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QUALITATIVE IMPACT ANALYSIS

FOR DYNAMIC SPATIAL SYSTEMS

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Abstract

This paper focuses attention on qualitative impact analysis for spatial dynamics. It aims at developing new analytical tools for describing absolute and relative dynamics of spatial evolution based on either a simple core-ring model or a more general core-ring-periphery model, based on a semi-ordinal sign analysis. Although the number of qualitative categories of possible states of an urban/regional system may be very large, the spatio-temporal evolution of such systems (based on the spatial cycle hypothesis) exhibits nevertheless fairly uniform patterns.

1. Introduction

In recent years, a great deal of attention has been paid to so-called qualitative calculus (see for overviews and applications among others Brouwer (1987), Brouwer et al (1987), Brouwer and Nijkamp (1984), Lancaster (1962), Maybee (1981), Maybee and Voogd (1984), Voogd (1982) and Quirk (1981)). Qualitative calculus is essentially an analytical tool for an impact assessment based on imprecise information, which aims at predicting the signs (or directions) of the impacts. Usually only a limited set of impact patterns of structural parameters is assumed, viz., positive (+), negligible (0) or negative (-) impacts. Assuming a qualitative stimulus-response model, the aim is then to assess the direction of influence on a certain state (or endogenous) variable as a result of a qualitative shift in a control (or exogenous) variable. An interesting recent application in the field of input-output analysis can be found in Bon (1986).

In the present paper the field of qualitative impact assessment will be further developed by in the context of spatial cycle analysis (cf. van den Berg et al, 1986) by:

- focussing the attention in particular on spatial sign analysis for qualitative changes in a core-ring system;
- concentrating on absolute and relative qualitative dynamics of a spatial system (i.e., shifts in the absolute and relative spatial shares of a state variable characterizing a certain spatial system, and inter-dependencies between the spatial behaviour of absolute and relative shares);
- extending qualitative impact analysis by allowing an semi-ordinal (i.e., binary) ranking of both positive and negative qualitative impacts. This means that positive impacts can be further subdivided into large impacts (++) and moderate impacts (+). The same holds true for negative impacts, so that it is possible to consider five rankings, viz. (++), (+), (0), (-), (--);
- constructing the qualitative categories of possible states of a spatial/regional system, and describing the spatial-temporal evolution of a system as the transfer from one qualitative category to another.

This paper will use by way of illustration a qualitative description of multi-regional/single population spatial impact models. It should be added, however, that this framework is also applicable to multi-population/single region models. In order to clarify the aim and scope of the present paper, we will first give a brief illustration of a spatial system based on the well-known core-ring model for spatial evolution.

2. Absolute Spatial Dynamics in a Core-Ring Model.

Assume a simple spatial system composed of an urban core and its surrounding area, the ring. The evolution of such a simplified system can be measured by means of shifts in state variables characterizing the socio-economic performance of the core and ring, for example, population size, employment, value added, etc. Thus, the absolute dynamics of this system can be analyzed by studying the absolute changes or shifts in these state variables.

Consider now the distribution of a given variable between the core and the ring within a simple spatial core-ring model in an otherwise open spatial system. The distribution of such a spatial variable based on the above mentioned rankings can be described by the distribution vector $\bar{z}$:

$$\bar{z} = (X,Y), \tag{2.1}$$

where X and Y are (positive) shares of the spatial variable (spatial substance) in the core and ring, respectively. The domain of all possible core-ring distributions of a spatial variable is described in Fig. 1. This domain is divided into two subdomains associated with the inequalities $X > Y$ and $X < Y$.

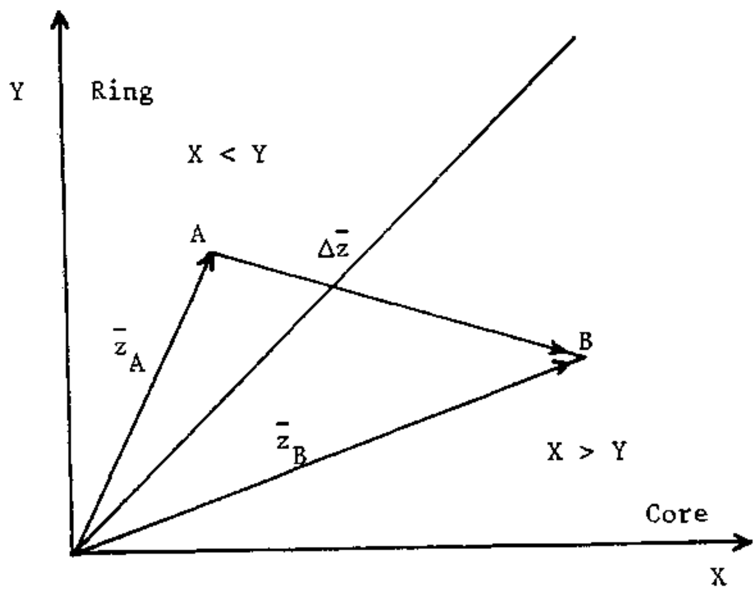

Figure 1. Domain of core-ring distributions

The evolution of such a system can be characterized by the absolute changes in the shares (X, Y), caused by the transition of the system from a state A, associated with a distribution $\bar{z}_A = (X_A, Y_A)$, to a state B, associated with a distribution $\bar{z}_B = (X_B, Y_B)$. The transitional redistribution of a state variable is described by the difference vector:

$$\Delta\bar{z} = \bar{z}_B - \bar{z}_A = (X_B - X_A, Y_B - Y_A) = (\Delta X, \Delta Y) \qquad (2.2)$$

A qualitative description of the transition from state A to B can be given by the sign vector:

$$\text{Sign } \Delta\bar{z} = (\text{Sign } \Delta X, \text{ Sign } \Delta Y) \qquad (2.3)$$

where

$$\text{Sign } \Delta X = \begin{cases} ++ & \text{if } \Delta X \text{ is a large positive increment,} \\ + & \text{if } \Delta X \text{ is a moderate positive increment,} \\ 0 & \text{if } \Delta X \text{ is a non-significant increment or decrement} \\ - & \text{if } \Delta X \text{ is a moderate negative decrement,} \\ -- & \text{if } \Delta X \text{ is a large negative decrement.} \end{cases}$$

Sign ΔY is defined analogously. This means that only 25 qualitative impact patterns (qualitative categories) are associated with the absolute dynamics in a core-ring model in the context of an open surrounding spatial system (see Table 1).

	Core	Ring		Core	Ring		Core	Ring
1	++	++	11	++	0	21	++	--
2	+	++	12	+	0	22	+	--
3	0	++	13	0	0	23	0	--
4	-	++	14	-	0	24	-	--
5	--	++	15	--	0	25	--	--
6	++	+	16	++	-			
7	+	+	17	+	-			
8	0	+	18	0	-			
9	-	+	19	-	-			
10	--	+	20	--	-			

Table 1. Qualitative categories (sign configurations) for absolute changes in a simplified core-ring model in the framework of an open surrounding spatial system.

The graphical presentation of the qualitative categories for possible absolute changes in the state variables (X,Y) is given in Fig. 2. The domain of qualitative categories can be divided into two subdomains associated with a total absolute core-ring growth ($\Delta X+\Delta Y>0$), and with a total absolute core-ring decline ($\Delta X+\Delta Y<0$). A more complicated division of the domain of absolute changes can be introduced with the help of qualitative interconnections between the absolute values of shares of spatial distributions such as, for example, $\Delta X>|\Delta Y|$, etc... Each such subdivision generates a definite qualitative category of spatial dynamics (see also Fig. 3).

The evolution of spatial systems can in general now be described with the help of possible transfers of the system from one qualitative category to another. As an example of spatial evolution we will consider here the hypothesis of spatial cycles within metropolitan areas (central city + suburbs), which is based on a consideration of four qualitative stages - viz. urbanization, suburbanization, desurbanization and reurbanization (see Klaassen et al, 1981, and Nijkamp, 1987). In a recent paper Kawashima (1987) presented this spatial cycle hypothesis with the help of Table 2. The graphical description of spatial cycles for the population change is given in Fig. 4, which is a diagram of the type Fig. 3 of absolute changes in population shares for a central city and its suburbs (see also Fig. 5).

Stage	Sub-stage (Qualitative category)	Population change			
		Central City X	Suburbs Y	Relative change	Metropolitan area
		Sign ΔX	Sign ΔY		Sign $(\Delta X+\Delta Y)$
Urbanization	1	+	-	X>Y	+
	2	+	+	X>Y	
Suburbanization	3	+	+	X<Y	+
	4	-	+	X<Y	
Desurbanization	5	-	+	X<Y	-
	6	-	-	X<Y	
Reurbanization	7	-	-	X>Y	-
	8	+	-	X>Y	

Table 2. Tabular presentation of configurations of spatial cycle hypothesis (Kawashima (1987)).

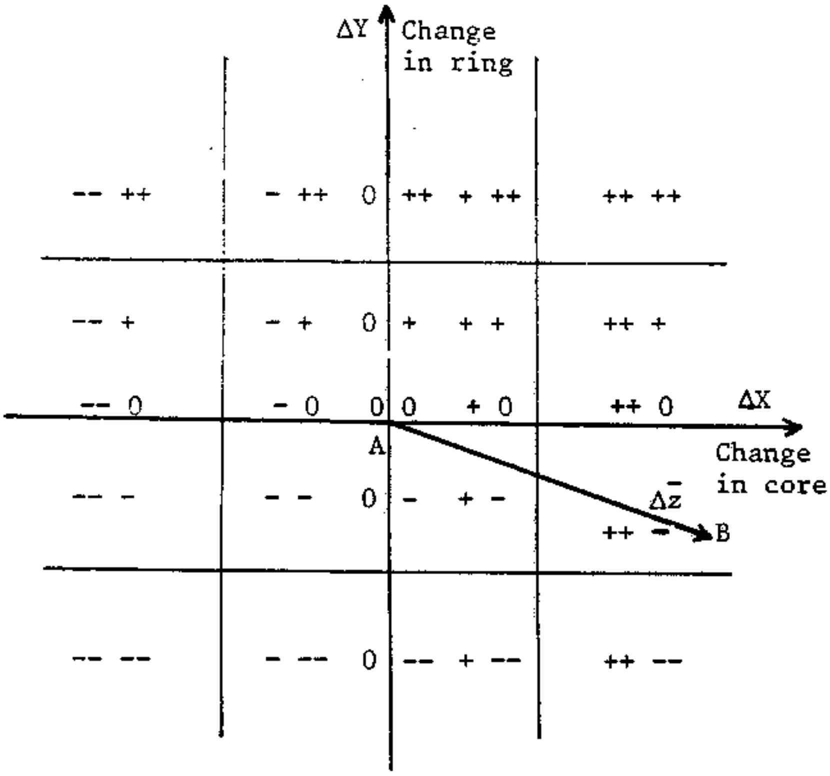

Figure 2. Tabular presentation of qualitative categories for absolute change

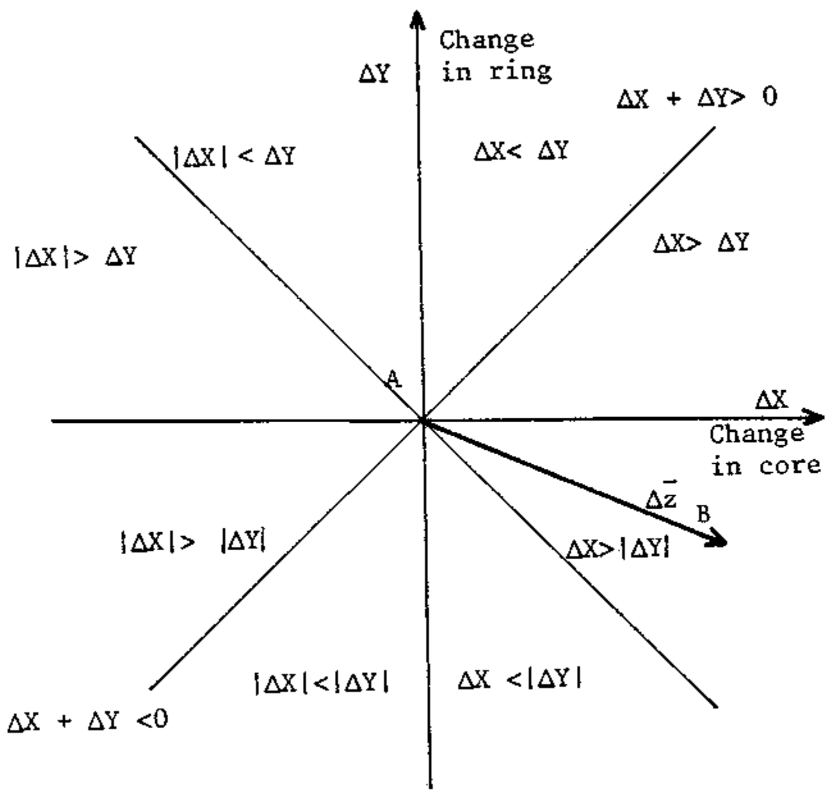

Figure 3. Graphical presentation of qualitative categories for absolute changes

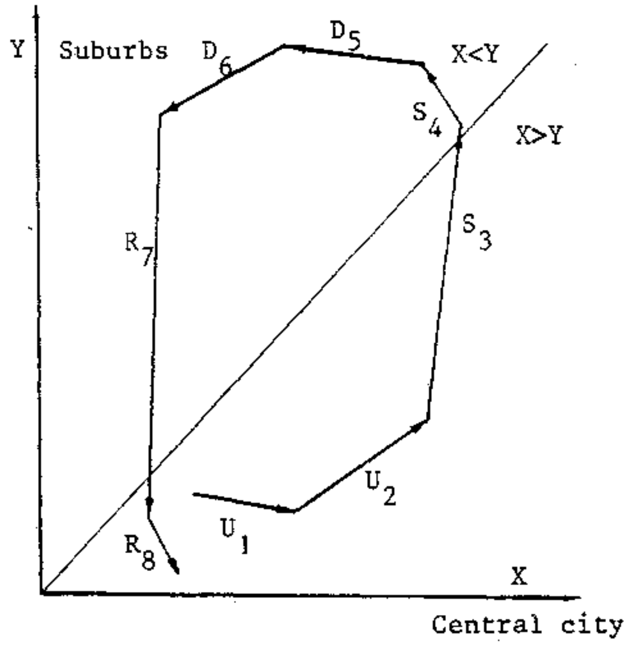

Figure 4. Graphical presentation of qualitative categories for the spatial cycle hypothesis.

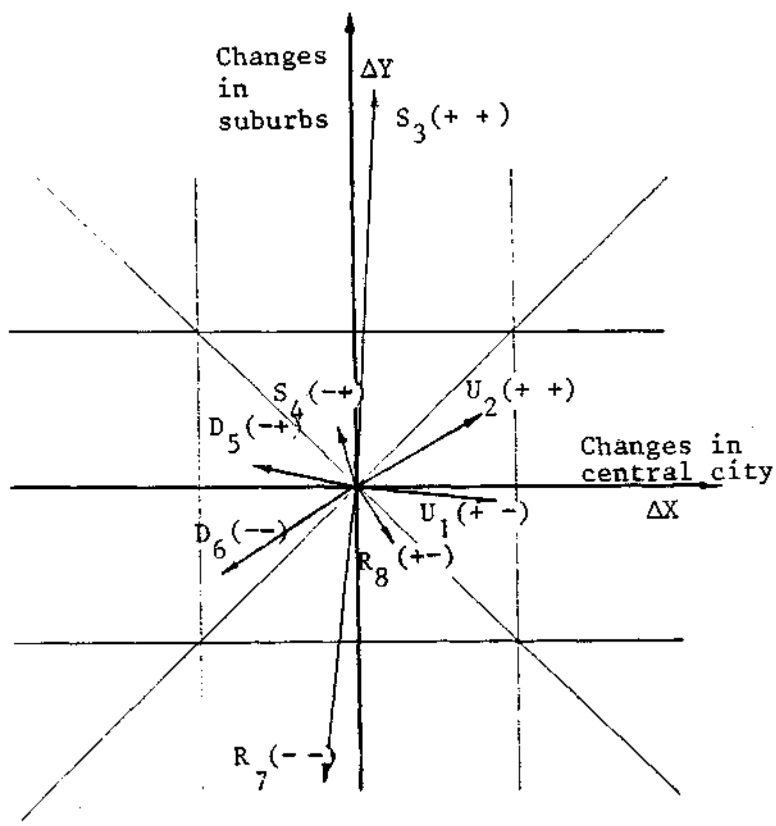

Figure 5. Graphical presentation of spatial evolution with the help of qualitative categories, of absolute changes in impact variables.

The introduction of the set of all possible qualitative categories requires a complete description of the set of evolutionary stages of a spatial system. It is important to emphasize that the Kawashima description of the spatial cycle hypothesis is only based on binary (+,-) sign configurations. Therefore, the introduction of a more heterogeneous set of ++,+,0,-,-- sign configurations is likely useful for generating a multiplicity of change patterns (including 'catastrophic' changes) for the processes of stagnation of population distribution within metropolitan areas. For example, Table 3 gives a more precise description of urbanization stages for a core-ring model.

Stage	Sub-stage	Population change				Metropolitan area Sign $(\Delta X+\Delta Y)$
		Central City Sign ΔX	Suburbs Sign ΔY	Relative change		
	U_1	++ ++ +	-- - -	$\Delta X>\lvert\Delta Y\rvert$		
Urbanization	Intermediate sub-stage	++ +	0 0		$X>Y$	+
	U_2	++ ++ +	++ + +	$\Delta X>\Delta Y$		
	S_3	++ + +	++ ++ +	$\Delta X<\Delta Y$		
Suburbanization	Intermediate sub-stage	0 0	++ +		$X<Y$	+
	S_4	-- - -	++ ++ +	$\lvert\Delta X\rvert<\Delta Y$		
	D_5	-- -- -	++ + +	$\lvert\Delta X\rvert>\Delta Y$		
Desurbanization	Intermediate sub-stage	-- -	0 0		$X<Y$	-
	D_6	-- - -	-- -- -	$\lvert\Delta X\rvert>\lvert\Delta Y\rvert$		
	R_7	-- - -	-- -- -	$\lvert\Delta X\rvert<\lvert\Delta Y\rvert$		
Reurbanization	Intermediate sub-stage	0 0	-- -		$X>Y$	-
	R_8	++ + +	-- -- -	$\Delta X<\lvert\Delta Y\rvert$		

Table 3. More precise description of the qualitative stages of the spatial cycle hypothesis.

3. General Framework of Qualitative Description of Multiregional Absolute Spatial Redistribution Dynamics

Let us now consider a spatial system (spatial wholeness) which is conceptually meaningful to view as comprising a finite number N of spatial units (or regions) (1,...,i,...N) and a distribution of 'spatial substance'. 'Spatial substance' can represent virtually anything of interest to the spatial analyst: territory, population, industry, conflicts, innovations, vegetation, temperature and other space - distributed geographical, economic, social, demographic, political, biological, climatological, geomorphologic or ecologic phenomena.

Each region i may be assumed to possess an amount X_i of a spatial substance such that the vector $\bar{z} = (X_1, X_2, \ldots, X_N)$ is a vector of (absolute) distribution of a spatial substance within the spatial system.

Next consider two different states A and B of the spatial system corresponding to the distribution vectors:

$$\begin{aligned} \bar{z}_A &= (X_1, X_2, \ldots, X_N) \\ \bar{z}_B &= (X'_1, X'_2, \ldots, X'_N) \end{aligned} \qquad (3.1)$$

The transition from state A to state B will be called the <u>absolute redistribution</u> of spatial substance. This redistribution can be associated with a vector of increments (see (2.2)):

$$\Delta\bar{z} = \bar{z}_B - \bar{z}_A = (X'_1 - X_1, X'_2 - X_2, \ldots, X'_N - X_N) = (\Delta X_1, \Delta X_2, \ldots, \Delta X_N) \qquad (3.2)$$

Its qualitative description is associated with a vector of signs of increments:

$$\text{Sign } \Delta\bar{z} = (\text{Sign } \Delta X_1, \text{ Sign } \Delta X_2, \ldots, \text{Sign } \Delta X_N) \qquad (3.3)$$

where

$$\text{Sign } \Delta X_i \begin{cases} + & \text{if } \Delta X_i > 0, \\ 0 & \text{if } \Delta X_i \approx 0, \\ - & \text{if } \Delta X_i < 0. \end{cases} \qquad (3.4)$$

It is clear that in case of complex spatial systems (for instance, territorial exclusion due to multiple urban cores) a wide variety of spatial qualitative categories (qualitative stages) can be determined, while the number of combinatorial possibilities increases even dramatically. Consequently, the usual theories on urban decay (for instance, the 'clean break' hypothesis) have to be dealt with very carefully, as the real-world spatial pattern may be much more complex than the simple spatial dichotomy (or dual spatial development) taken for granted in these theories suggests.

The construction of more substantial qualitative categories is related to additional constraints on the behaviour of increments for a chosen subset of regions, i.e., for example, to the behaviour of the Sign $(\Delta X_i+\Delta X_k+\ldots+\Delta X_r)$ for different subsets $i,k,\ldots,r$ of regions in the spatial system at hand. Thus spatial redistribution dynamics means the transfer from one qualitative category to another. This needs of course a comprehensive description of the (empirical and theoretical) regularities of such transformations. This issue will be further discussed in the next section.

4. Relative Spatial Dynamics in a Three Region Single Population Model.

In the present section we will assume a simple three region model for a spatial system composed of an urban core, a suburban ring and a rural periphery. The relative distribution of spatial substance (such as population or employment) can be measured by means of the successive relative shares (weights) p,q and r of the spatial substance in a core, ring and periphery in such a way that

$$p + q + r = 1;\ 0 \leq p,q,r \leq 1 \qquad (4.1)$$

In this case we will call the spatial substance the <u>relative population.</u>

The domain of possible changes in relative weights of the core, suburban ring and rural-periphery population can be presented geometrically by a so-called <u>triangram</u> or Möbius triangle (see Fig. 6) in such a way that the distribution $\bar{u} = (p,q,r)$ of relative population between the core, ring and periphery areas can be presented by a point $\bar{u}$ within this triangle. The weights p,q and r are called the <u>barycentric coordinates</u>, because it is possible to hang the weights p,q and r in the vertices of the Möbius triangle so that the point $\bar{u}$ will be a centre of gravity of the triangle (see Fig. 7).

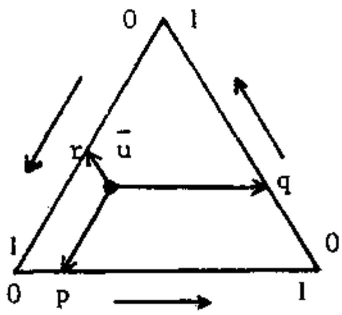

Figure 6. A triangram (or Möbius triangle)

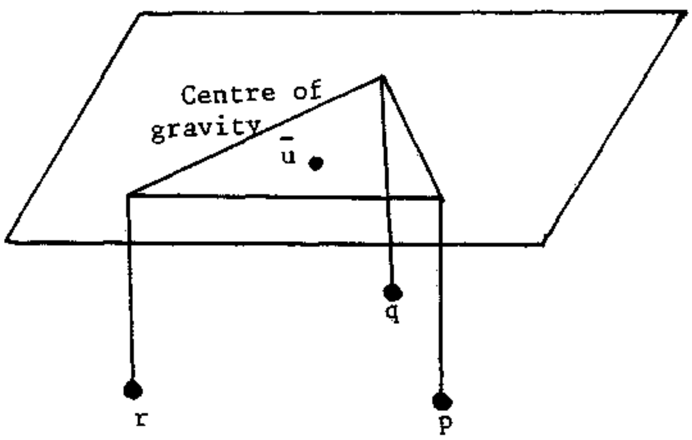

Figure 7. Representation by means of barycentric coordinates

The relative spatial dynamics adopts thus the form of the trajectory of the state $\bar{u}$ over time. Different forms of trajectories will thus represent different types of core-ring-periphery spatial dynamics.

In Fig. 8, points $\bar{u}_1, \bar{u}_2, \ldots, \bar{u}_7$ present a hypothetical relative dynamics of a core-ring-periphery system. The initial distribution $\bar{u}_1$ of relative population is associated with the absence of the relative population in a core and ring; the state $\bar{u}_2$ is associated with a population growth in a core at the expense of the periphery - the beginning of an urbanization process; the state $\bar{u}_3$ shows the beginning of suburbanization together with a relative concentration of population in the core; the states $\bar{u}_3, \bar{u}_4, \bar{u}_5, \bar{u}_6$ are those of suburbanization; and the states $\bar{u}_4, \bar{u}_5, \bar{u}_6, \bar{u}_7$ describe the gradual growth of the relative population in the periphery.

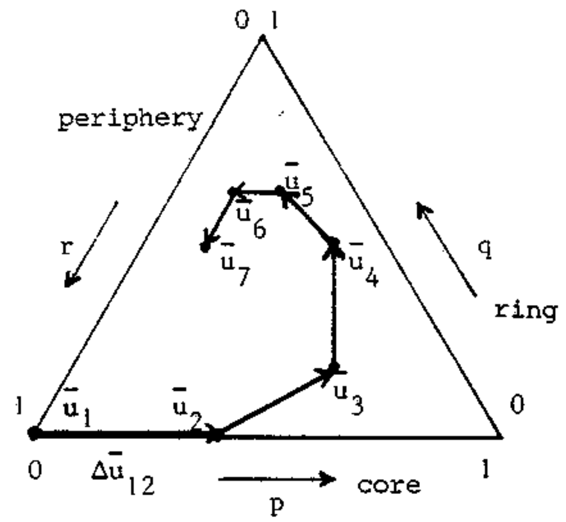

Figure 8. Illustration of relative dynamics in a core-ring periphery model

The relative spatial dynamics can be analysed by studying the changes or shifts in the relative weights p,q,r. The transition from the relative distribution $\bar{u}_1 = (p_1,q_1,r_1)$ to the distribution $\bar{u}_2 = (p_2,q_2,r_2)$ is measured by the difference

$$\Delta\,\bar{u}_{12} = \bar{u}_2 - \bar{u}_1 = (p_2-p_1, q_2-q_1, r_2-r_1) = (\Delta p,\ \Delta q,\ \Delta r) \qquad (4.2)$$

It is obvious that in a closed system:

$$\Delta p+\Delta q+\Delta r = 0; \quad -1\leq \Delta p, \Delta q, \Delta r \leq 1 \qquad (4.3)$$

The domain of all possible shifts in the relative weights p,q,r related to Fig. 8 can be presented by a hexagonal diagram or <u>hexagram</u> (see Fig. 9). The hexagram contains three coordinate axes $\Delta p, \Delta q, \Delta r$ originating from the diagram's central point with a 120° angle between the axes. Each of the axes represents the relative population change in a corresponding part of the core-ring-periphery regional system. Each vector $\bar{u}_{ik}$ in the hexagram, which starts from the origin, is a difference vector $\bar{u}_k - \bar{u}_i$ between two relative population distributions $\bar{u}_k$ and $\bar{u}_i$ of two different time periods k and i. The hexagram is divided into six sectors where each sector represents a different type of regional dynamics. The type of dynamics can be defined by the sign vector

$$\text{Sign}\ \Delta\bar{u} = (\text{Sign}\ \Delta p,\ \text{Sign}\ \Delta q,\ \text{Sign}\ \Delta r) \qquad (4.4)$$

Let us first consider the simple (+,-) impact model. Sector I, for example, is characterized by the sign vector (+ - -) indicating a pure urbanization process of relative growth of the (relative) core population at the expense of the ring and periphery; sector III indicates a pure suburbanization process of concentration of relative population within the ring, etc. These six sectors of the hexagram can, therefore, serve as a basis for classifying qualitatively possible changes in the relative population of the regional system (see Table 4). For example, we obtain three different forms of the urbanization process: I. pure urbanization; II. urbanization + suburbanization and VI. urbanization + sub-suburbanization.

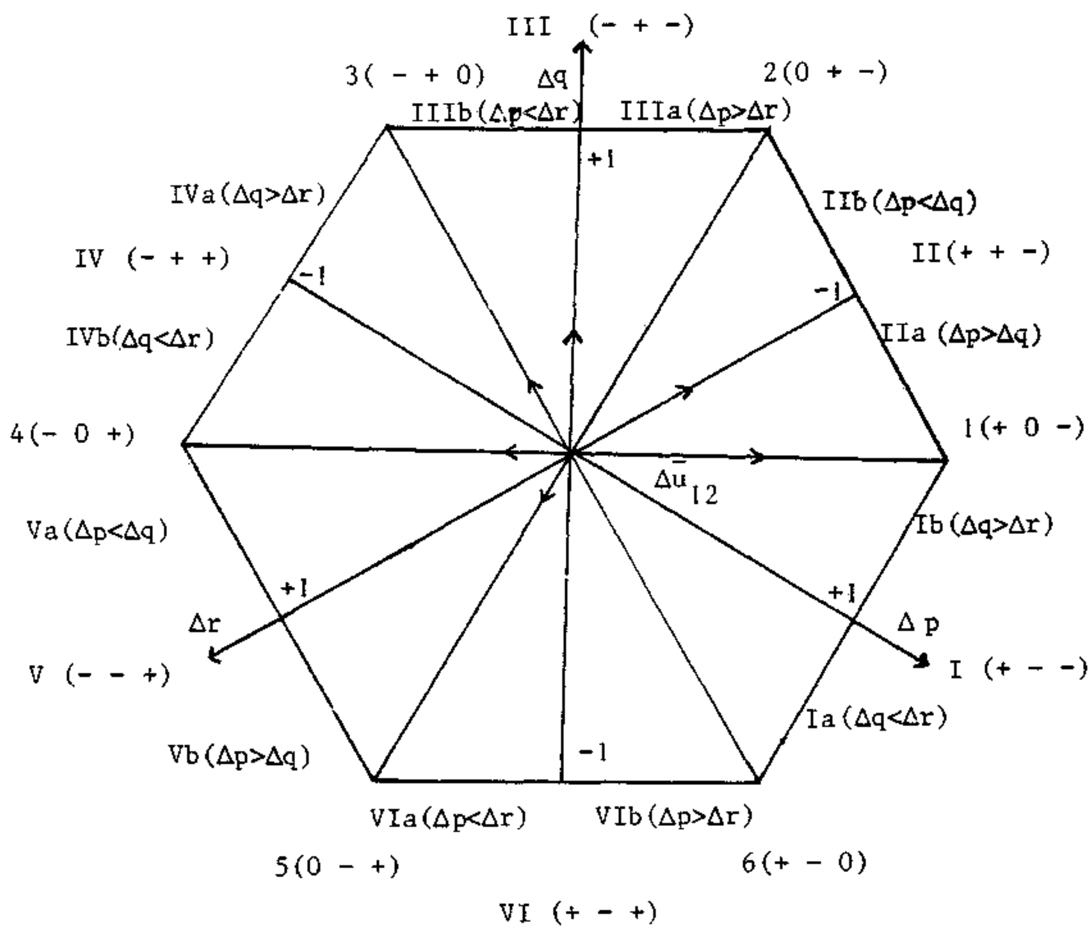

Figure 9. Hexagram of domain of shifts in relative weights

Next, it is possible to refine the qualitative classification of types of relative dynamics by subdividing each of the six sectors of hexagram into two (or more) subsectors, depending on the nature of the distribution of difference vectors $\Delta\bar{u}$. We will choose here a subdivision of sectors by means of three straight lines: $\Delta p = \Delta q$; $\Delta p = \Delta r$; $\Delta q = \Delta r$. This division generates 12 subsectors characterized by inequalities of the types $\Delta p \gtrless \Delta q$; $\Delta p \gtrless \Delta r$; $\Delta q \gtrless \Delta r$ (see Fig. 9). For example, subsector IIa represents the urbanization-suburbanization process with a stronger urbanization component ($\Delta p > \Delta q > 0$).

Sector in hexagram	Sign configurations and qualitative categories			Explanation
	Urbanization	Suburbanization	Sub-suburbanization	
I	+	-	-	Pure urbanization: increase in share of the core population at expense of ring and periphery population
II	+	+	-	Increase in share of core and ring population at expense of periphery population
III	-	+	-	Pure suburbanization: increase in share of ring population at expense of core and periphery population
IV	-	+	+	Increase in share of ring and periphery population at expense of core population
V	-	-	+	Pure sub-suburbanization: increase in share of periphery population at expense of core and ring population
VI	+	-	+	Increase in share of core and periphery population at expense of ring population

Table 4. Six qualitative categories of relative population dynamics in a core-ring periphery model.

For the (+,0,-) impact model we will obtain 13 different qualitative categories, geometrically including the six previous sectors, six additional straight line segments (or sides of sectors) associated with six sign configurations (+ 0 -), (0 + -), (- + 0), (- 0 +), (0 - +), and (+ - 0), and the point of origin with a sign configuration (0 0 0).

If we now assume a binary coding of both positive and negative impacts, we obtain a fairly complex combination problem for the relative changes of the state variables (see for a presentation of all possible cases Table 5).

a) Binary ranking (++,+,-,--) model

Sectors	I					II					III				
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Core	+	++	++	++	++	+	+	++	+	++	-	-	-	--	--
Ring	-	-	--	-	--	+	+	+	++	++	+	++	++	++	++
Periphery	-	-	-	--	--	-	--	--	--	--	-	-	--	-	--

Sectors	IV					V					VI				
	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Core	-	--	--	--	--	-	-	--	-	--	+	+	+	++	++
Ring	+	+	++	+	++	-	-	-	--	--	-	--	--	--	--
Periphery	+	+	+	++	++	+	++	++	++	++	+	+	++	+	++

b) Additional qualitative categories for a ternary (++,+,0,-,--) model

Sides	1		2		3		4		5		6		Origin
	31	32	33	34	35	36	37	38	39	40	41	42	43
Core	+	++	0	0	-	--	-	--	0	0	+	++	0
Ring	0	0	+	++	+	++	0	0	-	--	-	--	0
Periphery	-	--	-	--	0	0	+	++	+	++	0	0	0

Table 5. Sign configurations for relative dynamics in the binary and ternary ranking core-ring periphery model

Fig. 10 presents geometrically 30 possible qualitative categories on the hexagram of possible types of relative population dynamics.

The introduction of a ternary (++,+,0,-,--) impact model will give us 13 additional sign combinations (see also Table 5 and Fig. 10).

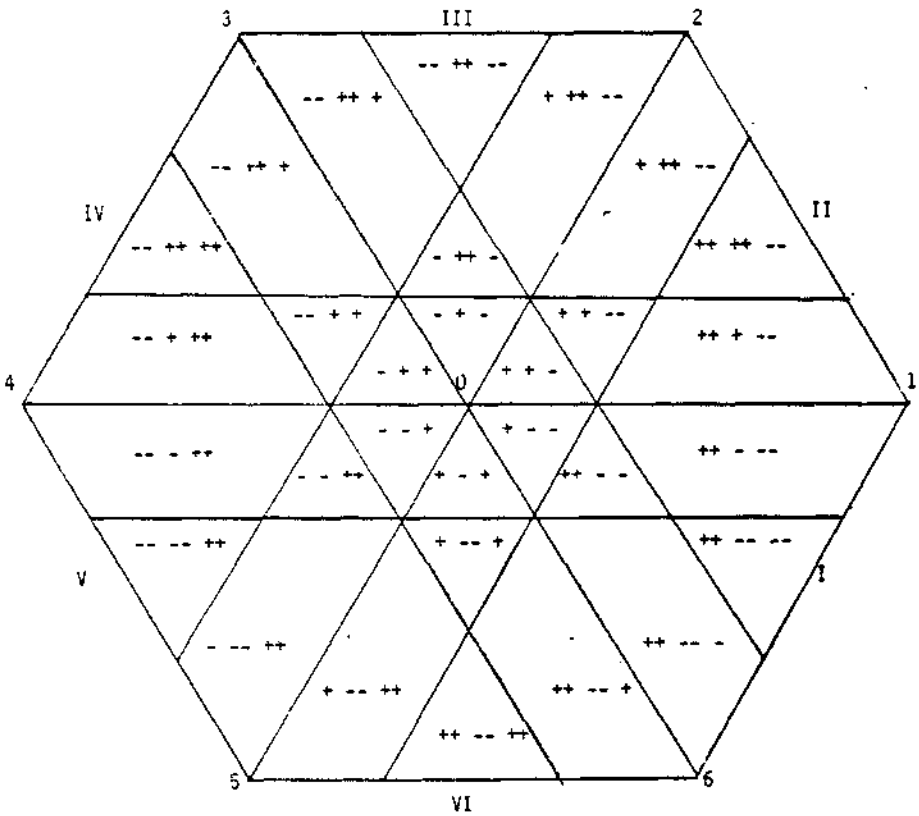

Figure 10. Graphical description of 30 qualitative categories for a binary ranking (++,+,-,--) impact model of relative population dynamics in a core-ring-periphery system.

5. General Framework of Qualitative Description of Relative Multiregional Dynamics

In a manner analogous to the case of absolute spatial dynamics, we will consider a spatial wholeness including N spatial units (regions) and a given spatial impact variable (or spatial substance).

The ith region can be assigned the relative weight z_i of a spatial substance such that

$$0 \leq z_i \leq 1$$
$$z_1+z_2 \ldots +z_N = 1 \tag{5.1}$$

The resulting vector $\bar{z} = (z_1,z_2,\ldots,z_N)$ will be called the vector of relative distribution of some given spatial substance within the spatial system. The weight z_i presents the frequency that a unit of spatial substance will be in the ith region of spatial wholeness.

The changes in the relative distribution vectors are the essence of the relative spatial redistribution process. Associated with this process are the following properties (or events):

(a) the possibility of movement of any unit of spatial substance from any region to any other region;

(b) a non-proportionate (or, as a special case, proportionate) entry or exit of spatial substance from and to the world outside of the spatial system to and from the various regions of the system;

(c) a non-proportionate (or, as a special case, proportionate) internal growth or decline of the shares of spatial substance among the regions of our spatial system;

(d) the ability to conceptually reclassify the regions into any M (or, as a special case, N) regions without altering the nature of the spatial substance; and

(e) the possibility of movement of any unit of spatial substance within a single region of the spatial system.

As can be seen, not all these properties are associated with actual spatial movements. Properties (a), (b), (e) (which might be termed inter-regional migration, 'inter-whole' migration and intra-regional migration, respectively) are clearly spatial in nature. However, property (c) (natural growth) occurs <u>in situ</u>; and property (d) (reclassification) results in a conditional redistribution, but not in actual movements.

To simplify the structure of the redistribution process, it is possible to view inter-whole migration (property (b)) and natural growth (property (c)) as constituting conditional redistributions as well. That is, rather than viewing units as entering the spatial system, they can be viewed as conceptually reclassified. It can then be argued, in summary, that the redistribution process, i.e., the transition from state A to state B, is created by a single process: the passage of the spatial substance from regions of the spatial wholeness to other regions.

Like in the case of absolute changes, this simplifying formulation allows the possibility of associating the transfer from state $\bar{z}_A$ to state $\bar{z}_B$, i.e., the redistribution of spatial substance, with the vector of increments

$$\Delta\bar{z} = \bar{z}_B - \bar{z}_A = (z'_1 - z_1, z'_2 - z_2, \ldots, z'_N - z_N) = (\Delta z_1, \Delta z_2, \ldots, \Delta z_N) \quad (5.2)$$

The qualitative description of the redistribution is then associated with a vector of signs of increments:

$$\text{Sign } \Delta\bar{z} = (\text{Sign } \Delta z_1, \text{ Sign } \Delta z_2, \ldots, \text{ Sign } \Delta z_N) \quad (5.3)$$

where

$$\text{Sign } \Delta z_i = \begin{cases} + \text{ if } \Delta z_i > 0, \\ 0 \text{ if } \Delta z_i = 0, \\ - \text{ if } \Delta z_i < 0. \end{cases} \quad (5.4)$$

The <u>first essential difference</u> between the absolute and relative distribution of the impact variable (spatial substance) emerges from the fact that vectors of relative distribution fall in the N-dimensional simplex defined by constraints;

$$z_1 + z_2 + \ldots + z_N = 1; \; 0 \le z_i \le 1; \; i=1,2,\ldots,N \quad (5.5)$$

This means that in the case of three regions the domain of possible relative distributions will be an equilateral triangle; in the case of four regions this domain will be a regular tetrahedron and in the case of N regions the domain will be a regular simplex.

The <u>second essential difference</u> between the absolute and relative distributions of the impact variable stems from the fact that the difference vector $\bar{z}_B - \bar{z}_A = \Delta\bar{z}$ has the property:

$$\Delta z_1+\Delta z_2+\ldots+\Delta z_N= 0, \ -1 \le \Delta z_i \le 1; \ i=1,2,\ldots,N \qquad (5.6)$$

This property puts strong constraints on the number of combinatorial sign possibilities: for instance, possibilities of type (+,+,+,...+), (+,0,+,...,+), (-,-,...,-), or (+,--,--,...,--) are impossible.

It is possible to demonstrate that the number n of sign configurations in the N region (+,-) impact model is equal to:

$$n = 2^N-2; \qquad (5.7)$$

in the (+,0,-) impact model:

$$n = 3^N-2(2^N-1); \qquad (5.8)$$

in the binary ranking (++,+,-,--) impact model:

$$n = 4^N- 2(2^N+ N(2^{N-1}-1)) = (2^N-2)(2^N-N); \qquad (5.9)$$

and in the ternary ranking (++,+,0,-,--) impact model:

$$n = 5^N -2(3^N-1 + N(3^{N-1}-2^{N-1})) \qquad (5.10)$$

6. Spatial Diversity, Temporal Dynamics and the Hypothesis of Unilinear Evolution

The urbanization spatial cycle hypothesis described in Section 2 means that each core-ring regional system passes periodically through the same sequence of urbanization stages: the stages of urbanization, suburbanization, desurbanization and reurbanization. In the set of different spatially distributed core-ring systems the stage of each such system depends on time: the same urbanization cycle generates the spatial diversity of urbanization stages. Essentially, the spatial cycle hypothesis is closely connected to the so-called universal hypothesis of unilinear evolution.

The hypothesis of unilinear evolution connects the spatial variations and temporal changes in the following way: the spatial distribution of the states of analogical spatial systems presents also the temporal stages through which any one spatial system is likely to pass.

Two main problems are connected with the hypothesis of unilinear evolution: the first problem is to find analytical interconnections between the empirical regularities of spatial distribution of quantitative categories of regions in a fixed point in time and the temporal dynamics of one fixed regional system, i.e., to evaluate the inner 'evolutionary' time for the dynamics of regional systems on the basis of empirical regularities of the spatial distribution of qualitative categories; the second problem is to find the cartographical presentation of spatial evolution of the ensembles of regional systems.

An illustration of the analytical and cartographical presentation of space-time relative evolution - by means of a three population/single regional spatial demographic system - can be found in Sonis (1981).

Finally, it is interesting to observe that two sides of the hypothesis of unilinear evolution - spatial and temporal - were the essence of the methods of Maxwell and Boltzmann. Max Planck expressed this as follows (see Planck (1931), p. 891):

'...there is some dissimilarity in the methods of Maxwell and Boltzmann. The first, for the obtaining of a definite statistical regularity in the case of complex compound mechanical system, considered simultaneously a multitude of specimen of this system in different states. Boltzmann, however, prefers to follow the diversity of changes of the same system for a very long time. Both these approaches, proceeded successively, lead to the same statistical laws.'

In its modern form, the ergodic theorem of Boltzmann-Gibbs resembles the hypothesis of unilinear evolution: for gas particles the mean velocity of the particles in the bounded volume is equal to the mean velocity of only one particle in time.

In the field of multiregional dynamics one may hypothesize that the concentric decrease in the density of some regional spatial substance around one or a few centres forms some evidence for the existence of the unilinear evolution of analogous spatial systems. Clearly, the hypothesis of unilinear evolution cannot be regarded as the exclusive foundation stone for the dynamic behaviour of a spatial system. Also singularities leading to bifurcations should be added as driving forces for complicated dynamics. Finally, it is noteworthy that unilinear evolution reflects essentially the dynamic pathways between different singularities or catastrophes. More rigorous research in this field supported by empirical evidence would no doubt mean an important contribution to the field of qualitative spatial analysis.

References

Berg, L. van den, L. Burns, and L. H. Klaassen eds.), Spatial Cycles, Gower, Aldershot, 1986.

Bon, R. (1986), Qualitative Input-Output Analysis, Paper North-American Meeting of Regional Science Association, Columbus, (mimeographed)

Brouwer, F. (1987), Integrated Environmental Modelling, Martinus Nijhoff, Dordrecht, the Netherlands.

Brouwer, F., P. Nijkamp and H. Voogd (1987), Mixed Qualitative Calculus and Economic Modelling, Systems Analysis Modelling and Simulation, vol.4, no.5, pp. 399-408.

Brouwer, F. and P. Nijkamp (1985), Qualitative Structure Analysis of Complex Systems, Measuring the Unmeasurable, (P. Nijkamp, H. Leitner and N. Wrigley, eds.), Martinus Nijhoff, Dordrecht, the Netherlands, pp. 509-532.

Kawashima, T. (1987), Population Decentralization in a System of Metropolitan Areas: Comparison between U.S. and Japan, Problems and Possibilities of Regional Decentralization (P. Friedrich and I. Masser, eds.), Nomos, Baden-Baden, pp. 153-168.

Klaassen, L.H., J.A. Bourdrez and J. Volmuller (1981), Transport and Reurbanization, Gower, Aldershot, U.K.

Lancaster K. (1962), The Scope of Qualitative Economics, Review of Economic Studies, vol. 29, pp. 99-123.

Maybee, J.S. and J.H. Voogd (1984), Qualitative Impact Analysis through Qualitative Calculus, Environment & Planning B, vol. 11, pp. 365-376.

Maybee, J.S. (1981), Sign Solvability, Computer Assisted Analysis and Model Simplification, (H.J. Greenberg and J.S. Maybee, eds.), Academic Press, New York, pp. 201-257.

Nijkamp, P. (1987), Transportation and Mobility in a Changing World, Transportation Planning in a Changing World, Gower, Aldershot, U.K., pp. 73-92.

Planck, M. (1931), James Clerk Maxwell in seiner Bedeutung fur die theoretische Physik in Deutschland, Naturwissenschaften, vol. 19, pp. 257-270.

Quirk, J. (1981), Qualitative Stability of Matrices and Economic Theory: A Survey Article, Computer Assisted Analysis and Model Simplification (H. J. Greenburg and J. Maybee, eds.), Academic Press, New York, pp. 113-164.

Sonis, M. (1981), Space and Time in the Geography of Aging, Dynamic Spatial Models, (D. Griffith and R, MacKinnon, eds.), Martinus Nijhoff, Dordrecht, the Netherlands, pp. 132-156.

Voogd, H. (1982), Spatial Impact Analysis through Qualitative Calculus: an Exploration. IIASA Working Paper WP-82-82, Laxenburg, Austria.