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Price adjustment in a two-country model

by

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1. Introduction

In this paper we consider price adjustments resulting from shocks in a two country model. For that purpose we present a price adjustment process that reaches an equilibrium price vector when starting from an arbitrarily chosen price vector. The latter vector can be interpreted as the equilibrium price prevailing in the economy before the shock took place.

The model we consider here is very simple. There are two countries supplying goods for the domestic market as well as for the international market. An equilibrium price vector is a vector of prices at which the demand for both the domestic goods and the goods traded on the common market equals their respective supplies. In other words, an equilibrium vector is a zero point of an appropriate excess demand vector.

Of course, economists are quite familiar with price adjustment processes for finding equilibrium prices. First of all we have the Walras tatonnement process. Here, the change in the price vector is related to the corresponding excess demand vector. Thus, prices of goods in excess demand (supply) are increased (decreased), while prices of goods in equilibrium are not adapted. The problem is that this process may not converge to an equilibrium. Strong conditions on the excess demand function are needed to guarantee convergence. Another well-known process is the Newton-like method of Smale. It adapts prices in such a way that the change in the excess demands is a negative factor times those excess demands. Thus, prices are adapted such that positive excess demands become smaller whereas negative excess demands increase. But also this process does not converge from any starting vector. Convergence is only guaranteed when the process starts at the boundary of the price space, while some boundary condition has to be satisfied.

Both processes only consider the current price vector and excess demand vector. This appears not to be sufficient to guarantee convergence. The process we present here also keeps in mind the starting vector. By doing this, and thus considering price changes (relative to the starting prices), it is guaranteed that the process converges from any starting price system to an equilibrium price system. The economic idea behind the process has some similarities with that of the Walras tatonnement process.

The international trade model we consider in this paper is the two country model in which both countries have domestic and international commodities (see Mansur and Whalley [3]). In fact they consider a similar model with an arbitrary number of countries. We only consider two countries because all the features of our process can be clarified in this setting. For applying our process we first have to rewrite the model. Here we follow the lines of van der Laan [1] who reformulated the model by making use of its specific structure. This reformulation introduces exchange rates between the domestic currencies and an international currency, and a balance of payments for each country. Thus, application of our process to the model not only guarantees convergence, but it also makes it possible to consider movements in balances of payments and exchange rates. In fact this makes our process very interesting from an economic viewpoint.

This paper consists of five sections. In Section 2 we present the model. Section 3 contains a formal description of the price adjustment process. In Section 4 we give an economic interpretation of that process. Finally, in Section 5 we present a simple numerical example. Readers that are only interested in the economic content of this paper can skip Section 3.

2. The model

As already mentioned, our model deals with two countries. Concerning the goods there are domestic goods that are produced and traded within one country, and internationally traded or common goods. The model describes a pure exchange economy. Initially, each agent in the economy possesses a bundle of goods. They exchange goods against equilibrium prices in such a way that they maximize their utilities.

Let us explain the model more formally. The number of domestic goods in country h, h = 1,2, equals n_h, and these goods are indexed by (h,k), where h denotes the country and k the good. The number of common goods is n_0+1 and they are indexed (0,k), $k = 0,1,\dots,n_0$. Thus, the total number of goods equals N+1, with $N = n_0+n_1+n_2$. A price vector $p \in \mathbb{R}_+^{N+1}/\{0\}$ can be written as $p = (p_0^\top,p_1^\top,p_2^\top)^\top$ with $p_0 \in \mathbb{R}_+^{n_0+1}$ and $p_h \in \mathbb{R}_+^{n_h}$, h = 1,2. An element p_{hk} denotes the price of good k on market h, h = 0,1,2, where the common market is regarded as market 0. The excess demand function z^h of country h, h = 1,2, is a continuous function from $\bar{R}^{N+1}$ into $\mathbb{R}^{N+1}$, where $\bar{R}^{N+1}$ is the subset of $\mathbb{R}_+^{N+1}/\{0\}$, such that for all $p \in \bar{R}^{N+1}$, $(p_0^\top,p_h^\top)^\top$ has at least one positive element for h = 1,2. More precisely, $z^h(p) = (z_0^h(p)^\top,z_1^h(p)^\top,z_2^h(p)^\top)^\top$, where $z_0^h(p) : \bar{R}^{N+1} \to \mathbb{R}^{n_0+1}$ denotes the total excess demand of country h for common goods whereas $z_k^h(p) : \bar{R}^{N+1} \to \mathbb{R}^{n_k}$ is the total excess demand of the consumers in country h for the domestic commodities of country k, with $k,h \in \{1,2\}$. Clearly $z_k^h(p) = 0$ if $k \notin \{h,0\}$. Because the excess demands in a country only depend on the domestic and the common goods prices we can write $z^h(p)$ as $z^h(p_0,p_h)$. Concerning z^h we make the following standard assumptions:

i) $z^h(\lambda p_0,\lambda p_h) = z^h(p_0,p_h)$, $\lambda > 0$ (homogeneity)

ii) $p^\top z^h(p) = p_0^\top z_0^h(p_0,p_h) + p_h^\top z_h^h(p_0,p_h) = 0$ (Walras' law) (2.1)

iii) Positive excess demand for goods with price zero (desirability).

At an equilibrium price vector $\bar{p} = (\bar{p}_0^\top,\bar{p}_1^\top,\bar{p}_2^\top) \in \mathbb{R}_+^{N+1}$ all markets have to be in equilibrium, i.e.

i) $z_0(\bar{p}) = z_0^1(\bar{p}_0,\bar{p}_1) + z_0^2(\bar{p}_0,\bar{p}_2) = 0$ (2.2)

ii) $z_h(\bar{p}) = z_h^h(\bar{p}_0,\bar{p}_h) = 0,\ h = 1,2.$

To apply our process we first reformulate the zero point problem on $\bar{R}^{N+1}$ into a zero point problem on the simplotope $S := S^{n_0} \times S^{n_1} \times S^{n_2}$, where $S^{n_h} := \{x \in \mathbb{R}_+^{n_h+1} \,|\, \Sigma_{j=1}^{n_h+1} x_j = 1\}$ (see van der Laan [1]). In the sequel we denote the set $\{(h,0),(h,1),\dots,(h,n_h)\}$ by $I(h)$, $h = 0,1,2$. Related to each element $q = (q_0,q_1,q_2)$ in S we define for both countries a price vector $\pi^h(q_0,q_h) = [(\pi_0^h(q_0,q_h))^\top,(\pi_h^h(q_0,q_h))^\top]^\top \in \mathbb{R}^{n_0+1} \times \mathbb{R}^{n_h}$, $h = 1,2$, by

$$\begin{aligned} \pi_{0k}^h(q_0,q_h) &= q_{h0}q_{0k} \quad , (0,k) \in I(0) \\ \pi_{hk}^h(q_0,q_h) &= q_{hk} \quad , (h,k) \in I(h)/\{(h,0)\}. \end{aligned} \tag{2.3}$$

So q_{hk}, $k \neq 0$, yields the price of the k-th commodity of country h. The elements of q_0 are multiplied by q_{h0} to get the prices of the international commodities for country h. The vectors q and $\pi^h(q)$, $h = 1,2$, are related in such a way that there is a 1-1 correspondence between a ray of price vectors in $\bar{R}^{N+1}$ and a vector q in S. More specific, the vector q yields the same excess demands as a vector of prices p on the corresponding ray. We illustrate this with an example.

<u>Example 2.1</u>. Let us assume that $(n_0,n_1,n_2) = (2,2,2)$, i.e. we have three common goods and two domestic goods for each country. Now consider the ray $R := \{y \in \bar{R}^7 | y = \lambda x,\ x = (5, 2, 3, 2, 1, 14, 6)^\top,\ \lambda > 0\}$. To derive the related q-vector we first normalize the sum of the common goods prices to one, i.e. we take $\bar{x} = (1/2,1/5,3/10,1/5,1/10,7/5,3/5)^\top$, with $q_0 = (1/2, 1/5,3/10)^\top$. Note that $z(y) = z(\bar{x})$ for all y in R. Next we find q_1 by solving the equations $q_{11}/q_{10} = \bar{x}_{11} = 1/5$, $q_{12}/q_{10} = \bar{x}_{12} = 1/10$ and $q_{10} + q_{11} + q_{12} = 1$. In this way we find $q_1 = (10/13,2/13,1/13)^\top$ and similar $q_2 = (1/3,7/15,1/5)^\top$. Clearly, $\forall y \in R$, $z^h(y) = z^h(\bar{x}_0,\bar{x}_h) = z^h(\pi^h(q_0,q_h))$, $h = 1,2$.

Next, we define $z(q) = (z_0^{\top}(q), z_1^{\top}(q), z_2^{\top}(q))^{\top} \in \mathbb{R}^{N+1}$ as the total excess demand vector at prices $\pi^h(q)$, $h = 1,2$, i.e.

$$z_0(q) = z_0^1(\pi^1(q_0,q_1)) + z_0^2(\pi^2(q_0,q_2))$$

$$z_h(q) = z_h^h(\pi^h(q_0,q_h)),\ h = 1,2.$$

Observe that q consists of $\Sigma_{h=0}^{2}(n_h+1)$ elements while $z(q)$ consists of $1 + \Sigma_{h=0}^{2} n_h$ elements. We now construct a function $\bar{z}$ relating to each q a vector with the same number of elements. Therefore, let
$\bar{z}(q) = (\bar{z}_0^{\top}(q), \bar{z}_1^{\top}(q), \bar{z}_2^{\top}(q))^{\top} \in \Pi_{h=0}^{2} \mathbb{R}^{n_h+1}$ be defined by

$$\bar{z}_{0k}(q) = z_{0k}(q) \qquad ,\ (0,k) \in I(0)$$

$$\bar{z}_{h0}(q) = \Sigma_{k=0}^{n_0} q_{0k} z_{0k}^h(\pi^h(q_0,q_h)) \qquad ,\ h = 1,2 \tag{2.4}$$

$$\bar{z}_{hk}(q) = z_{hk}(q) \qquad ,\ k = 1,\dots,n_h,\ h = 1,2.$$

Observe that for $h = 1,2$, and for all $q \in S$, (2.1) implies

$$q_h^{\top}\bar{z}_h(q) = \Sigma_{k=0}^{n_0}\pi_{0k}^h z_{0k}^h(\pi^h) + \Sigma_{k=1}^{n_h}\pi_{hk}^h z_{hk}^h(\pi^h) = 0. \tag{2.5a}$$

However, this complementarity condition does not hold for h=0. Then we get

$$q_0^{\top}\bar{z}_0(q) = \bar{z}_{10}(q) + \bar{z}_{20}(q). \tag{2.5b}$$

It follows straightforward from 2.1.i) and 2.1.iii) that $\bar{z}(q^*) = 0$ iff $(q_0^*, \bar{q}_1^*, \bar{q}_2^*) \in S^{n_0} \times \mathbb{R}^{n_1} \times \mathbb{R}^{n_2}$, with $\bar{q}_{hk}^* = q_{hk}^*/q_{h0}^*$ for $h = 1,2$, and $k = 1,\dots,n_h$, is an equilibrium price vector in $\bar{\mathbb{R}}^{N+1}$ (see also Example 2.1).

What about the interpretation of $\bar{z}$ and q? From (2.3) we see that q_{0k}, $(0,k) \in I(0)$, can be interpreted as the common good prices denoted in an international currency, e.g. ECU's, whereas the q_{hk}'s denote the prices of the domestic goods in the domestic currency. As we noted already, q_{h0} can be viewed upon as the value of one unit of the international currency in terms of the currency of country h. Thus, an increase of q_{h0} increases the common good prices in terms of the domestic currency. Moreover, because $\Sigma_{k\neq 0} q_{hk} = 1-q_{h0}$, an increase of q_{h0} also results in a decrease of the sum of the domestic good prices. With $1-q_{h0}$ as an index of the domestic price level, we can interprete $q_{h0}/(1-q_{h0})$ as the exchange rate. Thus, an increase (decrease) of q_{h0} yields a devaluation (revaluation) of the currency of country h.

Concerning the vector $\bar{z}(q)$, the elements $\bar{z}_{0k}(q)$ are the excess demands for the common goods, whereas $\bar{z}_{hk}(q)$, h = 1,2, and k ≠ 0, denote the excess demands for the domestic goods. From (2.4) we see that $\bar{z}_{h0}(q)$ is the value of the excess demand of country h for common goods, denoted in the international currency. In other words, it reveals the situation on the balance of payments. The element $\bar{z}_{h0}(q)$ being positive (negative) indicates that country h faces a deficit (surplus) on its balance.

Note that each excess demand $\bar{z}_{hk}$, (h,k) ≠ (1,0),(2,0), is related to a price q_{hk}, whereas the balance of the balance of payments of country h, h = 1,2, is related to q_{h0} being the component of q_h determining the exchange rate.

Thus, the reformulation of this model on the simplotope enriches the economic content of the model by the introduction of exchange rates and balances of payments. This enables us to pose a lot of questions that could not be dealt with in the standard model. For example, consider the situation in which a certain country has a fixed production capacity represented by its initial endowments. The country has to decide how to divide these endowments between the common market and its home market. With the reformulated model we can take for example the exchange rate as target variable. Then we can consider the problem how to divide the endowments in order to reach the target. In the sequel of the paper we discuss a price adjustment process for the model including exchange rates and balances. This process describes a way in which by adaptations of prices and exchange rates a shock in the economy is restored towards a new equilibrium.

3. The process

In this section we present a mathematical description of our price adjustment process. The process can start at any vector v in S and reaches a q^* at which $\bar{z}(q^*) = 0$ via a path of vectors q in S. The process is governed by v and the sign pattern of $\bar{z}(q)$ for vectors q on the path. Very roughly speaking, elements of q are increased (decreased) when the corresponding $\bar{z}$-element is positive (negative), while if an element of $\bar{z}$ is zero then the related element of q is adjusted such as to keep it in equilibrium. In the sequel we assume that v lies in the interior of S, i.e. $v_{hk} > 0$ for all $(h,k) \in I := I(0) \cup I(1) \cup I(2)$.

The sign vector related to x, notation sgn(x), is a vector s consisting of elements in $\{-1,0,+1\}$, where $s_i = +1$ (-1) if $x_i > 0$ (<0), while $s_i = 0$ if $x_i = 0$. Because of (2.5), the set $\mathcal{T}$ of feasible sign patterns of $\bar{z}$ is restricted. For example, it is impossible that the elements $\bar{z}_{hk}(q)$, $h = 1,2$, $k = 0,\ldots,n_h$, are all positive. In the sequel we first define $\mathcal{T}$ and related to each element in $\mathcal{T}$ we further define a primal and a dual set, both being subsets of S. A primal set states conditions on the location in S of its elements. It is here that the starting vector plays a major role. The corresponding dual set is induced by the sign pattern of the function values of its elements. The process then only considers vectors of S lying in the intersection of a primal and its corresponding dual set.

Let us formalize this. Each sign pattern of $\bar{z}$ is represented by a sign vector $s = (s_0^\top, s_1^\top, s_2^\top)^\top$ in $\Pi_{h=0}^{2} \mathbb{R}^{n_h+1}$. Thus, each element s_{hk}, $(h,k) \in I$, is an element in $\{-1,0,+1\}$. For each sign vector s and $h = 0,1,2$, we define the following subsets of indices,

$$I_h^-(s) = \{(h,k) \in I(h) \mid s_{hk} = -1\}$$

$$I_h^0(s) = \{(h,k) \in I(h) \mid s_{hk} = 0\}$$

$$I_h^+(s) = \{(h,k) \in I(h) \mid s_{hk} = +1\}.$$

Furthermore, we introduce some notation. With $s_h > 0$ we mean $s_{hk} > 0$ for all k in $\{0,1,\ldots,n_h\}$. If $s_{hk} \geq 0$ for all k and $s_{hk} > 0$ for at least one k we write $s_h \geq 0$, whereas $s_h \geqq 0$ indicates that s_h is a nonnegative vector. Accordingly, we define $<$, $\leq$ and $\leqq$. Now we are ready to define the set $\mathcal{T}$ of feasible nonzero sign vectors related to $\bar{z}$.

<u>Definition 3.1</u>. A sign vector s belongs to $\mathcal{T}$ if it satisfies the following conditions:

T1. there exists an $h \in \{0,1,2\}$ for which $I_h^+(s) \neq \emptyset$ and $I_h^-(s) \neq \emptyset$
T2. $I_h^+(s) = \emptyset$ iff $I_h^-(s) = \emptyset$, h = 1,2
T3. if $s_0 \geq 0$ $(s_0 \leq 0)$ then $s_{h0} = +1$ $(s_{h0} = -1)$ for at least one h in $\{1,2\}$
T4. if $s_0 = 0$ then $s_{h0} = 0$, h = 1,2.

The conditions T1, T2 and T3 reflect the conditions on $\bar{z}$ stated in (2.5) and the fact that $s \neq 0$. In fact, T2 reflects (2.5a) while T3 follows from (2.5b). Condition T1 results from T2, T3 and $s \neq 0$. Only condition T4 is imposed from outside the model. The reason for this will become clear later on. Observe that we implicitely consider only sign patterns of $\bar{z}(q)$ with q in the interior of S. For example, if q_1 equals (0,1,0) then the corresponding sign pattern of $z_1(q)$ equals (+1,0,+1) according to 2.1.iii) and (2.5a), which contradicts T2. In the sequel we show that our process never reaches a q with some element equal to zero.

Related to each $s \in \mathcal{T}$ we define a primal set P(s).

<u>Definition 3.2</u>. Let $s \in \mathcal{T}$ and $q \in P(s)$. Then, for some b, b_0 and a_h, h = 0,1,2, q satisfies the following conditions, where $0 \leq b \leq b_0 \leq 1$, and $a_h \geq 1$:

P1. for h = 1,2, and also for h = 0 if $s_0 \nleq 0$,

$$q_{hk} = a_h v_{hk} \qquad \text{if } s_{hk} = +1$$

$$bv_{hk} \leq q_{hk} \leq a_h v_{hk} \qquad \text{if } s_{hk} = 0$$

$$bv_{hk} = q_{hk} \qquad \text{if } s_{hk} = -1$$

P2. if $s_0 < 0$ then $q_0 = v_0$

P3. if $s_0 \leq 0$ and $s_0 \nless 0$,

$$b_0 v_{0k} \leq q_{0k} \leq a_0 v_{0k} \qquad \text{if } s_{0k} = 0$$

$$b_0 v_{0k} = q_{0k} \qquad \text{if } s_{0k} = -1.$$

To provide a better insight on these conditions we will make some remarks. If q is a vector in P(s) with $s_0 \nleq 0$ (P1) then all elements of q related to negative elements of s are, relative to v, minimal, i.e. $q_{hk}/v_{hk} = b = \min_I q_{i\ell}/v_{i\ell}$ for all (h,k) with $s_{hk} = -1$. In that case the elements q_{hk} with $s_{hk} = +1$ are, relative to v, maximal over all indices in I(h). More precisely, $q_{hk}/v_{hk} = a_h = \max_{I(h)} q_{h\ell}/v_{h\ell}$ if $s_{hk} = +1$. Elements of q related to zero sign vector elements vary between these bounds, i.e. $q_{hk}/v_{hk} \in [b, a_h]$ if $s_{hk} = 0$. From all this it implicitely follows that $q_0 = v_0$ if $s_{0k} = +1$ for all $(0,k) \in I(0)$. If q is a vector in P(s) with $s_0 < 0$ then q_0 equals v_0 (P2). In the case that $s_0 \leq 0$ while $s_0 \nless 0$ (P3), for all q in P(s) it holds that $q_{0k}/v_{0k} = b_0 = \min_{I(0)} q_{0\ell}/v_{0\ell}$ if $s_{0k} = -1$, whereas $q_{hk}/v_{hk} = b \leq b_0$ if $s_{hk} = -1$ for $(h,k) \in I(1) \cup I(2)$. We remark that b_0 may be larger than b. For example, when going from P2 to P3, i.e. one element of s_0 becomes zero, b_0 is equal to one. For illustration, compare the sets P((-1,-1),(1,-1),(-1,1)) and P((0,-1),(1,-1),(-1,1)) in Figure 3.1.

Now, suppose that for all h, $s_{hk} = +1$ for just one k, say k_h, while $s_{hj} = -1$ for all other $(h,j) \in I(h)$. Then the set P(s) is the line segment from v to the vertex R of S with $R_{hk_h} = 1$ and $R_{hj} = 0$ for all $j \neq k_h$, h = 0,1,2. Observe that along this ray the ratio between prices of commodities with corresponding negative s-component does not change, since all of them are decreased from v with the uniform factor 1-b. In fact, b decreases from 1 to zero when going from v to R. Of course we need a not uniform factor a_h to increase the prices of commodities $(h,k_h) \in I(h)$ in order to keep the sum of the components q_{hj}, $j = 0,\dots,n_h$, equal to one. When going from v to R, a_h increases from 1 to $1/v_{hk_h}$, h = 0,1,2.

Each sign vector s with $s_{hk} \neq 0$ for all $(h,k) \in I$ induces a ray. If neither $s_0 < 0$ nor $s_0 > 0$ such a ray points to a point $\bar{q}$ of S with

$$\bar{q}_{hk} = 0 \qquad \text{if } s_{hk} = -1$$

and

$$\bar{q}_{hk} = \bar{a}_h v_{hk} \qquad \text{if } s_{hk} = +1,$$

where $\bar{a}_h = (\Sigma_{(h,k)\in I_h^+(s)} v_{hk})^{-1}$. If $s_0 > 0$ or $s_0 < 0$ this remains true for all $\bar{q}_h$, $h \neq 0$, while $\bar{q}_0 = v_0$.

If $s_{hk} = 0$, the variable q_{hk} varies between the lower bound bv_{hk} (or $b_0 v_{hk}$) and the upper bound $a_h v_{hk}$. So, the dimension of P(s) increases with one if s_{hk} goes from nonzero to zero. However, if for some $h \neq 0$, $s_{h\ell}$ and s_{hp} become equal to zero while $s_{hj=0}$ for all $j \neq \ell, p$, the dimension increases with only one because we have the additional restriction $\Sigma_{k=0}^{n_h} q_{hk} = 1$. The related case for h = 0 is that the dimension does not increase if $s_{0\ell}$ becomes zero while $s_{0j} = 0$, $j \neq \ell$. So, we obtain that

$$\dim P(s) = 1 + \Sigma_{h=0}^{2}(|I_h^0(s)| - k_h(s)),$$

where $k_h(s) = 1$ if $|I_h^0(s)| = n_h + 1$ and $k_h(s) = 0$ otherwise. Here, $|A|$ denotes the cardinality of the set A.

It can happen that the primal sets corresponding to different sign vectors coincide. This is obvious for $s, \bar{s} \in \mathcal{T}$ with $s_0 > 0$, $\bar{s}_0 < 0$ while $s_h = \bar{s}_h$, $h = 1,2$. It also holds when $s \in \mathcal{T}$ is such that $s_{0\ell} = +1$ for just one ℓ, say ℓ_0 and $s_{0j} = 0$ for all $j \neq \ell_0$. Besides there is an $h \in \{1,2\}$ such that $s_{h0} = -1$. Then P1 says that for some q in P(s)

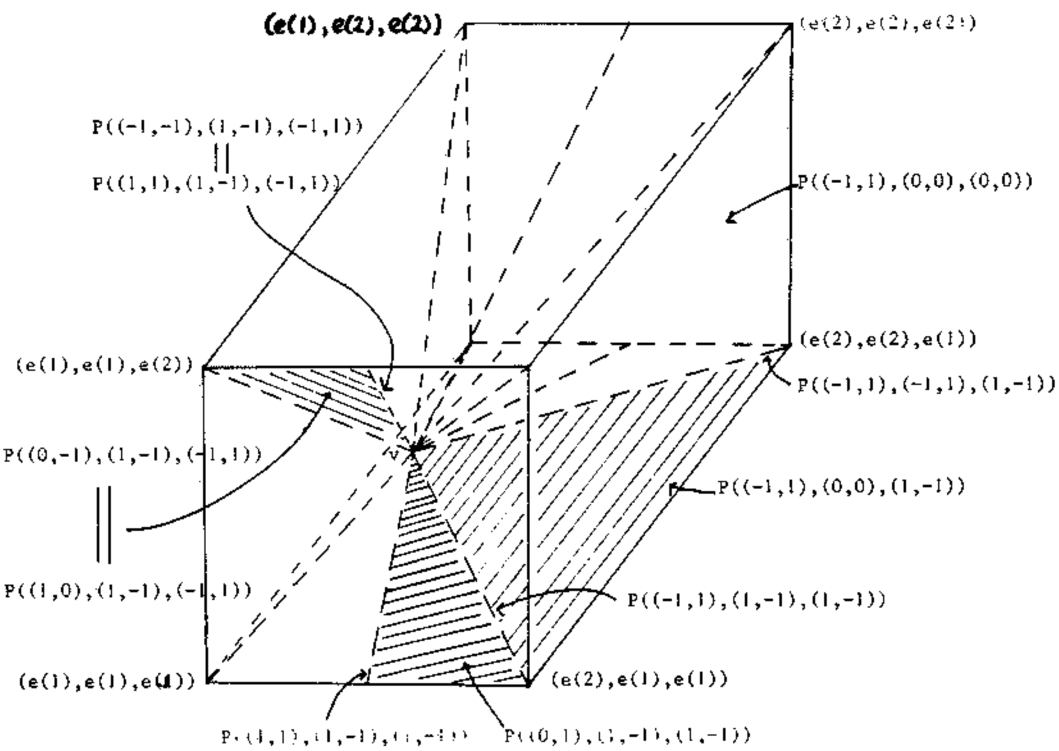
$$q_{0\ell_0} = a_0 v_{0\ell_0}$$

and

$$bv_{0j} \leq q_{0j} \leq a_0 v_{0j}, \; j \neq \ell_0.$$

Let λ_j, $j \neq \ell_0$ be such that $q_{0j} = \lambda_j v_{0j}$ and define $\lambda_k = \min_j \lambda_j$. Then with $b_0 = \lambda_k \geq b$ we have that q is also in $P(\bar{s})$ with $\bar{s}_{0k} = -1$, $\bar{s}_{0j} = 0$ for all $j \neq k$ and $\bar{s}_h = s_h$, h = 1,2 (compare P((0,-1),(1,-1),(-1,1)) and P((1,0),(1,-1)(-1,1)) in Figure 3.1).

Observe that the sets P(s) are completely determined by the starting vector v. In Figure 3.1 we give an illustration of some sets P(s), $s \in \mathcal{T}$, in case $S = S^1 \times S^1 \times S^1$, i.e. when there are two commonly traded goods and one domestic commodity for each country.



<u>Figure 3.1</u>. Illustration of sets P(s) in case $S = S^1 \times S^1 \times S^1$ and $v = ((1/2,1/2),(5/8,3/8),(2/3,1/3))^T$. e(1), e(2) denote the unit vectors (1,0) and (0,1) in $\mathbb{R}^2$.

Next we turn to the dual sets which are related to $\bar{z}$. More precisely, for each $s \in \mathcal{T}$ we define a dual subset $D(s)$ of S by

$$D(s) = C\ell(\{q \in S | \mathrm{sgn}(\bar{z}(q)) = s\}),$$

where $C\ell(A)$ denotes the closure of a set A. In the sequel we assume that $\bar{z}$ is a C^2-function. What about the dimension of $D(s)$, $s \in \mathcal{T}$? First recall that $\bar{z}$ is a C^2-function from S to $\Pi_{h=0}^{2} \mathbb{R}^{n_h+1}$ obeying three restrictions, i.e.

$$q_h^T \bar{z}_h(q) = 0 \qquad , h = 1,2$$

$$q_0^T \bar{z}_0(q) = \bar{z}_{10}(q) + \bar{z}_{20}(q).$$

Because of the first two restrictions we have that for $h = 1,2$, the (n_h+1) - vector $\bar{z}_h$ is fully determined by n_h of its elements. Moreover, by the last restriction, we have that the value of the excess demands for common goods must be equal to the sum of the balances of payments. Hence, we can regard $\bar{z}$ as an unconstrained function $\tilde{z}$ from S to the $\Sigma_{h=0}^{2} n_h$-dimensional set $\Pi_{h=0}^{2} \mathbb{R}^{n_h}$. Restricted to $D(s)$, we then may view upon $\tilde{z}$ as being a function from $D(s)$ to an $\Sigma_{h=0}^{2}(n_h - |I_h^0(s)| + k_h(s))$-dimensional subset of $\Pi_{h=0}^{2} \mathbb{R}^{n_h}$, say $\tilde{z}(D(s))$, where $k_h(s)$ is defined as before. The latter subset has a codimension equal to $\Sigma_{h=0}^{2}(|I_h^0(s)| - k_h(s))$. If $\tilde{z} : S \rightarrow \Pi_{h=0}^{2} \mathbb{R}^{n_h}$ is transversal to $\tilde{z}(D(s))$ then $D(s)$ is a C^2-manifold in S with a codimension in S which also equals $\Sigma_{h=0}^{2}(|I_h^0(s)| - k_h(s))$. Thus, under fairly general conditions $D(s)$ is a C^2-manifold of dimension $\Sigma_{h=0}^{2}(n_h - |I_h^0(s)| + k_h(s))$. Thus, the intersection $PD(s) := P(s) \cap D(s)$ is now either empty or has dimension equal to $\dim(P(s)) + \dim(D(s)) - \dim(S) = 1 + \Sigma_{h=0}^{2} (|I_h^0(s)| - k_h(s)) + \Sigma_{h=0}^{2}(n_h - |I_h^0(s)| + k_h(s)) - \Sigma_{h=0}^{2} n_h = 1$.

Thus, for some s the intersection of $P(s)$ and $D(s)$ not being empty is a 1-dimensional C^2-manifold, i.e. consists of disjoint paths and loops. In the rest of this section we argue that the set $\cup_{s \in \mathcal{T}} PD(s)$ contains under rather general conditions a piecewise C^2-path connecting v and a point q^* at which $\bar{z}(q^*) = 0$. By following this path we get a process of price adjusting. We note that each C^2-piece of the path lies in $PD(s)$ for some specific sign vector s and can be written as the solution curve to a system of differential equations. Thus, unlike the Walras' procedure and the method of Smale, we do not work with one differential system but instead we work with a sequence of such systems, one for each sign vector (see van der Laan and Talman [2]).

For the existence of the path we first require that $s^0 = \text{sgn}(\bar{z}(v))$ contains no zeros. This is not a great restriction. Note that $v \in PD(s^0)$ and $\dim(D(s^0)) = \dim(S)$. Thus, if $|I^0(s^0)| = 0$ then $PD(s^0)$ is a ray originating from v. Our process starts by following this ray from v.

In general, the process operates in a set PD(s), $s \in \mathcal{T}$, being a finite collection of paths and loops. In order to guarantee that the process never enters a loop we need that two sets PD(s), $PD(\bar{s})$, successively met on the path, intersect transversally. Also this is a generic property. Thus, under this condition the process either traverses a set PD(s) on the path along a curve segment connecting two points on the boundary of PD(s), or it stops in a point q^* with $\bar{z}(q^*) = 0$. In the former case we need that the boundary point via which PD(s) is left is the end point of a path in a set $PD(\bar{s})$, $\bar{s} \neq s$, $\bar{s} \in \mathcal{T}$. Because of the finite cardinality of $\mathcal{T}$ this guarantees the convergence of the process.

Let us become more precise. Note that the boundary of PD(s) can be written as

$$\text{bd}(PD(s)) = [\text{bd}(P(s)) \cap D(s)] \cup [P(s) \cap \text{bd}(D(s))].$$

When the process reaches a point q in the second part of bd(PD(s)) and hence reaches the boundary of D(s), we need that $q \in D(\bar{s})$, $\bar{s} \in \mathcal{T}$, with $\dim(D(\bar{s})) = \dim(D(s)) - 1$. Similarly, if $q \in \text{bd}(P(s))$, q has to lie in a set $P(\hat{s})$, $\hat{s} \in \mathcal{T}$, with dimension equal to $\dim(P(s)) - 1$. Note that a boundary point cannot lie in bd(S). This because $q \in P(s) \cap \text{bd}(S)$ implies $\bar{z}_{hk}(q) \leq 0$ and $q_{hk} = 0$ for some $(h,k) \in I$ which contradicts condition 2.1.iii).

Conditions guaranteeing that everything goes all-right when the process leaves PD(s) via a vector $\bar{q}$ with $\text{sgn}\, z(\bar{q}) = \bar{s}$ in bd(D(s)) can be spelled out in terms of conditions on the behaviour of $\bar{z}$ along the path. It is easy to verify that the following five possibilities exhaust the cases in which $\dim(D(\bar{s})) = \dim(D(s)) - 1$.

N1. There is an element $(0,p) \in I(0)$ s.t. $\bar{s}_{0p} = 0$ while $s_{0p} \in \{-1,+1\}$. Besides $\bar{s}_{hk} = s_{hk}$ for all $(h,k) \neq (0,p)$ with at least one element $(0,\ell) \in I(0)$ for which $\bar{s}_{0\ell} \neq 0$.

N2. There is an $h \in \{1,2\}$ and just one element $(h,p) \in I(h)$ s.t. $\bar{s}_{hp} = 0$, $s_{hp} \in \{-1,+1\}$, while $\bar{s}_{j\ell} = s_{j\ell}$ for all $(j,\ell) \neq (h,p)$. Besides there is a pair $\{(h,t),(h,r)\} \subset I(h)$ s.t. $\bar{s}_{ht} = +1$ and $\bar{s}_{hr} = -1$.

N3. There are an $h \in \{0,1,2\}$ and two elements (h,t), $(h,r) \in I(h)$ s.t. $\bar{s}_{ht} = \bar{s}_{hr} = 0$, $s_{ht} \cdot s_{hr} = -1$ while $\bar{s}_{h\ell} = s_{h\ell} = 0$ for all $\ell \neq t,r$, and $\bar{s}_{jk} = s_{jk}$ for all other $(j,k) \in I$. In case $h = 0$, $s_{j0} = 0$ for $j = 1,2$.

N4. There are an $h \in \{1,2\}$ with $s_{h0} \in \{-1,+1\}$ and an element $(0,p) \in I(0)$ with $s_{0p} = s_{h0}$ while $\bar{s}_{0p} = \bar{s}_{h0} = 0$ and $\bar{s}_{0\ell} = s_{0\ell} = 0$ for alle $\ell \neq p$, $\bar{s}_{j0} = s_{j0} = 0$ for $j \neq 0,h$, and $\bar{s}_{jk} = s_{jk}$ for all other elements $(j,k) \in I$. Besides, there are two elements (h,t), $(h,r) \in I(h)$ s.t. $s_{ht} \cdot s_{hr} = -1$.

N5. There is an element $(0,p) \in I(0)$ s.t. $s_{0p} \in \{-1,+1\}$, $\bar{s}_{0p} = 0$ and $s_{0k} = 0$ for all $k \neq p$, while $s_{10} \cdot s_{20} = -1$. For all elements $(h,k) \in I$ with $(h,k) \neq (0,p)$, $\bar{s}_{hk} = s_{hk}$.

Note that in the cases N1-N4 both s and $\bar{s}$ belong to $\mathcal{T}$. However, in case N5, $\bar{s}$ is not an element of $\mathcal{T}$, due to condition T4. Now we are ready to clarify the reason for that extra condition. Suppose we define $P(\bar{s})$ according to P1-P3. But then we have that $\dim(P(\bar{s})) = \dim(P(s))$, with $\bar{s}$ and s as in N5. Hence $PD(\bar{s})$ would be a collection of points instead of a 1-manifold. What to resolve this problem? When we discussed the definition of the primal sets we already argued that $\bar{q} \in bd(PD(s))$ with s as in N5 also lies in a set $P(\hat{s})$, with $\hat{s}_{0k} = -s_{0p}$ for some $k \neq p$, while $\hat{s}_{0\ell} = 0$, $\ell \neq k$. Because $\bar{q}$ also trivially lies in $bd(D(\hat{s}))$, the process continues from $\bar{q}$ via a path in $PD(\hat{s})$.

Next we consider the case when the vector $\bar{q}$ in $bd(PD(s))$ lies in $bd(P(s))$. We already argued that $\bar{q}$ has to lie in $P(\hat{s})$, for some $\hat{s}$ in $\mathcal{T}$, with $\dim(P(\hat{s})) = \dim(P(s)) - 1$. This is a kind of a nondegeneracy condition which has to be fulfilled on the path. Thus, it may not occur that a vector in a lower dimensional face of $bd(P(s))$ is generated. The cases that might occur are the reversals of those described in N1-N4.

4. Economic interpretation of the adjustments

In this section we describe how the prices along the path are adapted in order to reach an equilibrium situation. Broadly speaking, equilibrium on the domestic markets is achieved by adjustments of the domestic prices whereas equilibrium on the common market is reached via adjustments of both the international prices and the exchange rates. Adaptations of the exchange rates also restore equilibrium on the balances of payments. More precisely, an excess demand (supply) of a good induces an increase (decrease) of its price relative to the starting price. This tends to offset the imbalance. Similarly, a surplus (deficit) on the balance of payments leads to an appreciation (depreciation) of the currency in the related country. Such an appreciation (depreciation) makes for that country the common goods less (more) costly, which has also an impact on the common market. We remark that when we speak in the sequel about relative prices we mean relative to their initial values. The references P1,N1 etc. refer to Section 3.

At the starting vector v we assume that no balance or market is in equilibrium, i.e. all elements of $\bar{z}(v)$ differ from zero. From v, the prices of goods in excess supply are decreased while prices of goods in excess demand are increased. Besides, if a country has a surplus (deficit) on its balance of payments then its currency appreciates (depreciates) (see P1). In case all common goods reveal an excess supply (demand), their prices remain unaffected (P2 resp. P1). In that case the imbalances are attacked by appreciation (depreciation) of the national currencies.

In general, the relative prices of domestic goods in excess demand are maximal, whereas those of domestic goods in excess supply are minimal. Besides, the latter relative prices are equal among the countries. Relative prices of goods in equilibrium vary between these bounds (P1). All of this also holds for the common good prices except when no common good is in excess demand while at least one common good is not in equilibrium (see P2, P3). Although also in that case the relative prices of common goods in excess supply are equal, they might then be larger than the relative prices of domestic goods in excess supply. In the special case that all common goods are in excess supply, their prices equal the starting prices.

If along the path a good or balance becomes in equilibrium then it is kept in equilibrium by varying the corresponding relative price (exchange rate). In terms of Section 3 these are the cases N1 and N2 in which the boundary of a dual set is reached. However, if the relative price (exchange rate) of a good (balance) in equilibrium becomes equal to the relative prices of the goods in excess demand (supply) then the good (balance) is no longer kept in equilibrium. More precisely, its relative price (rate) is kept equal to the relative prices of the goods in excess demand (supply) and the good (balance) may become in excess demand (deficit) or excess supply (surplus). This case happens when the boundary of a primal set is reached.

Till sofar we described the basic behaviour of the price adjustment process. In the sequel we treat some special cases.

Concerning country h, h = 1,2, it can happen that two components of $\bar{z}_h$ simultaneously become equal to zero at the same q-vector. Under the nondegeneracy condition this can only occur when all other components of $\bar{z}_h$ are zero and hence country h becomes in equilibrium (N3). Also the reverse can occur. If a relative price (rate) in a country in equilibrium becomes equal to the relative prices of the goods in excess supply then the corresponding good (balance) is allowed to become in excess supply (surplus), while simultaneously the good (balance) with the highest relative price (rate) is allowed to become into excess demand (deficit). Similar things can occur on the common markets when both balances are in equilibrium. But on the common markets we can also have the situation in which some goods are in equilibrium while either all other goods are in excess supply (P3), or they are in excess demand (P1). In the latter case the adaptations are standard. In the former case the relative prices of the common goods in excess supply are lower than the relative prices of the common goods in equilibrium, but higher than the relative prices of the domestic goods in excess supply. If the relative prices of the common goods in excess supply become equal to the relative prices of the non common goods in excess supply, then the former relative prices are kept equal to the latter while the common good having the highest relative price may become into excess demand (change from P3 to P1). Of course the opposite situation occurs when the last common good in excess demand becomes in equilibrium (N1).

Next, we have to consider the cases in which, due to the connections between the common markets and the balances of the countries (2.5b), simultaneously things occur at the common markets and the balances. First, it can happen that the last common good and the last balance not being in equilibrium both become in equilibrium (N4). Then the corresponding price and rate are allowed to vary in order to keep them in equilibrium. Of course, also the opposite case is possible. In that case a rate becomes equal to the relative prices of the domestic goods in excess supply (demand). Then the rate is kept equal to this relative price and the balance may reveal a surplus (deficit). Simultaneously, the common good with the lowest (highest) relative price may become into excess supply (demand). Finally, it can occur that the last common good not being in equilibrium, becomes in equilibrium while not all balances are in equilibrium, i.e. there is a balance revealing a surplus while the other is in deficit (N5). When that common good was in excess supply (demand), the process continues by allowing the common good with the highest (lowest) relative price to become into excess demand (supply).

This completes the economic interpretation of the price adjustments made by our process. The most remarkable feature of our process in relation to other price adjustment processes is that it focusses on relative prices, i.e. prices relative to the starting price system. In this way the starting vector plays a very important role. At any point along the path the process keeps track of its position w.r.t. that starting vector. If necessary, this leads to disturbances of partial equilibria. The great progress resulting from this is that our process converges to an equilibrium under rather general conditions. Furthermore, it appears that our process describes price adjustments for a two country international trade model including adjustments of the exchange rates.

5. Numerical illustration

We consider a simple international trade model with two countries, 1 and 2. The consumers in each country are represented by one single consumer. Furthermore, each country has one domestic good (good (1,1) and (2,1) respectively), and there are two commonly traded goods (good (0,0) and (0,1)). The representative consumer in country 1 maximizes a Cobb-Douglas utility function $u^1(x_{00},x_{01},x_{11}) = x_{00}^{\alpha_1}x_{01}^{\alpha_2}x_{11}^{\alpha_3}$, $\alpha_i \geq 0$, $i = 1,2,3$, and $\alpha_1 + \alpha_2 + \alpha_3 = 1$. Here x_{00}, x_{01} and x_{11} denote the quantities consumed. Similarly, we have a utility function for the consumer in country 2, $u^2(x_{00},x_{01},x_{21}) = x_{00}^{\beta_1}x_{01}^{\beta_2}x_{21}^{\beta_3}$, $\beta_i \geq 0$, $i = 1,2,3$, and $\beta_1 + \beta_2 + \beta_3 = 1$. The budget constraints are induced by the initial endowments of the countries. For country 1 we have endowments $w^1 = (w^1_{00},w^1_{01},w_{11})^T$, and for country 2, $w^2 = (w^2_{00},w^2_{01},w_{21})^T$. In our numerical example we take $\alpha_i = \beta_i = 1/3$, $i = 1,2,3$.

On this model we performed two experiments. In the first we considered the way in which our process restored the equilibrium after the occurrence of a shock. We started the process in the old equilibrium price vector and considered the adaptations leading to the new equilibrium. After that we observed the consequences of the reversed shock. The most important issue is then the question whether the process follows the same path in reverse order.

The starting situation of the economy is the one with $w^1 = (100,60,80)^T$ and $w^2 = (0,40,20)^T$. The corresponding equilibrium price vector $q^* = (q^*_{00},q^*_{01},q^*_{10},q^*_{11},q^*_{20},q^*_{21})^T$ equals $(1/2,1/2,2/3,1/3,2/3,1/3)^T$. Now, a shock takes place in this economy resulting in a change in the initial endowments. More concrete, w^1 becomes $(0,60,180)^T$ whereas w^2 becomes $(50,40,50)^T$. Straightforward computation yields that the equilibrium price vector $\bar{q}$ corresponding to this new situation is equal to $\bar{q} = (2/3, 1/3,18/19,1/19,15/22,7/22) \cong (2/3,1/3, 0.947,0.053,0.682,0.318)^T$. How does our adjustment process bring the economy from q^* to $\bar{q}$? The process starts in the old equilibrium q^*. The excess demand vector $\bar{z}(q^*)$ after the shock equals $(\bar{z}_{00},\bar{z}_{01},\bar{z}_{10},\bar{z}_{11},\bar{z}_{20},\bar{z}_{21})^T = (76.54,26.54,49.92,-99.84,1.62, -3.24)^T$. At the start, both common goods are in excess demand, the balances are in deficit and the domestic goods are in excess supply. Thus, the process leaves q^* by devaluating the domestic currencies and decreasing equally the domestic good prices while the international prices of the common goods are kept fixed. The price adjustment process continues in this manner till the vector $(1/2,1/2,0.689,0.311,0.689,0.311)^T$ is reached at which country 2 becomes in equilibrium. The latter means that both the domestic market of country 2 and its balance become in equilibrium. From that vector on country 2 is kept in equilibrium, whereas the other adaptations are continued as before. Then, at the vector $(1/2,1/2,0.774, 0.226, 0.689,0.311)^T$, the second common good market becomes in equilibrium. The market for the first common good still reveals a surplus. Now, the process proceeds while also keeping the second common good in equilibrium and increasing the price of the first common good above that of the second till the new equilibrium vector is reached. At that latter vector country 1 becomes in equilibrium which also induces equilibrium on the first common market.

Next we considered the reverse case. We start from the situation with $w^1 = (0, 60,180)^T$, $w^2 = (50,40,50)^T$, and corresponding equilibrium $\bar{q} = (2/3,1/3,18/19,1/19,15/22,7/22)^T$ and assume a shock on this economy changing the endowments back to $w^1 = (100,60,80)^T$ and $w^2 = (0,40,20)^T$. How does our process adjust $\bar{q}$ to the new equilibrium $q^* = (1/2,1/2,2/3,1/3,2/3,1/3)^T$? The excess demand vector at $\bar{q}$ after the shock equals $\bar{z}(\bar{q}) = (-43.17,13.66,-25.99,467.76,1.76,-3.78)^T$. Thus, the shock results for country 1 in an excess demand situation on its domestic market whereas its balance turns into surplus. In country 2 the opposite occurs, i.e. its balance runs into deficit whereas the domestic market becomes in excess supply. Furthermore, the market for the first common good is in excess supply while the other common market is in excess demand. The process leaves $\bar{q}$ by decreasing relatively equal the first common good price and the domestic price in country 2 while relatively revaluating the currency of country 1 with the same factor. These adaptations are continued till country 2 becomes in equilibrium at the price vector $(0.5994,0.4006, 0.8516,0.1484,0.7141,0.2859)^T$. From that vector on country 2 is kept in equilibrium by adjusting the ratio between the price of the domestic good and its exchange rate whereas the other adjustments are continued as before. Next, the process reaches the vector $(0.585,0.415,0.832,0.168, 0.7055,0.2945)^T$ at which the market for the second common good reveals an equilibrium. By keeping this market and country 2 in equilibrium, increasing the second common good price relatively above the exchange rate in country 1, and simultaneously continuing with the previous adaptations the new equilibrium q^* is reached. At the latter vector also country 1 becomes in equilibrium and henceforth the market for the first common good.

It is interesting to observe from the experiments above that the path traced by the process from the second to the first equilibrium is not reverse to the path followed from the first to the second equilibrium situation. This appears to be a general feature of our process. It is mainly due to the fact that the price decreases are, relatively to the starting vector, equal among the markets. The differences occur because of the difference between the starting price vectors. Besides the sign patterns of the starting vectors are different.

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