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THEORETICAL FOUNDATIONS FOR
THE 3-C MODEL

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THEORETICAL FOUNDATIONS FOR THE 3-C MODEL

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Abstract - Alonso (1978) and Bikker (1982) independently introduce the Three Component (3-C) model. This is a model to explain and predict flows between origins and destinations. It generalizes many existing models, such as the gravity model. This generality is attained through so-called "systemic variables". Unfortunately, the interpretation of these variables, and hence that of the 3-C model, have remained somewhat elusive. In this paper, we derive *two* interpretations, by relating the 3-C model to economic and statistical theory respectively.

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1. Introduction

Alonso (1978) and Bikker (1982) independently introduce the Three Component (3-C) model (for references, see Nijkamp and Reggiani (1988)). This is a model to explain and predict flows between origins and destinations. It generalizes many existing models, such as the gravity model. This generality is attained through so-called "systemic variables". Unfortunately, the interpretation of these variables, and hence that of the 3-C model, have remained somewhat elusive (see e.g. Hua (1980) and Fotheringham and Dignan (1984)). In this paper, we derive *two* interpretations, by relating the 3-C model to economic and statistical theory respectively.

An economic interpretation of the 3-C model is provided (in section 3) by relating it to an extended version of the Armington demand model. All systemic variables are interpreted, in terms of prices and shadow prices. Since prices can be measured, and systemic variables cannot, the economic model has additional empirical content.

A statistical interpretation is provided (in section 5) by interpreting the systemic variables as (fixed or random) parameters. In its full generality, the 3-C model is either equally general or less general than a gravity model with dummy variables corresponding to the marginal totals. In empirical applications, additional identifying restrictions are imposed on the 3-C model. However, these identification restrictions critically depend on the *arbitrary* specification of one of the components of the 3-C model, and have little theoretical basis.

2. The 3-C model

In this section we briefly summarize the 3-C model, and illustrate it with two examples. The description we select is the one preferred by Hua (1980) and Bikker (1982).

The 3-C model derives its name from its three components, which are models explaining respectively (1) total outflows M_i ($=\Sigma_j M_{ij}$) from region i, (2) total inflows M_j* ($=\Sigma_i M_{ij}$) from region j, and (3) the allocation of these totals to specific values of M_{ij}. We assume that $M_{ij} \geq 0$ and M_i, $M_j > 0$ for all i and j. Maintaining the Alonso (1978) notation, the *allocation component* is:

$$M_{ij} = (M_i/D_i)\ (M_j/C_j)\ t_{ij} \qquad i=1,..,I\ ;\ j=1,..,J \tag{1}$$

In this component, marginal totals M_i and M_j are taken to be given, and t_{ij} is a function of exogenous variables. I is usually taken to be, but need not equal J. C_j and D_i are unknown "systemic" variables. The IJ equations (1) are overidentified. Since M_i and M_j are considered to be given, the consistency requirements $\Sigma_j M_{ij} = M_i$ and $\Sigma_i M_{ij} = M_j$ are implicit:

$$\Sigma_j\ (M_i/D_i)\ (M_j/C_j)\ t_{ij} = M_i \qquad i=1,..,I \tag{2}$$
$$\Sigma_i\ (M_i/D_i)\ (M_j/C_j)\ t_{ij} = M_j \qquad j=1,..,J \tag{3}$$

The I+J equations (2) and (3) are not independent. Since $\Sigma_i M_i = \Sigma_j M_j$ holds by assumption, they provide only I+J-1 *independent* restrictions. These restrictions limit the number of possible values which can be jointly taken by M_i, M_j, t_{ij}, C_j and D_i, even without considering any M_{ij}. Since M_i, M_j and t_{ij} are exogenous here, (2) and (3) endogenize C_j and D_i (up to a constant*).

Equations (1) to (3) apply if both M_i and M_j are exogenous. The last two components of the 3-C model allow us to endogenize these, by means of the *marginal outflow* and *marginal inflow* components:

$$M_i = V_i D_i^{\alpha_i} \qquad i=1,..,I \tag{4}$$
$$M_j = W_j C_j^{\beta_j} \qquad j=1,..,J \tag{5}$$

* In this paper, we do not use specific values for i and j, and there is no need for the more informative notation $M_{i.}$ and $M_{.j}$.

* The idea is analogous to the economic one of equilibrium (supply equals demand) determining prices up to a numeraire.

V_i and W_j are functions of exogenous variables measured at i and j respectively, and α_i and β_j are unknown parameters. Equations (4) and (5) consist of I+J equations, introducing I+J additional unknowns M_i and M_j. Equations (4) and (5) are overidentified. The consistency requirement $\Sigma_i M_i = \Sigma_j M_j$ is implicit:

$$\Sigma_i V_i D_i^{\alpha_i} = \Sigma_j W_j C_j^{\beta_j} \tag{6}$$

Restriction (6) does not appear to have been made explicit before. It can be used as the I+J-th independent equation needed to endogenize D_i and C_j uniquely. However, identification of the systemic variables can also be attained by imposing an additional restriction in equation (1) (see section 5). Equation (6) then consists of a restriction which must hold between V_i, W_j, α_i and β_j, and can be interpreted as endogenizing one of these.

Equations (1) to (6) define the 3-C model. Its name derives from the three building blocks (1), (4) and (5). The other three equations are restrictions imposing internal consistency. Note that M_i and M_j *can* be exogenous in the full 3-C model. This occurs if $\alpha_i = \beta_j = 0$. It also occurs in the unlikely case that $D_i = C_j = 1$ for all i and j is consistent with M_i, M_j and t_{ij} in (2) and (3) (see Anderson (1979)).

For concreteness, consider two examples: the gravity model and the RAS biproportional adjustment method.

The gravity model

The gravity model takes the form:

$$M_{ij} = V_i W_j t_{ij} \tag{7}$$

This model is introduced in the social sciences by Carey (1858) and Reilly (1929), and acquired its own place in this field through the so-called Social Physics School (e.g. Stewart (1948) and Zipf (1946)). For a review, see Hua and Porell (1979).

The gravity model is a special case of the 3-C model. If we set $\alpha_i=\beta_j=1$ for all i and j in (4) and (5), the gravity model results, for any V_i, W_j and t_{ij}. Observe that we use restriction (6) to identify C_j and D_i uniquely.

The RAS method

The RAS biproportional adjustment method (named after R.A. Stone, see Weber and Sen (1985), Günlük-Senesen and Bates (1986)) is a way of adjusting individual cells to *given marginal totals* on the basis of previous knowledge. The RAS method has usually been considered as a descriptive technique. An attempt to provide a theoretical basis for the RAS method has been provided by Evans (1973), who relates it to the gravity model. The existence of a relationship between the RAS method and the 3-C model is implicit in Fisch (1981) and Nijkamp and Poot (1987), but does not appear to have been formalized.

Suppose that our previous knowledge consists of cell values $M_{ij}{}^0$ at time period 0 and marginal totals $M_i{}^1$ and $M_j{}^1$ at time period 1, and that we wish to estimate $M_{ij}{}^1$. The RAS method uses as starting values

$$\hat{M}_{ij}{}^1 = M_{ij}{}^0 \; (M_i{}^1/M_i{}^0) \; (M_j{}^1/M_j{}^0) \; / \; (M^1/M^0) \qquad (8)$$

where $M=\Sigma_{ij}M_{ij}$. These initial values can be thought of as derived from the 3-C model. Since the marginal totals M_i and M_j are given, equations (4) and (5) are irrelevant, and the 3-C model consists only of equation (1). Let $C_j=D_i=\surd M$ in (1), i.e. let

$$M_{ij} = M_iM_j/M \; t_{ij} \qquad (9)$$

If t_{ij} is constant over time, $M_{ij}M/M_iM_j$ is constant over time, and (8) directly produces optimal values: $\hat{M}_{ij}{}^1=M_{ij}{}^1$. Constancy of t_{ij} holds in a regional context if, as is commonly assumed, t_{ij} is a function (constant over time) of the distance between i and j.

However, t_{ij} is rarely constant over time, and starting values $\hat{M}_{ij}^1$ do not usually add up to the given totals M_i^1 and M_j^1. Therefore these starting values are modified, by two sets of n proportionality factors (one only varying over i and the other only over j). These proportionality factors are computed iteratively from restrictions (2) and (3), and can be demonstrated to converge to an internally consistent solution.

This RAS modification of the starting values can thus also be derived from the first component of the 3-C model, but now allowing for consistency requirements (2) and (3). In particular, C_j and D_i in (1) should be allowed to take values *different* from those implicit in (9). If t_{ij} varies multiplicatively along i and j dimensions over time, the RAS adjustment method produces optimal values for M_{ij}.

Fisch (1981) appears to misinterpret the relationship between the various quantities considered by the RAS method and the 3-C model. He sees the proportionality factors as combinations of the systemic variables, *as well as of V_i and W_j*. In fact, V_i and W_j need not be specified to determine C_j and D_i from (1) (up to a constant).

3. An extended Armington model

In this section, we introduce an extended version of the Armington demand model.

The Armington demand model (Armington (1969)) is a special case of what Hua (1980) refers to as an economic-base demand-pull model. It represents a stylized economy, in which each country i produces one good, and each country a different one. Its general form is:

$$M_{ij} = P_i^{1-\sigma}(M_j/P_j^{1-\sigma})t_{ij} \tag{10}$$

where

- $\sigma>0$ is an unknown parameter, quantifying the ease of substitution between exporters i, assumed to be constant over j.
- p_i is the price of good i exported from country i.
- P_j ($P_j^{1-\sigma}=\Sigma_i t_{ij} p_i^{1-\sigma}$) is the shadow price of welfare of country j. If the underlying cardinal welfare function is taken to be linearly homogeneous, it equals both the average and the marginal cost of a unit of welfare.
- $t_{ij} = t_{ij}(\delta_{ij}, D_{ij})$, where δ is the contribution to the welfare of country j based on an additional unit of good imported from country i, and D_{ij} is the distance between countries i and j, and serves to multiply the export price p_i into the import price relevant for importer j.

In the origal Armington demand model, total imports M_j and prices p_i are exogenous. Let us extend the original model with two additional equations, endogenizing both imports and prices.

Suppose that import demand can be modelled as a two stage decision function, consisting of a first stage which decides on the total import budget, and a second stage which decides on the allocation of this budget (cf. section 3.2.2). In that case, the effect of import price levels p_i on total imports M_j can be summarized into one value P_j:

$$M_j = Q_j P_j = Q_j(P_j)P_j \tag{11}$$

where Q_j is the quantity of imports. In particular, suppose that (11) takes the form:

$$M_j = W_j P_j^{\beta_j'} \tag{12}$$

where W_j denotes functions of exogenous variables, and β_j' are parameters. $\beta_j'<1$ denotes a downward sloping quantity demand curve.

Let us also endogenize prices, by introducing a supply equation, say:

$$M_i = Q_i p_i = Q_i(p_i)p_i \tag{13}$$

where Q_i is the quantity of exports. For concreteness, suppose that (13) takes the form:

$$M_i = V_i' p_i^{\alpha_i'} \tag{14}$$

where V_i' denotes functions of exogenous variables measured at i, and α_i are parameters. $\alpha_i'>1$ denotes an upward sloping quantity supply curve.

For (12) and (14) to be consistent with optimization theory, they must be linearly homogeneous in prices. The right hand sides should therefore incorporate other prices. This can be done by interpreting prices as relative prices or by incorporating other prices into the exogenous variables or the parameters. For instance, a CES transformation curve describing combinations of products for the domestic market and products for the export market that can be produced with a given amount of resources results in (14) with prices interpreted as relative prices (in the same way as equation (10) was derived)). In particular, the balance of payment constraint strongly suggests that the export price level should appear in the import demand equation, and that the import price level should appear in the export supply equation.

By not distinguishing in our notation between potential demand (supply) and observed demand (supply), we implicitly assume market clearance. When all supply and demand equations are linearly homogeneous, this generally determines prices up to a numeraire. When we have completely specified supply and demand equations, and assume equilibrium, p_i need therefore not be measured for the model to be estimable.

4. The economic foundation

Let us now relate the 3-C model to the extended Armington model. This relationship is such that there is a one-to-one correspondence between inputs to and outputs from the two models.

If we equate:

α_i to $\alpha_i'/(\alpha_i'-(1-\sigma))$ (15)

β_j to $\beta_j'/(1-\sigma)$ (16)

V_i to $V_i'^{(1-\alpha_i)}$ (17)

it can be demonstrated that we have a a one-to-one relationship between the 3-C model and the extended Armington model, in which:

C_j equals $P_j^{1-\sigma}$ and (18)

D_i equals $V_i' p_i^{\alpha_i'-(1-\sigma)}$. (19)

The only difference between the Armington model and the 3-C model is that p_i can, but D_i cannot be measured independently of the model. The economic model therefore has more empirical content than the 3-C model.

Let us consider this relationship in more detail. First consider the parameters and the exogenous variables, then consider the systemic variables. Let us first consider the parameters: $\alpha_i = \alpha_i'/(\alpha_i'-(1-\sigma))$ and $\beta_j = \beta_j'/(1-\sigma)$.

If we have an upward sloping quantity supply curve ($\alpha_i'>1$), α_i must be non-negative (since $\sigma>0$). Little can be said about β_j. Note that changes in α_i' in the Armington model affect not only α_i, <u>but also V_i</u> (see section 5 for more discussion).

We have seen that $\alpha_i=\beta_j=1$ reduces the 3-C model to a gravity model. The extended Armington model allows an interpretation of this special case: $\alpha_i=1$ means that either $\alpha_i'=\infty$ or that $\sigma=1$, and $\beta_j=1$ means that $\beta_j'=1-\sigma$. The simplest case which results in the gravity model is $\sigma=1$ and $\beta_j'=0$. $\beta_j'=0$ implies that total import values are independent of prices. $\sigma=1$ implies that import value shares are also independent of prices. Thus *all* values are independent of prices. There is no value substitution of any form. This formalizes the critique of Poot (1986) and Bikker (1987), who prefer the 3-C model to the gravity model because it allows for substition.

The second case which results in the gravity model is $\alpha_i'=\infty$ and $\beta_j'=1-\sigma$. $\beta_j'=1-\sigma$ means that no import value substitution occurs: total import value is perfectly accomodating. $\alpha_i'=\infty$ means that supply is infinitely elastic. Prices do not introduce a value substitution between various importers. Total export value is also perfectly accomodating. Again, prices do not induce substitution between value flows.

Let us next consider the interpretation of the exogenous variables. W_j and t_{ij} in the 3-C model directly corresponds to W_j and t_{ij} in the extended Armington model, which is why we used the same notation in both models. V_i corresponds to $V_i'^{(1-\alpha_i)}$. Changes in V_i' thus affect V_i in a way *that requires knowledge of* α_i. V_i and α_i in the 3-C model cannot be considered separately (see section 5 for more discussion).

Let us finally consider the interpretation of the systemic variables. Although endogenous, they are unmeasured and only a given function of the exogenous variables. They merely serve as intermediates to describe the effect of exogenous variables on flows M_{ij}. Let us first consider D_i, which corresponds to $V_i'p_i^{\alpha_i'-(1-\sigma)}$. D_i can be thought of as consisting of a variable, p_i, and its effects:

(1) p_i: In the 3-C model, D_i is defined by restriction (2): $\Sigma_j M_{ij}=M_i$. This restriction can be interpreted economically as the equilibrium restriction. D_i is monotonically related to prices p_i, and fulfills the same role: it attains equilibrium. Incorrect values of D_i result in a failure of equilibrium to hold. However, unlike prices, which can be measured and can be shown to be too high or too low, the 3-C model does not allow an indication of incorrect systemic variables. The way in which equilibrium is attained depends on the two effects of p_i*:

(2a) $p_i^{1-\sigma}$: the *demand* effect of p_i on M_{ij} (multiplied to the remainder of (10)), and

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* For the role of V_i' in D_i, see section 5.

(2b) $p_i^{\alpha_i'}$: the *supply* effect of p_i on M_i (multiplied to V_i' in (14))

Note that we must be careful in interpreting the effects (2a) and (2b). We cannot change p_i (and D_i), keeping everything else constant, since both p_i and D_i are endogenous.

Let us finally consider C_j, which corresponds to $P_j^{1-\sigma}$. Similar to the interpretation of D_i, we can think of C_j as consisting of a variable, P_j, and its effects:

(1) P_j: In the 3-C model, C_j is defined by restriction (3): $\Sigma_i M_{ij}=M_j$. Restriction (3) can be interpreted economically as the budget constraint. C_j is monotonically related to prices P_j, and fulfills the same role: it attains budget restriction. Incorrect values of C_j result in a failure of the the restriction to hold. However, neither C_j nor P_j can be measured independently of the model, and neither model therefore allows an indication of incorrect values. The way in which the budget restriction is attained depends on the two effects of P_j:

(2a) $P_j^{1-\sigma}$: the effect of P_j on *expenditures* M_{ij} (taking the other variables in (10) as given), and

(2b) $P_j^{\beta_j'}$: the effect of P_j on the *budget* M_j (taking W_j in (12) as given).

We identified a one-to-one correspondence between an extended version of the Armington model and the 3-C model. The parameters α_i and β_j characterize the generalization of the 3-C model over the gravity model, and play a critical role. In the next section, we examine this role in more detail.

5. A statistical foundation

Up till now, we introduced the 3-C model, and discussed its interpretation from an economic perspective. In this section, we maintain the assumption that the systemic variables are not directly measurable. Statistical theory has developed a large body of theory around unmeasured variables, which are referred to as (fixed or random) parameters.

The systemic variables are not directly measurable. Their effect on flows M_{ij} cannot be separated from the effect of a) other systemic variables, b) exogenous variables and/or c) parameters. There are thus several ways of writing the allocation component (1) in an empirically equivalent way. Each leads to a different definition for the systemic variables, thereby affecting the allocation equations (4) and (5).

We first consider the three identification issues a), b) and c). We next consider their effect on the definition of the systemic variables, and hence the allocation equations.

Identification of systemic variables (a)

The effect of the unmeasurable systemic variables are, without additional information, empirically indistinguishable from the effect of other systemic variables in the same equation. From a single observation from (1), D_i and C_j cannot be identified (without additional information). Because the same systemic variables appear in several observations from (1), identification is facilitated. It appears that we only need to impose *one* additional identification restriction. There are two main types of identification restriction (which can be combined). Let us briefly discuss them.

The first type of identification restriction considers parameters fixed, and imposes a prior deterministic restriction on the parameters. There are many restrictions which can be imposed. Bikker (1982) requires the geometrical average of C_j, denoted by C_*, to be one. This restriction loses the symmetry in the original formulation of the problem. In section 2, we noted that the symmetric consistency requirement (6) can be used to identify the systemic variables uniquely. An appealing symmetric alternative arises if we introduce an additional "systemic constant" k in allocation equation (1):

$$M_{ij} = k \, (M_i/D_i) \, (M_j/C_j) \, t_{ij} \tag{20}$$

This was the first "extension" of 3-C model (Anselin and Isard (1979)) and is also used by Bikker (1982), Fotheringham and Dignan (1984) and Nijkamp and Poot (1987). We now require not just one, but *two* identifying restrictions. The direct analogue of familiar statistical ANalysis Of VAriance (ANOVA) restrictions requires the geometrical average of M_i/D_i and that of M_j/C_j to be one. If all M_{ij}'s would be known, and the model is perfect, these deterministic restrictions allow us to compute D_i and C_j in a closed form (analogous to Bikker (1982, p.64)).

The second type of identification restriction considers parameters random, and chooses some estimation criterion. For instance, the systemic variables could be considered as drawn from a distribution with unit mean (in equation (20)), and we could choose the maximum likelihood estimation criterion to estimate them. This allows us to examine the sensitivity of other fixed parameters in the model with respect to the precise value of these random parameters.

The two types of restrictions can also be combined, as is demonstrated by De Vos and De Vries (1987). To illustrate their restriction, consider the standard linear regression model

$$Y = X\beta + u \tag{21}$$

Identification of fixed β can arise from imposing a "fixed parameter" type of singularity of the error term:

$$X'u = 0 \tag{22}$$

Although the error terms are still random, this fixed parameter type of restriction allows identification of β. Characteristic of this kind of restriction is that it is not possible to estimate the variance of the parameter estimates (the parameter estimates are not even a function of the error term). This principle can be applied to identify the *systemic variables* by specifying an analogous "*fixed* parameter" type of singularity in the distribution of the *random* systemic variables.

Identification of systemic variables (b)

The effect of the unmeasured systemic variables are, without additional information, empirically indistinguishable from the effect of exogenous variables. The point here is the usual difficulty with error terms (and dummy variables). If error terms are (in the random specification) correlated or (in the fixed specification) collinear with the exogenous variables, we cannot distinguish empirically between the error term and (the effect of) an exogenous variable. The definition (and interpretation) of the error term depends critically on the exogenous variables put into the model.

For instance, the following allocation equation, which omits M_i and M_j from (1) (e.g. by incorporating their inverse in t_{ij}) has the same empirical content as equation (1):

$$M_{ij} = D_i' \; C_j' \; t_{ij} \tag{23}$$

(with restriction analogous to equations 2 and 3). Nevertheless, the values of D_i' and C_j' are different from the original values of D_i and C_j. A similar effect occurs if we change t_{ij} to $t_{ij}'=t_{ij}t_{**}/t_{i*}t_{*j}$. The same allocation model results, but the effects of t_{ij} disappear from D_i and C_j. Analogously, the systemic variables, but not the explanatory power of the model, will be affected if we incorporate elements of V_i or W_j in t_{ij}.

Identification of systemic variables (c)

Unmeasured systemic variables are, without additional information, empirically indistinguishable from their effects. For instance, if in (1) we append parameters to the systemic variables, we obtain empirically equivalent models:

$$M_{ij} = (M_i/D_i^{1/\alpha_i})\ (M_j/C_j^{1/\beta_j})\ t_{ij} \qquad (24)$$

for non-zero α_i and β_j.

Impact on the allocation equation

In resolving the identification issues, we have to specify a way of writing the allocation equation. However, the particular choice made (arbitrarily) affects the definition of the systemic variables, and hence the specification of the marginal equations (1) and (2) (cf. section 4). Consider each identification difficulty in turn.

In the first identification difficulty (distinguishing systemic variables from other systemic variables) restrictions were chosen arbitrarily*. However, if we choose a different normalization constant, the functional form of the marginal equations is changed. For instance, the usual assumptions of $\alpha_i=\alpha$ and $\beta_j=\beta$ are not invariant under the choice of normalization constant.

The resolution of the second identification difficulty (distinguishing systemic variables from exogenous variables) has similar impacts on the allocation equation. Depending on the representation of the allocation equation, the parameters α and β will be a function of M_i, V_i and t_{ij}, and cannot be considered constant over time.

The resolution of the third identification difficulty (distinguishing systemic variables from their effects) also has consequences for the allocation equation. For instance, if we arbitrarily choose to write

* The idea is analogous to the economic one of choosing a reference commodity arbitrarily.

allocation equation in the form (24), all parameters in the marginal equations equal one identically.

These identification difficulties are not purely theoretical. Different presentations of the 3-C model differ in the choice of identification restriction(s), the appearance of parameters α_i and β_j in the allocation equation, and the presence of V_i and W_j variables in t_{ij}. They therefore cause different definitions of the systemic variables, see Alonso (1978), Bikker (1982), and Nijkamp and Poot (1987).

3-C model

We are now in a position to examine the whole 3-C model. Equations (1), (4) and (5) are a recursive simultaneous equation system. In the allocation equation, we find variables (M_i and M_j) which are explained elsewhere in the model. There are cross equation parameter restrictions (arising from C_j and D_i). The "reduced form" equations are:

$$M_{ij} = V_i \; W_j \; D_i^{\alpha_i - 1} \; C_j^{\beta_j - 1} \; t_{ij} \tag{25}$$

with the following I+J+1 (dependent) constraints on the right hand side (cf. equations 2, 3 and 6):

$$C_j = \Sigma_i \; V_i D_i^{\alpha_i - 1} t_{ij} \tag{26}$$
$$D_i = \Sigma_j \; W_j C_j^{\beta_j - 1} t_{ij} \tag{27}$$
$$\Sigma_i \; V_i D_i^{\alpha_i} = \Sigma_j \; W_j C_j^{\beta_j} \tag{28}$$

Equations (26) to (28) impose restrictions on equation (25). As discussed in section 2, these restrictions can be interpreted as endogenizing D_i and C_j. It is an open problem whether these restrictions allow us to attain all non-negative combinations of $D_i^{\alpha_i - 1}$ and $C_j^{\beta_j - 1}$ by varying α_i and β_j (for all V_i, W_j and t_{ij}). If this is so, and if α_i and β_j are unknown, $D_i^{\alpha_i - 1}$ and $C_j^{\beta_j - 1}$ have the same effect as dummy variables. The 3-C model then represents a gravity model with dummy variables corresponding to all marginal totals. This can be clarified by examining equations (4) and (5) directly. These contain I+J equations with I+J parameters α_i and β_j. Even if D_i and C_j were measured (and not equal to one), (4) and (5) cannot be falsified, and have no empirical content.

If not all combinations of $D_i^{\alpha_i-1}$ and $C_j^{\beta_j-1}$ can be attained, or if we impose additional identifying restrictions on the parameters (such as $\alpha_i=\alpha$ and $\beta_j=\beta$) or the systemic variables, the 3-C model is *less general* than the gravity model with dummy variables corresponding to all marginal totals. The 3-C model has additional empirical content, although its interpretation remains ambiguous, in view of the arbitrary specification of the allocation equation.

6. Concluding comments

In the full generality of the 3-C model, the marginal inflow and outflow components are not falsifiable, are empirically empty. To make these equations estimable, additional restrictions need to be imposed. Unfortunately, the nature of these restrictions depends critically on the arbitrary specification of the allocation equation. As a result, they are selected arbitrarily, which may result in non-constant parameters.

The economic interpretation exposes a limitation of the 3-C model. The functional form of the marginal equations depends on the choice of identifying constant in the allocation equation. The economic interpretation is that the marginal equations are not necessarily linearly homogeneous in prices. There is no internal factor (analogous to domestic prices) balancing the external (import) prices for the inflow equation. Similarly, there is no domestic market competing for export supply in the outflow equation. Most importantly, there is no exchange rate mechanism, to achieve equilibrium between imports and exports. If we ignore money illusion, alternative uses of resources are not explicitly considered. The marginal equations are at most interpretable in a partial equilibrium context.

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