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"STOP = RECIRCULATE"
FOR EXPONENTIAL PRODUCT FORM
QUEUEING NETWORKS WITH DEPARTURE BLOCKING

Nico M. van Dijk

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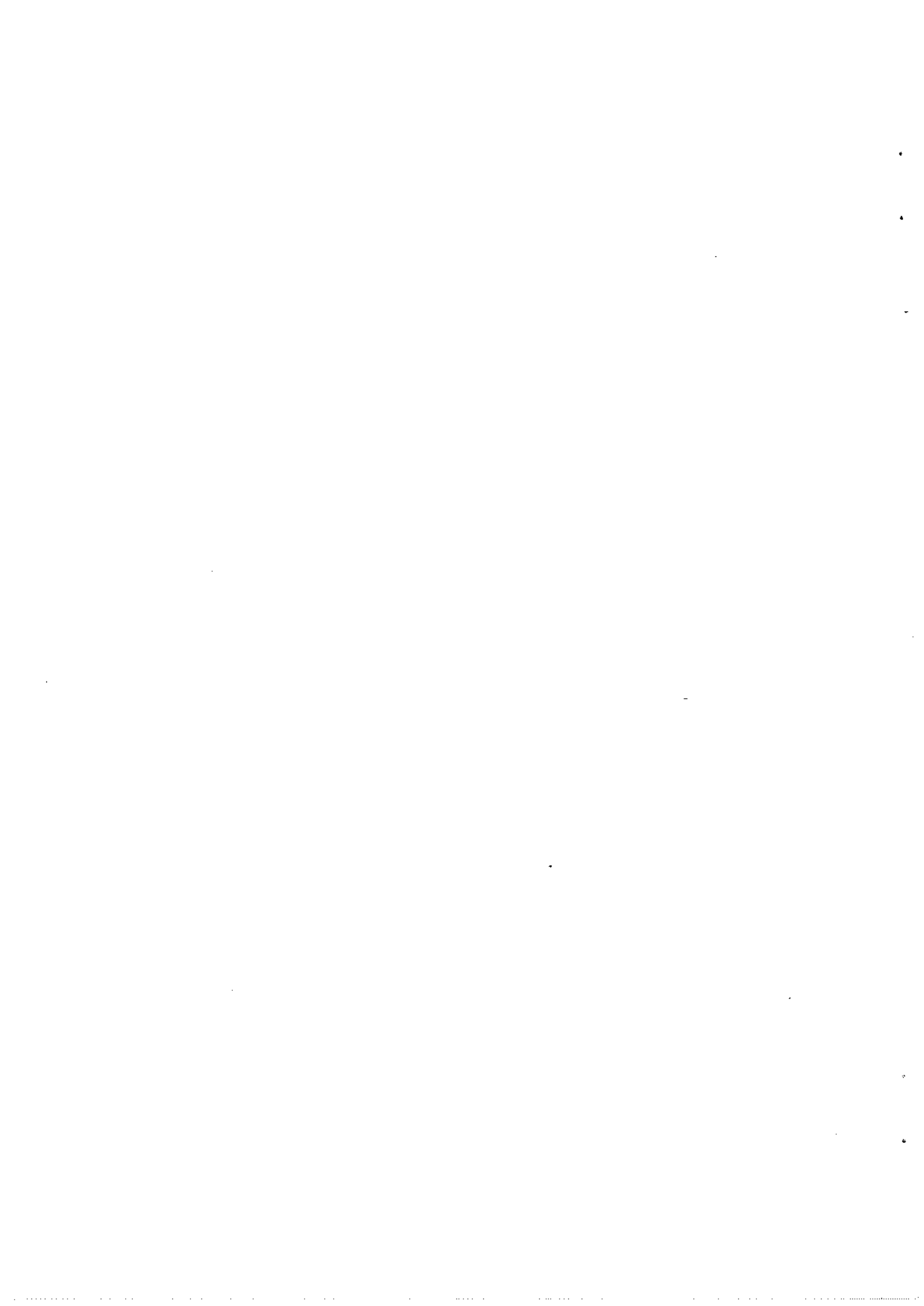


"Stop = Recirculate"
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Nico M. van Dijk
Free University, The Netherlands

Abstract The blocking protocols of "stopping service" and of "recirculating" jobs through an entire system are shown to be equivalent for Jacksonian queueing networks with blocked departures.

Queueing networks * product-form * stopping and recirculating blocking protocol * partial balance.



1 Introduction

Blocking phenomena in queueing networks arise most naturally in applications such as in telecommunication due to restricted links, in computer systems due to shared resources and in manufacturing due to finite storage buffers. Most notably among practical protocols in order are the "stop (or service)" and "repeating (or rejection)" communication protocol, in which services such as message transmissions are stopped (interrupted) respectively repeated upon blocking, and the "production (transfer or manufacturing)" protocol in which jobs continue their service and wait upon blocking (cf. [1], [13], [21]). In the exponential case equivalencies between these different protocols have been established (cf. [13]) and product form results for the stationary joint queue length distribution have been widely reported (e.g. [6], [7], [9], [14], [16], [17], [18], [20], [21]).

In practice, however, a total service may involve a number of service phases and is therefore no longer of an exponential form. For example, the total service may consist of a number of services at different service stations such as along an assembly line. Another protocol introduced therefore, which is to be seen as an extension of the repeating communication protocol to multi-stage services, is the "recirculate" protocol under which a blocked job is recirculated as a newly arriving job which has to undergo (repeat) a number of service stages, such as throughout an entire network. In his historical paper [10] Jackson already showed that his product form result was retained under the "recirculate" protocol (formulated by lost and triggered arrivals) when imposing a total network size constraint. This result was extended in [11] to multi-class queueing networks and class-interdependent blocking. For the "stop" protocol, however, no such results have been reported.

This note aims to illustrate that the "stop" and "recirculate" protocol are effectively the same when the system exhibits a product form. This result is of interest as:

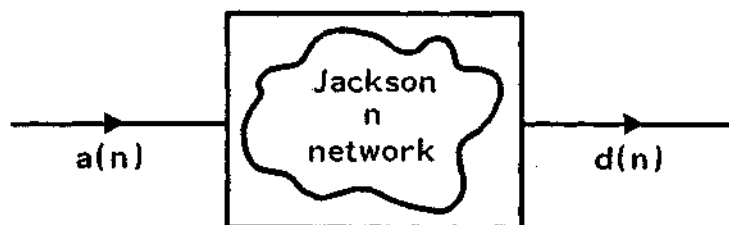
- (i) the "stop" protocol seems more practical
- (ii) the "stop" protocol simplifies product form verifications
- (iii) it formalizes the intuitive equivalence of state dependent global delay or geometrical repetition of multi-stage services.
- (iv) the "stop" protocol naturally leads to necessary blocking conditions to conclude a product form (see remarks 3.2 and 3.5).

More precisely, for a Jackson network with departure blocking we show:

1. the "stop" and "recirculate" protocol yield equal product forms.
2. station balance is responsible for this.

The insight that station or partial balance is responsible directly suggests generalizations. Two extending examples will be given: (i) the multi-class network with arrival and departure blocking from [11], and (ii) a network that consists of three finite Jacksonian clusters that are interconnected in a non-reversible manner.

2 Standard model



Consider a Jackson network with N service stations and:

- . exponential arrival rates $a(n)$ when n jobs are present
- . departure blocking function $d(n)$ when n jobs are present
- . routing probabilities p_{0j} for arriving jobs to route to station j
- . routing probabilities p_{ij} for jobs to route from station i to $j \leq N$
- . exponential single services with parameter μ_i at each station $i \leq N$.

In view of the departure blocking function two protocols are considered:

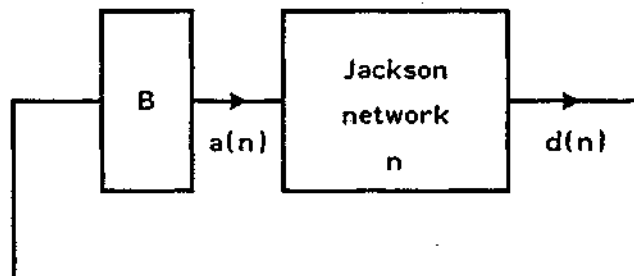
P1 (Stop) The service rate of each station is delayed by a factor $d(n) \leq 1$ whenever n jobs are present. Upon completion of service at station i a job leaves the system with probability $p_{i0} = 1 - [p_{i1} + \dots + p_{iN}]$. Particularly, for $d(n) = 0$ this means that all stations are stopped and can resume service only after an arrival.

P2 (Recirculate) When a job completes its service at station i it leaves the system with probability $p_{i0} d(n)$ while it routes to station $j \leq N$ with probability

$$p_{ij} + p_{i0} [1 - d(n)] p_{0j}.$$

The P2 protocol can be seen as the "triggering" protocol from [10] and [11] where a departure from the system triggers an instantaneous new arrival with probability $[1 - d(n)]$ when $n-1$ jobs are left behind.

Example



To illustrate the departure blocking function, let the Jackson network described above be connected to a service station (e.g. representing a finite source input), numbered station 0, as illustrated above. The total network contains a fixed total number of M jobs. Station 0 cannot contain more than B jobs and services at a rate $a(n)$ when it contains $M-n$ jobs. Then, with $1(A)$ denoting the indicator of an event A and n the total number of jobs in the Jackson network, the parametrization given above applies with

$$d(n) = 1(n \geq M - B).$$

Assumptions Without restriction of generality assume that there exists:

(i) a unique solution of the traffic equations

$$(1) \quad \begin{cases} \lambda_0 = \lambda \\ \lambda_j = \sum_{i=0}^N \lambda_i P_{ij} \end{cases}$$

(ii) unique stationary distributions $\{\pi_1(\tilde{n})\}$ and $\{\pi_2(\tilde{n})\}$ under protocol P1 and P2 respectively, restricted to some irreducible set S of feasible states $\tilde{n}=(n_1, n_2, \dots, n_N)$, denoting the number n_i at stations $i=1, \dots, N$, which with $n = n_1 + \dots + n_N$ is of the form

$$(2) \quad S = \{\tilde{n} \mid L \leq n \leq U\}$$

for certain numbers L and U, so that necessarily

$$a(U) = 0 \quad \text{if } U < \infty \quad \text{and} \quad a(n) > 0 \quad \text{for } L \leq n < U$$

$$d(L) = 0 \quad \text{if } L > 0 \quad \text{and} \quad d(n) > 0 \quad \text{for } L < n \leq U.$$

Theorem With c a normalizing constant, we have

$$(3) \quad \pi_1(\tilde{n}) = \pi_2(\tilde{n}) = c \prod_{k=L}^{n-1} [a(k)/d(k+1)] \prod_{j=1}^N (\lambda_j/\mu_j)^{n_j} \quad (\tilde{n} \in S)$$

Proof By $\tilde{n}+e_i$ or $\tilde{n}-e_i$ we denote the state equal to \tilde{n} with one job more or less, provided $n_i > 0$, at station i respectively. In order to better highlight the differences we will deal with both protocols simultaneously. To this end, let p denote the protocol under consideration with $p=1$ for protocol P1 and $p=2$ for protocol P2. As standard it suffices to verify the global balance equations which in turn are verified by showing that for each station separately:

"the rate out of a state due to a "departure" at that station =
the rate into that state due to an "arrival" at that station.

Here, for notational convenience below we consider a transition from a station into itself as both a departure and an arrival. Now fix a state $\bar{n} \in S$ and station j . Then, the rate out of this state due to a departure at station j equals

$$(4) \quad \begin{cases} \pi_1(\bar{n}) \mu_j d(n) & \text{for } p=1 \\ \pi_2(\bar{n}) \mu_j & \text{for } p=2 \end{cases}$$

while the rate into this state due to an arrival at station j is given by

$$(5) \quad \begin{cases} \pi_p(\bar{n}-e_j) a(n) p_{0j} + \\ 1(p=1) \sum_{i=1}^N \pi_1(\bar{n}-e_j+e_i) \mu_i d(n) p_{ij} + \\ 1(p=2) \sum_{i=1}^N \pi_2(\bar{n}-e_j+e_i) \mu_i (p_{ij} + p_{i0} [1-d(n)] p_{0j}) \end{cases}$$

By substituting

$$(6) \quad \pi_p(\bar{n}-e_j+e_i) = \pi_p(\bar{n}) [\mu_j/\mu_i] [\lambda_i/\lambda_j]$$

$$(7) \quad \pi_p(\bar{n}-e_j) a(n) = \pi_p(\bar{n}) [\mu_j/\lambda_j] d(n)$$

as according to (3), where $a(n)=0$ is excluded as we assumed $\pi(\bar{n})>0$, one directly concludes from the traffic equations (1) that for $d(n)>0$:

$$(8) \quad \pi_p(\bar{n}-e_j) a(n) p_{0j} / d(n) + \sum_{i=1}^N \pi_p(\bar{n}-e_j+e_i) \mu_i p_{ij} = \pi_p(\bar{n}) \mu_j.$$

For protocol 1, equality of (4) and (5) is hereby directly proven in any state \bar{n} with $d(n)>0$, while for a state \bar{n} with $d(n)=0$ it holds trivially as $\bar{n}-e_j$ is not reachable so that $\pi_1(\bar{n}-e_j)=0$.

For protocol 2, equality of (4) and (5) also follows from (8) provided

$$(9) \quad \sum_{i=1}^N \pi_2(\bar{n}-e_j+e_i) \mu_i p_{i0} [1-d(n)] = \pi_2(\bar{n}-e_j) a(n) [1/d(n)-1].$$

for any state \bar{n} with $d(n)>0$, as for \bar{n} with $d(n)=0$ and thus $\pi_2(\bar{n}-e_j)=0$:

$$(10) \quad \sum_{i=1}^N \pi_2(\bar{n}-e_j+e_i) \mu_i (p_{ij} + p_{i0} p_j) = \pi_2(\bar{n}) \mu_j$$

by virtue of (1) and (6) and the traffic relation, following from (1):

$$(11) \quad \sum_{i=1}^N \lambda_i P_{i0} = \sum_{j=1}^N \lambda_j - \sum_{j=1}^N \sum_{i=1}^N \lambda_i P_{ij} = \sum_{j=1}^N \lambda_0 P_{0j} = \lambda_0 = 1.$$

Relation (9), however, directly results from combining (6) and (7), noting that $1/d(n)-1 = [1-d(n)]/d(n)$ and using the traffic relation (11) again. \square

Remark 2.1 (Standard result?) Expression $\pi_2(\cdot)$ is a slight extension of the results by [10] and [11] as departure blocking probabilities rather than 0-1 values are allowed. Expression $\pi_1(\cdot)$ may seem a standard application of a Kelly network (cf. [9], [18], [20]) with state dependent service capacity functions of the form $\varphi(\bar{n}-e_i)/\varphi(\bar{n})$ at station i . However, in these as well as related references (e.g. [4], [12]) such capacity functions are implicitly assumed to be strictly positive for all feasible states with jobs present (see [18], p.193, theorem 2.2, p.194, p.195 or [20], p.119, fact. 3.7.4). Though the present result can be obtained from these frameworks if one carefully allows 0 capacity values in boundary states, no mentioning in any such direction has been made in these references. In contrast, theorem 3.4, p.200 in [20] states that "partial balance of the form 5", which is equivalent to the station balance employed above, "is inconsistent with the phenomenon of blocking". Proposition 3.5.5 in [18], furthermore, which does deal with networks with delays, just proves an opposite result when a fixed service delay d tends to 0. Beyond the rather technical proof of this proposition, our result is of the form $d \rightarrow \infty$, where d is state dependent, while the proof is straightforward.

Finally, for $d(\cdot)=0$ the particular example of this section can be seen as a reversible state space restriction if one regards the whole network as one station. However, as this network does not have a reversible routing itself, the truncation results from [9], section 1.6, [14] or [18], definition 3.7.2, which concern reversible systems do not apply. Moreover, as per the example in the next section, the product form and equivalence result extend to clusters connected by non-reversible routing.

Remark 2.2 (Partial balance and protocol equivalence) The equivalence proof is essentially based upon the same station or partial balance relations (8) for both protocols and the fact that the relations (9) and (10) are satisfied with $\pi_2(\cdot) = \pi_1(\cdot)$ substituted. These latter two relations, in turn, also come down to a station or partial balance interpretation reading that the rate into the exterior (to be seen as a station) is equal to the rate out of the exterior (that station). As partial balance notions are generally known to be responsible for product form type results, cf. [2], [3], [4], [7], [9], [12], [20], the equivalence result seems to be extendable to more complex product form networks. In the next section we will simply present two of such examples of special interest.

Remark 2.3 Note that the proof of the product form by equating (3) and (4) is simpler under the "stop" protocol. In this case it simply comes down to the standard balance equations for a network without blocking up to a scaling factor $d(n)$. Clearly, the complexity of the equations under the recirculate protocol will grow for more complex networks.

3 Two further examples

Rather than investigating to which extent the preceding equivalence result generalizes, which would require an extensive analysis of product form results, this section will simply present two more examples. These examples further support the relation of product forms or relatedly notions of partial balance and equivalence of stopping and recirculating protocols also in more complex situations. The proofs are omitted as these can be given along similar lines of substitution.

3.1 Multiple-classes (cf. [11])

Consider a Jackson network with N service stations and R fixed job-classes. Jobs of different classes arrive according to independent Poisson processes with parameter $\lambda^r(m^r)$ for class r when m^r jobs of this class are already present, where $\lambda^r(m) > 0$ for $m > 0$. The class-type of a job will be fixed throughout its residence in the system. Upon acceptance, a class- r job routes to station j with probability p_{0j}^r and after a service

completion at station i it routes to another station j with probability p_{ij}^r or attempts to leave the system with probability $1 - \sum_{j=1}^N p_{ij}^r$. The service stations are infinite server stations. A job of class r requires an exponential amount of service with parameter μ_i^r at station i .

To describe the arrival and departure blocking, let $\vec{m} = (m^1, \dots, m^R)$ denote the numbers m^r of class- r jobs present and introduce functions $A^r(\vec{m})$ and $D^r(\vec{m})$ for any class r , which can only take on values 0 and 1.

Arrival and departure protocol With \vec{m} denoting the configuration of jobs currently present, an arriving class- r job is:

$$\begin{cases} \text{rejected and lost} & \text{when } A^r(\vec{m}) = 0 \\ \text{accepted} & \text{when } A^r(\vec{m}) = 1. \end{cases}$$

and either one of the following departure protocols, in analogy with section 1, is employed:

P_1 (Stopping protocol) Servicing of class- r jobs is stopped throughout the entire network as long as $D^r(\vec{m}) = 0$.

P_2 (Recirculating protocol) Upon completing a service at station i a job of class r routes to station j with probability:

$$p_{ij}^r + p_{i0}^r [1 - D^r(\vec{m})] p_{0j}^r$$

while it leaves the system with probability

$$p_{i0}^r D^r(\vec{m}).$$

Assumptions In analogy with section 1 we make the assumptions:

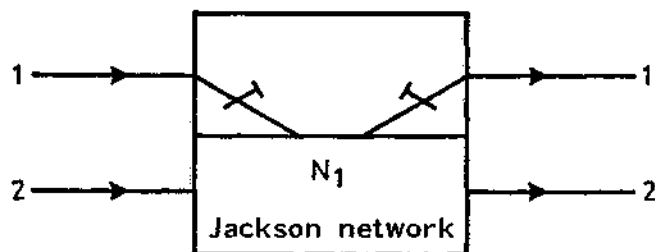
(i) For each $r=1, \dots, R$ there exists a unique solution $(\lambda_0^r, \dots, \lambda_N^r)$ of:

$$(12) \quad \lambda_j^r = \sum_{i=0}^N \lambda_i^r p_{ij}^r, \quad \lambda_0^r = 1.$$

(ii) There exist unique stationary distributions $\{\pi_1(\bar{n})\}$ and $\{\pi_2(\bar{n})\}$ under protocol P_1 and P_2 respectively, restricted to some irreducible set S of feasible states $\bar{n}=(\bar{n}_1, \dots, \bar{n}_N)$ where $\bar{n}_i=(n_i^1, \dots, n_i^R)$ denotes the number n_i^r of class- r jobs at station i , such that with $m^r=n_1^r+\dots+n_N^r$ for all r , $\bar{m}-e^r$ the vector equal to \bar{m} with one class- r job less and $V=(\bar{m}|\bar{n}\in S)$:

$$(13) \quad A^r(\bar{m}-e^r) = 0 \iff D^r(\bar{m}-e^r) = 0 \quad (\bar{m}, \bar{m}-e^r \in V).$$

Example. As a special example illustrating (13) consider 2 jobclasses where class-2 jobs are stopped to be served (as lower priority jobs) but also rejected upon arrival, when the number of class-1 jobs exceeds some threshold T . Illustratively,



Class-1 jobs thus receive strong priority over class-2 jobs when their number becomes too large. The corresponding parametrization is:

$$\begin{cases} A^1(\cdot) = D^1(\cdot) = 1 \\ A^2(\bar{m}-e^2) = D^2(\bar{m}) = 1(m^1 \leq T) \end{cases}$$

Result 2 With c a normalizing constant, we have for all $\bar{n} \in S$:

$$(14) \quad \pi_1(\bar{n})_i = \pi_2(\bar{n}) = c \prod_{r=1}^R \left\{ \sum_{k=0}^{n_i^r-1} \lambda_i^r(k) \right\} \left\{ \prod_{i=1}^N \frac{1}{n_i^r!} \left[\frac{\lambda_i^r}{\mu_i^r} \right]^{n_i^r} \right\}$$

Remark 3.1 (Extension) Under the recirculating protocol P_2 the above result is proven in [11] in a slightly more general form by allowing jobs to change their classnumber. This extension can easily be included but is excluded here so as not to distract the attention from the novelty: the protocol equivalence. Similarly, as in [11], processor sharing or last-come first-served preemptive disciplines could have been included as these preserve the balance principle per class at a given station. Only first-come first-served stations, also covered in [11], would require a somewhat more detailed specification as different job-classes at these stations have to be indistinguishable. The equivalence result, however, can be given also in that case.

Remark 3.2 (Stop protocol + condition (13)) The underlying insight of the stopping protocol is: "If at some service stage jobs are blocked to continue and thus to get out of a station, one should also avoid jobs to get into that station so as to preserve its station balance (both the station in- and outrate then become 0)". This principle repeats at each preceding station so that eventually along a total station trajectory out- and inrates are to be avoided. In particular, exterior arrivals are then to be excluded also. We thus conclude: $D^F(\vec{m})=0 \Rightarrow A^F(\vec{m}-e^x)=0$. A similar arguing holds for the reverse direction. Roughly speaking, by employing the stopping principle necessary (and usually sufficient) conditions are thus concluded naturally.

3.2 Closed networks of finite Jackson clusters (cf. [17])

Consider a single-class closed queueing network with M jobs and N service stations that are partitioned in P fixed station clusters, labeled C_1, \dots, C_p , such that cluster C_p cannot contain more than N_p jobs.

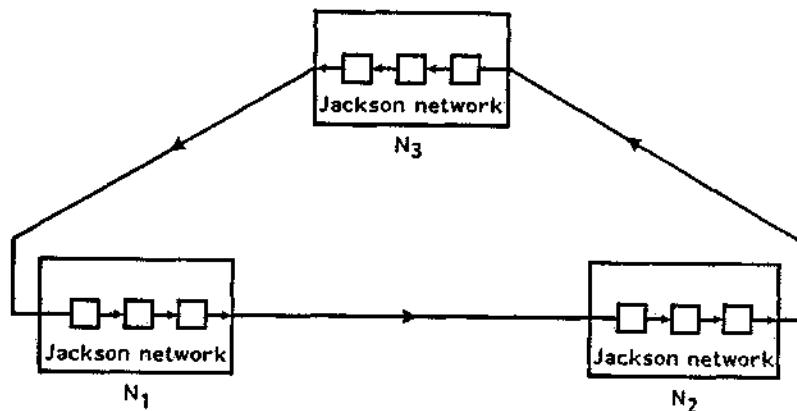
The scheduled routing probabilities (that is, disregarding blocking consequences as described below) from stations i to j are given by

$$(15) \quad p_{ij} = \begin{cases} p_{ij}^p & i, j \in C_p \\ p_{i0}^p R_{pq} p_{0j}^q & i \in C_p, j \in C_q \text{ with } p \neq q \end{cases}$$

where p_{ij}^p and p_{oj}^q are arbitrary probabilities and $p_{io}^p = [1 - \sum_{j \in C_p} p_{ij}^p]$. In words that is, routing from one cluster to another is station independent, while upon arrival at and within a cluster arbitrary routing is allowed.

The services are assumed to be exponential with parameter μ_i at station i while each station contains a single server. The illustrative example below, in which the routing between the clusters is cyclic, visualizes that not only the routing within a cluster (as in a standard Jackson network) but also in between clusters is allowed to be non-reversible.

Example Consider a network of 3 clusters with finite capacity N_p for cluster $p=1,2,3$. Upon leaving a cluster p a job routes to the next cluster $p+1$, i.e. $R_{12} = R_{21} = R_{31} = 1$.



Protocols In view of the finite capacity limitations the following protocols are considered.

P₁ (Stop) As long as one of the clusters is saturated, the servicing at all stations outside this cluster is stopped.

P₂ (Recirculate) As long as one of the clusters is saturated a job which completes its service at some station i within another cluster, say q , cannot leave this cluster and routes to station j within this cluster q with probability:

$$P_{ij}^a + P_{i0}^a P_{0j}^a.$$

For illustration, in the cyclic 3-cluster example above, upon saturation of cluster 3 when $m_3=N_3$, under P_1 the servicing at any station of clusters 1 and 2 is stopped, while under P_2 servicing at these clusters is continued but a job wishing to leave its cluster is recirculated as a newly arriving job at that cluster. Note that either protocol avoids two (or more) clusters to become saturated at the same time.

Assumption Let the traffic equation (1) with $\lambda_0=0$ have a unique probability solution $(\lambda_1, \dots, \lambda_N)$.

Denote by S the set of feasible states (which is the same under either protocol) and by $\{\pi_1(\vec{n})\}$ and $\{\pi_2(\vec{n})\}$ the unique stationary distributions under P_1 and P_2 respectively, with $\vec{n}=(n_1, \dots, n_N)$ the vector of queue lengths.

Result 3 With c a normalizing constant, we have:

$$(16) \quad \pi_1(\vec{n}) = \pi_2(\vec{n}) = c \prod_{i=1}^N (\lambda_i/\mu_i)^{n_i} \quad (\vec{n} \in S).$$

Remark 3.3 (Literature) Under P_1 the result can be concluded directly from [17] and indirectly from [7] and [16]. Under P_2 the result seems to be totally new.

Remark 3.4 (Extensions) Clearly, extensions where in addition to the protocols above each cluster itself can have a mechanism as in section 2 or 3.1 are possible. As another extension, mixing of protocols P_1 and P_2 for different clusters is allowed without affecting the product form. For instance in the above example, stopping cluster 1 and recirculating jobs at cluster 2 when cluster 3 is saturated will retain the product form. Finally, similar results can be provided for open versions.

Remark 3.5 (Stop protocol \rightarrow blocking condition) Once again, also see remark 3.2, note that the station balance principle in combination with the stopping protocol directly leads to the condition that in the example above not only cluster 2 but also cluster 1 should effectively be stopped when cluster 3 is saturated.

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