptive statistics of the c.d.f of each alternative.

	Alternative	Mean	St. dev.
1	Bath	.593	.071
2	Borssele	.555	.086
3	Eems	.809	.064
4	Flevo	.669	.062
5	Ketel	.655	.053
6	Maasvlak	.589	.090
7	Moerdijk	.419	.093
8	NO Polder	.771	.057
9	Wiering	.735	.054

Table 3 Mean and standard deviation of 9 alternatives
(n = 1000)

Looking just at the means suggests that four different clusters can be distinguished. The cluster with the most attractive alternatives consists of {3, 8, 9}. The other clusters are {4,5}, {1,2,6} and {7}.

When a decision-maker is risk averse, he prefers the alternative with the lower standard deviation if the means are equal. Further, when the means are not equal, the alternative with the lowest mean is said to be dominated if it has a larger variance than the alternative with the highest mean. In the case of Table 3, this would imply that the alternatives 3, 5, 8 and 9 are non-dominated.

Next we consider the complete c.d.f. by using the concept of stochastic dominance (SD). The results are summarized in Table 4. The upper-triangular part of the table (above the main diagonal) contains information about the first order stochastic dominance (see section 5). The lower triangular part of the table is dealing with second order stochastic dominance. Whenever there is a '+' this means that the row-alternative is stochastically dominating the column variable, whereas a '-' means the reverse. Undecided relationships have been represented by a dot '.'. Remember that SD_1 implies SD_2 but not vice versa. This is reflected by the observation that a '+' in the upper right corner implies a '-' in the lower left corner but not vice versa. Thus the '+' for the SD_1 relation between 2 and 7 implies a '-' for the SD_2 relation between these two alternatives, but the '-' for the SD_2 relation of 2 and 1 has no implication for their SD_1 relation. The alternatives are rearranged so as to reach a table with as many '+''s as possible in the upper right corner and as many '-''s as possible in the lower left corner. Note that it is always possible to rearrange the alternatives in such a way that there is no '-' in the upper right corner, nor any '+' in the lower left corner. This is because of the transitivity of the SD relations.

Alternative	3	8	9	4	5	1	6	2	7
3		.	.	+	+	+	+	+	+
8	.		+	+	+	+	+	+	+
9	.	-		+	+	+	+	+	+
4	-	-	-		.	+	+	+	+
5	-	-	-	.		+	+	+	+
1	-	-	-	-	-		.	.	+
6	-	-	-	-	-	-		.	+
2	-	-	-	-	-	-	.		+
7	-	-	-	-	-	-	-	-	

Table 4 Stochastic dominance relationships (upper right corner: SD_1; lower left corner SD_2)

Table 4 displays a relatively large number of '+''s and '-''s. This means that for the given example SD is a powerful decision criterion. Next we see that there are six relations undecided in the SD_1 for two of which there exists an SD_2 relation (the pairs 1,6 and 1,2).

The conclusions reached with mean-variance dominance are not entirely consistent with stochastic dominance. For example, alternative 9 is stochastically dominated by alternative 8, whereas with mean-variance analysis this dominance relationship does not apply.

The methods presented thus far do not take into account the correlation between the alternatives. No attention is given to the relative rankings of the alternatives each time a score vector is calculated.Therefore we constructed Tables 5 and 6. Table 5 contains information on how often a certain alternative gets a particular place in the ranking of alternatives. Thus in 76% of all cases alternative 3 scores best, and in 13% it is second best. Alternative 7 is worst in 98% of all cases. Table 6 looks at the scores of the alternatives in still another way. This table relates to the number of times one alternative scores better than the other. Thus alternative 3 has a higher score than 8 for 79% of the times and alternative 9 is in 10% of the cases better than 3.

The sequence in which the alternatives are presented in Table 6 is based on the mean performance values presented in Table 3. All elements in the upper right part of Table 6 are higher than 50%. This means that in this example the probability that any alternative performs better than another alternative with a lower mean value is higher than 50%. Such an outcome does not necessarily hold true in all cases; it shows that in the present case a detailed analysis of pairwise relationships between alternatives does not add much information compared with the outcomes presented in Tables 3 and 4.

Alternative	Rank: 1	2	3	4	5	6	7	8	9
3	76	13	8	2	1	0	0	0	0
8	20	71	7	2	1	0	0	0	0
9	0	9	73	9	5	2	1	0	0
4	1	3	5	53	26	7	4	1	0
5	0	0	1	22	50	15	8	4	0
1	0	0	1	4	7	43	21	23	0
6	3	4	4	7	8	23	35	15	1
2	0	0	1	2	3	9	30	55	1
7	0	0	0	0	0	0	1	2	98

Table 5 Probability matrix that alternative i gets position n in final ranking, for all i and n.

Alternative	3	8	9	4	5	1	6	2	7
3		79	90	98	99	100	97	100	100
8	21		100	96	100	100	93	98	100
9	10	0		89	99	98	89	96	100
4	2	4	11		71	89	81	95	100
5	1	0	1	30		88	73	89	100
1	0	1	2	11	12		51	72	100
6	3	8	12	19	27	49		82	99
2	0	2	4	5	11	28	18		99
7	0	0	0	0	0	0	1	1	

Table 6 Pairwise comparison matrix with probability that alternative i performs better than i'.

All results discussed so far stem from an 'equal to' interpretation of ties. Given the great number of ties in the impact matrix this means that there is little source of variation between the alternatives. This might explain the quite strong results of the SD. This point may be best illustrated by looking at the impact scores of alternatives 8 and 9 (Table 2). Both alternatives have the same impact on all but two alternatives both of which show a more favourable impact for alternative 8. Thus while we interpret a tie in terms of 'equality' 8 is definitely judged the better. The difference is small but inevitable.

The results of alternative interpretations of ties are given in Appendix III and IV. It appears that the difference between an interpretation of ties in terms of 'exactly equal' and 'approximately equal' observations is rather small in this case. However, the outcomes with the 'incomparability' interpretation are rather different. As shown in Table IV.1 the uncertainty intervals of the performance of alternatives are strongly overlapping. This implies that the position the alternatives may get in the final ranking varies strongly (cf. Table IV.3). Nevertheless, even in this case the number of undecided outcomes according to stochastic dominance is limited.

7. Concluding remarks

We have developed a multicriteria method with the following features.

1. The method can deal with ordinal information on criterion scores, weights and the combination of both.
2. The method can also be used in the case of mixed (ordinal/cardinal) data.
3. The method can be applied in the case of ordinal data with degrees of difference.
4. The method can deal with various types of ties.

The method leads to random distributions of performance indicators of the alternatives. In the present paper we propose to analyse these distributions by means of the stochastic dominance concept, although also other approaches might be adopted to derive a ranking of alternatives.

This paper contains an application of the method to the problem of siting nuclear power plants. It should be noted, however, that the stochastic interpretation of ordinal data developed in this paper can also be used in contexts to outside multicrtiteria analysis, for example: decision making under uncertainty (cf. Rietveld, 1989). Also outside the realm of decision making the stochastic approach is of potential relevance, for example in the case of economic modeling with qualitative information on parameters. We refer to Nijkamp et al. (1990) for an example in the field of qualitative input-output analysis.

Notes

1) For example, in the case of $0 \leq_2 \lambda_1 \leq_1 \lambda_2 = \lambda_3 \leq_1 \lambda_4$, one has $m_1=2$, $m_2=1$, $m_3=0$, $m_4=1$ and the following random numbers are drawn: $0 \leq \mu_{11} \leq \mu_{12} \leq \mu_{21} \leq \mu_{41}$. The values assigned to the weights λ_1, λ_2, λ_3, λ_4 are: μ_{12}, μ_{21}, μ_{21}, μ_{41}.

2) For precise definitions refer to Fishburn and Vickson (1978).

3) For the first criterion (population) a stochastic approach is not necessary because quantitative values are available. In order to arrive at a standardisation for this criterion which is consistent with the scaling of the random values of the other criteria, the following transformation is used: $s_{1i} = \min_{i'} (p_{1i'})/p_{1i}$.

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APPENDIX 1. Generating random values for weights and criterion scores

1. Generating weights

The starting point is the joint density function of weights:

$$f(\lambda_1,\ldots,\lambda_{J-1}) = \qquad \begin{array}{l} 0 \leq \lambda_1 \leq 1/J \\ \lambda_1 \leq \lambda_2 \leq (1-\lambda_1)/(J-1) \\ \lambda_2 \leq \lambda_3 \leq (1-\lambda_1-\lambda_2/(J-2) \\ \quad \vdots \\ \lambda_{J-2} \leq \lambda_{J-1} \leq 1-\lambda_1,-\ldots-\lambda_{J-2})/2 \end{array} \qquad (2)$$

$$= 0 \text{ elsewhere}$$

where $c = (J-1)!J!$

Then the marginal density function of λ_1 can be derived as:

$$\begin{array}{ll} f(\lambda_1)=(J-1)J(1-J\lambda_1)^{J-2} & \text{for } 0\leq \lambda_1 \leq (1/J) \\ = 0 & \text{elsewhere} \end{array}$$

Further, the conditional density functions can be shown to read as follows for $j = 2,\ldots,J-1$

$$f(\lambda_j|\lambda_1,\ldots,\lambda_{J-1})$$

$$= (J-j)(J-j+1)\,\frac{[1-\lambda_1-\ldots-\lambda_{j-1}\quad-(J-j+1)\lambda_j]^{J-j-1}}{[1-\lambda_1-\ldots-\;(J-j+2)\lambda_{j-1}]^{J-j}}$$

where $\lambda_{J-1} \leq \lambda_J \leq (1-\lambda_1 - \ldots - \lambda_{J-1})/(J-j+1)$

Then, a random weight vector can be generated by drawing a value for λ_1 on the basis of $f(\lambda_1)$, followed by drawing a value for λ_2 on the basis of $f(\lambda_2|\lambda_1)$, etc. Finally, λ_J can be computer as $1-\lambda_1 \ldots - \lambda_{J-1}$.

The conditional distributions mentioned above are not included in standard statistical packages. Therefore, random weight vectors cannot be directly created by means of random generators. A solution for this problem is given by the theorem which says that if F(x) is the distribution function of x, then $u=F(x)$ is uniformly distributed on the interval $0 \leq u \leq 1$ (Hogg and Craig, 1970, p. 349). For the latter uniform distribution, standard random generatiors are available. Then, if u_1 is uniformly distributed on the interval $[0,1]$, $\lambda_1=F^{-1}(u_1)$ can be shown to be distributed according to the density function ($f(\lambda_1)$ corresponding with the distribution function $F(\lambda_1)$. Thus, random values for λ_1 can be found by using the following transformation:

$$\lambda_1 = \frac{1}{J}\,[1-(1-u_1)^{1/(J-1)}]$$

For $\lambda_2,\ldots,\lambda_{J-1}$ the following transformation has to be used:

$$\lambda_j = [(1-\lambda_1-\ldots-\lambda_{j-1})-(1-\lambda_1-\ldots-\lambda_{j-2}-(J-j+2)\lambda_{j-1})(1-u_j)^{1/(J-j)}]/(J-j+1)$$

Finally, λ_J can be computed as $1-\lambda_1-\ldots-\lambda_{J-1}$.

2. Generating criterion scores

As indicated by Mood and Graybill (1963), it is not difficult to generate random values for $p_1,\ldots,p_{I-1}$. They show by means of order statistics that one can start with drawing I-1 numbers from the uniform distribution on $[0,1]$, after which p_1 is assigned the smallest number, p_2 the one but smallest number, etc. An alternative approach would be to follow the procedure described above for weights after the necessary adjustments. Taking (5) as a starting point, it can be shown that:

$$g(p_1) = (I-1)(1-p_1)^{I-2} \qquad , \qquad 0 \leq p_1 \leq 1,$$

and

$$g(p_i|p_1,\ldots,p_{i-1})=(I-i)(1-p_i)^{I-1-i}/(1-p_{i-1})^{I-i}$$

where

$$p_{i-1} \leq p_i \leq 1 \qquad , \qquad i = 2,\ldots,I-1.$$

Let again u_i denote a number drawn from the uniform distribution on the interval $[0,1]$. Then it can be shown that the following transformation has to be used to generate random values of x_1:

$$p_1 = 1 - (1-u_1)^{\frac{1}{I-1}}$$

For $i=2,\ldots,$ I-1 the following transformation has to be used:

$$p_i = 1 - (1-p_{i-1})\,(1-u_i)^{\frac{1}{I-i}}$$

3. Generating weights in the case of rankings with degrees of difference.

Consider the probability density function (11) under the constraints in (10). Let M be equal to $\sum_j m_j$. Then the conditional distribution of λ_j ($=\mu_{jm_j}$) reads:

$$f(\lambda_j | \lambda_1, \ldots, \lambda_{j-1}, \mu_{j,m_j-1}) =$$

$$(M-\sum_{k=1}^{j} m_k)(J+1-j)\,\frac{[1-\sum_{k=1}^{j-1}\lambda_k-(J+1-j)\lambda_j]^{M-\sum_{k=1}^{j} m_k-1}}{[1-\sum_{k=1}^{j-1}\lambda_k-(J+1-j)\mu_{j,m_j-1}]^{M-\sum_{k=1}^{j} m_k}}$$

where $\mu_{j,m_j-1} \le \lambda_j \le (1-\lambda_1- \ldots -\lambda_{j-1})/(J-j+1)$

For the distribution of μ_{j,m_j-1} a similar formula can be derived.

Let v denote a random number drawn from the uniform distribution on the interval [0,1]. Then it can be shown that the following distribution has to be used to generate random values for λ_j:

$$\lambda_j = [1-\sum_{k=1}^{j-1}\lambda_k - (1-\sum_{k=1}^{j-1}\lambda_k - (J+1-j)\mu_{j,m_j-1}).(1-v)^{1/(M-\sum_{k=1}^{j} m_k)}]/(J+1-j)$$

APPENDIX 2

Definition of evaluation criteria

Population	To calculate this score a weighted sum of population around a location was calculated. The weight designed decreases with distance. The result is standardized by division through the maximum score. The criterion is a cost criterion.
Evacuation	The score reflects the availability of sufficient transport infrastructure.
Industry/Agriculture at risk	This score reflects the size and importance of industry/agriculture near the location.
Fresh water at risk	This score reflects the quantity of fresh water that may be affected by a nuclear plant at each location.
Cool-water quantity	This score represents the quantity of available water for cooling the nuclear plant.
Cool-water quality	This score represents the capacity of coolant to flush out pollution originating from a nuclear plant at each location.
Air pollution	It is assumed that the nuclear plant is an alternative to a conventional coal power plant. This is assumed to have the most beneficial effects at the most polluted location.
Thermal pollution	The amount of pollution is lower if users of the heat generated are available. The score reflects the availability of such users.
Indirect land-use	This score reflects limitations on the potential land-uses around nuclear plant.
Landscape	This score reflects the visual effects the landscape round the landscape and the extent to which a nuclear plant fits in with existing activities.
Natural environment	This score reflects expected damage to the natural environment
National grid	This score reflects the availability near of at the location of high voltage lines and connector stations.
Infrastructure	This score reflects the availability of transport and other infrastructure around the location.
Coal-location	It is assumed that the nuclear plant is an alternative to a conventional coal power plant. The score reflects the cost of the lost opportunity to build a coal plant at the location if a nuclear plant is constructed.

APPENDIX III Evaluation results if ties are interpreted as approximately equal outcomes

	Alternative	Mean	St. dev.
1	Bath	.550	.081
2	Borssele	.512	.114
3	Eems	.786	.069
4	Flevo	.629	.064
5	Ketel	.610	.052
6	Maasvlak	.540	.104
7	Moerdijk	.382	.112
8	NO Polder	.729	.065
9	Wiering	.686	.059

Table III.1 Mean and standard deviation of 9 alternatives (n = 1000)

Alternative	3	8	9	4	5	1	6	2	7
3		+	+	+	+	+	+	+	+
8	-		+	+	+	+	+	+	+
9	-	-		+	+	+	+	+	+
4	-	-	-		.	+	+	+	+
5	-	-	-	.		+	.	+	+
1	-	-	-	-	-		.	.	+
6	-	-	-	-	-	-		+	+
2	-	-	-	-	-	-	-		+
7	-	-	-	-	-	-	-	-	

Table III.2 Stochastic dominance relationships (upper right corner: SD_1; lower left corner SD_2)

Rank:

Alternative	1	2	3	4	5	6	7	8	9
3	86	10	3	2	0	0	0	0	0
8	11	66	9	6	4	2	1	0	0
9	0	5	67	9	6	5	5	3	0
4	0	7	9	48	24	8	3	1	0
5	1	2	3	22	48	8	8	9	0
1	0	1	2	4	8	45	22	18	0
6	2	6	4	6	7	23	44	9	1
2	0	4	4	4	4	9	15	59	0
7	0	0	0	0	0	0	1	1	99

Table III.3 Probability matrix that alternative i gets position n in final ranking, for all i and n.

Alternative	3	8	9	4	5	1	6	2	7
3		88	97	99	98	99	98	99	100
8	12		94	86	96	97	88	90	100
9	3	6		77	88	91	82	85	100
4	1	4	23		72	89	83	89	100
5	2	4	12	28		81	74	80	100
1	1	3	10	11	19		54	68	100
6	2	12	18	17	26	46		79	99
2	1	10	15	11	20	32	21		99
7	0	0	0	0	0	0	1	1	

Table III.4 Pairwise comparison matrix with probability that alternative i performs better than i'.

APPENDIX IV Evaluation results if ties are interpreted as incomparable outcomes

	Alternative	Mean	St. dev.
1	Bath	.505	.066
2	Borssele	.522	.076
3	Eems	.672	.065
4	Flevo	.562	.066
5	Ketel	.545	.063
6	Maasvlak	.572	.058
7	Moerdijk	.322	.062
8	NO Polder	.599	.067
9	Wiering	.535	.070

Table IV.1 Mean and standard deviation of 9 alternatives (n = 1000)

Alternative	3	8	6	4	5	9	2	1	7
3		+	+	+	+	+	+	+	+
8	-		.	+	+	+	+	+	+
6	-	.		.	.	.	.	+	+
4	-	-	-		+	+	+	+	+
5	-	-	-	-		.	.	+	+
9	-	-	-	-	-		.	+	+
2	-	-	-	-	-	-		.	+
1	-	-	-	-	-	-	.		+
7	-	-	-	-	-	-	-	-	

Table IV.2 Stochastic dominance relationships (upper right corner: SD_1; lower left corner SD_2)

Rank:

Alternative	1	2	3	4	5	6	7	8	9
3	71	17	7	3	1	1	0	0	0
8	9	31	18	16	12	8	5	1	0
6	9	17	19	16	14	16	9	2	0
4	4	13	15	20	19	14	10	5	0
5	2	6	12	16	20	19	14	10	0
9	2	5	15	12	12	14	20	20	0
2	3	9	8	10	10	14	18	28	1
1	1	3	6	7	12	14	23	33	1
7	0	0	0	0	0	0	0	1	99

Table IV.3 Probability matrix that alternative i gets position n in final ranking, for all i and n.

Alternative	3	8	6	4	5	9	2	1	7
3		86	87	93	96	96	94	98	100
8	14		58	67	77	87	74	86	100
6	13	42		54	63	67	78	81	100
4	7	33	46		63	61	68	76	100
5	4	24	37	37		57	59	70	100
9	4	13	33	39	44		53	64	100
2	6	26	22	32	41	47		57	100
1	2	14	19	24	30	36	43		99
7	0	0	0	0	0	0	0	1	

Table IV.4 Pairwise comparison matrix with probability that alternative i performs better than i'.

1988-1	H. Visser	Austrian thinking on international economics
1988-2	A.H.Q.M. Merkies T. van der Meer	Theoretical foundations for the 3-C model
1988-3	H.J. Bierens J. Hartog	Nonlinear regression with discrete explanatory variables, with an application to the earnings function
1988-4	N.M. van Dijk	On Jackson's product form with 'jump-over' blocking
1988-5	N.M. van Dijk M. Rumsewicz	Networks of queues with service anticipating routing
1988-6	H. Linneman C.P. van Beers	Commodity Composition of Trade in Manufactures and South-South Trade Potential
1988-7	N.M. van Dijk	A LCFS finite buffer model with batch input and non-exponential sevices
1988-8	J.C.W. van Ommeren	Simple approximations for the batch-arrival $M^X/G/1$ queue
1988-9	H.C. Tijms	Algorithms and approximations for bath-arrival queues
1988-10	J.P. de Groot H. Clemens	Export Agriculture and Labour Market in Nicaragua
1988-11	H. Verbruggen J. Wuijts	Patterns of South-South trade in manufactures
1988-12	H.C. Tijms J.C.W. van Ommeren	Asymptotic analysis for buffer behaviour in communication systems
1988-13	N.M. van Dijk E. Smeitink	A non-exponential queueing system with batch servicing
1988-14	J. Rouwendal	Existence and uniqueness of stochastic price equilibria in heterogeneous markets
1988-15	H. Verbruggen	GSTP, the structure of protection and South-South trade in manufactures
1988-16	Mevr. H. Weijland Mevr. R. Herweijer J. de Groot	Female participation in agriculture in the Dominican Republic
1988-17	N.M. van Dijk	Product Forms for Random Access Schemes
1988-18	A.H.Q.M. Merkies I.J. Steyn	Adaptive Forecasting with Hyperfilters
1988-19	J. Rouwendal	Specification and Estimation of a Logit Model for Housing Choice in the Netherlands
1988-20	J.C.W. van Ommeren R.D. Nobel	An elementary proof of a basic result for the GI/G/1 queue
1988-21	H. Kool	A Note on Consistent Estimation of Heterostedastic and Autocorrelated Covariance Matrices
1988-22	C.P.J. Burger	Risk Aversion and the Family Farm
1988-23	N. van Dijk I.F. Akyildiz	Networks with mixed processor sharing parallel queues and common pools
1988-24	D.J.F. Kamann P. Nijkamp	Technogenesis: Incubation and Diffusion
1988-25	P. Nijkamp L. van Wissen A. Rima	A Household Life Cycle Model For the Housing Market
1988-26	P. Nijkamp M. Sonis	Qualitative Impact Analysis For Dynamic Spatial Systems
1988-27	R. Janssen P. Nijkamp	Interactive Multicriteria Decision Support For Environmental Management
1988-28	J. Rouwendal	Stochastic Market Equilibria With Rationing and Limited Price Flexibility
1988-29	P. Nijkamp A. Reggiani	Theory of Chaos in a Space-Time Perspective
1988-30	P. Nijkamp J. Poot J. Rouwendal	R & D Policy in Space and Time
1988-31	P. Nijkamp F. Soeteman	Dynamics in Land Use Patterns Socio-Economic and Environmental Aspects of the Second Agricultural Land Use Revolution
1988-32	J. Rouwendal P. Nijkamp	Endogenous Production of R & D and Stable Economic Development
1988-33	J.A. Hartog E. Hinloopen P. Nijkamp	Multicriteria Methoden: Een gevoeligheidsanalyse aan de hand van de vestigingsplaatsproblematiek van kerncentrales
1988-34	R. van der Mark P. Nijkamp	The Development Potential of High Tech Forms in Backward Areas - A Case study for the Northern Part of The Netherlands
1988-35	E.R.K. Spoor J.W.B. Vermeulen	Principes en gebruik van Envisage
1988-36	C. Gorter P. Nijkamp P. Rietveld	The Duration of Unemployment: Stocks and Flows on Regional Labour Markets in the Netherlands
1988-37	M. Hofkes	Parametrization of simplicial algorithms with an application to an empirical general equilibrium model
1988-38	J. van Daal A.H.Q.M. Merkies	A Note on the Quadratic Expenditure Model