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ON MULTIDIMENSIONAL
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Piet Rietveld

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On Multidimensional Inequality Comparisons

Piet Rietveld

Department of Economics
Free University
P.O. Box 7161
1007 MC Amsterdam

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Abstract

This paper is addressed to comparing inequality in distributions of two or more variables. Based on an extended version of the Lorenz Curve criterion, a theorem is proved which says that total income is always less unequally distributed than the most unequally distributed income component. As an illustration, a decomposition of the coefficient of variation of total income is given in terms of coefficients of variation of the income component. The possibilities are discussed to transfer the results of income inequality to the field of welfare inequality. An empirical illustration is given for interregional welfare inequalities in the Netherlands.

1. Introduction

The literature on inequality analysis shows that economists usually study inequality in terms of only one variable, mostly income. Inequality in the context of more than one variable is seldom studied. This is not entirely satisfactory, if one wants to link income and welfare, since there are several other relevant determinants of welfare, for example: health, housing, quality of natural environment and accessibility of public facilities. If one wants to avoid a one-sided view of the relative situation of low income groups, also the other welfare components have to be considered. Of course, much depends on the correlations between the various components. In the case of high and positive correlations between components, one may expect an aggravating effect on aggregate welfare inequality, whereas in the case of low or negative correlations, a mitigating influence will occur. This paper is addressed among others to clarifying these issues.

The literature on multidimensional inequality comparisons is rather thin. A short review is given by Marshall and Olkin (1979). They indicate that various definitions of inequality are possible. For example, Kolm (1977) presents results for the case where interrelations between welfare components are assumed to be irrelevant in inequality comparisons. On the other hand, these interrelationships play a central role in a paper written by Atkinson and Bourguignon (1982) who study inequality comparisons by means of stochastic dominance. In the references mentioned above, inequality comparisons relate to <u>two</u> different distributions of the same total bundle of commodities. The difference with the present paper is that here for <u>one</u> particular distribution, inequality in the distribution of the elements of the bundle is compared with inequality in the distribution of some aggregate indicator of the elements.

In section 2, some results will be given of one-dimensional inequality analysis. Sections 3 and 4 are devoted to a special case of multidimensional inequality, i.e. when the relationship between the aggregate variable and the elements of the bundle is known to be additively separable by definition. In section 3 the relationship between these two is analysed without making use of a specific inequality measure. In section 4, results are presented for some specific inequality measures. Section 5 is addressed to the more general case of a welfare function which is not necessarily separable. Empirical results are presented in section 6.

2. One-dimensional Inequality

The inequality among N persons (n=1,...,N) according to a certain variable (for example: income) can be studied as follows. Let $x=(x_1,...,x_N)$ be the allocation of a given amount of income among the N persons. Assume that the vector x is ordered so that $x_1 \geq x_2 \geq ... \geq x_N$. Similarly, let $y_1=(y_1,...,y_N)$ be another allocation of the same amount with $y_1 \geq y_2 \geq ... \geq y_N$.

Then one can define inequality as follows.

Definition 1. Distribution x is less unequal than distribution y (or x is majorized by y, denoted as xLy) if:

$$\begin{cases} x_1 + ... + x_n \leq y_1 + ... + y_n & \text{for } n=1,...,N-1 \\ x_1 + ... + x_N = y_1 + ... + y_N & \end{cases} \qquad (1)$$

This definition means that when two distributions are compared by Lorenz curves, the upper curve respresents a less unequal distribution than the lower curve.

According to the principle of transfers, as formulated by Dalton (1920), the inequality in a certain distribution y can be reduced by transferring a (small) amount from individual m to n if $y_n < y_m$. Repeated use of this principle leads to the conclusion that every vector x of which the elements can be written as a weighted average of the elements of y is less unequal than y. Such a weighted average can be written as:

$$x_n = p_{1n} y_1 + ... + p_{Nn} y_N \qquad \text{for } n=1,...,N \qquad (2)$$

where the weights satisfy the following constraints:

$$\begin{cases} p_{mn} \geq 0 & \text{for all } m, n \\ p_{1n} + ... + p_{Nn} = 1 & \text{for all } n \\ p_{m1} + ... + p_{mN} = 1 & \text{for all } m \end{cases} \qquad (3)$$

The $N \times N$ matrix P with elements p_{mn} satisfying (3) is called doubly stochastic.

The following theorem has been proved about doubly stochastic matrices (Hardy, Littlewood and Polya, 1934):

Theorem 1. A necessary and sufficient condition that xLy is that there exists a doubly stochastic matrix P such that x=yP.

Another theorem on inequalities owing to Hardy, Littlewood and Polya (1934) reads as follows:

Theorem 2. A necessary and sufficient condition that xLy is that for all continuous concave functions $g: R \to R$:

$$g(x_1) + \dots + g(x_N) \geq g(y_1) + \dots + g(y_N) \qquad (4)$$

If the function g in (4) is interpreted as an individual welfare function, Theorem 2 implies that the sum of individual welfare levels is increased when the income distribution becomes more equal (see Atkinson, 1970).

For more results on the properties and measurement of inequality, refer to e.g. Rothschild and Stiglitz (1973), Dasgupta et al. (1973) and Marshall and Olkin (1979).

3. Inequality of the Whole versus the Parts

Consider a variable such as income, which can be decomposed into two or more parts, for example: labour versus non-labour income, income from agricultural versus non-agricultural activities, household income earned by female versus male household members. To understand the nature of income inequality observed, it may be illuminating to know how inequality in the various income components relates to aggregate inequality. Does the aggregation of income components lead to a mitigation of componentwise inequalities, or may it have an aggravating effect? To answer such questions, one needs a basis to compare inequalities among various income components. Definition 1 cannot serve as such a basis, since it only deals with inequality comparisons between two different distributions of the same amount of income. A slightly modified definition of inequality will therefore be used. Consider N observations on the variables u $(u_1,\dots,u_N)$ and $v(v_1,\dots,v_N)$. Assume, that the observations have been ranked in decreasing order: $u_1 \geq \dots \geq u_N$ and $v_1 \geq \dots \geq v_N$, and that their sums are positive. Then the following definition can be formulated:

Definition 2. The distribution of variable u is less unequal than the distribution of variable v (denoted as uVv) if:

$$(u_1+\dots+u_n)/(u_1+\dots+u_N) \leq (v_1+\dots+v_n)/(v_1+\dots+v_N)$$
$$\text{for } n=1,\dots,N-1, \text{ where } u_1+\dots+u_N>0 \text{ and } v_1+\dots+v_N>0 \qquad (5)$$

In the present context, u and v may stand for any pair of income components, but also for any combination of total income and one of the income components.

The following lemma can easily be proved:

Lemma 1. Consider the variables u, with $u_1+\ldots+u_N>0$, and v, with $v_1+\ldots+v_N>0$. Let s_n be defined as $u_n/(u_1+\ldots+u_N)$ and t_n as $v_n/(v_1+\ldots+v_N)$, for $n=1,\ldots,N$. Then uVv if and only if sLt.

In this lemma the equivalence is formulated between inequality comparisons in terms of V for unstandardized variables u and v, and inequality comparisons in terms of L for their standardized counterparts s and t. The use of Lemma 1 is that Theorems 1 and 2 can be made applicable to unstandardized variables.

Consider total income $u_.$, divided into J components for N individuals:

$$u_{.n} = u_{1n}+ \ldots + u_{Jn} \qquad (n=1,\ldots,N) \qquad (6)$$

Let u_j denote the row vector $(u_{j1}, \ldots, u_{jN})$, representing the distribution of component j. In the following theorem, a relationship between inequality in total income and inequality in the income components is formulated.

Theorem 3. Let u_jVu_1 for $j=2,\ldots,J$. Then for total income $u_.$ as defined in (6): $u_.Vu_1$.

Proof. Let the standardized value of u_{jn} be denoted as t_{jn}, so that for all j and n:

$$t_{jn}= u_{jn}/(u_{j1}+\ldots+u_{jN}) \qquad (7)$$

Further, $\bar{u}_j$ is used to denote the mean value of $u_{j1},\ldots, u_{jN}$ and $\bar{u}_.$ denotes the mean value of $u_{.1}, \ldots, u_{.N}$. Then $t_{.n}$, the standardized value of $u_{.n}$, can be expressed as a weighted mean of the t_{jn}:

$$t_{.n}=(\bar{u}_1/\bar{u}_.)t_{1n}+\ldots+(\bar{u}_J/\bar{u}_.)t_{JN} \qquad n=1,\ldots,N \qquad (8)$$

When u_jVu_1 for all $j\neq1$, Lemma 1 implies that t_jLt_1 for all $j\neq1$, where t_j denotes the row vector $(t_{j1},\ldots,t_{jN})$. Then, according to Theorem 1, there is a doubly stochastic matrix P_j for each $j\neq1$ such that $t_j= t_1P_j$. When this result is substituted into (8), one obtains for $t_.= (t_{.t},\ldots, t_{.N})$:

$$t_.=t_1[(\bar{u}_1/\bar{u}_.)I+(\bar{u}_2/\bar{u}_.)P_2+(\bar{u}_3/\bar{u}_.)P_3+\ldots(\bar{u}_J/\bar{u}_.)P_J], \qquad (9)$$

where I is the unit matrix. It can easily be verified that the matrix between square brackets in (9) is a doubly stochastic matrix. Therefore, one may conclude by means of Theorem 1 that $t_{.}Lt_{1}$, which in its turn implies that $u_{.}Vu_{1}$ because of Lemma 1.

Theorem 3 says that inequality in total income is smaller than in the most unequal income component. This conclusion holds true irrespective of the inequality measure used. Thus, taking several income components together has a mitigating effect on total income inequality.

A limitation of Theorem 3 is that it is based on the assumption that one of the income components is more unequally distributed than all of the other components. In practice, one will often find that Lorenz curves intersect so that two distributions are obtained, of which the inequalities are incomparable. The usual way of dealing with this incompleteness problem in L and V is the use of one or more inequality measures. This allows for inequality comparisons, even when the Lorenz criterion is indecisive (see e.g. Fields and Fei, 1978). This will be the subject of the next section.

4. Decomposition of Total Income Inequality

In this section, a decomposition of total income inequality will be given, using the coefficient of variation. Some attention will also be given to the Gini-coefficient.

The coefficient of variation for income component j is defined as:

$$cv_j = \sqrt{(1/N)\sum_n (u_{jn}-\bar{u}_j)^2}/\bar{u}_j \tag{10}$$

where $\bar{u}_j$, the mean value of income component j, is assumed to be positive. Similarly, the coefficient of variation for total income is:

$$cv_. = \sqrt{(1/n)\sum_n (u_{.n}-\bar{u}_.)^2}/\bar{u}_. \tag{11}$$

where mean income $\bar{u}_.$ is assumed to be positive. Substitution of (6) and (10) into (11) leads to:

$$cv_. = \sqrt{[(\sum_j \bar{u}_j cv_j)^2 - H]}/\bar{u}_. \tag{12}$$

where H is defined as follows:

$$H = 2 \sum_{j'=j+1}^{J} \sum_{j=1}^{J-1} \bar{u}_j \bar{u}_{j'} (1-r_{jj'}) cv_j cv_{j'} \tag{13}$$

with $r_{jj'}$ being the Pearson correlation coefficient between components j and j'. If all mutual correlations between the welfare components are equal to 1, H would vanish in (12), so that $cv_.$ can simply be written as a weighted mean of all individual cv_j, the weights being equal to $\bar{u}_j/\bar{u}_.$. If some of these correlations are smaller than 1, H is positive, which has a mitigating effect on total income inequality as measured by the coefficient of variation.

It is not difficult to check that (12) is consistent with Theorem 3, since if $cv_j \leq cv_1$ for all $j \neq 1$, then $cv_. \leq cv_1$. To investigate more deeply how inequality in total income depends on the correlations between income components, assume that inequality in all components is equal to $\overline{cv}$. Further, let the average share of each income component $(\bar{u}_j/\bar{u}_.)$ be equal to $(1/J)$, and assume that the correlations between all components are equal: $r_{jj'} = r$ for all $j \neq j'$. Then (12) can be re-written as:

$$cv_. = \overline{cv}\,\sqrt{(1/J) + r(J-1)/J} \qquad (12')$$

J \ r	0	.25	.50	.75	1.00
1	1.000	1.000	1.000	1.000	1.000
2	.707	.791	.866	.935	1.000
5	.447	.632	.773	.894	1.000
10	.316	.570	.742	.880	1.000
100	.100	.507	.711	.867	1.000
∞	.000	.500	.707	.866	1.000

Table 1. Coefficient of variation cv of total income relative to cv of income components as a function of intercomponent correlation r given the number of components J.

Table 1, displaying the relationship between $cv_./\overline{cv}$ and the correlation coefficient r for some values of J, shows how strongly the aggregate inequality depends on r for larger values of J. Correlation coefficients around zero and may give rise to a very strong reduction of aggregate inequality versus componentwise income inequality.

Along similar lines, it can be shown that for the Gini coefficient G the following result holds true:

$$G_. = (\sum_j \bar{u}_j\, G_j - H)/u_. \qquad (14)$$

where

$$H = 2 \sum_j \sum_n u_{jn}\, w_{jn}/N^2 \qquad (15)$$

The factors w_{jn} in (15) indicate the differences in rank of u_j and $u_.$ in the following way. Let rank 1 be assigned to the highest value, rank 2 to the one but highest value, etc. Let for a certain j and n the ranks of u_j and $u_.$ be k and l, respectively. Then

$w_{jn} = l-k$ (note: $\sum_n w_{jn} = 0$). Thus, the factors w_{jn} are related to Spearman's rank correlation coefficient. When the rank correlation between the various u_j and $u_.$ is perfect, all w_{jn} will be zero and thus H=0. In the case of non-perfect rank correlation, H can be shown to be positive, which has a mitigating influence on total income inequality as measured by the Gini coefficient.

It is not difficult to check that the results for both inequality measures discussed are consistent with Theorem 3. Also in other respects, the results for the coefficient of variation and the Gini coefficient are similar. In the case of perfect correlation between the income components, total income inequality can be decomposed as a weighted mean of inequality of individual income components, the weights being $\bar{u}_j/\bar{u}_.$. The correction terms H depend on correlations among the income components and/or total income, although the correlation coefficients used differ. With cv, use is made of the product moment correlation coefficient, whereas for G a correlation coefficient is used which is related to Spearman's rank correlation.

5. Decomposition of Welfare Inequality

The above sections only deal with decomposition of variables such as income, which is related to its components by means of a linear additive relation which holds true by definition. One may wonder whether the results obtained are also valid in the more general context of welfare inequality. Thus, consider a welfare function $g: R^J \rightarrow R$ assigning a real value z_n to the vector of welfare components $v_{1n}, \ldots, v_{Jn}$ such as income, health, accessibility of public services, etc.:

$$z_n = g(v_{1n}, \ldots, v_{Jn}) \qquad n=1,\ldots,N \qquad (16)$$

More specifically, assume that g is an additive separable function:

$$z_n = \alpha_1 f_1(v_{1n}) + \ldots + \alpha_J f_J(v_{Jn}) \qquad n=1,\ldots,N \qquad (17)$$

where each f_j is a utility function over the j-th component, $f_j(v_{jn})$ is scaled from zero to one, and the weights α_j sum to one (cf. Keeney and Raiffa, 1976). In this case, the following reformulation of Theorem 3 holds true:

<u>Theorem 3'</u>. Let $f_j(v_j) V f_1(v_1)$ for $j=2,\ldots,J$.
Then $z V f_1(v_1)$.

<u>Proof</u>. The proof runs along similar lines with Theorem 3. The only essential difference is that in (9), the weight applied to the doubly stochastic matrix P_j does not only depend on the mean value of component j, relative to the aggregate value, but also on the factor α_j; i.e. $\bar{u}_j/u_.$ is replaced by $\alpha_j \bar{f}_j(v_j)/\bar{z}$.
Thus, given the welfare function (17), the simultaneous consideration of welfare components leads to a degree of inequality which is smaller

than the inequality in the most unequal component. Table 1 implies in this context that low or zero correlation coefficients between the welfare components lead to a substantial reduction of aggregate welfare inequality compared with componentwise welfare inequality.

The mitigation of inequality which takes place with the additive separable welfare function is due to the infinite substitution elasticity of this function: bad scores for a certain welfare component can be fully compensated by high scores for another welfare component. Thus, one may expect a mitigation of inequality when the welfare components are not perfectly correlated.

The following example shows that with a lower substitution elasticity, mitigation of inequality is no longer guaranteed. Let z be a welfare function with zero substitution elasticity:

$$z_n = \min_j v_{jn} \qquad n=1,\ldots,N \tag{18}$$

Consider the following observations for three individuals and two welfare components (N=3; J=2): $v_1=(1,1,0)$ and $v_2=(.8,\ .2,\ 1.0)$. Thus, inequality in component 1 is greater than in component 2, according to Definitions 1 and 2. For z one obtains: $z=(.8,\ .2,\ .0)$, so that according to Definition 2, z is more unequal than both v_1 and v_2. Thus, with welfare function (18), the joint consideration of various welfare components may lead to a more serious diagnosis of inequality than the individual welfare indicators would suggest.

6. Example: Interregional Welfare Inequalities

An empirical example of multidimensional inequality comparisons will be given for Dutch regional data from 1976-1978 (see Van Veenendaal, 1981 for details). Data have been collected for 40 regions and 13 welfare components (N=40, J=13), relating to socio-economic conditions, environmental quality and infrastructure. The location of the regions has been depicted in Figure 1. The mean population size of the regions is approximately 350,000 inhabitants.

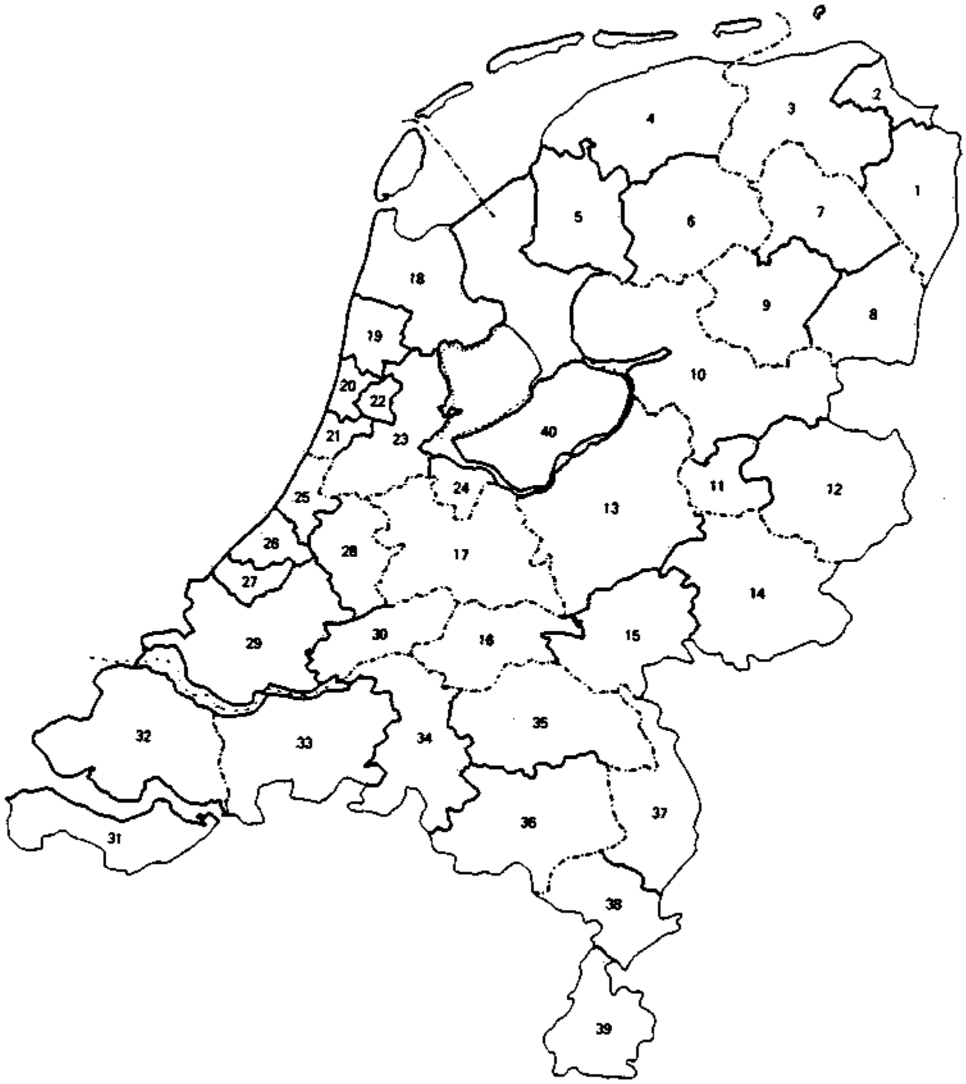
The socio-economic variables are:

1. fiscal income per capita
2. unemployment rate
3. wealth per capita
4. index of cost of living.

The environmental variables are:

5. population density

Figure 1. Location of 40 regions in the Netherlands



6. size of natural environment as percentage of total regional area
7. index of industrialiation related to regional area
8. index of the emission of pollutants related to regional area.

The infrastructural variables are;

9. density of transport network
10. index of cultural centres and sport accomodation per capita
11. index of number of schools of various types per capita
12. distance to the centre of the Netherlands
13. index of various medical services per capita.

The outcomes of the coefficient of variation for the 13 welfare components are represented in Table 2. The table indicates that the interregional inequality in the socio-economic variables is relatively small, while the inequality in the environmental variables is relatively large. For the infrastructural variables we find in most cases intermediate positions.

sub-profile	component j	componentwise coefficient of variation cv_j	aggregate cv per subprofile
	1	.08	
socio-economic	2	.41	.17
	3	.34	
	4	.07	
	5	.88	
environment	6	.70	.63
	7	.76	
	8	.94	
	9	.42	
	10	.44	
infrastructure	11	.17	.31
	12	1.24	
	13	.43	

Table 2. Coefficients of variation for 13 welfare components and 3 sub-profiles.

The above statements hold for the variables in the three sub-profiles independently. It is also interesting to know the degree of inequality for a composite variable representing a whole sub-profile. These composite variables have been constructed by calculating the unweighted average of the normalized variables in each profile. For the three composite variables, we find as outcomes for the coefficient of variation cv (.17, .63, .31). When we compare this outcome with the mean values of cv in Table 2 per sub-profile (.22, .82, .54), we note that the rank order is the same and that in all cases the former is smaller than the latter. The relative and absolute decrease is largest for the infrastructure variable, which means that inequality mitigation occurs to a larger extent in this sub-profile than in the other sub-profiles. From (12) and (13) one knows that the degree of inequality mitigation depends among others on the correlation coefficients per sub-profile. As can be seen from Table 3, the correlation coefficients among variables in the infrastructure block are relatively low so that it is no surprise to find a relatively high degree of inequality mitigation for this sub-profile.

1	2	3	4	5	6	7	8	9	10	11	12	13
1.00	.57	.60	-.17	-.84	.30	-.63	-.83	.77	.05	.15	.62	.55
.57	1.00	.49	-.13	-.40	.31	-.56	-.37	.25	-.15	.17	.72	.24
.60	.49	1.00	.06	-.39	.29	-.11	-.36	.36	.12	.15	.28	.46
-.17	-.13	.06	1.00	.04	-.24	-.09	.01	-.01	.17	-.06	-.24	.04
-.84	-.40	-.39	0.04	1.00	-.19	.76	.97	-.94	-.10	-.01	-.52	-.47
.30	.31	.29	-.24	-.19	1.00	-.17	-.09	.15	-.04	.04	.25	.19
-.63	-.56	-.11	-.09	.76	-.17	1.00	.69	-.68	-.03	.03	-.55	-.49
-.83	-.37	-.36	.01	.97	-.09	.69	1.00	-.91	-.11	-.01	-.52	-.47
.77	.25	.36	-.01	-.94	.15	-.68	-.91	1.00	.04	-.02	.46	.38
.05	-.15	.12	.17	-.10	-.04	-.03	-.11	.04	1.00	.01	-.17	.09
.15	.17	.15	-.06	-.01	.04	.03	-.01	-.02	.01	1.00	.12	.27
.62	.72	.28	-.24	-.52	.25	-.55	-.52	.46	-.17	.12	1.00	.18
.55	.24	.46	.04	-.47	.19	-.49	-.47	.38	.09	.27	.18	1.00

Table 3. Correlation matrix for 13 welfare components.[1)]

An even stronger reduction of inequality is obtained when an overall welfare indicator is formulated, based on the three sub-profiles mentioned above. The reason is that there is a rather strong negative correlation between the environmental index (E) on the one hand, and the socio-economic (SE) as well as the infrastructure (I) index on the other hand (see Table 4). For example, if a linear indicator of total welfare is formulated based on the three sub-profile indices with weights α_{SE}= .6, α_E= .2 and α_I= .2, one obtains for the cv of total welfare a value of .13. This is much lower than the weighted average of the cv of the three individual sub-profiles (.29). Of course, this result depends on the weights used. In Table 5 the outcomes of cv are given for some alternative combinations of weights.

	SE	E	I
socio-economic (SE)	1.00	-.35	.62
environment (E)	-.35	1.00	-.63
infrastructure (I)	.62	-.63	1.00

Table 4. Correlation coefficients for 3 sub-profile indices.

weights α_{SE}	α_E	α_I	coefficient of variation cv
1.0	.0	.0	.173
.8	.2	.0	.149
.8	.0	.2	.181
.6	.4	.0	.213
.6	.2	.2	.135
.6	.0	.4	.200
.4	.6	.0	.329
.4	.4	.2	.187
.4	.2	.4	.137
.4	.0	.6	.229
.2	.8	.0	.471
.2	.6	.2	.303
.2	.4	.4	.170
.2	.2	.6	.155
.2	.0	.8	.265
.0	1.0	.0	.633
.0	.8	.2	.447
.0	.6	.4	.281
.0	.4	.6	.165
.0	.2	.8	.187
.0	.0	1.0	.308

Table 5. Dependence of coefficient of variation of welfare on weights of welfare function.

In the present case it may be concluded that a linear welfare function leads to a degree of welfare inequality which is much lower than the average inequality in the individual welfare components. Comparing interregional inequality in total welfare with interregional income inequality (variable 1) one finds that the two are rather near, when using Table 2 as a frame of reference. Most individual variables display inequalities which are much higher. Yet, it is interesting to note that in the present case despite all mitigating effects, interregional welfare inequality is still higher than interregional income inequality. An implication is that even if interregional income inequalities are relatively small, governments may have good reasons not to ignore regional specific policies in view of broader based welfare inequalities among regions.

Note

1) Variables 2, 4, 5, 7, 8 and 12 have been multiplied with a factor of -1 so that for all variables a larger value is preferred to a smaller one.

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