Faculteit der Economische Wetenschappen en Econometrie

Serie Research Memoranda

Efficiency Wage Theory and General Equilibrium

M.W. Hofkes

Research Memorandum 1990-48
September 1990

vrije Universiteit *amsterdam*

EFFICIENCY WAGE THEORY AND GENERAL EQUILIBRIUM[1)]

by

M.W. Hofkes[2)]

September 1990

---

1) *This research is part of the VF-program "Competition and Cooperation"*

2) *Department of Econometrics*
Vrije Universiteit
De Boelelaan 1105
1081 HV Amsterdam
The Netherlands

Abstract

In this paper we incorporate the efficiency wage theory in a general equilibrium model. In such a model wage rigidities resulting from the behaviour of rational agents can be endogenized, while at the same time general equilibrium effects can be taken into account. These wage rigidities can give rise to either unemployment or overemployment. We will give sufficient conditions for the existence of a quantity constrained equilibrium in a general equilibrium model with production. Furthermore, our model will be illustrated with a simple example.

Acknowledgement

The author wishes to thank Gerard van der Laan for some helpful comments.

1. Introduction

Unemployment is nowadays one of the major issues of economics. Several theories have been developed explaining not only the existence of involuntary unemployment but also its form and persistence. By and large, there exist two sets of micro-theories which explain unemployment by explaining wage and price rigidities. On the one hand there are the so-called implicit contract theories and on the other hand there are the efficiency wage theories (see e.g. Stiglitz (1986)).

Wage and price rigidities have already been recognized by Keynesian economists to account for unemployment, and this idea has been formalized in so-called disequilibrium models (see Malinvaud (1977)). One of the major drawbacks, however, of these disequilibrium models is that the rigidities are simply assumed. On the other hand, the modern micro-theories of unemployment are mostly of a partial equilibrium nature.

The purpose of this paper is to incorporate the efficiency wage theory in a general equilibrium model. In such a model price rigidities resulting from the behaviour of rational agents can be endogenized, while at the same time general equilibrium effects can be taken into account. Furthermore, these price rigidities can give rise to rationing on the corresponding markets resulting in quantity constrained equilibria.

The existence of quantity constrained equilibria in exchange economies with exogenous price rigidities has been proved by Benassy (1975) and Drèze (1975). Dehez and Drèze (1984) have investigated constrained equilibria in economies with production, again in the case of exogenous price rigidities. We will give sufficient conditions for the existence of a quantity constrained equilibrium in a general equilibrium model with production where rationing can occur on both sides of the market due to endogenous price rigidities.

This paper is organized as follows. Section 2 briefly discusses the efficiency wage theory. In section 3 a general equilibrium model is presented of an economy with a constrained labour market resulting from an endogenously arising wage rigidity. In section 4 the existence of an equilibrium in the described economy is proved, while in section 5 a simple example is given to illustrate the model. Finally, section 6 gives some concluding remarks.

2. Efficiency Wage Theory

The basic hypothesis of the efficiency wage theory is that labour productivity is a function of the wage paid. Now, a rational profit maximizing firm may in the presence of unemployment choose to keep the wage above the market clearing, competitive level, because lowering the wage would result in a more than proportionate decrease in productivity.

Up to now we referred to the wage not indicating whether we meant the nominal or the real wage. In a general equilibrium model without money there is no money illusion, i.e. only relative prices matter and it seems logical that we assume the productivity to depend upon the real wage. This means that not only the nominal wage enters the labour productivity function, but also prices of other commodities in the economy in the form of some price index.

The efficiency wage theory can be formalized as follows. Let $Q^j(\lambda(p_\ell/P).L^j)$ be the production function of producer j, with p_ℓ the nominal wage paid for one unit of labour L^j, P some price index and λ some function of the real wage, taking values between zero and one, indicating the labour productivity of one unit of labour. Now, producer j maximizes his profits $p_j.Q^j-p_\ell.L^j$ over p_ℓ and L^j subject to his production function, with p_j the price of one unit of the good produced by producer j. So his first order conditions are:

$$p_j.(\partial Q^j/\partial(\lambda L^j)).L^j.\lambda'.1/P = L^j$$

and

$$p_j.(\partial Q^j/\partial(\lambda L^j)).\lambda = p_\ell$$

By dividing the second equation by the first we get $\lambda/\lambda' = p_\ell/P$. Grafically this means that the producer chooses the wage at which the ray from the origin through the corresponding point on the labour productivity curve is tangent to this labour productivity curve (see also figure 1).

It must be noted, however, that this result is only valid when the producer is not rationed in his demand for labour. Although the efficiency wage theory seeks to provide an explanation for unemployment it cannot be ruled out beforehand that the resulting wage induces overemployment. If this is indeed the case the producer will no longer set the real wage p_ℓ/P equal to λ/λ'. Taking account of a quantity constraint $L^j \leq L^{max}$, the first order conditions of the producer now become:

$$p_j . (\partial Q^j/\partial(\lambda L^j)) . L^j . \lambda' . 1/P = L^j$$

and

$$p_j . (\partial Q^j/\partial(\lambda L^j)) . \lambda = p_\ell + \mu^j$$

with μ^j the Lagrange multiplier of the constraint producer j faces.

So, $\lambda/\lambda' = (p_\ell+\mu^j)/P$. Now, as $\mu^j \geq 0$ the optimal p_ℓ/P will be larger than (or equal to) the optimal p_ℓ/P in the unconstrained case (see figure 1). In other words, whenever the labour productivity maximizing wage is that low that the producer is rationed in his demand for labour he will increase the real wage, relative to the optimal wage when no rationing occurs, in his own interest. Note that this does not mean that the real wage will be increased until the producer will no longer be rationed.

Whenever the supply of labour is larger than the demand, i.e. the producers are not rationed in their demand for labour and all producers face the same labour productivity function, $\mu^j = 0$ for all j and a uniform wage is set. However, when there is overemployment the producers will be rationed and in general the wages offered will differ from firm to firm depending upon the degree of rationing. In this case it will be necessary to make some additional assumptions with respect to the preferences of the consumers in order to be able to assign the labour-supply of a specific consumer to a specific producer and thus to determine which consumer earns what wage. Possible assumptions to make are that all consumers are identical or that there is a continuum of consumers. For the sake of simplicity and of clarity of the exposition, however, we will in the sequel assume that there is only one producer and that the consumers are uniformly rationed.

3. Model

Let us consider a private ownership economy with 1 producer, m consumers, indexed by $i=1,\ldots,m$, and k commodities, indexed by $h=1,\ldots,k$. Consumer i is characterized by a consumption set $X^i \subset R^k_+$, a utility function $u^i(x^i)$ on X^i, a vector of initial endowments $w^i \in R^k_+$ and a share $\vartheta_i \subset R_+$ in the profit made by the producer, where $\Sigma_i\vartheta_i = 1$. (The inner product of two vectors a and b, $a,b \in R^\ell$, is denoted by $a.b = \Sigma_{i=1}^{\ell} a_i b_i$ and $(a \le b) \perp (c \le d)$ means that $a \le b$, $c \le d$ and $(a-b).(c-d) = 0$).

The following assumptions are made on consumption.

(A.1) X^i, $i=1,\ldots,m$, is a closed, convex subset of R^k_+, containing $\{0\}$.

(A.2) $u^i(x^i)$ is a strictly quasi-concave function, $i=1,\ldots,m$.

(A.3) $u^i(x^i)$ satisfies local nonsatiation, i.e. for all $x^i \in X^i$, for all $\epsilon > 0$, there is a consumption bundle $x \in X^i \cap B(x^i,\epsilon)$ such that $u^i(x^i) < u^i(x)$, $i=1,\ldots,m$.

(A.4) $w^i > 0$, $i=1,\ldots,m$.

The producer is characterized by a production possibilities set $Y \subset R^k \times R_+$, with R^k the space of production vectors, where inputs are measured negatively and outputs are measured positively, and R_+ the space of wages. Note, that since the producer has one choice variable more than the consumers, i.e. the wage, the production possibilities set Y is of higher dimension than the consumption sets X^i of the consumers. The following assumptions are made on production.

(A.5) Y is a closed and strict convex subset of $R^k \times R_+$ and contains $\{0\}$.

(A.6) The free disposal assumption holds, i.e. $Y - (R^k_+ \times \{0\}) \subset Y$.

(A.7) There is no free production, i.e. $Y \cap (R^k_+ \times \{0\}) = \{0\}$.

Now, let us assume that the producer produces a single output Q, using k-1 inputs (k>1), one of these inputs being labour, denoted by L, the other inputs being represented by a (k-2)-vector K. Now, with the production function given by $Q = Q(\lambda(p_\ell/P(p)).L,K)$ we have that $Y = \{(Q,K,L,p_\ell) \mid Q \le Q(\lambda(p_\ell/P(p)).L,K),\ p_\ell \in R_+,\ L \in R_-,\ K \in R^{k-2}_-\}$. The producer is price setter on the labour market, while all other markets are competitive. Note, that as inputs are measured negatively we have that $L \in R_-$ and $K \in R^{k-2}_-$. The market price of the output, of labour and the vector of market prices of the other k-2 inputs are denoted respectively by $p^O \in R_+$, $p_\ell \in R_+$ and $p^I \in R^{k-2}_+$ and let $p=(p^O,p^{IT})^T$. Let (l,s) be a uniform rationing scheme, with $|l|$ $(l \leq 0)$ the maximum labour input of the producer and s (≥ 0) the maximum labour supply of a consumer.

The producer maximizes his profit, $p^O.Q - p_\ell.L - p^{IT}.K$, subject to his production function, which is given by $Q = Q(\lambda(p_\ell/P(p)).L,K)$, and the quantity constraint $L \geq l$, where p_ℓ is the wage paid for one unit of labour and $P(p)$ is a price index, which is homogeneous of degree one in p. λ is some continuous function of the real wage $p_\ell/P(p)$ taking values between zero and one, indicating the productivity of one unit of labour.

The m consumers have initial endowments $\underline{Q}_i \in R_+$, $\underline{K}_i \in R^{k-2}_+$ and $\underline{L}_i \in R_+$, $i=1,..,m$ and maximize their utility function $u^i(Q_i,K_{i1},\ldots, K_{i,k-2},\underline{L}_i-L_i)$ subject to their budget constraints and the quantity constraint $L_i \leq s$, with Q_i and K_{ij} the demand by consumer i for the output Q respectively input j, $j=1,\ldots,k-2$, and L_i the labour supply of consumer i. The final assumption we make on production is the following.

(A.8) The production function $Q(\lambda(p_\ell/P(p)).L,K)$ and the labour productivity function $\lambda(p_\ell/P(p))$ are continuously differentiable functions.

As the producer is a wage setter, the labour market does not necessarily clear. The real wage follows from the profit maximization problem of the producer. The real wage is determined by the equation $(p_\ell+\mu)/P(p) = \lambda(p_\ell/P(p))/\lambda'(p_\ell/P(p))$ (see also section 2). Note, that the nominal wage can be derived from the profit maximizing real wage for given prices of the other commodities. In other words the profit maximizing nominal wage is a function of all the other prices, i.e. $p_\ell = p_\ell(p)$. This relationship between nominal wage and other prices induces general equilibrium effects the model can take account of. If for example due to some tax reform, prices of other commodities in the economy change, the nominal wage will as a consequence also change and consequently the level of rationing on the labour market will change.

Definition 3.1

An efficiency wage equilibrium is a set of consumption plans $x^{i*} = (Q_i^*, K_i^*, \underline{L}_i - L_i^*)$, $i=1,\ldots,m$, a production plan $y^* = (Q^*, K^*, L^*)$, a vector of prices $(p^{O*}, p^{I*}, p_\ell^*)$, and a (uniform) rationing scheme $(1^*, s^*)$ such that:

(i) x^{i*} maximizes $u^i(Q_i, K_i, \underline{L}_i - L_i) = u^i(x^i)$ on X^i subject to $p^{I*}.K_i + p_\ell^*(\underline{L}_i - L_i) + p^{O*}Q_i \le p^{O*}\underline{Q}_i + p^{I*}.\underline{K}_i + p_\ell^*\underline{L}_i + \vartheta_i.\pi(p^*)$ and $L_i^* \le s^*$, $i=1,\ldots,m$, with $\pi(p^*) = (p^{O*}, p^{I*T}, p_\ell^*).(Q^*, K^{*T}, L^*)^T$.

(ii) (y^*, p_ℓ^*) maximizes profit $(p^{O*}, p^{I*T}, p_\ell)^T.y$ on Y subject to $Q = Q(\lambda(p_\ell/P(p^*)).L, K)$ and $L^* \ge 1^*$.

(iii) $(\Sigma_i Q_i^* \le Q^* + \Sigma_i \underline{Q}_i) \perp (p^{O*} \ge 0)$.

(iv) $(\Sigma_i K_{ih}^* - K^*_h \le \Sigma_i \underline{K}_{ih}) \perp (p^{I*}_h \ge 0)$, $h=1,\ldots,k-2$.

(v) $-L^* = \Sigma_i L_i^*$.

(vi) if for some i: $L_i^* = s^*$ then $L^* > 1^*$ and if $L^* = 1^*$ then $L_i^* < s^*$ for all i.

So at an efficiency wage equilibrium consumers maximize their utility, the producer maximizes profits over inputs and wage, exchange is voluntarily, agents at the short side of the market realize their plans and actually bought and sold amounts of goods are equal.

4. Existence of equilibrium

In this section we will prove the existence of an efficiency wage equilibrium. Following Debreu (1959) we define the set of attainable states $A = \{(x^i), y, p_\ell \mid \Sigma_i x^i - y \le w,\ p_\ell \le \underline{p}_\ell^{sup},\ x^i \in X^i,\ (y, p_\ell) \in Y\}$, with $x^i = (Q_i, K_{i1}, \ldots, K_{i,k-2}, \underline{L}_i - L_i)$, $y = (Q, K_1, \ldots, K_{k-2}, L)$, $w = (\Sigma_i \underline{Q}_i, \Sigma_i \underline{K}_i, \Sigma_i \underline{L}_i)$ and $\underline{p}_\ell^{sup}$ some number larger than zero. Let $\hat{X}^i$ and $\hat{Y}$ be the projections of A respectively on X^i and Y and let $\tilde{X}^i = F \cap X^i$ and $\tilde{Y} = G \cap Y$, with F and G compact convex sets containing respectively $\hat{X}^i$ and $\hat{Y}$ in their interiors. Now, as the production function is continuously differentiable, $\partial Q/\partial(\lambda L)$ has a maximum value on the compact set $\tilde{Y}$. We will call this maximum value $\underline{p}_\ell^{max}$. For the existence proof it will be necessary that $\underline{p}_\ell^{max} \le \underline{p}_\ell^{sup}$. It can easily be seen that we can always find a $\underline{p}_\ell^{sup}$ such that the corresponding $\underline{p}_\ell^{max}$ satisfies

$\underline{p}_\ell^{max} \le \underline{p}_\ell^{sup}$. The strict convexity of the production possibilities set requires a strict concave production function and consequently $\partial^2 Q/\partial(\lambda L)\partial p_\ell = \partial^2 Q/(\partial(\lambda L))^2 . L . \partial\lambda/\partial p_\ell \le 0$, in other words $\partial Q/\partial(\lambda L)$ must be non-increasing in p_ℓ. So for increasing $\underline{p}_\ell^{sup}$, $\underline{p}_\ell^{max}$ will be non-increasing and for large enough $\underline{p}_\ell^{sup}$ we will find $\underline{p}_\ell^{max} \le \underline{p}_\ell^{sup}$. Since the demand and supply functions which follow from the utility maximization problems of the consumers respectively the profit maximization problem of the producer are homogeneous of degree one in the prices, we have one degree of freedom. We will use this degree of freedom to normalize the prices of the output good of the producer and of the k-2 inputs other than labour as to sum to one.

We define the set $T = \{q \in S^{k-2},\ r \in R_+,\ \underline{p}_\ell \in R_+ \mid 0 \le r \le 2\underline{p}_\ell^{max},\ 0 \le \underline{p}_\ell \le \underline{p}_\ell^{max}\}$, with S^{k-2} the (k-2)-dimensional unit simplex given by $S^{k-2} = \{q \in R^{k-1}{}_+ \mid \Sigma_i\, q_i = 1\}$. On T we define the functions $p(q,r,\underline{p}_\ell)$, $l(q,r,\underline{p}_\ell)$ and $s(q,r,\underline{p}_\ell)$ by:

$$p_i(q,r,\underline{p}_\ell) = \begin{array}{ll} q_i & \text{for } i=1,\dots,k-1 \\ \underline{p}_\ell & \text{for } i=k, \end{array}$$

$$l(q,r,\underline{p}_\ell) = \begin{cases} -\max[0,\min\{1,2-r/\underline{p}_\ell\}].(\Sigma\underline{L}_i+1) & \text{if } \underline{p}_\ell > 0 \\ 0 & \text{if } \underline{p}_\ell = 0 \end{cases}$$

$$s(q,r,\underline{p}_\ell) = \begin{cases} \min\{1,r/\underline{p}_\ell\}.(\Sigma\underline{L}_i+1) & \text{if } \underline{p}_\ell > 0 \\ \Sigma\underline{L}_i+1 & \text{if } \underline{p}_\ell = 0 \end{cases}$$

Note that when $r \le \underline{p}_\ell$ then $l(q,r,\underline{p}_\ell) = -(\Sigma\underline{L}_i+1)$, when $r \ge \underline{p}_\ell$ then $s(q,r,\underline{p}_\ell) = (\Sigma\underline{L}_i+1)$ and $r \ge 2\underline{p}_\ell$ implies $l(q,r,\underline{p}_\ell) = 0$ (see also figure 2). Furthermore, $p^O(q,r,\underline{p}_\ell) = p_1(q,r,\underline{p}_\ell)$ and $p^I(q,r,\underline{p}_\ell) = (p_2(q,r,\underline{p}_\ell), \dots, p_{k-1}(q,r,\underline{p}_\ell))^T$.

Let the function $z : S^{k-2} \times R_+ \times R_+ \to R^{k+1}$ be given by:

$$z(q,r,\underline{p}_\ell) = \begin{pmatrix} \Sigma_i x^i(q,r,\underline{p}_\ell) - y(q,r,\underline{p}_\ell) - w \\ p_\ell(q,r,\underline{p}_\ell) - \underline{p}_\ell \end{pmatrix} = \begin{pmatrix} \Sigma_i Q_i - Q - \Sigma_i \underline{Q}_i \\ \Sigma_i K_i - K - \Sigma_i \underline{K}_i \\ -\Sigma_i L_i - L \\ p_\ell(q,r,\underline{p}_\ell) - \underline{p}_\ell \end{pmatrix},$$

with $x^i(q,r,\underline{p}_\ell) = \{\text{argmax } u^i(x^i) \mid x^i \in \underline{X}^i,$
$$p^O(q,r,\underline{p}_\ell)Q_i + p^I(q,r,\underline{p}_\ell).K_i + p_k(q,r,\underline{p}_\ell)(\underline{L}_i - L_i) \le p^O(q,r,\underline{p}_\ell)\underline{Q}_i + p^I(q,r,\underline{p}_\ell).\underline{K}_i + p_k(q,r,\underline{p}_\ell)\underline{L}_i + \vartheta_i.\pi(q,r,\underline{p}_\ell),\ L_i \le s(q,r,\underline{p}_\ell)\}$$

and $(y(q,r,\underline{p}_\ell),p_\ell(q,r,\underline{p}_\ell)) = \{\text{argmax } (p^O(q,r,\underline{p}_\ell),(p^I(q,r,\underline{p}_\ell))^\top,p_\ell).y \mid (y,p_\ell)\in\underline{Y}, L \ge l(q,r,\underline{p}_\ell)\}.$

Now, $p(q,r,\underline{p}_\ell)$, $s(q,r,\underline{p}_\ell)$ and $l(q,r,\underline{p}_\ell)$ are continuous in $(q,r,\underline{p}_\ell)$. From the continuity of $l(q,r,\underline{p}_\ell)$ it follows that the correspondence on T defined by $\{(y,p_\ell)\in\underline{Y} \mid L\ge l(q,r,\underline{p}_\ell)\}$ is continuous on T. So, since Y is strictly convex, $(y(q,r,\underline{p}_\ell),p_\ell(q,r,\underline{p}_\ell))$ is a continuous function on T. Furthermore, $\pi(q,r,\underline{p}_\ell)= (p^O(q,r,\underline{p}_\ell),(p^I(q,r,\underline{p}_\ell))^\top,p_\ell(q,r,\underline{p}_\ell))^\top.y(q,r,\underline{p}_\ell)$ is a continuous function. Finally, as the budget correspondences of the consumers are continuous in p, π and s (see Drèze (1975)) and $p(q,r,\underline{p}_\ell)$, $\pi(q,r,\underline{p}_\ell)$ and $s(q,r,\underline{p}_\ell)$ are continuous on T, the budget correspondences are also continuous on T and consequently, using the strict quasi-concavity of $u^i(x^i)$, we can derive that $x^i(q,r,\underline{p}_\ell)$ is a continuous function on T. Now, $z(q,r,\underline{p}_\ell)$ is a continuous function on T, by the continuity of $x^i(q,r,\underline{p}_\ell)$, $y(q,r,\underline{p}_\ell)$ and $p_\ell(q,r,\underline{p}_\ell)$.

Lemma 4.1

$p_\ell(q,r,\underline{p}_\ell) \le \underline{p}_\ell^{max}$

Proof

In section 2 we have seen that the producer sets a wage p_ℓ^{opt} with $\lambda/\lambda' = p_\ell^{opt}$ if he is not rationed. From the first order conditions of the maximization problem of the producer we have that p_ℓ^{opt} must satisfy the equation $p_\ell^{opt} = p.\partial Q/\partial(\lambda L).\lambda$. So, as $p\le 1$ and $\lambda\le 1$ we have that $p_\ell^{opt} \le \partial Q/\partial(\lambda L) \le \underline{p}_\ell^{max}$. If the producer is rationed the wage set must satisfy the equation $p.\partial Q/\partial(\lambda L).\lambda = p_\ell + \mu$, with μ the Lagrange multiplier of the constraint $L \ge l(q,r,\underline{p}_\ell)$. But then also $p_\ell^{opt} \le \underline{p}_\ell^{max}$ as $\mu \ge 0$.

□

It can easily be seen that the following theorem holds.

Theorem 4.2

$\Sigma_{i=1}^{k} p_i(q,r,\underline{p}_\ell).z_i(q,r,\underline{p}_\ell) = (p_\ell(q,r,\underline{p}_\ell) - \underline{p}_\ell).L$

Theorem 4.3

Under Assumptions (A.1) to (A.8) there exists an efficiency wage equilibrium.

Proof

Define the function $f: S^{k-2} \times [0, 2p_\ell^{max}] \times [0, p_\ell^{max}] \to S^{k-2} \times [0, 2p_\ell^{max}] \times [0, p_\ell^{max}]$ by:

$$f_h(q,r,p_\ell) = (q_h + \max[0, z_h(q,r,p_\ell)])/(1+\Sigma_{j=1}^{k-1} \max[0, z_j(q,r,p_\ell)]) \qquad h=1,\ldots,k-1$$

$$f_k(q,r,p_\ell) = \max[0, \min(2p_\ell^{max}, r+z_k(q,r,p_\ell))]$$

$$f_{k+1}(q,r,p_\ell) = \max[0, \min(p_\ell^{max}, p_\ell+z_{k+1}(q,r,p_\ell))]$$

Now, according to Brouwer's theorem f has a fixed point (q^{*T}, r^*, p_ℓ^*). So, $f(q^{*T}, r^*, p_\ell^*) = (q^{*T}, r^*, p_\ell^*)^T$.

At (q^{*T}, r^*, p_ℓ^*) we have:

(i) $f_{k+1}(q^*, r^*, p_\ell^*) = p_\ell^*$

Now, either $p_\ell^* = 0$, $p_\ell^* = p_\ell^{max}$ or $p_\ell^* = p_\ell^* + z_{k+1}(q^*, r^*, p_\ell^*)$.

In the latter case we have that $z_{k+1}(q^*, r^*, p_\ell^*) = 0$ straight away.

If $p_\ell^* = 0$ then $p_\ell^* + z_{k+1}(q^*, r^*, p_\ell^*) \le 0$ and consequently $z_{k+1}(q^*, r^*, p_\ell^*) \le 0$. On the other hand $z_{k+1}(q^*, r^*, p_\ell^*) = p_\ell(q^*, r^*, p_\ell^*) - p_\ell^* \ge 0$. So, $z_{k+1}(q^*, r^*, p_\ell^*) = 0$.

Finally, if $p_\ell^* = p_\ell^{max}$ then $p_\ell^{max} \le p_\ell^* + z_{k+1}(q^*, r^*, p_\ell^*)$. So, $z_{k+1}(q^*, r^*, p_\ell^*) \ge 0$. On the other hand, $z_{k+1}(q^*, r^*, p_\ell^*) = p_\ell(q^*, r^*, p_\ell^*) - p_\ell^* = p_\ell(q^*, r^*, p_\ell^*) - p_\ell^{max} \le 0$.

So, $z_{k+1}(q^*, r^*, p_\ell^*) = 0$.

(ii) $f_k(q^*, r^*, p_\ell^*) = r^*$

Now, either $r^* = 0$, $r^* = 2p_\ell^{max}$ or $r^* = r^* + z_k(q^*, r^*, p_\ell^*)$.

In the latter case we have again straight away that $z_k(q^*, r^*, p_\ell^*) = 0$.

If $r^* = 0$ we must have that $r^* + z_k(q^*, r^*, \underline{p}_\ell^*) \leq 0$. But then also $z_k(q^*, r^*, \underline{p}_\ell^*) \leq 0$. But on the other hand, $r^* = 0$ implies $s(q^*, r^*, \underline{p}_\ell^*) = 0$. So, $L_i = 0$ and consequently, $z_k(q^*, r^*, \underline{p}_\ell^*) = -L \geq 0$. But then we must have $z_k(q^*, r^*, \underline{p}_\ell^*) = 0$.

If $r^* = 2\underline{p}_\ell^{max}$ then $2\underline{p}_\ell^{max} \leq r^* + z_k(q^*, r^*, \underline{p}_\ell^*)$. So, $z_k(q^*, r^*, \underline{p}_\ell^*) \geq 0$. On the other hand, $l(q^*, r^*, \underline{p}_\ell^*) = 0$ and consequently $L = 0$ and $z_k(q^*, r^*, \underline{p}_\ell^*) \leq 0$.

So, $z_k(q^*, r^*, \underline{p}_\ell^*) = 0$ must hold.

(iii) $f_h(q^*, r^*, \underline{p}_\ell^*) = q_h^*$, $h=1,\ldots,k-1$

Now, $z_h(q^*, r^*, \underline{p}_\ell^*) \leq 0$ holds for $h=1,\ldots,k-1$.

Suppose, $z_h(q^*, r^*, \underline{p}_\ell^*) > 0$ for some h, $h=1,\ldots,k-1$. Then $\Sigma_j \max[0, z_j(q^*, r^*, \underline{p}_\ell^*)] > 0$. But then $z_h(q^*, r^*, \underline{p}_\ell^*) > 0$ for all $h=1,\ldots,k-1$ with $q_h^* = p_h(q^*, r^*, \underline{p}_\ell^*) > 0$. But as $z_{k+1}(q^*, r^*, \underline{p}_\ell^*) = 0$ and consequently $p_\ell(q^*, r^*, \underline{p}_\ell^*) = \underline{p}_\ell^*$ this cannot be the case, as according to theorem 4.2 $\Sigma_{i=1}^{k} p_i(q^*, r^*, \underline{p}_\ell^*) . z_i(q^*, r^*, \underline{p}_\ell^*) = (p_\ell(q^*, r^*, \underline{p}_\ell^*) - p_\ell^*) . L = 0$ and $z_k(q^*, r^*, \underline{p}_\ell^*) = 0$ and $p^* = (p_1^*, \ldots, p_{k-1}^*)^T \in S^{k-2}$.

So $z_h(q^*, r^*, \underline{p}_\ell^*) \leq 0$ for all h must hold.

Finally, it can easily be seen that a vector $(q^{*T}, r^*, \underline{p}_\ell^*)$, for which $z_h(q^{*T}, r^*, \underline{p}_\ell^*) \leq 0$ for $h=1,\ldots,k-1$, $z_k(q^{*T}, r^*, \underline{p}_\ell^*) = z_{k+1}(q^{*T}, r^*, \underline{p}_\ell^*) = 0$ holds, induces an efficiency wage equilibrium. Firstly, using a standard argument (see Debreu (1959)) one shows that conditions (i) and (ii) of definition 3.1 are actually satisfied. Furthermore, it is straightforward that conditions (iii), (iv) and (v) are satisfied. Finally, from the definitions of $l(q^T, r, \underline{p}_\ell)$ and $s(q^T, r, \underline{p}_\ell)$ it is immediately clear that also condition (vi) is satisfied, as for all i we have $L_i \leq \underline{L}_i$. So, $-L^* = \Sigma_i L_i^* \leq \Sigma_i \underline{L}_i < \Sigma_i \underline{L}_i + 1$ or $L^* > -(\Sigma_i \underline{L}_i + 1)$ and $L_i \leq \underline{L}_i < \Sigma_i \underline{L}_i + 1$. In other words, whenever $L_i^* = s(q^{*T}, r^*, \underline{p}_\ell^*)$, it must be the case that $r^* < \underline{p}_\ell$, but then $l(q^{*T}, r^*, \underline{p}_\ell^*) = -(\Sigma_i \underline{L}_i + 1)$ and consequently $L^* > l(q^{*T}, r^*, \underline{p}_\ell^*)$ and if $L^* = l(q^{*T}, r^*, \underline{p}_\ell^*)$, it must be the case that $r^* > \underline{p}_\ell$, but then $s(q^{*T}, r^*, \underline{p}_\ell^*) = \Sigma_i \underline{L}_i + 1$ and consequently $L_i^* < s(q^{*T}, r^*, \underline{p}_\ell^*)$.

□

5. An Example

In this section we will give a simple example to illustrate our model. The construction of the existence proof given in section 4 allows for a direct application of a simplicial (fixed point) algorithm which operates in $S^{k-2} \times R^2_+$ (for a detailed description of such an algorithm see Hofkes (1990)) to actually compute an equilibrium of the described economy. It must be noted that, although the assumption of convexity of the production possibilities set (A.5) implicitly gives restrictions on λ, in practise a labour productivity function of the form as given in figure 1, i.e. of the form $\lambda = \{1/(1+\exp((-p_\ell/P(p))/c+a)) - 1/(1+\exp(a))\}.\{(1+\exp(a))/\exp(a)\}$ can be used , since the algorithm always finds an equilibrium in the convex part of λ, i.e. for a value of p_ℓ larger than or equal to the value of p_ℓ given by $\lambda/\lambda' = p_\ell$.

Let us assume that we have an economy with one producer and two consumers. The producer has a Cobb Douglas production function given by $Q = K^{1/4}\ L^{1/2}.\lambda^{1/2}$, with $\lambda = \{1/(1+\exp(-p_\ell/c+3))-1/(1+\exp(3))\}.\{(1+\exp(3))/\exp(3)\}$, so P(p) is taken to be identical to 1. The consumers have a Cobb Douglas utility function given by $u = Q^{1/3}.K^{1/3}.(\underline{L} - L)^{1/3}$.

Now, equilibria have been computed for various values of c and various initial endowments of the consumers (see table 1). It appears that for a low value of c (c=0.05) the producer is rationed in his demand for labour ($\mu>0$) and the wage offered is indeed larger than the optimal wage in case he would not be rationed (i.e. the wage given by $\lambda/\lambda' = p_\ell$, i.e. $p_\ell = 0.21$, see also figure 3). For higher values of c (e.g. c = 0.5) the producer is not rationed and the labour productivity maximizing wage given by $\lambda/\lambda' = p_\ell$ is set (see figure 4). Now, the consumers are rationed in their supply of labour and unemployment occurs.

Note, that for low values of c the labour productivity is larger for a given value of p_ℓ than for higher values of c. The parameter c can be seen as an indicator of working conditions. When c is small, working conditions are good and low wages induce a high labour productivity. When c is large, working conditions are not so good and wages have to be higher to induce a high labour productivity.

Table 1. equilibrium prices

$\underline{Q}_1$	$\underline{K}_1$	$\underline{L}_1$	$\underline{Q}_2$	$\underline{K}_2$	$\underline{L}_2$	ϑ_1	ϑ_2	c	p	p_k	p_ℓ	μ
3	5	8	4	3	8	0.75	0.25	0.05	0.53	0.47	0.23	0.05
3	5	8	4	3	8	0.75	0.25	0.5	0.53	0.47	2.13	0
3	5	8	4	3	8	0.75	0.25	1	0.53	0.47	4.26	0
25	17	12	20	10	12	0.75	0.25	0.1	0.37	0.63	0.57	0.98
25	17	12	20	10	12	0.75	0.25	0.5	0.37	0.63	2.13	0

6. Concluding Remarks

In this paper we have incorporated the efficiency wage theory in a general equilibrium framework. In our model a wage rigidity arises endogenously from the profit maximizing behaviour of the producer by incorporating a labour productivity function in the production function.

The existence of a quantity constrained equilibrium is proved and a simple example is given. This example illustrates that the model not only covers the case of unemployment, i.e. the case where consumers are rationed in their supply of labour, but also the case of overemployment.

References

- Benassy, J.P (1975), Neo-Keynesian Disequilibrium Theory in a Monetary Economy, *Review of Economic Studies*, 42, pp. 503-523.
- Debreu, G. (1959), *Theory of Value: an axiomatic analysis of economic equilibrium*, Yale University Press, New Haven.
- Dehez, P. and J.H. Drèze (1984), On Supply-Constrained Equilibria, *Journal of Economic Theory*, 33, pp. 172-182.
- Drèze, J.H. (1975), Existence of an Exchange Equilibrium under Price Rigidities, *International Economic Review*, 16, pp. 310-320.
- Hofkes, M.W. (1990), A Simplicial Algorithm to Solve the Nonlinear Complementarity Problem on $S^n \times R^m_+$, *Journal of Optimization Theory and Applications*, forthcoming.
- Malinvaud, E. (1977), *The Theory of Unemployment Reconsidered*, Oxford, Basil Backwell.
- Stiglitz, J.E. (1986), Theories of Wage Rigidity, in *Keynes' Economic Legacy*, ed. by J.L. Butkiewicz, K.J. Koford and J.B. Miller.

Figure 1. the labour productivity function

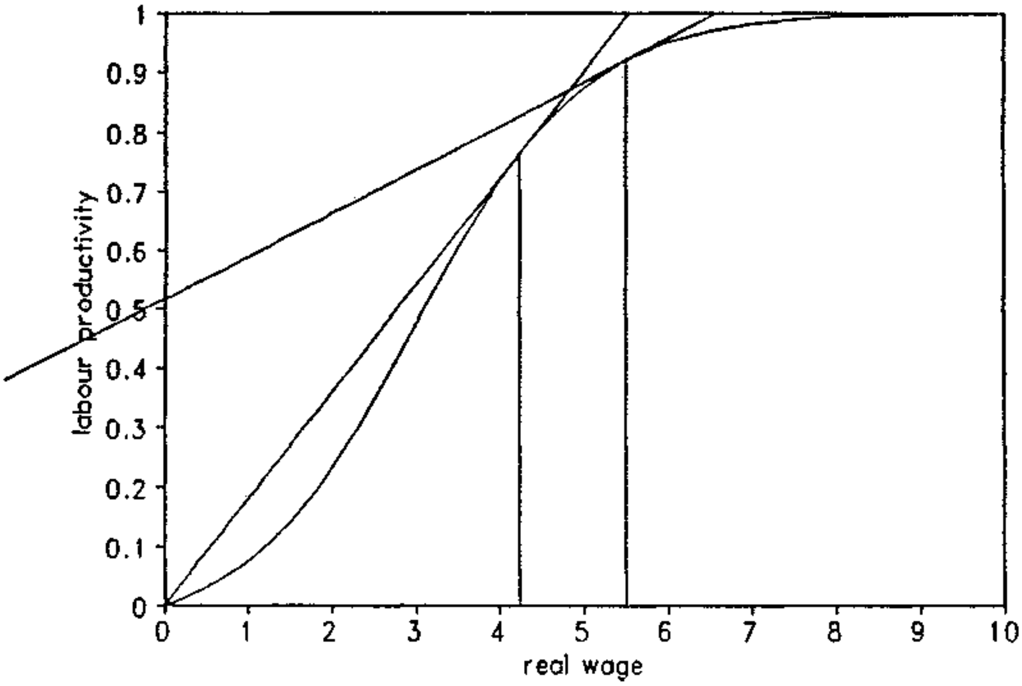


Figure 2. the rationing schemes

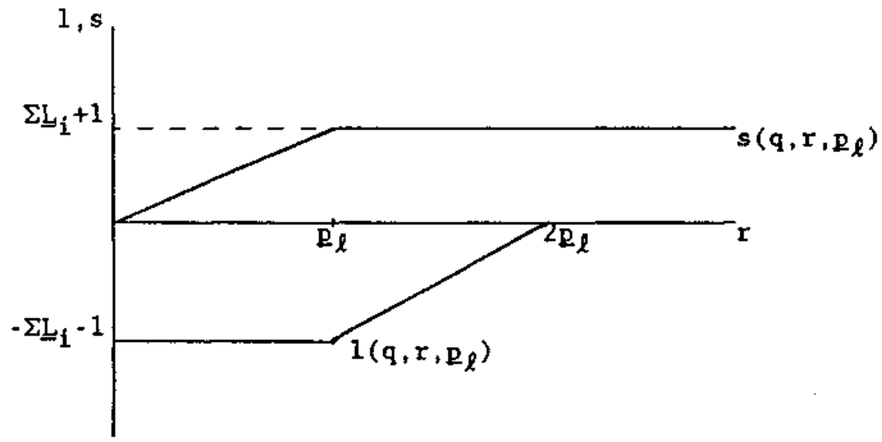


Figure 3. <u>the labour productivity function; c = 0.05</u>

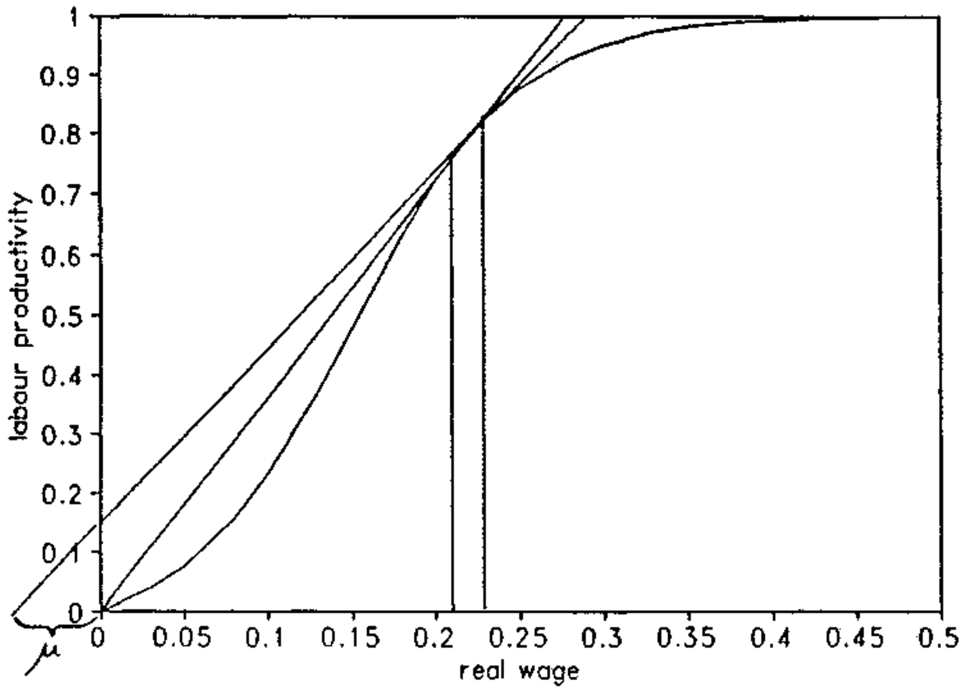
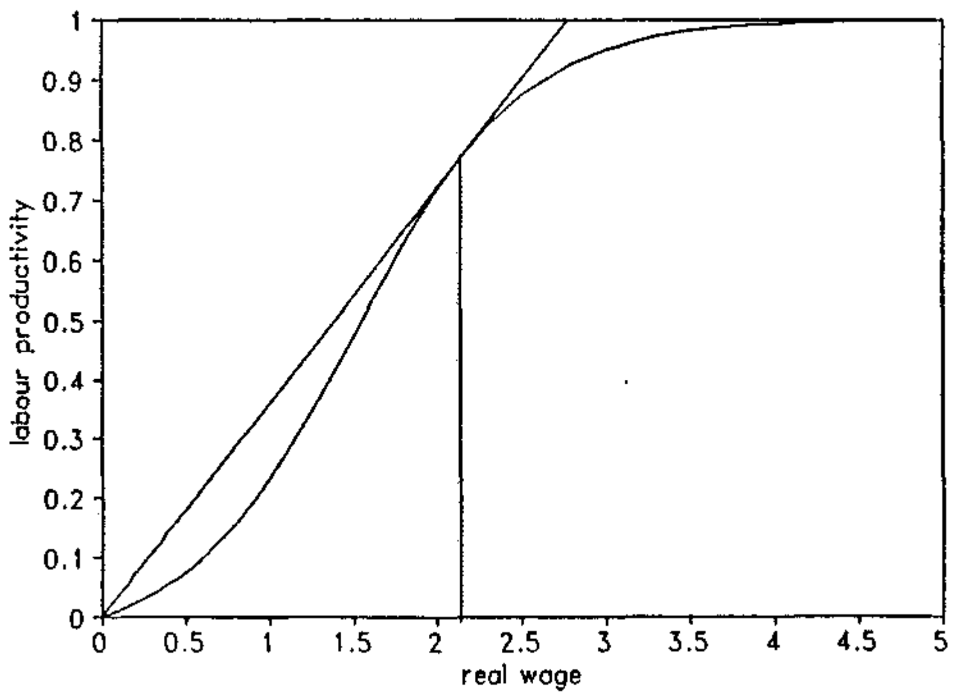


Figure 4. <u>the labour productivity function; c = 0.5</u>



1990-1	B. Vogelvang	Testing For Co-Integration with Spot Prices of Some Related Agricultural Commodities
1990-2	J.C.J.M. van den Bergh P. Nijkamp	Ecologically Sustainable Economic Development Concepts and Model Implications
1990-3	J.C.J.M. van den Bergh P. Nijkamp	Ecologically Sustainable Economic Development in a Regional System: A Case Study in Agri cultural Development Planning in the Netherlands
1990-4	C.Gorter P.Nijkamp P.Rietveld	Employers' Recruitment Behaviour and Re-Employment Probabilities of Unemployed
1990-5	K.Burger	Off-farm income and the farm-household the case of Kenyan smallholders
1990-6	H. Visser	Crowding out and the Government Budget
1990-7	P. Rietveld	Ordinal Data in Multicriteria Decision Making, a Stochastic Dominance Approach to Siting Nuclear Power Plants
1990-8	G. van der Laan P.H.M. Ruys D.J.J. Talman	Signaling devices for the supply of semi-public goods
1990-9	F.A.G. den Butter	Labour Productivity Slowdown and Technical Progress: An empirical analysis for The Netherlands
1990-10	R.W. van Zijp	Neo-Austrian Business Cycle Theory
1990-11	J.C. van Ours	Matching Unemployment and Vacancies: The Efficiency of the Dutch Labour Market
1990-12	B. Vogelvang	Hypotheses Testing Concerning Relationships between Spot Prices of Various Types of Coffee
1990-13	A.F. de Vos I.J. Steyn	Stochastic Nonlinearity: A Firm Basis for the Flexible Functional Form
1990-14	Y.H. van Emmerik D. de Jong W.W.A. Zuurmond D.N. Dukkers-van Emden	Opereren in overleg: geprotocolleerde samenwerking 1e-2e-lijn bij dagchirurgie
1990-15	T.J.J.B. Wolters	Mediation and Collective Bargaining: A Diagnostic Approach
1990-16	E.M.A. Scholten J. Koelewijn	Financieringsproblematiek van startende ondernemingen: een mogelijke verklaring op basis van empirisch onderzoek.
1990-17	E. Hüner H.P. Smit	Saturation and Model Specification of Passen ger car Ownership
1990-18	F.A.G. den Butter	Sociale zekerheid, de wig en economische groei