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TESTING STATIONARITY AGAINST
THE UNIT ROOT HYPOTHESIS

by

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1. INTRODUCTION

In this paper we propose a simple nonparametric test for a unit root in a univariate time series. The tests proposed in the literature all test the unit root hypothesis against the alternative that the time series involved is stationary. See Fuller (1976), Dickey and Fuller (1979, 1981), Evans and Savin (1981, 1984), Phillips (1987). For further related references, see Phillips (1987) and Haldrup and Hylleberg (1989, Table 1). In this paper we take the stationarity hypothesis as the null and the unit root hypothesis as the alternative, i.e., denoting the time series process by y_t we test the null hypothesis

(1.1) $H_0: y_t = \mu + u_t$ with $\mu = E[y_t]$

against the alternative

(1.2) $H_1: \Delta y_t = u_t$,

where u_t is a stationary process. The other difference with the tests in the literature is that the asymptotic null distribution is of a well-known type, namely standard Cauchy (c.q. the Student distribution with one degree of freedom). Under H_1 the test statistic, divided by the sample size n, converges weakly to a continuous distribution, so that the absolute value of this test statistic converges in probability to infinity (at order n). The test involved is therefore consistent against the unit root hypothesis (1.2).

2. LINEAR TIME TREND REGRESSION

The intuition behind our test is that under H_1 the process y_t has a stochastic trend and therefore behaves (more or less) as if there is a deterministic linear trend. This suggests to regress y_t on time t, i.e., estimate the auxiliary "model" $y_t = \alpha + \beta t + v_t$ on the basis of the observations t=1,...,n, and use the least squares estimate of β,

$$(2.1) \quad \beta_n = [\Sigma_{t=1}^{n}(t - \bar{t})(y_t - \bar{y})] \,/\, [\Sigma_{t=1}^{n}(t - \bar{t})^2],$$

with

$$(2.2) \quad \bar{t} = (1/n)\Sigma_{t=1}^{n} t = \tfrac{1}{2}(n+1) \text{ and } \bar{y} = (1/n)\Sigma_{t=1}^{n} y_t,$$

as a basis for a test statistic. The further intuition is that the rate of convergence in distribution of β_n is different under H_0 and H_1, and that this difference can be exploited to distinguish between H_0 and H_1.

As in Phillips (1987) we shall not assume a specific model for u_t, except that it is Gaussian and that its covariance function $\gamma(m) = E[u_t u_{t+m}]$ vanishes at an exponential rate. The former assumption is not strictly necessary, but eases the argument. It may be replaced by mixing conditions like the α-mixing condition employed by Phillips (1985), or any other condition that ensures the applicability of a functional central limit theorem. The condition that $\gamma(m)$ is exponentially decreasing typically holds if u_t is an ARMA process with invertible AR lag polynomial. Thus:

ASSUMPTION 1: The process u_t is a stationary Gaussian process with exponentially vanishing covariance function $\gamma(m) = E[u_t u_{t+m}]$.

Now denote:

$$(2.3) \quad \sigma^2 = \lim_{n\to\infty} E\{[(1/\sqrt{n})\Sigma_{t=1}^{n} u_t]^2\} = \gamma(0) + 2\Sigma_{m=1}^{\infty}\gamma(m)$$

and

$$(2.4) \quad r_n = \{[(n+1)^3 - 3(n+1)^2 + 2(n+1)]/12\}^{\frac{1}{2}}.$$

Then:

LEMMA 1: *Under Assumption 1 and* H_0, $r_n\beta_n \Rightarrow N(0,\sigma^2)$.

PROOF: Observe that

$$(2.5)\quad \beta_n = \frac{\Sigma_{t=1}^{n}(t - \tfrac{1}{2}(n+1))u_t}{[(n+1)^3 - 3(n+1)^2 + 2(n+1)]/12} = \frac{\eta_n}{r_n{}^2},$$

say. Since u_t is Gaussian, β_n is normally distributed. The asymptotic variance of β_n follows from

$$(2.6)\quad E(\eta_n{}^2/r_n{}^2) = \gamma(0) + 2(1/r_n^2)\Sigma_{t=1}^{n-1}\Sigma_{m=1}^{n-t}(t-\tfrac{1}{2}(n+1))(t+m-\tfrac{1}{2}(n+1))\gamma(m)$$

$$= \gamma(0) + 2(1/r_n^2)\Sigma_{t=1}^{n-1}(t-\tfrac{1}{2}(n+1))^2\Sigma_{m=1}^{n-t}\gamma(m)$$

$$+ 2(1/r_n^2)\Sigma_{t=1}^{n-1}(t-\tfrac{1}{2}(n+1))\Sigma_{m=1}^{n-t}m\gamma(m)$$

$$= \gamma(0) + 2\cdot\Sigma_{m=1}^{\infty}\gamma(m) - 2(1/r_n^2)\Sigma_{t=1}^{n-1}(t-\tfrac{1}{2}(n+1))^2\Sigma_{m=n-t+1}^{\infty}\gamma(m)$$

$$+ 2(1/r_n^2)\Sigma_{t=1}^{n-1}(t-\tfrac{1}{2}(n+1))\Sigma_{m=1}^{n-t}m\gamma(m)$$

$$= \gamma(0) + 2\cdot\Sigma_{m=1}^{\infty}\gamma(m) + O(n^{-1}).$$

The latter result follows from the fact that $\Sigma_{t=1}^{\infty}\Sigma_{m=t}^{\infty}|\gamma(m)|$ and $\Sigma_{m=1}^{\infty}m|\gamma(m)|$ are convergent series. Q.E.D.

Next, denote for $\lambda \in [0,1]$,

$$(2.7)\quad W_n(\lambda) = (1/\surd n)\Sigma_{j=1}^{[\lambda n]}u_j/\sigma \text{ if } \lambda n \geq 1,\ W_n(\lambda) = 0 \text{ if } \lambda n \leq 1,$$

where σ is defined in (2.3) and $[x]$ means truncation to the nearest integer $\leq x$. Then $W_n(\lambda)$ is a stochastic element of the metric space $D[0,1]$ of functions on $[0,1]$ with countably many discontinuities. The metric involved is the sup norm

(2.8) $\rho(f,g) = \sup_{0\leq\lambda\leq 1}|f(\lambda)-g(\lambda)|$.

It is well-known [cf. Billingsley (1968)] that W_n converges weakly to a standard Wiener process W (denoted by $W_n \Rightarrow W$), which is a stochastic element of the metric space C[0,1] with norm (2.8) of continuous functions on [0,1] such that for $0 \leq \lambda \leq 1$ and $0 \leq \delta \leq 1-\lambda$,

(2.9) $W(\lambda) \sim N(0,\lambda)$, $W(\lambda+\delta)-W(\lambda) \sim N(0,\delta)$,
$W(\lambda)$ and $W(\lambda+\delta)-W(\lambda)$ are mutually independent.

Moreover, for any continuous mapping Φ from D[0,1] into C[0,1] we have $\Phi(W_n) \Rightarrow \Phi(W)$. A special case of such a mapping is the integral $\Phi(W_n) = \int_0^1 \lambda^m W_n(\lambda)d\lambda$; $m \geq 0$. Furthermore, $(1/n)\Sigma_{t=1}^n (t/n)^m W_n(t/n) = \int_0^1 \lambda^m W_n(\lambda)d\lambda + O_p(1/n)$, hence

(2.10a) $(1/n)\Sigma_{t=1}^n (t/n)^m W_n(t/n) \Rightarrow \int_0^1 \lambda^m W(\lambda)d\lambda$,

and similarly for $0 < \mu \leq 1$,

(2.10b) $(1/n)\Sigma_{t=1}^{[\mu n]} (t/n)^m W_n(t/n) \Rightarrow \int_0^\mu \lambda^m W(\lambda)d\lambda$,

(2.10c) $(1/n)\Sigma_{t=[\mu n]+1}^n (t/n)^m W_n(t/n) \Rightarrow \int_\mu^1 \lambda^m W(\lambda)d\lambda$.

With these results at hand we can now prove:

LEMMA 2: *Under Assumption 1 and* H_1, $(r_n/n)\beta_n \Rightarrow N(0,\sigma^2/10)$.

PROOF: Observe that

$$(2.11)\quad \Sigma_{t=1}^n (t-\bar{t})(y_t-\bar{y}) = \Sigma_{t=1}^n t\Sigma_{j=1}^t u_j - \tfrac{1}{2}(n+1)\Sigma_{t=1}^n \Sigma_{j=1}^t u_j$$

$$= \sigma n^2\sqrt{n}\{(1/n)\Sigma_{t=1}^n (t/n)[(1/\sqrt{n})\Sigma_{j=1}^{[(t/n)n]} u_j/\sigma]\}$$

$$- \sigma\tfrac{1}{2}(n+1)n\sqrt{n}\{(1/n)\Sigma_{t=1}^n [(1/\sqrt{n})\Sigma_{j=1}^{[(t/n)n]} u_j/\sigma]\}$$

$$= \sigma n^2\sqrt{n}\{(1/n)\Sigma_{t=1}^n (t/n)W_n(t/n) - \sigma\tfrac{1}{2}(n+1)n\sqrt{n}\{(1/n)\Sigma_{t=1}^n W_n(t/n).$$

Hence by (2.10a),

(2.12) $[r_n^2/(n^2\surd n)]\beta_n \Rightarrow \sigma\int_0^1(\lambda-\frac{1}{2})W(\lambda)d\lambda$,

where r_n is defined in (2.4).

Since the limiting distribution in (2.12) has been emerged from a sequence of linear functionals of Gaussian random variates, it is normal itself. Clearly, its expectation is zero. Its variance is:

(2.13) $$E[\int_0^1(\lambda-\frac{1}{2})W(\lambda)d\lambda]^2 = \int_0^1\int_0^1(\lambda_1-\frac{1}{2})(\lambda_2-\frac{1}{2})E[W(\lambda_1)W(\lambda_2)]d\lambda_1 d\lambda_2$$

$$= \int_0^1\int_0^1(\lambda_1-\frac{1}{2})(\lambda_2-\frac{1}{2})\min[\lambda_1,\lambda_2]d\lambda_1 d\lambda_2 = 1/120.$$

Observing that

(2.14) $12r_n^2/n^3 \to 1$,

it follows from (2.12 and (2.13) that

(2.15) $(r_n/n)\beta_n \Rightarrow \sigma\surd 12\int_0^1(\lambda-\frac{1}{2})W(\lambda)d\lambda \sim N(0,\sigma^2/10)$.

Q.E.D.

Comparing the results in Lemmas 1 and 2 we see that under H_0 the asymptotic rate of convergence in distribution of β_n is of order $n\surd n$, whereas under H_1 the asymptotic rate of convergence is of order $\surd n$. Thus, if σ^2 would be known the test $r_n\beta_n/\sigma$ is a consistent standard normal test of the stationarity hypothesis against the unit root hypothesis, for $|r_n\beta_n/\sigma| \to \infty$ in probability under H_1. However, in practice we cannot use this test statistic because the variance σ^2 is unknown.

3. A CAUCHY TEST

We may think of estimating σ^2 in a similar way as in White and Domowitz (1984), Newey and West (1987) and Phillips (1987), but here we shall use a more elegant approach. The idea is to construct a statistic with *equal* rates of convergence under H_0 and H_1 that also depends on σ in a similar way as above. Then taking the ratio of $r_n\beta_n$ with this this statistic σ will cancel out. The statistic involved is based on:

$$(3.1)\quad \xi_n = (1/n)\Sigma_{t=1}^{n} y_t - (1/[\tfrac{1}{2}n])\Sigma_{t=1}^{[\frac{1}{2}n]} y_t .$$

LEMMA 3: *Under Assumption 1 and* H_0,

$$(3.2)\quad (\surd n)\xi_n \Rightarrow N(0,\sigma^2), \textit{ and}$$

(3.3) $r_n\beta_n$ *and* $(\surd n)\xi_n$ *are asymptotically independent.*

PROOF: We have

$$(3.4)\quad (\surd n)\xi_n = \sigma[W_n(1) - 2([\tfrac{1}{2}n]/(\tfrac{1}{2}n))W_n(\tfrac{1}{2})] \Rightarrow \sigma[W_*(1)-2W_*(\tfrac{1}{2})],$$

where W_* is a standard Wiener process. But $W_*(1)-2W_*(\frac{1}{2}) = [W_*(1)-W_*(\frac{1}{2})] - W_*(\frac{1}{2})$ is the difference of two independend $N(0,\frac{1}{2})$ variates, hence $W_*(1)-2W_*(\frac{1}{2}) \sim N(0,1)$. This proves (3.2). For proving (3.3) it suffices to show that $E[\eta_n\xi_n/n] \to 0$, where η_n is defined in (2.5). Now observe that

$$(3.5)\quad E[\eta_n\xi_n/n] = (1/n^2)\Sigma_{t=1}^{n}(t - \tfrac{1}{2}(n+1))\Sigma_{m=1}^{n}\gamma(t-m)$$

$$- (2/n^2)\Sigma_{t=1}^{n}(t - \tfrac{1}{2}(n+1))\Sigma_{m=1}^{[\frac{1}{2}n]}\gamma(t-m)$$

$$= (1/n^2)\Sigma_{t=1}^{n}(t - \tfrac{1}{2}(n+1))[\Sigma_{m=1}^{t-1}\gamma(m) + \gamma(0) + \Sigma_{m=1}^{n-t}\gamma(m)]$$

$$- (2/n^2)\Sigma_{t=1}^{n}(t - \tfrac{1}{2}(n+1))\{I([\tfrac{1}{2}n]-t\geq 1)\Sigma_{m=1}^{[\frac{1}{2}n]-t}\gamma(m)$$

$$+ I([\tfrac{1}{2}n]=t)\gamma(0) + I([\tfrac{1}{2}n]-t\leq -1)\Sigma_{m=1}^{t-1}\gamma(m)\}$$

$$= -(1/n^2)\Sigma_{t=1}^{n}(t - \tfrac{1}{2}(n+1))\Sigma_{m=t}^{\infty}\gamma(m)$$

$$-(1/n^2)\Sigma_{t=1}^{n}(t - \tfrac{1}{2}(n+1))\Sigma_{m=n-t+1}^{\infty}\gamma(m)$$

$$+ (2/n^2)\Sigma_{t=1}^{[\frac{1}{2}n]-1}(t - \tfrac{1}{2}(n+1))\Sigma_{m=[\frac{1}{2}n]-t+1}^{\infty}\gamma(m)$$

$$+ (2/n^2)\Sigma_{[\frac{1}{2}n]+1}^{n}(t - \tfrac{1}{2}(n+1))\Sigma_{m=t}^{\infty}\gamma(m) + O(n^{-1})$$

$$= -(1/n^2)\Sigma_{t=1}^{n}(t - \tfrac{1}{2}(n+1))\Sigma_{m=n-t+1}^{\infty}\gamma(m)$$

$$+ (2/n^2)\Sigma_{t=1}^{[\frac{1}{2}n]-1}(t - \tfrac{1}{2}(n+1))\Sigma_{m=[\frac{1}{2}n]-t+1}^{\infty}\gamma(m) + O(n^{-1})$$

The third equality follows by substituting expressions of the form

$$(3.6)\quad \Sigma_{m=1}^{t-1}\gamma(m) = \Sigma_{m=1}^{\infty}\gamma(m) - \Sigma_{m=t}^{\infty}\gamma(m),$$

and the last equality follows from the fact that $\gamma(m)$ is exponentially vanishing, by which

(3.7) $\Sigma_{t=1}^{\infty} t\Sigma_{m=t}^{\infty}\gamma(m)$ and $\Sigma_{t=1}^{\infty}\Sigma_{m=t}^{\infty}\gamma(m)$ are convergent series.

Now let $k(n)$ be an integer function of n such that $k(n)/n \to (1-\varepsilon)$ for some $\varepsilon \in (0,\tfrac{1}{2})$. Then

$$(3.8)\quad \left|(1/n^2)\Sigma_{t=1}^{n}(t - \tfrac{1}{2}(n+1))\Sigma_{m=n-t+1}^{\infty}\gamma(m)\right|$$

$$\le \left|(1/n^2)\Sigma_{t=k(n)+1}^{n}(t - \tfrac{1}{2}(n+1))\Sigma_{m=n-t+1}^{\infty}\gamma(m)\right|$$

$$+ \left|(1/n^2)\Sigma_{t=1}^{k(n)}(t - \tfrac{1}{2}(n+1))\Sigma_{m=n-t+1}^{\infty}\gamma(m)\right|$$

$$\le \left|(1/n^2)\Sigma_{t=k(n)+1}^{n} t\right|\Sigma_{m=1}^{\infty}\left|\gamma(m)\right|$$

$$+ \tfrac{1}{2}(n+1)n^{-1}\left|(1/n)\Sigma_{t=k(n)+1}^{n} 1\right|\Sigma_{m=1}^{\infty}\left|\gamma(m)\right|$$

$$+ \left|(1/n^2)\Sigma_{t=1}^{k(n)}(t - \tfrac{1}{2}(n+1))\Sigma_{m=n-k(n)+1}^{\infty}\gamma(m)\right|$$

$$\leq \tfrac{1}{2}\{[n(n+1)-k(n)(k(n)+1) + (n-k(n))(n+1)]n^{-2}\}\Sigma_{m=1}^{\infty}|\gamma(m)|$$

$$+ 2\Sigma_{m=n-k(n)+1}^{\infty}|\gamma(m)|$$

$$\rightarrow \tfrac{1}{2}(1-(1-\varepsilon)^2 + \varepsilon)\Sigma_{m=1}^{\infty}|\gamma(m)| \text{ as } n \rightarrow \infty.$$

Letting $\varepsilon \rightarrow 0$, it follows that

$$(3.9) \quad (1/n^2)\Sigma_{t=1}^{n}(t - \tfrac{1}{2}(n+1))\Sigma_{m=n-t+1}^{\infty}\gamma(m) = o(1).$$

Along similar lines it follows that

$$(3.10) \quad (2/n^2)\Sigma_{t=1}^{[\frac{1}{2}n]-1}(t - \tfrac{1}{2}(n+1))\Sigma_{m=[\frac{1}{2}n]-t+1}^{\infty}\gamma(m) = o(1).$$

Combining (3.5), (3.9) and (3.10), (3.3) follows. Q.E.D.

LEMMA 4: *Under Assumption 1 and* H_1, $((r_n/n)\beta_n, \xi_n/\sqrt{n})' \Rightarrow \sigma(z_1,z_2)'$, *where*

$$(3.11) \quad (z_1,z_2)' \sim N_2\left(\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}1/10 & 5\sqrt{3}/96\\ 5\sqrt{3}/96 & 1/12\end{pmatrix}\right).$$

PROOF: Observe that

$$(3.12) \quad \xi_n = (1/n)\Sigma_{t=1}^{n}\Sigma_{j=1}^{t}u_j - (1/[\tfrac{1}{2}n])\Sigma_{t=1}^{[\frac{1}{2}n]}\Sigma_{j=1}^{t}u_j$$

$$\approx (\sigma\sqrt{n})\{(1/n)\Sigma_{t=1}^{n}W_n(t/n) - 2(1/n)\Sigma_{t=1}^{[\frac{1}{2}n]}W_n(t/n)\}$$

$$= (\sigma\sqrt{n})\{(1/n)\Sigma_{t=[\frac{1}{2}n]+1}^{n}W_n(t/n) - (1/n))\Sigma_{t=1}^{[\frac{1}{2}n]}W_n(t/n)\},$$

hence by (2.10b-c),

$$(3.13) \quad \xi_n/\sqrt{n} \Rightarrow \sigma\{2\int_{\frac{1}{2}}^{1}W(\lambda)d\lambda - \int_0^1 W(\lambda)d\lambda\}.$$

The relation between the limiting distributions in (2.13) and (3.13) is the following. Denote

(3.14) $z_1 = \sqrt{12}\int_0^1(\lambda-\frac{1}{2})W(\lambda)d\lambda$; $z_2 = 2\int_{\frac{1}{2}}^1 W(\lambda)d\lambda - \int_0^1 W(\lambda)d\lambda$.

It is easy to verify that z_1 and z_2 are jointly normally distributed with zero means. We already have seen in (2.13) that $E[z_1{}^2] = 1/10$. Along similar lines we can verify that $E[z_2{}^2] = 1/12$ and $E[z_1 z_2] = 5\sqrt{3}/96$. Q.E.D.

Now if we would use $r_n\beta_n/|\sqrt{n}\xi_n|$ as a test statistic then the rate of convergence under H_0 and H_1 is the same so that the test has hardly any power. However, the following direct corollary of Lemmas 3 and 4 provides a solution to this problem.

LEMMA 5: *Let Assumption 1 hold. Under* H_0,

(3.15) $\xi_n\sqrt{n}/(1 + \xi_n^2) \Rightarrow N(0,\sigma^2)$,

whereas under H_1,

(3.16) $[\xi_n\sqrt{n}/(1 + \xi_n^2)]^{-1} \Rightarrow \sigma z_2$

where z_2 *is defined in (3.11).*

Consequently, denoting

(3.17) $S_n = r_n\beta_n(1 + \xi_n^2)/|\sqrt{n}\xi_n|$,

it follows easily from Lemmas 1 through 5:

THEOREM 1: *Let Assumption 1 hold. Under* H_0, $S_n \Rightarrow$ *Cauchy*(0,1), *whereas under* H_1, $S_n/n \Rightarrow \sigma^2 z_1|z_2|$, *where* (z_1,z_2) *is defined in (3.11).*

Note that the limiting distribution of S_n/n under H_1 is continuous, hence under H_1 we have,

(3.20) $\text{plim}_{n\to\infty}|S_n| = \infty$.

This result implies that the test involved is consistent against the unit root hypothesis.

4. CONSISTENCY AGAINST OTHER ALTERNATIVES

Our test is not only consistent against the unit root hypothesis (1.2) but also against other deviations from stationarity. Consider the following alternatives. Let H_1 be either

(4.1a) $H_1: \Delta y_t = \beta + u_t$,

or

(4.1b) $H_1: y_t = \alpha + \beta t + u_t$,

where β is non-zero. Then it is easy to verify that under (4.1),

(4.2) $\text{plim}_{n\to\infty}\beta_n = \beta$ and $\text{plim}_{n\to\infty}\xi_n/n = \beta/4$.

It follows therefore from (2.14) and (3.17) that under (4.1)

(4.3) $\text{plim}_{n\to\infty}S_n/n^2 = \beta|\beta|/(8\sqrt{3})$.

The similarity of the two hypotheses (4.1a) and (4.1b) with respect to the power of our test suggests that these hypotheses are hardly distinguisable. This is confirmed by Haldrup and Hylleberg (1989) and Haldrup (1989) who constructed and applied a test of the null hypothesis (4.1a) against the alternative hypothesis (4.1b).

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