SERIE RESEARCH MEMORANDA

R & D POLICY IN SPACE AND TIME

A Nonlinear Evolutionary Growth Model

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1. Introduction

In the recent past much attention has been devoted to the economic foundations and impacts of technological innovations (cf. Stoneman, 1983). Various behavioural paradigms have been proposed in this context, such as the depression trigger hypothesis, the demand pull hypothesis and the technology push hypothesis. In each of these paradigms, research and development (R&D) plays a crucial role in enhancing the efficiency and the competitive position of firms or regions. However, R&D expenditures generate, like investment in capital goods, an intertemporal allocation problem: more R&D may lead to a higher long-run productivity and profitability, but requires lower short-run consumption, and vice versa. This issue has been considered for capital accumulation in general extensively in traditional growth theory, both for economies on a steady state growth path ('golden rule of accumulation') and as an intertemporal welfare optimisation problem (optimal control theory); see e.g. Jones (1975) and Rasmanathan (1982). With respect to R&D expenditure, two interesting questions emerge. The first question is what the behaviour of the growth path of the economy would be in an integrated consumption, production, investment and R&D system, in particular when the system is facing capacity limits in terms of congestion, other diseconomies of scale, or depletion of exhaustible resources. Secondly, the introduction of spatial considerations generates a non-trivial dimension in that new technology in any region may be obtained by R&D internally, or by acquisition from external sources. In the latter case, the boost to productivity may be delayed and less effective since the adopted technology is not likely to be 'custom-made'. The question arises therefore what the optimal balance should be between adopting technology from external technology-leaders, and generating technology by means of own R&D expenditure.

In this paper we shall explore these questions by means of a multi-regional dynamic (discrete-time) model. It will be shown that the system can generate a wide range of dynamic behaviour, including - for certain parameters - the dynamic evolutions occurring in models of population biology (notably the so-called May type of models, see e.g. May (1976)). The May model is a particular type of model generating chaotic behaviour in dynamic systems. The theory of chaos as such has gained increasing interest in economics (see among others Benhabib and Day, 1981, 1982, Pohjola, 1981, or Stutzer, 1980). It is well-known that the periodic or chaotic behaviour of a May model is the result of specification of the

model in difference equation form (see e.g. Barentsen and Nijkamp, 1988), but it will be shown that the model developed here generates bounded non-linear dynamics even in differential equation form. Finally, the steering possibilities of the system (e.g. by means op optimized control) will be touched upon.

2. A Prototype Model for Economic Dynamics

In this section a simple prototype model for economic development will be formulated and presented stepwise. We commence with the assumption that each of the regional economies under consideration is operating under the following production function regime (in difference equation form):

$$Y_t - f_t(K_t) \qquad , \qquad (2.1)$$

where Y_t is actual production (or output) during the period (t, t+1), K_t the installed capital stock at the beginning of period t, and f_t a time-dependent (i.e., varying in parameters) production function.

For the sake of simplicity, we will for the moment assume a simple production technology, i.e.,

$$Y_t = \epsilon_t K_t \qquad , \qquad (2.2)$$

where ϵ_t is a technological coefficient representing average capital productivity during the period (t-1,t). However, this assumption is not as restrictive as it seems, since in a sense we may consider (2.2) an identity in which ϵ_t includes all factors which influence capital productivity. Thus, rather than making the a priori assumption that the elasticity of substitution between capital and other production factors is zero, we shall show below that the time trajectory of ϵ can incorporate both movements along the production frontier as well as shifts in this frontier.

With respect to capital accumulation, the following equation holds:

$$K_{t+1} = (1-\delta) K_t + I_t \qquad (2.3)$$

where I_t stands for gross investment during period (t,t+1) and δ is the

rate of physical depreciation of the capital stock. We assume the following simple investment function for capital expansion (or widening):

$$I_t = \sigma_1 Y_t \qquad , \qquad (2.4)$$

where σ_1 is the fixed average savings rate. Clearly, this relationship takes for granted the existence of equilibrium between savings and capital increase. Assuming a given savings behaviour, it is evident that σ_1 acts as one of the driving forces or key parameters of our dynamic system.

It is now easily seen from (2.2) that if any growth rate is expressed as: $g_t^X = (X_{t+1} - X_t)/X_t$, the growth rate of income (i.e., g_t^Y) is (approximately) equal to the sum of the growth rate of capital (i.e., g_t^K) and the growth rate of capital productivity (i.e., g_t^ϵ):

$$g_t^Y = g_t^K + g_t^\epsilon \qquad (2.5)$$

and that - by means of (2.3) and (2.4) - we find that

$$g_t^Y = (\sigma_1 \epsilon_t - \delta) + g_t^\epsilon \qquad (2.6)$$

The latter relationship implies the obvious result that - in case of a negligible change in the production technology in period $(t, t+1)$, i.e. $g_t^\epsilon = 0$ - the growth in income g_t^Y would be equal to the savings rate σ_1 times the capital coefficient ϵ_t (i.e. Harrod's conventional warranted growth rate) minus the rate of capital depreciation.

Since we have taken for granted period-by-period macroeconomic equilibrium with respect to investment and savings, it follows that

$$Y_t = C_t + I_t \qquad , \qquad (2.7)$$

and we can easily derive that consumption C_t is equal to:

$$C_t = (1 - \sigma_1) Y_t \qquad , \qquad (2.8)$$

so that consumption, investment and income all follow the same growth path, i.e.

$$g_t^C = g_t^I = g_t^Y = \sigma_1 \epsilon_t - \delta \qquad (2.9)$$

Naturally, the case of a variable propensity to save violates (2.9); this will be considered later in Annex A, where the savings rate will be regarded as a control variable in an optimal control model.

Having presented now the basic elements of a simple growth model, we will introduce in the next section in a more detailed way the causes and consequences of changes in capital productivity.

3. Variable Capital Productivity

Here we assume that the production efficiency can be enhanced by means of R&D efforts embodied in the production technology. Consequently, the production function has to be adjusted, as R&D investments will imply a growth in efficiency due to a change in the capital coefficient (see Baumol and Wolff, 1984; Mansfield, 1980; Nelson, 1981). Therefore, we may assume that - in order to develop a new 'technological regime' (cf. Nelson and Winter, 1982) - R&D expenditures will exert a positive impact on the production efficiency parameter ϵ_t. Thus the productivity of capital can be improved through capital deepening, which is in general a function of R&D. The effect may be represented by the variable ν_t, which measures the impact on productivity of a unit of expenditure in R&D. Hence

$$\Delta \epsilon_t = \epsilon_{t+1} - \epsilon_t = \nu_t R_t \qquad (3.1)$$

where R_t represents the R&D investments per capita during period $(t, t+1)$ amd ν_t the R&D impact parameter for the capital coefficient for the same period. We introduce the following equation for R_t, which defines the savings rate for R&D investments in a way analogous to (2.4):

$$R_t = \sigma_2 Y_t \qquad (3.2)$$

Thus consumption is now equal to:

$$C_t = (1 - \sigma_1 - \sigma_2) Y_t \quad , \qquad (3.3)$$

while substitution of (3.2) into (3.1) leads to the following result:

$$\Delta \epsilon_t = \nu_t \sigma_2 Y_t \qquad (3.4)$$

Using (3.4), (2.2) and (2.6) it can be easily seen that

$$g_t^Y = (\sigma_1 \epsilon_t - \delta) + \nu_t \sigma_2 K_t \qquad (3.5)$$

The latter equation suggests that, if ν_t would be constant over time, capital accumulation generates ever-increasing growth in output and capital productivity. This unrealistic outcome suggests that it is plausible to assume that ν_t becomes less when output increases, which simply implies that the marginal efficiency of R&D expenditure declines when output grows. Under a given 'technological regime', ultimately a 'saturation' level of output may exist at which further R&D expenditure has no longer an impact on productivity. Such a saturation level may arise from capacity limits (technological, social, economic) and reflects - for a given production technology - a 'limits to growth' phenomenon, stemming from congestion, lack of natural resources or labour input.

Calling this level of output Y_t^C, it follows that $\nu_t - 0$ when $Y_t \geq Y_t^C$. Naturally, the limits to growth themselves may be shifting, so that Y_t^C will increase with time and, as bottlenecks are overcome, further R&D expenditure may again have a positive effect on productivity. We may therefore assume the following specification for an adjusted (i.e., time-dependent) R&D impact parameter:

$$\nu_t - \max \{\nu^* (1 - Y_t/Y_t^C), \; 0\} \qquad (3.6)$$

This relationship is depicted in Figure 1. It is clear that an analogous result might be reached by imposing a saturation level for ϵ_t.

In our case, it is easily seen that not only would R&D expenditure
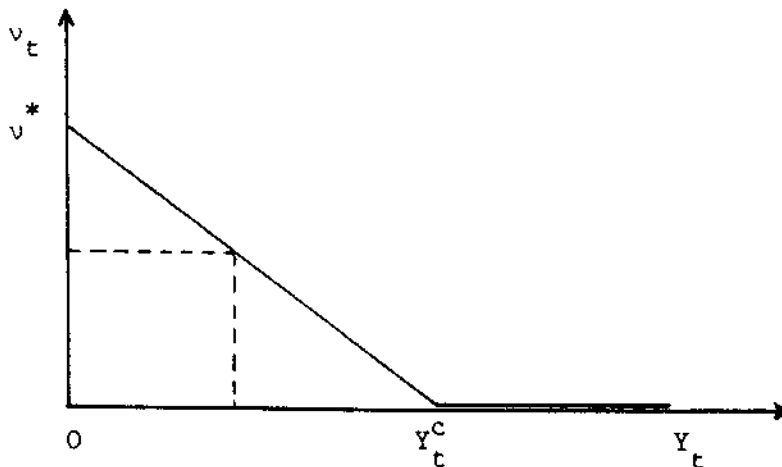

Figure 1. Decreasing marginal benefits of R&D expenditures.

become ineffective if output expands beyond Y_t^c, but it may also be expected that congestion and other external diseconomies set in which reduce capital productivity. In a broader framework, it may also be plausible to assume that the notion of a limited system capacity also refers to increasing labour scarcity along the growth path, which would tend to lead to substitution of capital for labour in response to higher real wages. Capital accumulation would therefore proceed, resulting in an increase in capital intensity (or a reduction in capital productivity).

The previous remarks suggest that we may replace (3.1) by the following simple relationship:

$$\Delta \epsilon_t = \nu_t \, R_t - \mu_t \, Y_t \qquad\qquad (3.1')$$

in which μ_t measures the congestion etc. effects on productivity when output exceeds Y_t^c and, thus:

$$\mu_t = \max \, (\mu^* \, (Y_t/Y_t^c - 1), \, 0) \qquad\qquad (3.7)$$

Substituting (3.7) and (3.6) into (3.1) and recalling (2.2) and (2.3), the motion in the system can be described by the following set of non-linear difference equations:

$$K_{t+1} = (1-\delta) \, K_t + \sigma_1 \, \epsilon_t \, K_t \qquad\qquad (3.8)$$

$$\epsilon_{t+1} = \epsilon_t + [\sigma_2 \nu^* \max \, (1-Y_t/Y_t^c, 0) -$$
$$\qquad\qquad (3.9)$$
$$- \mu^* \max \, (Y_t/Y_t^c - 1, 0)] \, \epsilon_t \, K_t$$

$$Y_{t+1} = \epsilon_{t+1} \, K_{t+1} \qquad\qquad (3.10)$$

$$Y_{t+1}^c = f \, (Y_t^c) \qquad\qquad (3.11)$$

From (3.11) it can be seen that the time-trajectory of Y_t^c is considered exogenous to the system under consideration. However, instead of an autonomous trajectory of Y_t^c (e.g., based on a fixed technological progress leading to a permanent upward shift of the system's bottlenecks), it might also be possible to relate this upward shift of

Y_t^c to the average change in the production efficiency parameter in previous periods. The linkages between the variables in the stock-flow system are depicted graphically in Figure 2.
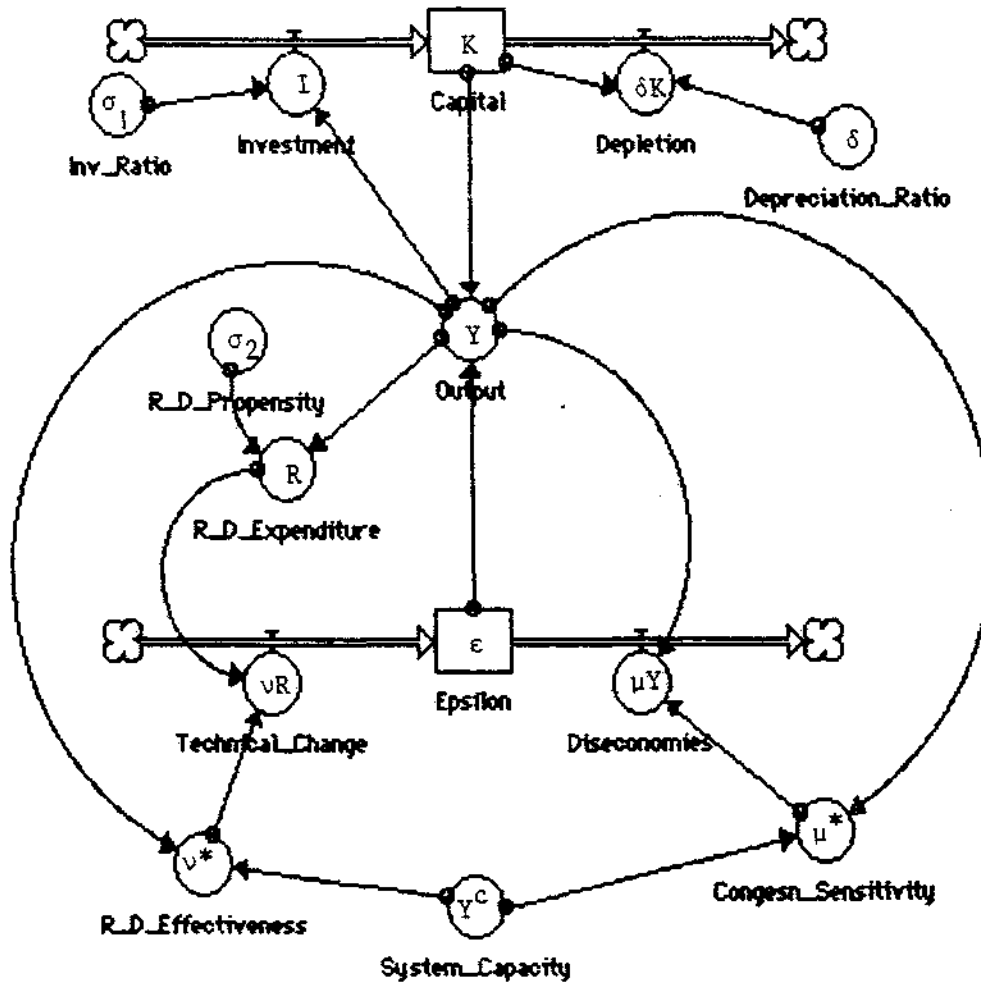

Figure 2. A representation of the simple growth model.

It is obvious that for any given initialisation (K_o, G_o, Y_o, Y_o^c), the system (3.8)-(3.11) can exhibit a wide range of time trajectories dependent on parameter values. It is therefore useful to consider some special cases.

Firstly, if the capacity of the system Y_t^c is constant over time, i.e. if $Y_t^c = \bar{Y}^c$ for all t, a non-trivial zero growth economy exists, which satisfies (3.8)-(3-10) and for which $\bar{Y} = \bar{Y}^c$, $\bar{\epsilon} = \delta/\sigma_1$ and $\bar{K} = \sigma_1 \bar{Y}^c/\delta$ (this trivial solution has zero capital and output).

Is this stationary state stable? The answer depends on the values of the parameters in the equation for capital productivity. It can be easily seen that this equation can be written as:

$$\epsilon_{t+1} = \epsilon_t + [\sigma_2 \nu^* \max(1-\epsilon_t K_t/\overline{Y}^C, 0) +$$

$$+ \mu^* \min(1-\epsilon_t K_t/\overline{Y}^C, 0)] \epsilon_t K_t \tag{3.12}$$

If the depreciation of capital δ and/or the savings ratio σ_1 can vary over time and they are such that $\underline{net}$ investment is zero, (3.9) reduces to a well-known non-linear difference equation when $\sigma_2 \nu^* = \mu^*$. In this case:

$$\epsilon_{t+1} = \epsilon_t + \mu^* \overline{K}(1-\epsilon_t \overline{K}/\overline{Y}^C) \epsilon_t \tag{3.13}$$

Equation (3.13) is the standard May type model (from population dynamics), which may generate any type of dynamic behaviour ranging from stable growth to chaotic fluctuations, depending on the value of $\mu^* \overline{K}$ (see for further details among others Brouwer and Nijkamp, 1985; Goh and Jennings, 1977; Jeffries, 1979; Parker, 1975 and Pimm, 1982). In particular, if $\mu^* \overline{K} > 2.57$, such a May-type of model may exhibit wild fluctuations (May, 1974). For further discussions on the May model in the context of chaos theory, the reader is referred to Nijkamp and Reggiani (1988b).

Nonetheless, $\overline{\epsilon} = \overline{Y}^C/\overline{K}$ is $\underline{locally}$ stable when $\mu^* \overline{K} < 2$. Moreover, if we replace the 'non-overlapping generations' approach of the above mentioned difference equation by the corresponding differential equation, it can be easily derived that the general solution is

$$\epsilon(t) = \frac{1}{\dfrac{\overline{K}}{\overline{Y}^C} + \left(\dfrac{1}{\epsilon(0)} - \dfrac{\overline{K}}{\overline{Y}^C} \right) e^{-\mu^* \overline{K} t}} \tag{3.14}$$

which represents the well known $\underline{logistic}$ growth with a $\underline{globally}$ stable solution $\overline{\epsilon} = \overline{Y}^C/\overline{K}$, since $\mu^* \overline{K} > 0$ always.

More generally, when $\sigma_2 \nu^* \neq \mu^*$ and both δ and σ_1 are constant so that the capital stock can vary over time, inspection of the local stability of the system is complicated by the fact that (3.12) is not differentiable with respect to ϵ_t in the neighbourhood of $\overline{Y}^C/\overline{K}$. Moreover, it is obvious that the system (3.8)-(3.11) is a generalisation of the May-type of model and may generate unpredictable behaviour. However, for empirically plausible values of ν^* and μ^* such that for Y_t not far from Y^C, the change in productivity would remain bounded (i.e. ϵ would remain non-negative), we would expect the system to converge to

the zero growth state, with $(\bar{\epsilon}, \bar{K}) = (\delta/\sigma_1, \sigma_1 \bar{Y}^c/\delta)$ being a <u>node</u> type of singular point (e.g., Gandolfo, 1980, pp. 428-459). Further expositions on the existence of such steady state points in an optimal control framework can be found in Figure A1 in Annex A.

Moreover, simulation experiments show that when Y_t^c is no longer constant, but increases at a constant rate n per period, this growth rate becomes the 'natural' growth rate of the system. In this case, average capital productivity settles at a value of $(n+\delta)/\sigma_1$ and output and capital will grow at rate n, with Y_t being identical to Y_t^c. Starting with $Y_o<Y_o^c$, the time it takes to reach the capacity constraint decreases when either the investment ratio (σ_1), or the R&D propensity (σ_2), or the R&D effectiveness (ν^*) increases, but increases when the growth rate of Y^c is larger. Naturally, when output exceeds the capacity of the system, productivity will decrease the faster, the stronger the congestion effect μ^*.

These results are illustrated by means of Figures 3-5. Figure 3 is based on the assumption that $K_o=1000$ and the capital-output ratio equals 5, so that $\epsilon_o=0.2$ and $Y_o=200$. The saving ratio is 20 percent, 2 percent of the capital stock becomes obsolete each period and 2 percent of income is spent on R&D. Hence, $\sigma_1=0.20$ and $\delta=\sigma_2=0.02$. The <u>sustainable</u> output capacity $Y_o^c=1000$ and grows at 1 percent p.a. Moreover, $\mu^*=0.0001$ and $\nu^*=0.001$. Since $5\sigma_2\nu^*=\mu^*$, the productivity response is five times as elastic when $Y_t>Y_t^c$ than when $Y_t<Y_t^c$, and of opposite sign. Clearly, the evolution of Y_t^c might in principle differ for each region.

Figure 3 shows that growth in the system is under these conditions initially <u>accelerating</u>, but the growth rate of capital productivity reaches a maximum at t=25 and subsequently declines until Y_t reaches the capacity level Y_t^c at t=37. At this point, the growth rate of capital accumulation reaches a maximum. Beyond t=37, Y_t will remain above Y_t^c but will converge to the latter. Consequently, capital productivity becomes constant at a rate of $(n+\delta)/\sigma_1=0.15$ and capital and output grow at a steady state rate of 1 percent.
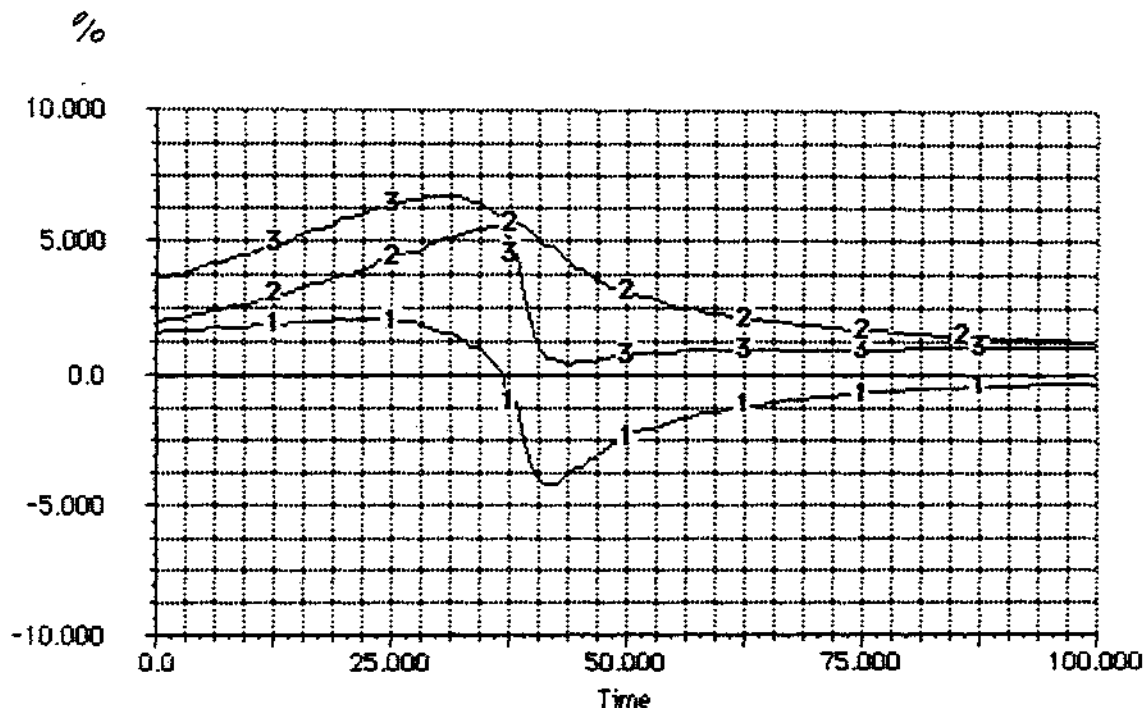
Figure 3. Growth converging to a steady state.
Legend: 1 : growth in capital productivity
2 : capital growth rate
3 : income growth rate

In figure 4 all parameters are the same as in Figure 1, but μ^* has been increased to five times its former value. Consequently, the congestion and other diseconomics effects are now sufficiently strong to push Y_t at times below Y_t^c so that growth cycles are generated with a variable periodicity but with decreasing amplitude. The system eventually converges again to a steady-state growth of 1 percent.

It must be emphasized that the cyclical behaviour in Figure 4 is entirely due to the difference equation specification of system (3.8). When this system is written in differential equation form and the solution is computed by means of the efficient Runge-Kutta integration method, Figure 5 results for parameters identical to those in Figure 4. Hence growth rates adjust relatively smoothly to the steady-state levels.
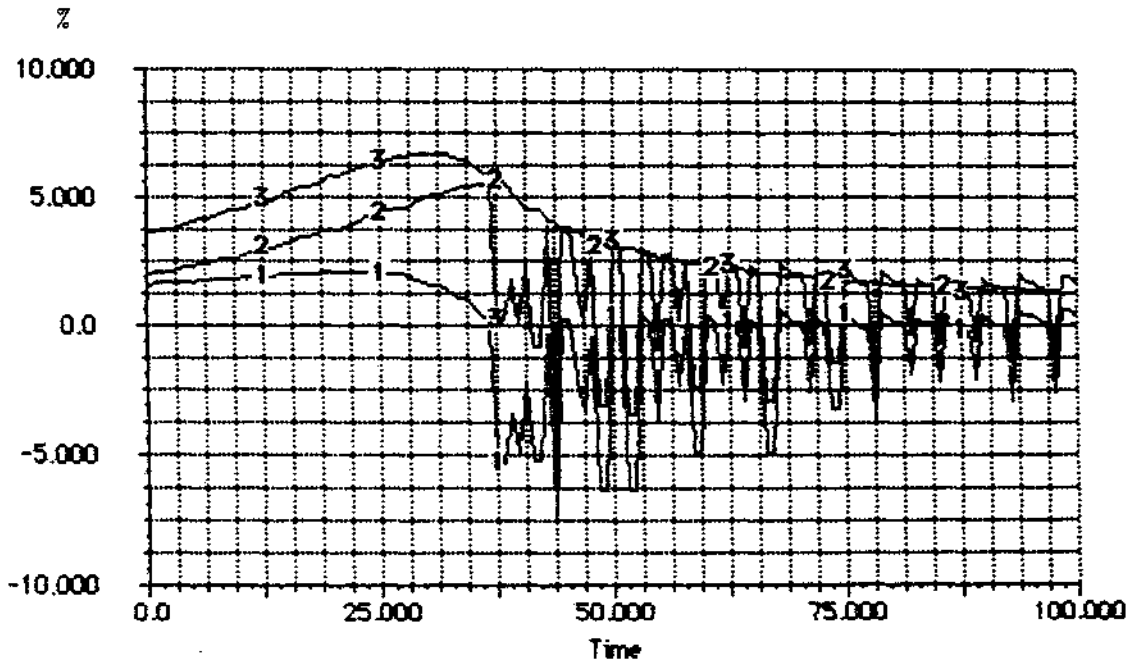
Figure 4. Growth cycles generated by strong external
diseconomies.

Thus far we have focussed exclusively on the dynamic properties of
an economic growth system in isolation. In the next section we shall
consider the consequences of allowing for spatial interaction in the
form of diffusion and adoption of new technology generated by R&D in a
multiregional system. It will be shown that such a system can generate
growth patterns which have no tendency to converge to a steady state,
even if the parameters are chosen such that the regions in isolation
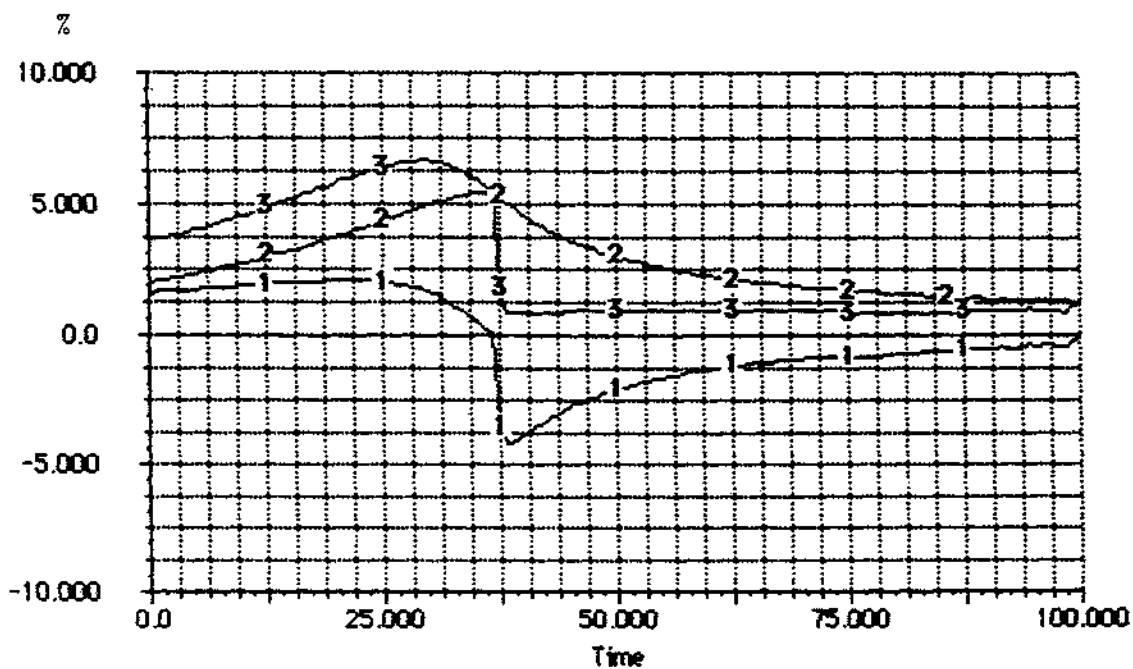would do so.

Figure 5. Simulation with a differential equation structure.

4. A Multiregional Dynamic Model

In a system of regions, technology transfers from any one region would exert an impact on the R&D efficiency of other regions (see also Kamien and Schwartz, 1982, Nijkamp, 1985, and Scherer, 1980). Such interregional spill-over phenomena may be taken into account by introducing a certain spatial R&D transfer function, which incorporates spill-over effects from R&D investments in other regions upon the regional production efficiency. However, as in the single region case, the effect of R&D on productivity would depend on how close the level of production is to the capacity level at which applications of the new technology have been exhausted and bottlenecks and other constraints prevent further increases in productivity. When capital accumulation generates output beyond this capacity level, productivity declines as a result of diminishing returns, congestion and other diseconomies effects.

Denoting regions by an index r (r=1,2,...,R), the process of spatial diffusion and adoption of technology described by

$$\epsilon_{t+1}^r = \epsilon_t^r + \sum_{i=1}^{R} \nu^{ir} \max (1-Y_t^r/Y_t^{cr}, 0) \; \sigma_2^i \; Y_t^i$$

$$- \mu^r \max (Y_t^r/Y_t^{cr} - 1, 0) \; Y_t^r \qquad (4.1)$$

$$r=1,2,...,R$$

in which ν^{ir} represents the marginal efficiency of R&D expenditure in region i when the technology is adopted by region r. It is obvious that the dynamic behaviour of the system depends crucially on the R&D diffusion and adoption matrix:

$$N = \begin{bmatrix} \nu^{11} & \cdots & \nu^{1R} \\ \vdots & & \vdots \\ \nu^{R1} & \cdots & \nu^{RR} \end{bmatrix} \qquad (4.2)$$

Naturally, the model discussed in the previous section is a special case in which N is a diagonal matrix and spatial linkages are absent.

Generally, we would expect the off-diagonal elements of N to be non-negative with larger values for transmission between contiguous rather

than non-contiguous regions. This may be reflected by a distance decay function

$$\nu^{ir} = \nu^{rr} e^{-cd_{ir}} \qquad (4.3)$$

in which c is a constant and d_{ir} measures the distance (or cost) of diffusion of technology from i to r.

Using that $Y_t^r = \epsilon_t^r K_t^r$ for r = 1,2,...,R, equation (4.1) can be written in matrix form as

$$
\begin{bmatrix} \epsilon_{t+1}^1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \epsilon_{t+1}^R \end{bmatrix} = \begin{bmatrix} \epsilon_t^1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \epsilon_t^R \end{bmatrix} + \begin{bmatrix} \max \ (1-\epsilon_t^1 K_t^1/Y_t^{c1},0) \ 0 \text{-} \cdots \text{-} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 0 \text{-} \cdots \text{-} 0 \ \max \ (1-\epsilon_t^R K_t^R/Y_t^{cR},0) \end{bmatrix} N^T \begin{bmatrix} \sigma_2^1 \ \epsilon_t^1 \ K_t^1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \sigma_2^R \ \epsilon_t^R \ K_t^R \end{bmatrix}
$$

$$
- \begin{bmatrix} \mu^1 \ \max(\epsilon_t^1 K_t^1/Y_t^{c1}-1,0)\epsilon_t^1 K_t^1 \\ \cdot \\ \cdot \\ \cdot \\ \mu^R \ \max(\epsilon_t^R K_t^R/Y_t^{cR}-1,0)\epsilon_t^1 K_t^1 \end{bmatrix} \qquad (4.4)
$$

Qualitatively, the spatial interaction between regions in system (4.4) is characterised by a **positive feedback loop**: R&D expenditure in any one region leads to higher growth in other regions, which - in turn - boost R&D expenditure in those regions, giving a further impulse to growth in the original region. Thus, given a 'pooling' model for inter-regional technology transfer, a new evolutionary pattern of regional growth may emerge. However, the presence of capacity constraints implies that the system can again exhibit the great many types of dynamic behaviour of the May model.

It can be easily verified that $(\bar{\epsilon}^r, \bar{K}^r) = ((\delta^r+n^r)/\sigma_1^r, \sigma_1^r \ Y^{cr}/\delta^r)$ is, as in the single region case, a singular point, when Y^{cr} grows at a rate of n^r (r=1,2,...,R).

However, stability of this steady state is extremely unlikely, in particular when the system consists of a mixture of large and small regions, with varying capacity growth rates n^r. In that case, explosive time trajectories would be common.

These results can again be illustrated by means of simulation. We shall consider here one case-study of three regions with one 'large' region $(K_o^1=2000)$ and two 'small' ones $(K_o^2 = K_o^3 = 500)$. In all other respects (initial technology, savings propensities etc.) the regions are identical. The technology adoption matrix N is given by:

$$ N = \begin{bmatrix} 0.001 & 0.0005 & 0 \\ 0.0001 & 0.001 & 0.0001 \\ 0.0001 & 0.0001 & 0.001 \end{bmatrix} $$

Hence region 3 does not adopt technology from region 1, while region 2 readily adopts this technology. It is also assumed that $Y_o^{cr}= 1000$ for $r = 1,2,3$ and grows at 1 percent per period. For simulation, (4.4) was replaced by its differential equation equivalent and the Runge-Kutta method was used for integration. Figure 6 shows that under these conditions regions 1 and 2 converge to a steady-state growth process, in which capital productivity is constant and output (and capital) grows at the 'natural' rate of 1 percent. In contrast, region 3 remains behind in production efficiency until $t=25$, but subsequently 'overtakes' both other regions. Moreover, while there is a tendency for productivity to come close to its steady-state value, at that stage persisting cycles emerge.

Differences in the growth paths between the three regions suggest that even if consumption per capita would be identical initially, this would not remain so due to differences in R&D expenditure and investment between regions. This can be seen from figure 7. It has been assumed that populations are such that initially consumption per capita is equal across regions. Population growth in all regions is assumed to be 1 percent per period. Consumption per capita in regions 1 and 2 settles down at steady-state values (but at a much higher level for the latter). However, consumption per capita reaches an even higher level in region 3, but this level cannot be sustained.
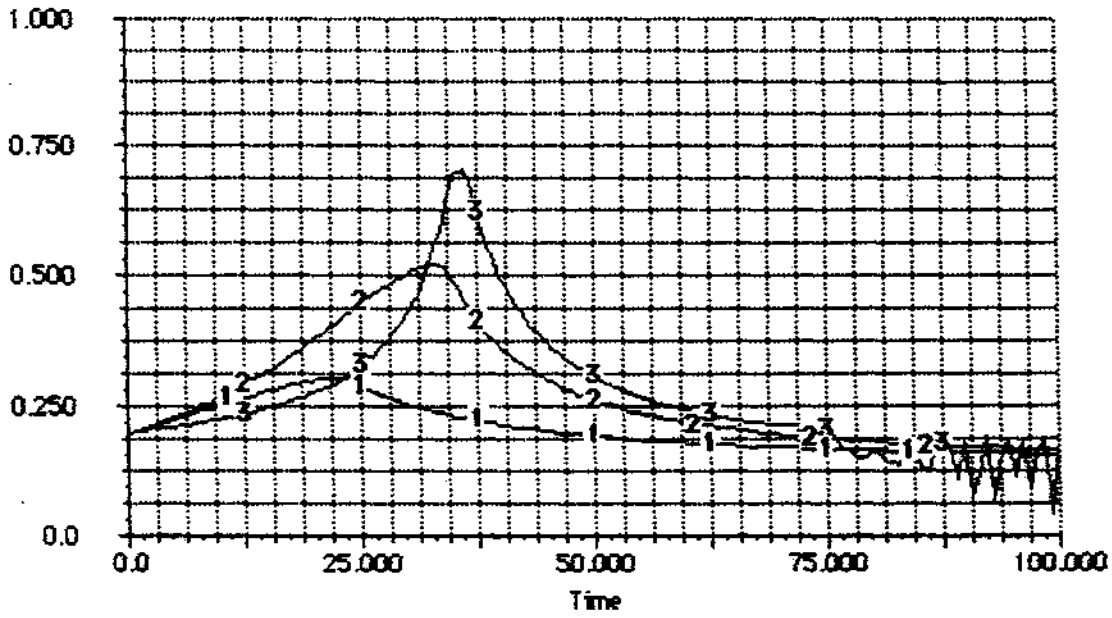
Figure 6. Capital productivity growth in a multiregional
system with technology transfers
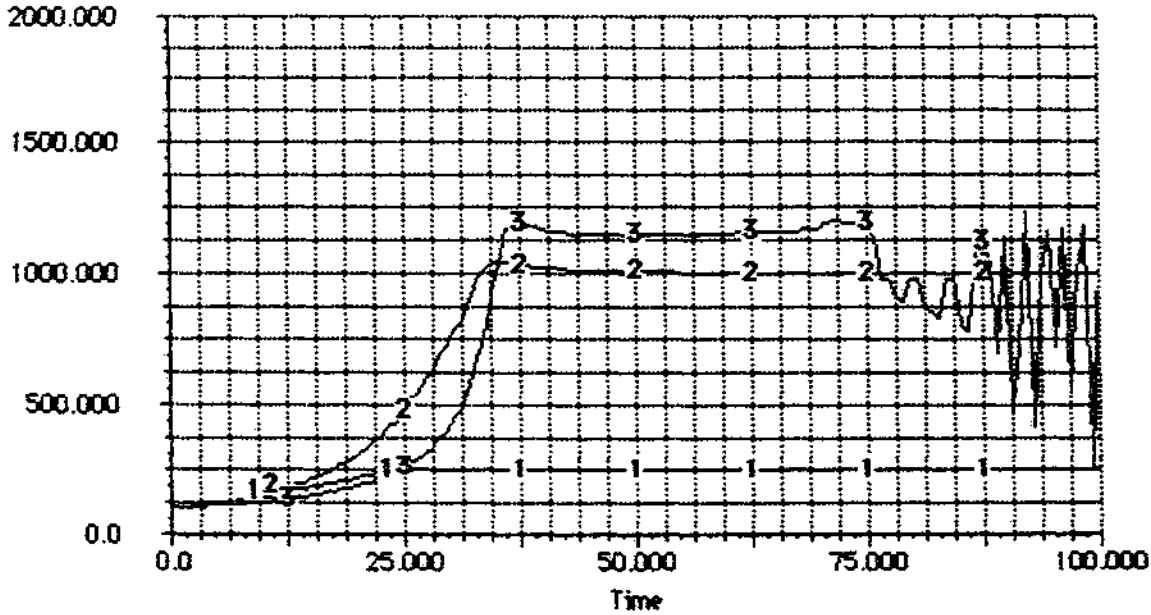Legend: 1: region 1
2: region 2
3: region 3

Figure 7. Consumption per capita in a multiregional
system with technology transfers

In terms of the trade-off between consumption and investment dis-
cussed in section 1, the question arises whether region 2 would be able
to reach the same (or even higher) levels of welfare with solely import-
ing new technology rather, than developing such technology itself (i.e.
$\sigma_2^2=0$). The answer is, for the parameter values chosen here, affirmative.
This can be seen from Figure 8.

Comparing Figures 7 and 8, several conclusions emerge. First, the
absence of any R&D expenditure in region 2 has - as expected - no impact
on region 1, which is the 'technology' leader which reaches the capacity
constraint the fastest. Secondly, without its own R&D expenditure,
region 2 takes longer to maximize consumption per capita, but its
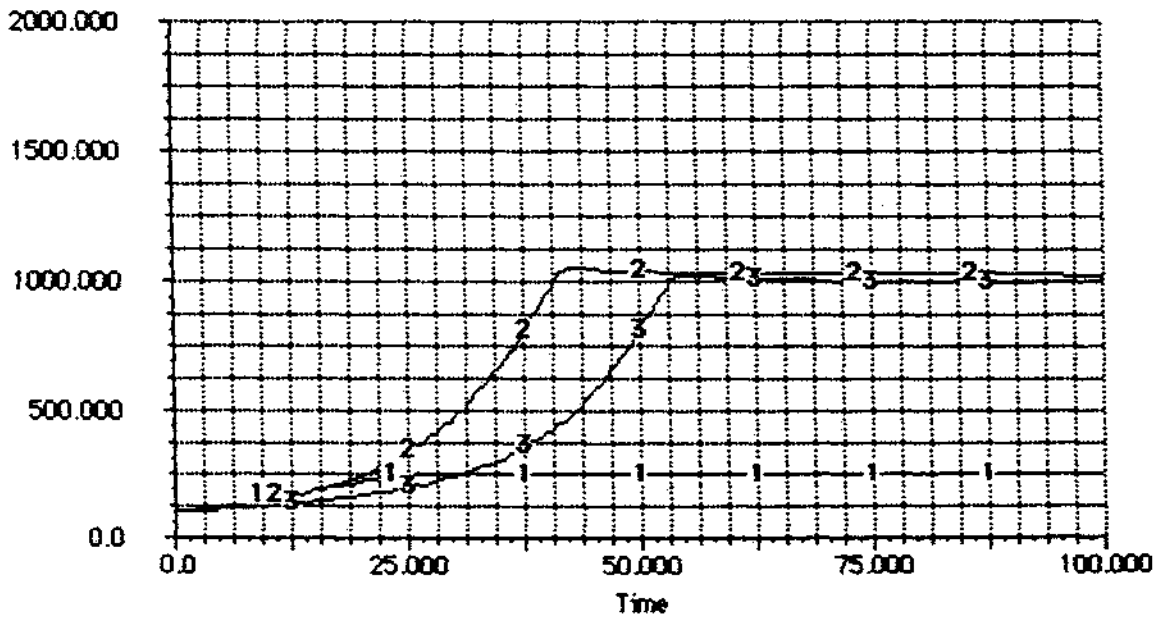steady-state level of consumption per capita is ultimately somewhat
higher.

Figure 8. Consumption per capita in a multi-regional
system with one technology-follower

Thirdly, the absence of R&D expenditure in region 2 has a detrimental effect on the time-trajectory of consumption per capita in region 3, but it does enable region 3 to reach a stable steady-state situation.

While the simulation results appear plausible for the given parameter values, it must be emphasised that different parameter values may generate quantitatively and qualitatively different time trajectories. Thus can be illustrated by using the same data as for Figure 8, with one exception, viz. a difference in output capacities Y_t^{cr}. It we assume $Y_0^{c1} = 2000$, $Y_0^{c2} = 500$ and $Y_0^{c3} = 500$, a different pattern emerges (see Figures 9 and 10). Then the highest capital productivity is reached in region 1, while all regions converge to a steady state growth path with capital productivity equal to 0.15. Furthermore, it is interesting that - despite differences in output capacity -, consumption per capita in all regions tends toward 500.
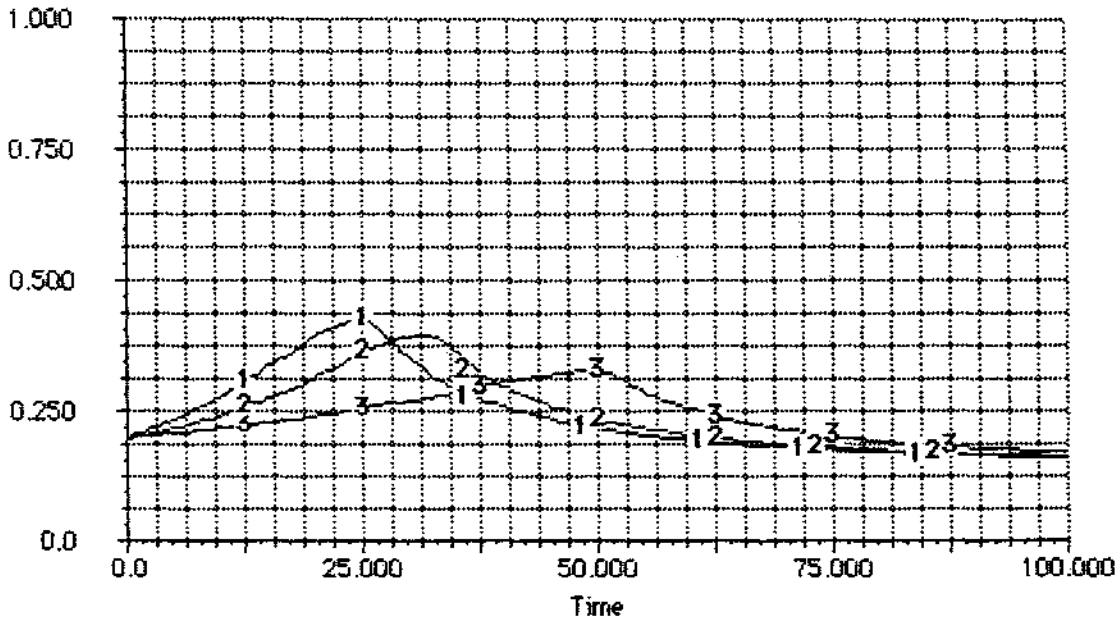
Figure 9. Capital productivity in a multiregional
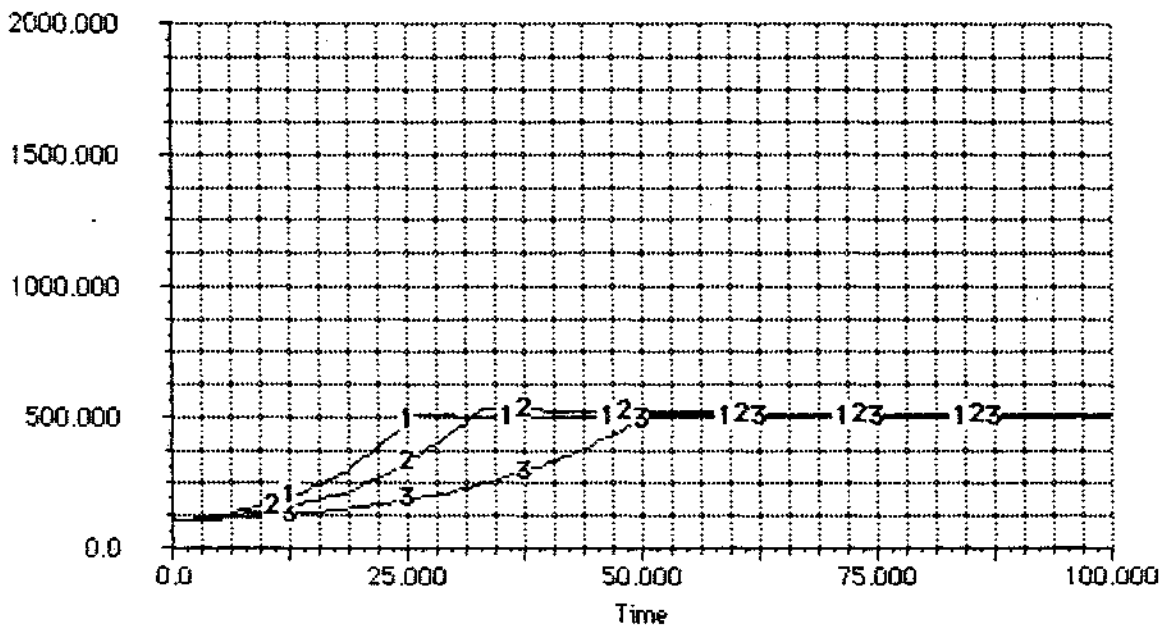system with interregional differentials
in output capacity

Figure 10. Convergence of consumption per capita
in a multiregional system with inter-
regional differences in output capacity.

5. Concluding Remarks

Several extensions of the multiregional system described above can be suggested which enhance the realism of the system. For example, the spatial interaction considered hitherto has consisted solely of a positive feedback loop through the diffusion of technological advances generated by means of R&D. Other forms of spatial interaction could be postulated, such as capital mobility in response to spatial differentials in capital productivity or negative spillover effects. In the presence of negative feedback loops between the growth path of one region and others, Lotka-Volterra type dynamics, possibly with stable cycles, may be generated. Other extensions would include a varying savings rate for R&D investments which would respond to change in economic conditions over time (depression trigger hypothesis) or to spatial differentials in economic well-being (space trigger hypothesis).

The model discussed above was a descriptive dynamic model. Given a set of parameters and of initial conditions, the model was able to generate a trajectory of the multiregional system concerned, given its structure as reflected in the specified equations. If one would regard this model as a policy model, it would be necessary to introduce a certain objective (or welfare) function encompassing a trade-off between relevant welfare arguments. A dynamic programming or optimal control formulation may then be desirable (see Kendrick, 1981, and Nijkamp and Reggiani, 1988a, 1988b). Such a constrained dynamic optimization might then in principle prevent the variety of chaotic fluctuations inherent in the nonlinear dynamics of an interdependent multiregional growth system. The formal treatment of such an optimal control model is given in Annex A, in which the assumption is made that each region tries to maximize (the net present value of) an overall welfare function by means of a proper choice of the savings rates for both capital investments and R&D investments.

Annex A

Capital Accumulation, Endogenous Technical Change and Optimal Control.

In this Annex we will analyze the implications of introducing the savings rates σ_1 and σ_2 as control parameter in an adjusted optimal control version of the model from sections 3 and 4.

Let capital accumulation be given by:

$$\dot{K} = I - \sigma K \tag{A1}$$

where I is gross investment and σ is the rate of depreciation. The investment function is again equal to:

$$I = \sigma_1 Y \tag{A2}$$

with Y being output; R&D expenditure is also proportioned to output, i.e.:

$$R = \sigma_2 Y \tag{A3}$$

The effect of R on capital productivity $\epsilon = Y/K$

is given by:

$$\dot{\epsilon} = \nu^* R + E \quad , \tag{A4}$$

where E measures the adoption of externally generated innovations. However, ν^* is not constant. The closer $Y = \epsilon K$ gets to a capacity level Y^c, the lower the marginal efficiency of R&D expenditure. When $Y > Y^c$, capital productivity can even decrease as a result of congestion and other external diseconomies effects. We assume here the following simple relationship:

$$\nu^* = \nu \quad (1 - Y/Y^c) \tag{A5}$$

Combining (1) - (5), we get the equations of motion for ϵ and K:

$$\dot{\epsilon} = \sigma_2 \nu \left(1 - \frac{\epsilon K}{Y^c}\right) \epsilon K \tag{A6a}$$

$$\dot{K} - \sigma_1 \epsilon K - \delta K \qquad\qquad\qquad\qquad (A6b)$$

It is relatively straightforward to see that, for a given Y^c, $(\bar{\epsilon}, \dot{K})$ $= (\delta/\sigma_1 , \sigma_1 Y^c/\delta)$ is a locally stable equilibrium. This can be illustrated by means of the phase plane below:
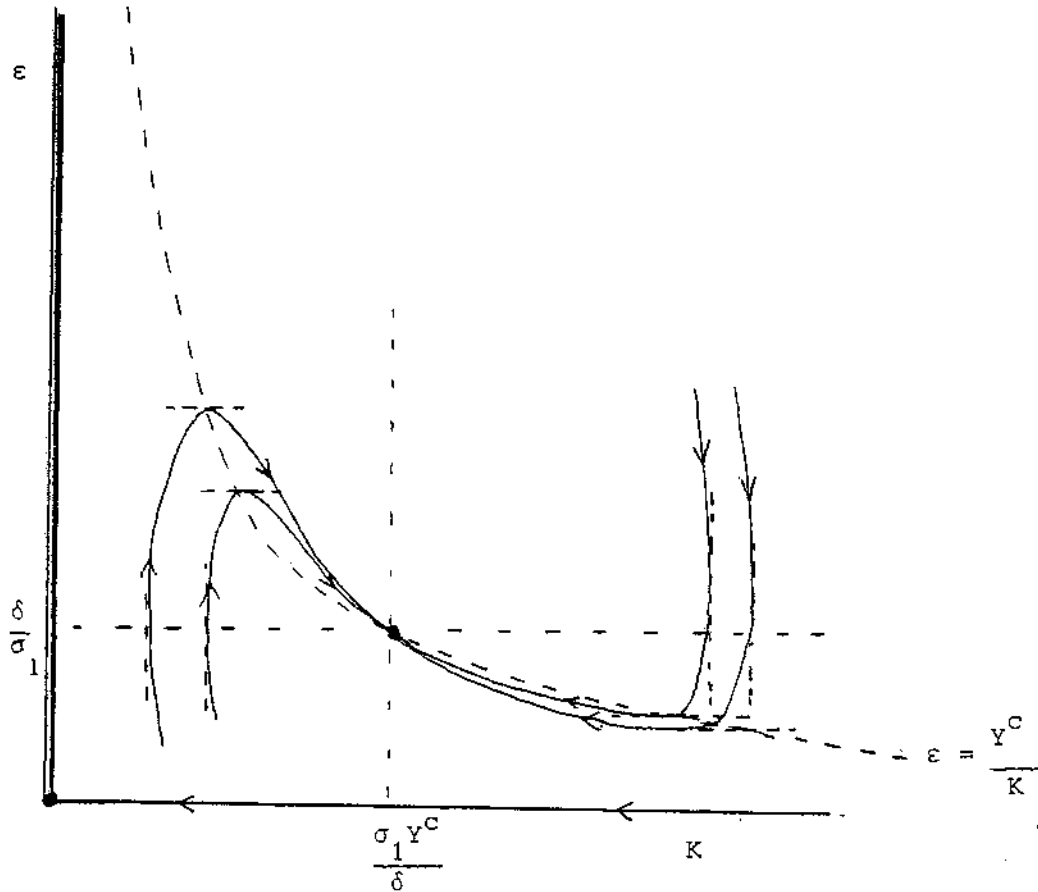

Figure Al. The time trajectory of ϵ and K for a fixed system capacity Y^c

At any point in time, the level of consumption is given by

$$C - Y - I - R - (1-\sigma_1 - \sigma_2) Y \qquad\qquad (A7)$$

Hence, here the classical Ramsey-type optimal control problem arises in which σ_1 and σ_2 are to be chosen such that the present value of welfare of the system is maximized.

Let ρ be the proper discount rate and U(.) an appropriate welfare function. This function is assumed quasi-concave, as usual, to ensure that the second-order conditions are fulfilled.

The optimal control problem can now be formulated as follows:

$$\text{Max } I \; [\epsilon, K, \sigma_1, \; \sigma_2] = \int_0^\infty U \; [(1-\sigma_1-\sigma_2)\epsilon K] \; e^{-\rho t} \; dt \qquad \text{(A8)}$$

subject to (A6a) and (A6b), with ϵ, K being <u>state</u> variables and σ_1 and σ_2 being <u>controls</u>. The initial values are $K(0) = K_o$ and $\epsilon(0) = \epsilon_o$. Moreover, ρ, δ, E, ν and Y^c are constants.

The Hamiltonian related to (A8) is (see e.g. Miller (1979, p.104)):

$$H \; (t, \epsilon, K, \sigma_1, \sigma_2, \lambda_1, \lambda_2) =$$

$$U \; [(1-\sigma_1-\sigma_2)\epsilon K] \; e^{\rho t} \; +$$

$$\lambda_1 \; \{\sigma_2\nu \; (1 - \frac{\epsilon K}{Y^c}) \; \epsilon K + E\} + \lambda_2 \; (\sigma_1 \epsilon K - \delta K) \qquad \text{(A9)}$$

The first-order conditions are

$$\frac{\partial H}{\partial \epsilon} = U_c \cdot (1-\sigma_1-\sigma_2)K \cdot e^{-\rho t} + \lambda_1 \sigma_2 \nu K$$

$$- \lambda_1 \; \sigma_2 \; \nu \; 2\epsilon \; \frac{K^2}{Y^c} + \lambda_2 \; \sigma_1 \; K = -\dot{\lambda}_1 \qquad \text{(A10)}$$

$$\frac{\partial H}{\partial K} = U_c \; (1-\sigma_1-\sigma_2) \; \epsilon \; e^{-\rho t} + \lambda_1 \sigma_2 \nu \epsilon - \lambda_1 \sigma_2 \nu 2 \; K\frac{\epsilon^2}{Y^c}$$

$$+ \lambda_2 \sigma_1 \epsilon - \lambda_2 \delta = -\dot{\lambda}_2 \qquad \text{(A11)}$$

$$\frac{\partial H}{\partial \sigma_1} = U_c \cdot - \epsilon K \cdot e^{-\rho t} + \lambda_2 \epsilon K = 0 \qquad \text{(A12)}$$

$$\frac{\partial H}{\partial \sigma_2} = U_c \cdot - \epsilon K \cdot e^{-\rho t} + \lambda_1 \nu \epsilon K = 0 \qquad \text{(A13)}$$

$$\frac{\partial H}{\partial \lambda_1} = \sigma_2 \nu \; (1-\frac{\epsilon K}{Y^c}) \; \epsilon K + E = \dot{\epsilon} \qquad \text{(A14)}$$

$$\frac{\partial H}{\partial \lambda_2} = \sigma_1 \epsilon K - \delta K = \dot{K} \qquad \text{(A15)}$$

It follows immediately from (A12) that $\lambda_2 = U_c e^{-\rho t}$ (i.e., the discounted marginal utility at time t). From (A12) and (A13) we see that $\lambda_1 = \lambda_2/\nu$ (i.e. the discounted marginal utility at time t per unit of the responsiveness of productivity to R&D expenditure).

Following Dorfman (1969), equation (A11) shows that the loss to society that would be incurred if the acquisition of a unit of capital were postponed for a short time, is equal to the sum of the contributions of that unit of capital to, firstly, the present value of welfare; secondly, the change in productivity, and, thirdly, the change in the capital stock itself. Similarly, the loss to society of postponing productivity, growth for a short time is equal to the sum of the contributions of such an increment in productivity to the present value of welfare, the change in productivity itself and the effect on capital accumulation.

In the very simple case in which $U(c) = C$, we find $U_c = 1$ and $\lambda_2 = e^{-\rho t}$, while $\lambda_1 = e^{-\rho t}/\nu$. If, in addition, we restrict ourselves to the steady-state growth path with K and Y^c growing at rate n, we recall that $\bar{\epsilon} = (n+\delta)/\sigma_1$. Equations (A10) and (A11) now become:

$$(1-\sigma_1-\sigma_2)\frac{\sigma_1 Y^c}{\delta}e^{-\rho t} + e^{-\rho t}\sigma_1\sigma_2\frac{Y^c}{\delta} - e^{-\rho t}2\epsilon\,\sigma_1^2\,\sigma_2\frac{Y^c}{\delta^2} +$$

$$e^{-\rho t}\sigma_1^2\frac{Y^c}{\delta} = \frac{\rho e^{-\rho t}}{\nu} \qquad (A16)$$

and

$$(1-\sigma_1-\sigma_2)\frac{(n+\delta)}{\sigma_1}e^{-\rho t} + e^{-\rho t}\sigma_2\frac{(n+\delta)}{\sigma_1} - e^{-\rho t}2\frac{\sigma_1\sigma_2}{\delta}(\frac{n+\delta}{\sigma_1})^2 +$$

$$e^{-\rho t}(n+\delta) - \delta e^{-\rho t} = \rho e^{-\rho t} \qquad (A17)$$

Equations (A16) and (A17) are two non-linear equations in the two variables σ_1 and σ_2. Since both equations can be divided by $e^{-\rho t}$, the solution is independent of the choice of a discount rate ρ. However, the solution would depend on the value of Y^c. Hence, the values of σ_1 and σ_2 would tend to vary over time (see also Nijkamp and Reggiani, 1988a). In the general case, the solution to the differential equation system (A10-A15) in the variables ϵ, K, σ_1, σ_2, λ_1 and λ_2 would not appear analytically tractable. Then simulation runs would have to be made to examine the possibility of stable trajectories.

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