1904 - 91

SERIE RESEARCH MEMORANDA

AN INSENSITIVE PRODUCT FORM FOR DISCRETE-TIME COMMUNICATION NETWORKS

by

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Research Memorandum 1989-91

December 1989

VRIJE UNIVERSITEIT
Faculteit der Economische Wetenschappen en Econometrie
AMSTERDAM

AN INSENSITIVE PRODUCT FORM FOR DISCRETE-TIME COMMUNUCATION NETWORKS

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Abstract

Discrete-time communication networks are studied as due to digital transmissions. Simultaneous multiple transmission requests and completions as well as blocking are hereby involved. Scheduling times and packet lengths are allowed to have a general discrete distribution.

A discrete-time product form result is obtained for the steady-state busy source distribution. Particularly, this form is shown to be insensitive, i.e. to depend only on means, in analogy with continuous-time results. The prooftechnique is of interest in itself as a multiple transition extension of continuous-time results. Typical applications include:

. CSMA, BTMA and Rude-CSMA -protocols
. Circuit switching structures
. MAN-allocation schemes.

Keywords Random access protocols * product form * insensitivity * Markov chain * CSMA/BTMA/MAN/Circuit switching.

1. Introduction

Random access protocols for communication networks have been widely introduced over the last decade for modeling purposes of actual communication protocols such as CSMA, BTMA or Rude-CSMA in interconnection networks, end-to-end access rules in circuit switching, or circuit allocation in metropolitan area networks. Closed product form results have hereby been detected or derived for various protocol classes.

In [7] standard circuit switching with limited trunkgroups is shown to have a product form. Earlier papers as [12] and [14] are of a related nature. In [4], [5] and [6] an extensive study is performed to determine necessary and sufficient product form conditions for multihop packet radio networks with CSMA/BTMA-type protocols. Non-exponential transmissions are hereby allowed. Also for multihop radio networks, in [15] the Rude-CSMA protocol is introduced with randomized blocking to take into account "hidden terminal effects" while in [2] and [9] link selective characteristics are allowed. These latter references provide a product form under exponentiality assumptions. In [18] a product form is reported for MAN which is also closely related to [12] and [14]. Recently, in [25] all the above results were unified and extended to a framework of interconnected sources with randomized blocking, delays and non exponential distributions.

Without exception though, all these results heavily rely upon a continuous-time modeling assumption. This enables one to use global or partial balance equations in which only one source can change its modus (idle in busy or vice versa) at a time. Present day communication, however, becomes more and more digitized. A discrete- rather than continuous-time modeling is thus required. This distinction will not generally be just a matter of some approximation order Δ, where Δ is the length of the time-slot, as a transmission itself may be of that order, say requiring only one or a small number of time slots.

This paper will provide a discrete-time closed product form expression in analogy with the above continuous-time results. This extension is non-trivial as simultaneous transitions are now to be taken into account so that standard partial balance principles do no longer apply.

Various discrete-time analogs of Jackson's product form have been reported over the last couple of years (cf. [1], [3], [8], [10], [16], [17], [20], [21], [26]). However, blocking or interference phenomena, which are crucial in random access protocols, are hereby excluded. Recently, in [3] the issue of batch movements in single-class queueing networks with blocking has been analyzed in an abstract continuous-time exponential framework. Product forms are hereby concluded provided the transition rates are of a special structure. Application of these results to random access schemes would at least require a multi-jobclass extension and discrete-time transformation, both of which are far from obvious.

Most notably, however, discrete-time (or batch movement) insensitivity results seem to be restricted to [8] and [10]. In these papers a discrete-time insensitive product form is reported for a Jackson network without blocking and a two station (central processor) model with a total capacity restriction. Further, the strong condition is thereby imposed that no more than one arrival or departure at a station can take place at the same time.

The present paper deals with source (job) interdependencies (blocking) and allows multiple transmission starts (arrivals) and completions (departures) at the same time. The proof in this paper therefore is of interest in itself but also for extension to more complex communication structures.

A variety of random access scheme examples for which the product form result applies are already given in [25]. For the purpose of illustration, however, some basic selected examples are briefly reviewed or added involving:

. CSMA, BTMA and Rude-CSMA protocols
. Circuit switching networks, and
. MAN-systems.

First in section 2 the model is described and the essential blocking condition is presented. The product form result is given in section 3, while the examples are given in section 4.

2 Model and condition

This section presents the model in a somewhat abstract or artificial formulation so as to avoid technical issues as well as to allow different interpretations at more technically detailed levels in a later stage as will be illustrated in section 4.

State description Consider a set of N transmitters, such as satellites, terminals or in/output devices, which will be called sources hereafter. Each source is alternatively in an "idle" (non-transmitting/scheduling) and "busy" (transmitting) mode as according to the protocol described below. A state $H=\{h_1,\dots,h_n\}$ represents that sources $h_1,\dots,h_n$ are busy. Write $\bar{H} = \{h|h\notin H\}$, $H+h = H \cup \{h\}$, $H-h = H/\{h\}$, $H+G = H \cup G$ and $H-G = H/G$ and denote by $\emptyset$ the state in which none of the sources is busy.

Idle-busy mechanism The time is slotted in fixed intervals of length Δ. Sources can change their status only at the end of a time slot as follows. When source h becomes busy it will require a "busy service" of k units with probability $q_h(k)$, $k\geq 0$. During one time slot it will actually have one of these units provided (or worked off) with a (success) probability ν_h. Similarly, when source h becomes idle it will require an "idle service" of k units with probability $p_h(k)$, $k\geq 0$. During one time slot it will actually have one of these units provided (or worked off) with (success) probability γ_h.

Note that the above description allows multiple sources to receive a unit service at the same time. Particularly, multiple sources may thus complete an "idle or busy service" at the same time and thus attempt to change their mode simultaneously. This, however, may give rise to blocking depending on the current state and source attempts as will be described in detail below. When blocking occurs all attempts are lost or equivalently all sources that complete an "idle service" will have to undergo or restart a totally new "idle service" and similarly for busy sources.

Blocking mechanism Suppose that at the end of a time-slot the group of busy sources is H, that a subgroup $G\subset H$ of these complete their "busy service" while a subgroup $G'\subset\bar{H}$ of idle sources complete their "idle service". Then with probability

$A(G'|H-G)$

all of these attempts are successful, so that the busy source configuration changes in $H-G+G'$, while with probability

$1 - A(G'|H-G)$

all of these attempts are blocked, so that the busy source configuration remains unchanged, that is H.

Blocking condition A condition upon the blocking function $A(.|.)$ is required. As this condition will be used in a two-fold manner later on two versions will be given. These versions can be shown to be equivalent similarly to the equivalence of reversibility and the Kolmogorov criterion as in [13].

(i) For all H and $G+h \subset \hat{H}$:

(1) $$A(G+h|H) = A(G|H)\, A(h|G+H)$$

(ii) For any H and $G = \{h_1, \ldots, h_m\} \subset \hat{H}$:

(2) $$A(G|H) = \prod_{k=1}^{m} A(h_{i_k}|h_i + h_{i_1} + h_{i_2} + \ldots + h_{i_{k-1}})$$

for all possible permutations $(i_1, \ldots, i_m) \in (1, \ldots, m)$.

Remark 2.1 Relatedly to the equivalence of the Kolmogorov criterion and reversibility as in [13], in order to verify (1) or (2) it suffices to find a function $P(.)$ such that for all H and $H+h$:

(3) $$P(H+h) = P(H)\, A(h|H)$$

Example (Coordinate Convex) A wide class of examples satisfying the blocking condition is given by

$$A(h+H) = 1(H+h \in C)$$

where $1(A)$ is 1 if event A is satisfied and 0 otherwise and where C is a set of states such that

(4) $$H \in C \Rightarrow H-h \in C \quad (\forall\, h \in H).$$

The set of admissible busy source configurations is hereby restricted to C. This type of blocking is known in the literature as "coordinate convex" (cf. [12], [14]). Sector 4 contains some concrete examples of this category. Particulary, note that in this case (3) is satisfied by:

$$P(H) = A(H|\emptyset) = \begin{cases} 1 & , H \in C, \\ 0 & , H \notin C. \end{cases}$$

Remark 2.2 (Possible extensions) In analogy with [22], also blocking probabilities $D(G|H)$ can be included which makes the "idle-busy" mechanism completely symmetric. Particularly, features as priority messages or breakdowns could hereby be modeled in certain situations. As another extension in analogy with [25], the idle and busy probabilities γ_h and ν_h can be made state dependent provided a condition similar to (1) or (2) is included. The present mechanism is kept restricted so as to concentrate on merely the essential novel aspect of discrete-time analysis.

Remark 2.3 (Total group retransmission) The protocol that upon blocking of any of the attempts, all attempts are blocked may naturally reflect for instance that only collisions but not the individual sources causing them can be detected by the system (e.g. as in slotted ALOHA or CSMA).

3 Product form result

Let

$$[(S_1,R_1),\ (S_2,R_2)]$$

with

$$(S_1,R_1) = ((g_1,s_1),\ldots,(g_m,s_m))$$

$$(S_2,R_2) = ((h_1,t_1),\ldots,(h_n,t_n))$$

be a generic notation for the state in which sources $S_1 = \{g_1,\ldots,g_m\}$ are currently idle with for source g_i still s_i units of idle service required up to completion of its current "idle service", $i = 1,\ldots,m$ and in which sources $S_2 = \{h_1,\ldots,h_n\}$ are currently busy with for source h_j still t_j units of "busy service" required up to completion, $j = 1,\ldots,n$. The state $[S_1,S_2]$ is the obvious notation for merely idle and busy source specification. Here one may note that $S_2 = H$ as per the notation of section 2. Further, we use the notation

$$[(S_1,R_1);(S_2,R_2)] - [(\alpha_1,R_1),(\alpha_2,R_2)] + [(\beta_1,R_1');(\beta_2,R_2')]$$

to denote that subgroups $(\alpha_1,R_1)\subset(S_1,R_1)$ and $(\alpha_2,R_2)\subset(S_2,R_2)$ are replaced by new groups (β_1,R_1') and (β_2,R_2') respectively. Particularly, we write $R_j' = 1$ if all components of R_j' are equal to 1 and symbolize by $R_j' = R_j+1$ that all components of R_j are increased by 1. Again, a notation $[S_1,S_2] - [\alpha_1,\alpha_2] + [\beta_1,\beta_2]$ is the obvious restriction to merely source specification. (One may note here that in this case necessarily either: $\beta_1 = \alpha_2$ and $\beta_2 = \alpha_1$, or: $[\beta_1,\beta_2] = [\alpha_1,\alpha_2]$). Let

$$\text{(5)}\qquad \begin{aligned} r_h(s) &= \gamma_h^{-1}\ \Sigma_{j=s}^{\infty}\ p_h(j) \\ v_h(t) &= \nu_h^{-1}\ \Sigma_{j=t}^{\infty}\ q_h(j) \end{aligned}$$

and note by standard calculus that

$$\text{(6)}\qquad \begin{aligned} \sigma_h &= \gamma_h^{-1}\ \Sigma_s\ s\ p_h(s) = \gamma_h^{-1}\ \Sigma_s\ r_h(s) \\ \tau_h &= \nu_h^{-1}\ \Sigma_t\ t\ p_h(t) = \nu_h^{-1}\ \Sigma_t\ r_h(t) \end{aligned}$$

We are now able to present the main result. To this end, first observe that the underlying probability structure implies that the process which keeps track of the number of residual "idle and busy service" requirements of all sources at time points $0, \Delta, 2\Delta, \ldots$ constitutes a discrete-time Markov chain. (In fact, one can also think of these residual numbers as numbers of residual phases where each phase itself has a geomeric distribution). Without loss of generality, assume that this Markov chain is irreducible at some set of states. (Clearly, this set is determined by the $A(.|.)$ access mechanism for busy source configurations). Two results will now be proven. The first is the technical one. The second is the practical consequence revealing an insensitive product form. For $H = (h_1, \ldots, h_n)$ let

$$(7) \qquad P(H) = \prod_{k=1}^{n} A(h_{i_k} | h_{i_1} + \ldots + h_{i_{k-1}})$$

for arbitrary permutation $(i_1, \ldots, i_n) \in (1, \ldots, n)$ as justified by (2). Further for $h = g_i \in S_1$ let $s_h = s_i$ and for $h = h_j \in S_2$ let $t_h = t_j$.

Result 1 *(Detailed product form) With c a normalizing constant*

$$(8) \qquad \pi([(S_1, R_1), (S_2, R_2)]) = c\, P(S_2) \prod_{h \in S_1} r_h(s_h) \prod_{h \in S_2} v_h(t_h)$$

Proof It suffices to verify the global balance equations:

$$(9) \qquad \pi(i)\, \Sigma_{j \neq i}\, p(i,j) = \Sigma_{j \neq i}\, \pi(j)\, p(j,i)$$

where i and j symbolize the different possible states and where $p(.,.)$ represents the corresponding one-step transition probabilities. Herein, transitions from a state into itself are excluded as these would equally contribute to both hand sides. To specify these equations in more detail, the following notation is used for any $[S_1, S_2]$, subset $\alpha \subset S_1$ and subset $\beta \subset S_2$:

$$(10) \qquad \begin{aligned} L(\alpha | S_1) &= \prod_{h \in \alpha} \gamma_h \prod_{h \in S_1 - \alpha} [1 - \gamma_h] \\ L(\beta | S_2) &= \prod_{h \in \beta} \nu_h \prod_{h \in S_2 - \beta} [1 - \nu_h] \end{aligned}$$

Consider a fixed state $[(S_1, R_1), (S_2, R_2)]$ as symbolized by i in (9). The left hand side (probability flux out state $[(S_1, R_1), (S_2, R_2)]$) of (9) then becomes:

$$(11) \qquad \sum_{G_1 \subset S_1, G_2 \subset S_2} \pi([(S_1, R_1), (S_2, R_2)])\, L(G_1)\, M(G_2),$$

while the right hand side (probability flux into state $[(S_1, R_1), (S_2, R_2)]$) equals:

(12)
$$\sum_{\substack{G_1 \subset S_1 \\ G_2 \subset S_2}} \sum_{\substack{\alpha_1 \subset G_1,\ \beta_1 = G_1 - \alpha_1 \\ \alpha_2 \subset G_2,\ \beta_2 = G_2 - \alpha_2}}$$

$$\Big\{ \pi([(S_1,R_1),(S_2,R_2)] - [(\alpha_1+\beta_1,R_1),(\alpha_2+\beta_2,R_2)]$$

$$+ [(\alpha_2,1),(\alpha_1,1)] + [(\beta_1,R_1+1),(\beta_2,R_2+1)]) \times$$

$$L(\alpha_2+\beta_1 | S_1-\alpha_1+\alpha_2)\ M(\alpha_1+\beta_2 | S_2-\alpha_2+\alpha_1)\ A(\alpha_2 | S_2-\alpha_2) \prod_{h\in\alpha_1} p_h(s_h) \prod_{h\in\alpha_2} q_h(t_h)$$

$$+$$

$$\pi([(S_1,R_1),(S_2,R_2)] - [(\alpha_1+\beta_1,R_1),(\alpha_2+\beta_2,R_2)]$$

$$+ [(\alpha_1,1),(\alpha_2,1)] + [(\beta_1,R_1+1),(\beta_2,R_2+1)]) \times$$

$$L(\alpha_1+\beta_1 | S_1)\ M(\alpha_2+\beta_2 | S_2)[1-A(\alpha_1 | S_2-\alpha_2)] \prod_{h\in\alpha_1} p_h(s_h) \prod_{h\in\alpha_2} q_h(t_h) \Big\}$$

By substituting (8) this can be rewritten as:

(13)
$$\sum_{\substack{G_1 \subset S_1 \\ G_2 \subset S_2}} \sum_{\substack{\alpha_1 \subset G_1,\ \beta_1 = G_1 - \alpha_1 \\ \alpha_2 \subset G_2,\ \beta_2 = G_2 - \alpha_2}}$$

$$\Big\{ P(S_2-\alpha_2+\alpha_1)[\prod_{h\in\beta_1} r_h(s_h+1) \prod_{h\in\beta_2} v_h(t_h+1) \prod_{h\in\alpha_1} v_h(1) \prod_{h\in\alpha_2} r_h(1) \times$$

$$\prod_{h\in S_1-\alpha_1-\beta_1} r_h(s_h) \prod_{h\in S_2-\alpha_2-\beta_2} v_h(t_h)] \times$$

$$L(\alpha_2+\beta_1 | S_1-\alpha_1+\alpha_2)\ M(\alpha_1+\beta_2 | S_2-\alpha_2+\alpha_1)\ A(\alpha_2 | S_2-\alpha_2) \prod_{h\in\alpha_1} p_h(s_h) \prod_{h\in\alpha_2} q_h(t_h)$$

$$+$$

$$P(S_2)[\prod_{h\in\beta_1} r_h(s_h+1) \prod_{h\in\beta_2} v_h(t_h+1) \prod_{h\in\alpha_1} r_h(1) \prod_{h\in\alpha_2} v_h(1) \times$$

$$\prod_{h\in S_1-\alpha_1-\beta_1} r_h(s_h) \prod_{h\in S_2-\alpha_2-\beta_2} v_h(t_h)] \times$$

$$L(\alpha_1+\beta_1 | S_1)\ M(\alpha_2+\beta_2 | S_2)[1-A(\alpha_1 | S_2-\alpha_2)] \prod_{h\in\alpha_1} p_h(s_h) \prod_{h\in\alpha_2} q_h(t_h) \Big\}$$

Now first conclude from (2) and (9):

$$P(S_2-\alpha_2+\alpha_1) = P(S_2)\ A(\alpha_1|S_2-\alpha_2)/A(\alpha_2|S_2-\alpha_2)$$

provided $A(\alpha_2|S_2-\alpha_2)>0$. This, however, follows from the fact that (S_1,S_2) is assumed to be an admissible configuration (otherwise global balance doesn't need to be verified in this state), so that the configuration $(S_1+\alpha_2,S_2-\alpha_2)$ is also admissible and thus, as the blocking condition requires $P(S_2) = A(S_2|\emptyset) = A(S_2-\alpha_2|\emptyset)\ A(\alpha_2|S_2-\alpha_2) > 0$, that $A(\alpha_2|S_2-\alpha_2) > 0$. Further, by (5):

$$r_h(1) = \gamma_h^{-1}$$

$$v_h(1) = \nu_h^{-1}$$

while by (10):

$$L(\alpha_2+\beta_1|S_1-\alpha_1+\alpha_2) = L(\alpha_1+\beta_1|S_1)[\prod_{h\in\alpha_2}\gamma_h][\prod_{h\in\alpha_1}\gamma_h]^{-1}$$

$$M(\alpha_1+\beta_2|S_2-\alpha_2+\alpha_1) = M(\alpha_2+\beta_2|S_2)[\prod_{h\in\alpha_2}\nu_h][\prod_{h\in\alpha_2}\nu_h]^{-1}$$

By substituting these expressions in (13), noting that $\alpha_1+\beta_1=G_1$, $\alpha_2+\beta_2=G_2$ and collecting the terms $A(\alpha_1|S_2-\alpha_2)+[1-A(\alpha_1|S_2-\alpha_2)]=1$ we obtain:

$$\Sigma_{G_1\subset S_1}\ P(S_1)\ L(G_1|S_1)\ M(G_2|S_2) \prod_{h\in S_1-G_1} r_h(s_h) \prod_{h\in S_2-G_2} v_h(t_h) \times$$

$$\Sigma_{\substack{\alpha_1\subset G_1,\ \beta_1=G_1-\alpha_1\\ \alpha_2\subset G_2,\ \beta_2=G_2-\alpha_2}} \prod_{h\in\alpha_1}[p_h(s_h)\ \gamma_h^{-1}] \prod_{h\in\alpha_2}[q_h(t_h)\ \nu_h^{-1}] \times$$

$$\prod_{h\in\beta_1} r_h(s_h+1) \prod_{h\in\beta_2} v_h(t_h+1)\ .$$

Noting that by (5):

$$[p_h(s_h)\gamma_h^{-1}] + [r_h(s_h+1)] = r_h(s_h)$$

$$[q_h(t_h)\nu_h^{-1}] + [v_h(t_h+1)] = v_h(t_h)$$

and interchanging summation and factorization, the summation in the last expression over all $\alpha_1\subset G_1$ and $\alpha_2\subset G_2$ can be rewritten as:

$$\prod_{h\in G_1} r_h(s_h) \prod_{h\in G_2} v_h(t_h)$$

By also substituting (8) in (11) we have thus proven that both (11) (the left hand side of (9)) and (12) (the right hand side of (9)) are equal to:

$$c\ P(S_2)\ \Sigma_{G_1 \subset S_1, G_2 \subset S_2} L(G_1|S_1)\ M(G_2|S_2) \prod_{h\in S_1} r_h(s_h) \prod_{h \in S_2} v_h(t_h) \qquad \square$$

Result 2 *(Insensitive busy source distribution). With $\bar{c}$ a normalizing constant:*

(14) $\pi(H) = \bar{c}\ P(H) \prod_{h\in H} \{\tau_h/\sigma_h\}$

Proof This follows by summing (8) over all possible residual numbers s_h and t_h for all sources h, recalling (6) and substituting $\bar{c} = c\{\sigma_1, \sigma_2, \ldots \sigma_N\}^{-1}$. $\square$

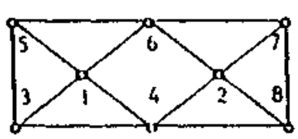
Remark (Group balance) One may note that the proof has actually been established by showing that for each group (G_1, G_2) separately the corresponding terms in (11) and (12) were equal. This observation is of interest as a multiple extension of the standard local- or job-local-balance in continuous-time systems which is known to be responsible for insensitivity results. The idea of group balance has been introduced in [21] and exploited in [3] and [10] as a responsible factor for product form results.

4. Applications

To illustrate the possible practical applications of the abstract model description and the blocking condition of section 2, this section provides some examples of present-day interest. For none of these a discrete-time insensitive product form has been reported in the literature. For each of them, however, the discrete-time product form result (25) turns out to have a similar form as their continuous-time analogues. The examples 4.1(i), (ii), 4.2(i) and 4.3(i) are all coordinate convex, so that P(.)=1 and S=C. Examples 4.1(iii), 4.2(ii) and 4.3(ii) involve a randomized blocking and P(.) as according to (7) takes a special form.

4.1 CSMA-protocols (cf. [2], [4], [5], [6], [9], [15], [23], [24], [25])

(i) CSMA Let the sources correspond to transmitters that can be graphically represented such that adjacent sources (neighbors) cannot be busy (transmit) at the same time. In practice this is achieved by the so-called "Carrier Sense Multiple Access" (CSMA)-scheme in which a transmitter senses the state of its channels just prior to starting a transmission and where upon sensing a busy channel from a neighbor the transmission is aborted (inhibited). For example, in the figure below a transmission from source 1 prohibits any source 3...6 to start a transmission.



With N(h) the set of neighbors of source h, the coordinate convexity condition (4) is guaranteed by

(15) $C = \{H\ |h_2 \notin N(h_1) \text{ for all } h_1, h_2 \in H\}$

(ii) BTMA (cf. [22]) In the above example the sources 1 and 2 which are outside hearing range can transmit at the same time. This will lead to a collision at nodes 4 and 6 which in turn will result in lost messages. This is known as the "hidden terminal problem". To eliminate this problem, the so-called Busy Tone Multiple Access (BTMA)-scheme has been introduced (cf. [22]). Under BTMA a node which senses a busy channel (in other words, which hears a transmitting neighbor) broadcasts a busy tone to all its neighbors to prevent idle neighbors from starting a transmission.

The set C from (15) now still applies (i.e., is coordinate convex), provided we replace N(h) by the set of all one and two-link neighbors (e.g. $N(5) = \{2,\ldots,7\}$).

In continuous-time the corresponding solution (25), along with necessary and sufficient conditions for arbitrary 0-1 CSMA blocking protocols to have this form, can be found in [4], [5], [6].

(iii) Rude-CSMA (cf. [15]) Another way to take into account the hidden terminal problem is introduced in [15] under the name of "rude-CSMA". In "Rude-CSMA" the access mechanism is randomized as according to

$$(16)\qquad A(h|H) = x^{N_0^h(H)}\, y^{N_1^h(H)}$$

where $N_0^h(H)$ and $N_1^h(H)$ are the numbers of idle (not transmitting) and busy (transmitting) neighbors from h when the sytstem is in state H and where x and y are given system parameters, with $0 \le x,y \le 1$. For instance $x=1$, $y=1$ corresponds to the ALOHA-protocol (no collisions), $x=1$, $y=0$ models the standard CSMA protocol (example 4.1(i)) and other values of x and y may reflect for instance that sensing of channels is not always reliable (cf. [25]). Condition (3) is easily verified with:

$$(17)\qquad P(H) = x^{-B_0(H)}\, y^{B_1(H)}$$

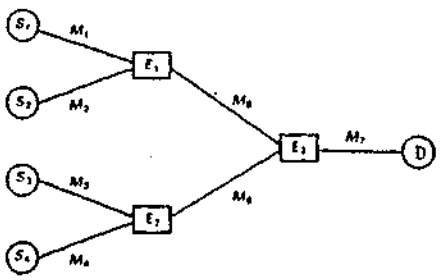
where

$$(18)\qquad \begin{array}{l} B_0(H)\text{: number of idle pairs of neighbors in state H} \\ B_1(H)\text{: number of busy pairs of neighbors in state H.} \end{array}$$

In continuous-time this solution was provided in [15] under exponentiality assumptions and source independent characteristics, extended in [9] to source dependent parameters generalized in [25] to non-exponential idle (scheduling) and busy (transmission) times.

4.2 Circuit switching

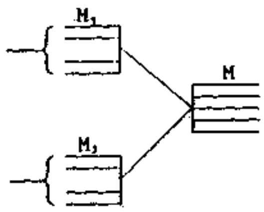
(i) Restricted trunkgroups (cf. [7], [14]) Consider a circuit switching network with 4 different types of sources with a fixed path along which a message from that source to the destination is to be transmitted. This transmission requires one trunk from each trunkgroup along this path. Interference thus arises with limited trunkgroups and messages using the same trunkgroups.



With M_i the number of trunks in trunkgroup i and n_i the number of busy sources of type i, the coordinate convexity condition is satisfied by C the set of states H such that

$$
\begin{aligned}
&n_i \le M_i \qquad (i=1,\dots,4)\\
&n_1 + n_2 \le M_5\\
(19) \qquad &n_3 + n_4 \le M_6\\
&n_1 + n_2 + n_3 + n_4 \le M_7 .
\end{aligned}
$$

<u>(ii) (Random gradings)</u> In analogy with the classical Engset random grading, assume a circuit switching as depicted below with two types of sources, M_i input channels for sources of type i and M common output channels.



Upon transmission request by a group of sources G with g_j sources of type j while the system is in state H with n_j sources of type j transmitting, this total group request is accepted with probability

$$(20) \qquad A(G|H) = \prod_{j=1,2} (M_j - n_j)\dots(M_j - n_j - g_j + 1)\ M_j^{-g_j}\, 1(n_1+n_2+g_1+g_2 \le M)$$

as corresponding to individual random selection probabilities of the form:

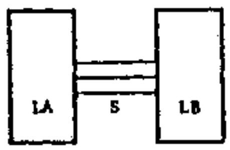
$$(21) \qquad A(h|H) = [(M_j - n_j)/M_j]\ 1(n_1+n_2<M)$$

for a source h of type j, where 1(A) = 1 if an event A is satisfied and 1(A) = 0 otherwise. Condition (1) or (2) is readily verified. Hence

$$(22) \qquad P(H) = A(H|\emptyset)$$

4.3 Interconnected Metropolitan Area Networks (MAN's) (cf. [16])

Consider a communication system with two groups of subscribers, say a group A and B with M and N subscribers, such as representing two metropolitan or local area networks. Both within a group and in between the groups communication between subscribers might be possible. To this end, number all subscribers 1,...,M+N and identify each possible connection from a source subscriber m to a destination subscriber n as a source (m,n). The description of section 2 now applies by saying that a connection is busy when a transmission along this connection takes place and idle otherwise.



(i) (Limited total number of circuits) (cf. [18]) For a given state H of busy connections let n_A, n_B and $n_{A,B}$ denote the number of busy connections within A, within B and in between A and B respectively. Assume finite numbers of LA and LB local circuits within A and B and S circuits in between A and B. Then the continuous-time model of [18] is extended to a coordinate convex discrete-time model by:

(23) $$C = \{H \mid n_A \le LA,\ n_B \le LB,\ n_{A,B} \le S\}$$

for the "dedicated allocation policy" with separate circuits for local and long-distance transmissions and by

(24) $$C = \{H \mid n_A \le LA+S,\ n_B \le LB+S,\ 0 \le n_{A,B} \le S-(n_A-LA)^+-(n_B-LB)^+\}$$

where $(y)^+=0$ for $y\le 0$ and y^+ for $y>0$, for the "shared allocation policy" in which the inter MAN circuits are shared among local and long-distance calls. As another shared allocation policy, each long-distance connection may require a local circuit within each local area, which is reflected by

(25) $$C = \{H \mid n_A+n_{A,B} \le LA,\ n_B+n_{A,B} \le LB,\ n_{A,B} \le S\}$$

(ii) (Error detection) In the examples (i) above, each long distance transmission may have to be registered before it can be started. However, the registration of each source separately, assumed to take place in negligible time, may lead to an error, say with probability p, in which case the total transmission request is to be rescheduled. Condition (3) for this example is directly verified by

$$A(h|H) = \begin{cases} p \text{ if } h=(m,n) \text{ with } m\in A,\ n\in B \text{ or } m\in B \text{ and } H+h\in C \\ 1 \text{ if } h=(m,n) \text{ with } m,n\in A \text{ or } m,n\in B \text{ and } H+h\in C \\ 0 \text{ otherwise, and} \end{cases}$$

(27) $$P(H) = p^{n_{A-B}}$$

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