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A DISCRETE-TIME PRODUCT FORM
FOR RANDOM ACCESS PROTOCOLS

by

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A DISCRETE-TIME
PRODUCT FORM FOR RANDOM ACCESS PROTOCOLS

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Abstract A discrete-time extension is obtained of recent product form results for random access schemes. Applications include:

. CSMA/BTMA and Rude-CSMA protocols
. Circuit switching structures
. MAN-systems.

1 Introduction

Since the introduction of the ALOHA-system in the early seventies, a large variety of random access protocols for (tele)communication networks has been proposed and implemented over the last decade (cf. [4], [19], [20], [21]). Particularizing to CSMA-protocols and variants as BTMA (cf. [19]) or rude-CSMA (cf. [13]) which take into account the "hidden terminal problem", explicit product form results have been established (cf. [2], [4], [5], [6]). Recently, by introducing randomized blocking functions and under a so-called protocol invariance condition, these results were unified and generalized in [22].

All these product form results, however, have been obtained under a continuous time modeling assumption. This has the simplifying consequence that only one source can change its status at a time. Present-day communication in contrast becomes more and more digitized and is thus actually to be analyzed in discrete-time. This distinction is most crucial as a digital transmission may require only one or a few time slots. The maximum throughput of the standard slotted ALOHA-protocol, for example, is known to be doubled by halving the duration of time slots. The major complication here is that more than one source may wish to change its status at the end of one and the same time slot, so that collisions for instance may arise.

This note will give a discrete-time extension of the results in [22]. As simultaneous transitions are to be taken into account, this extension is non-trivial since standard partial balance principles for continuous-time analysis do no longer apply. Several discrete-time analogues of Jackson's celebrated product form have been reported over the last couple of years (cf. [14], [15], [18], [23]). Most essentially, however, blocking or interference phenomena, which are most essential in random access schemes, have hereby remained untouched. Only recently, the issue of multiple transitions and blocking has been addressed by the author in a joint report but in a continuous-time setting (cf. [3]). Product form results are herein concluded provided the transition rates exhibit a particular functional form. Whether a particular concrete random access protocol such as rude-CSMA has this form, and whether and how the results transform to a discrete-time setting, as of interest in this paper, hereby remains unanswered.

More precisely, to the best of knowledge, the prooftechnique that will be followed is new in that the global balance equations, which are much more complicated that the continuous-time analogues, are verified by inductively proving a multiple partial balance notion. An illustration of present-day applications is given. Particularly, randomized protocols are hereby included and explicit discrete-time product form analogues of continuous-time results are obtained. Applications are included of:

. CSMA-protocols such as BTMA and Rude-CSMA
. Circuit switching structures, and
. MAN-systems.

2 Model and result

This section presents the main result in an abstract formulation so as to avoid technicalissues and to allow different interpretations later on. Illustration of practical applications will be given in section 3.

State description Consider a set of N transmitters, such as satellites, terminals or in/output devices, which will be called sources hereafter. Each source is alternatively in an "idle" (non-transmitting/scheduling) and "busy" (transmitting) mode as according to the protocol described below. A state $H=\{h_1,\ldots,h_n\}$ represents that currently sources $h_1,\ldots,h_n$ are busy. Write $\bar{H} = \{h|h\notin H\}$, $H+h = H \cup \{h\}$, $H-h = H/\{h\}$, $H+G = H \cup G$ and $H-G = H/G$ and denote by $\emptyset$ the state in which none of the sources is busy.

Idle-busy mechanism The time is slotted in fixed intervals of length Δ and a change of state is possible only at the end of a time slot. More precisely, at the end of a time slot an idle source h attempts to become busy with probability λ_h while a busy source h attempts to become idle with probability μ_h, independently of the other sources. Particularly, note that more than one source is thus allowed to change its mode at the same time. These attempts, however, can be blocked as per the description below.

Blocking mechanism Assume that in state H a group of idle sources $G' \subset \bar{H}$ attempts to become busy, which occurs with probability

$$L(G') = \prod_{\{h \in G'\}} \lambda_h \prod_{\{h \in \bar{H}-G'\}} [1-\lambda_h]$$

while a group $G \subset H$ of busy sources attempts to become idle, which occurs with probability

$$M(G) = \prod_{\{h \in G\}} \mu_h \prod_{\{g \in H-G\}} [1-\mu_h].$$

Then with probability

$$A(G'|H-G)$$

all these attempts are successful so that the state changes in H-G+G' while with probability

$$1 - A(G'|H-G)$$

all of these attempts are blocked in which case the state remains unchanged, that is H.

Blocking condition For all H and $G \subset \bar{H}$, we have

$$A(G|H) = A(h|H)\, A(G-h|H+h) \qquad (h \in G) \tag{2.1}$$

Remark 2.1 (Condition 2.1) Condition (2.1) reflects that the success probability for a group of sources to become busy is actually determined by letting the sources attempt to become busy one after the other in a given order. This probability furthermore has to be independent of the order. More precisely, similarly to the Kolmogorov criterion for reversibility (cf. Kelly [11]) one easily verifies that (2.1) is equivalent to:

Invariance condition For any $H, G=(g_1,\dots,g_m) \subset \bar{H}$ and all permutations $(k_1,\dots,k_m) \in (1,\dots,m)$, we have:

$$A(G|H) = \prod_{i=1}^{m} A(g_{k_i} | H+g_{k_1}+\dots+g_{k_{i-1}}) \tag{2.2}$$

Special case 2.1 (Coordinate convex) As an important class of examples satisfying condition (2.1) let

$$A(h|H) = 1(H+h \in C)$$

where C is a set of states such that (2.3)

$$H \in C \Rightarrow H-h \in C \qquad (h \in H).$$

As a consequence, the admissible states are restricted to the set C. In correspondence with literature (cf. [10], [12]), this set or blocking protocol is called "coordinate convex". In the next section various concrete "coordinate convex" examples will be given. □

Without restriction of generality, now assume that the system is irreducible at some set S with unique stationary distribution $\{\pi(H) | H \in S\}$ and define

$$P(H) = \prod_{i=1}^{n} A(h_{k_i} | h_{k_1}+\dots+h_{k_{i-1}}) \tag{2.4}$$

which is well-defined regardless of the chosen permutation $(k_1,\dots,k_n) \in (1,\dots,n)$ by virtue of condition (2.1) or equivalently (2.2). The following key-result is then obtained. Its proof will be given at the end of the section.

Theorem 2.1 *With c a normalizing constant, we have for all* $H \in S$:

$$\pi(H) = c\ P(H) \prod_{(h \in H)} \left[\frac{\lambda_h\ [1-\mu_h]}{\mu_h\ [1-\lambda_h]} \right] \tag{2.5}$$

Special case 2.2 (Coordinate convex) Under (2.3), we have S=C and

$$P(H) = 1 \qquad (H \in C) \tag{2.6}$$

Remark 2.2 (Possible extensions) In analogy with [22] also blocking probabilities $D(G|H)$ can be included which makes the "idle-busy" mechanism completely symmetrical. Particularly, features as priority messages or breakdowns could hereby be modelled in certain situations. As another extension in analogy with [22], the idle and busy probabilities λ_h and μ_h can be made state dependent provided a condition similar to (2.1) or (2.2) is included. The present mechanism is kept restricted so as to concentrate on merely the essential novel aspect of discrete-time analysis.

Remark 2.3 (Total group retransmission) The protocol that upon blocking of any of the attempts, all attempts are blocked may naturally reflect for instance that only collisions but not the individual sources causing them can be detected by the system (e.g. as in slotted ALOHA or CSMA).

Proof of the theorem We need to verify the global balance (or forward Kolmogorov) equations. For a given state $H \in S$ and recalling the shorthand notation $M(G)$ and $L(G')$, these are given by

$$\pi(H) \Sigma_{G \subset H, G' \subset \bar{H}} \; M(G) \; L(G') \; A(G'|H-G) =$$

$$\Sigma_{G' \subset \bar{H}, G \subset H} \; \pi(H-G+G') \; M(G') \; L(G) \; A(G|H-G), \tag{2.7}$$

where the expressions corresponding to a blocking are deleted as they would contribute equally to both the left and right hand side of (2.7). These equations in turn are verified by showing that for each pair of groups $G \subset H$ and $G' \subset \hat{H}$ separately:

$$\pi(H) \; M(G) \; L(G') \; A(G'|H-G) = \pi(H-G+G') \; M(G') \; L(G) \; A(G|H-G). \tag{2.8}$$

This will be proven by induction to $s=n(G)+n(G')$ with $n(M)$ the cardinality of M. Clearly, (2.8) holds for s=0. Assume that (2.8) holds for all G and G' with $s=n(G)+n(G')=m$. As $n(G)$ or $n(G')$ can be equal to 0, in order to prove (2.8) for $s=m+1$ we need to distinguish the two situations

(i) $n(G+h) + n(G') = m + 1$ for some G, G' and $h \in H-G$.

(ii) $n(G) + n(G'+h) = m + 1$ for some G, G' and $h \in \bar{H}-G'$.

(i) ($G \to G+h$) From the definition of $M(G)$ and expressions (2.4) and (2.5), we obtain

$$\begin{aligned} L(G+h) &= L(G)\ \lambda_h [1-\lambda_h]^{-1} \\ M(G+h) &= M(G)\ \mu_h [1-\mu_h]^{-1} \end{aligned} \tag{2.9}$$

$$\pi(H) = \pi(H-h)\ A(h|H-h)\ \lambda_h [1-\mu_h]\ \mu_h^{-1} [1-\lambda_h]^{-1}. \tag{2.10}$$

By substituting these relations, applying the induction hypothesis (2.8) for G and G' replaced by $H-h$ and recalling condition (2.1), we find

$$\begin{aligned} &\pi(H)\ M(G+h)\ L(G')\ A(G'|H-(G+h)) = \\ &\pi(H-h)\ A(h|H-h)\ \lambda_h [1-\lambda_h]^{-1}\ M(G)\ L(G')\ A(G'|H-G-h) = \\ &\pi(H-G-h+G')\ \lambda_h [1-\lambda_h]^{-1}\ L(G)\ M(G')\ A(G|H-G-h)\ A(h|H-h) = \\ &\pi(H-G-h+G')\ M(G')\ L(G+h)\ A(G+h|H-G-h) \end{aligned} \tag{2.11}$$

which proves (2.8) with G replaced by $G+h$ and where $n(G+h)+n(G') = m+1$.

(ii) ($G' \to G+h$) Similarly, by directly using condition (2.1), substituting (2.9) with G replaced by G' as well as

$$\pi(H-G+G'+h) = \pi(H-G+G')\ A(h|H-G+G')\ \lambda_h [1-\mu_h] \mu_h^{-1} [1-\lambda_h]^{-1}, \tag{2.12}$$

and applying the induction hypothesis (2.8) for G and G', we also obtain

$$\pi(H)\ M(G)\ L(G'+h)\ A(G'+h|H-G) =$$

$$\pi(H)\ M(G)\ L(G')\ \lambda_h[1-\lambda_h]^{-1}\ A(G'|H-G)\ A(h|H-G+G')$$

$$\pi(H-G+G')\ M(G')\ L(G)\ A(G|H-G)\ A(h|H-G+G')\ \lambda_h[1-\lambda_h]^{-1}$$

$$\pi(H-G+G'+h)\ \mu_h[1-\mu_h]^{-1}\ M(G')\ L(G)\ A(G|H-G)$$

$$\pi(H-G+G'+h)\ M(G'+h)\ L(G)\ A(G|H-G) \tag{2.13}$$

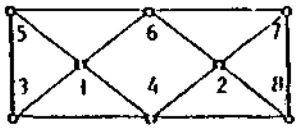
which proves (2.8) with G' replaced by $G'+h$ where $n(G)+n(G'+h) = m+1$. As the induction hypothesis (2.8) is hereby proven for all s, the proof is completed. □

3 Applications

To illustrate the possible practical applications of the abstract description and the blocking or invariance condition of section 2, this section provides some examples of present-day interest. For none of these a discrete-time product form has been reported in the literature. For each of them, however, the discrete-time product form result (25) turns out to have a similar form as their continuous-time analogues. The examples 3.1(i), (ii), 3.2(i) and 3.3(ii) are all coordinate convex, so that $P(.)=1$ and $S=C$. Examples 3.1(iii), 3.2(ii) and 3.3(ii) are randomized.

3.1 CSMA-protocols (cf. [2], [4], [5], [6], [8], [13], [20], [21], [22])

(i) CSMA Let the sources correspond to transmitters that can be graphically represented such that adjacent sources (neighbors) cannot be busy (transmit) at the same time. In practice this is achieved by the so-called "Carrier Sense Multiple Access" (CSMA)-scheme in which a transmitter senses the state of its channels just prior to starting a transmission and where upon sensing a busy channel from a neighbor the transmission is aborted (inhibited). For example, in the figure below a transmission from source 1 prohibits any source 3...6 to start a transmission.



With N(h) the set of all neighbors of source h, the coordinate convexity condition (2.3) is guaranteed by

$$C = \{H \mid h_2 \notin N(h_1) \text{ for all } h_1, h_2 \in H\} \tag{3.1}$$

(ii) BTMA (cf. [19] In the example above sources 1 and 2, for example, which are outside hearing range can transmit at the same time. This will lead to a collision at nodes 4 and 6 which in turn will result in lost messages. This is known as the "hidden terminal problem". To eliminate this problem, the so-called Busy Tone Multiple Access (BTMA)-scheme has been introduced (cf. [19]). Under BTMA a node which senses a busy channel (in other words, which hears a transmitting neighbor) broadcasts a busy tone to all its neighbors to prevent idle neighbors from starting a transmission.

The set C from (3.1) now still applies (i.e., satisfies (2.3)), provided we replace N(h) by the set of all one and two-link neighbors (e.g. N(5) = {2,...,7}).

In continuous-time the corresponding solution (25) along with necessary and sufficient conditions for arbitrary 0-1 CSMA blocking protocols to have this form can be found in [4], [5] and [6].

(iii) Rude-CSMA (cf. [13]) Another way to take into account the hidden terminal problem, which is introduced in [13] under the name of "rude-CSMA", is to let the access mechanism be randomized as according to

$$A(h|H) = x^{N_0^h(H)}\, y^{N_1^h(H)} \tag{3.2}$$

where $N_0^h(H)$ and $N_1^h(H)$ are the numbers of idle (not transmitting) and busy (transmitting) neighbors from h in state H and where x and y are given system parameters, with $0 \le x, y \le 1$. For instance x=1, y=1 corresponds to the ALOHA-protocol (no collisions), x=1, y=0 models the standard CSMA protocol of example 2.1 and other values of x and y may reflect for instance that sensing of channels is not always reliable (cf. [22]). Condition (2.1) and (2.4) are easily verified with:

$$P(H) = x^{-B_0(H)}\, y^{B_1(H)}$$

where

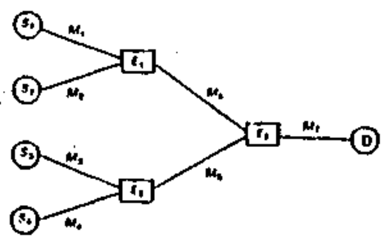
$B_0(H)$: number of idle pairs of neighbors in state H

$B_1(H)$: number of busy pairs of neighbors in state H. (3.3)

In continuous-time this solution was provided in [13] under exponentiality assumptions and source independent characteristics, extended in [8] to source dependent parameters and in [22] generalized to non-exponential idle (scheduling) and busy (transmission) times.

3.2 Circuit switching

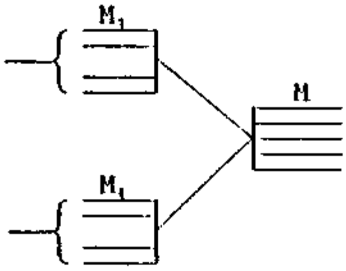
(i) Restricted trunkgroups (cf. [12]) Consider a circuit switching network with 4 different types of sources with a fixed path along which a message from that source to the destination is to be transmitted. This transmission requires one trunk from each trunkgroup along this path. Interference thus arises with limited trunkgroups and messages using the same trunkgroups.



With M_i the number of trunks in trunkgroup i and n_i the number of busy sources of type i, the coordinate convexity condition (2.3) is satisfied by C the set of states H such that:

$$n_i \le M_i \qquad (i=1,\dots,4)$$
$$n_1 + n_2 \le M_5$$
$$n_3 + n_4 \le M_6$$
$$n_1 + n_2 + n_3 + n_4 \le M_7 . \qquad (3.4)$$

(ii) (Random gradings) In analogy with the classical Engset random grading, assume a circuit switching as depicted below with two types of sources, M_i input channels for sources of type i and M common output channels.



Upon transmission request by a group of sources G with g_j sources of type j while the system is in state H with n_j sources of type j transmitting, this total group request is accepted with probability

$$A(G|H) = \prod_{j=1,2} (M_j - n_j)\ldots(M_j - n_j - g_j + 1)\, M_j^{-g_j}\, 1(n_1+n_2+g_1+g_2 \leq M) \tag{3.5}$$

as corresponding to individual random selection probabilities

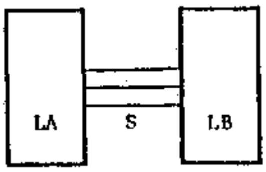
$$A(h|H) = [(M_j - n_j)/M_j]\, 1(n_1+n_2 < M) \tag{3.6}$$

for a source h of type j. Condition (2.1) is satisfied and

$$P(H) = A(H|\emptyset) \tag{3.7}$$

3.3 Interconnected Metropolitan Area Networks (MAN's) (cf. [16])

Consider a communication system with two groups of subscribers, say a group A and B with M and N subscribers, such as representing two metropolitan or local area networks. Both within a group and in between the groups communication between subscribers might be possible. To this end, number all subscribers 1,...,M+N and identify each possible connection from a source subscriber m to a destination subscriber n as a source (m,n). The description of section 2 now applies by saying that a connection is busy when a transmission along this connection takes place and idle otherwise and assuming some circuit allocation policy which restricts the feasible busy configuration, to some "coordinate convex" region C. We give some examples below.



(i) (Limited total number of circuits) (cf. [16]) For a given state H of busy connections let n_A, n_B and $n_{A,B}$ denote the number of busy connections within A, within B and in between A and B respectively. Assume finite numbers of LA and LB local circuits within A and B and S circuits in between A and B. Then the continuous-time model of [16] is extended to discrete-time by

$$C = \{H \mid n_A \le LA,\ n_B \le LB,\ n_{A,B} \le S\} \tag{3.8}$$

for the "dedicated allocation policy" with separate circuits for local and long-distance transmissions and by

$$C = \{H \mid n_A \le LA+S,\ n_B \le LB+S,\ 0 \le n_{A,B} \le S-(n_A-LA)^+-(n_B-LB)^+\} \tag{3.9}$$

where $(y)^+=0$ for $y\le 0$ and y^+ for $y>0$, for the "shared allocation policy" in which the inter-MAN circuits are shared among local and long-distance calls. As another shared allocation policy, each long-distance connection may require a local circuit within each local area, which is reflected by

$$C = \{H \mid n_A+n_{A,B} \le LA,\ n_B+n_{A,B} \le LB,\ n_{A,B} \le S\} \tag{3.10}$$

(ii) (Excluding connections) Certain connections may have to be excluded to be busy at the same time. For example, exclusion of busy connections (m,n) and (n,m) at the same time reflects one-way communication systems such as in air traffic. The corresponding set of admissible states is "coordinate convex" by:

$$1((m,n)\in H) + 1((n,m)\in H) \le 1 \qquad (\forall (n,m)) \tag{3.11}$$

(iii) (Error detection) In the examples (i) above, each long distance transmission may have to be registered first before it can be started. However, the registration of each source separately, assumed to take place in negligable time, may lead to an error, say with probability p, in which case the total transmission request is to be rescheduled. Condition (2.1) for this example is directly verified by

$$\begin{cases} A(h|H) = p \\ P(H) = p^n \end{cases} \tag{3.12}$$

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