23
ET
05348
1987

SERIE RESEARCH MEMORANDA

A NEW PERSPECTIVE ON PRICE INDICES

Tjemme van der Meer

Researchmemorandum 1987-23 April '87

VRIJE UNIVERSITEIT
Faculteit der Economische Wetenschappen en Econometrie
AMSTERDAM

5348 -

A NEW PERSPECTIVE ON PRICE INDICES

Tjemme van der MEER *
Free University, Amsterdam, The Netherlands

We propose a new definition of price indices. If the utility function is homothetic, it reduces to the usual definitions. If not, it satisfies all of Fishers (1922) tests.

1. Introduction

We propose a new definition of price indices. If the utility function is homothetic, it reduces to the usual definitions, but with a different interpretation. If the utility function is not homothetic, it requires no arbitrary reference welfare level, can be consistently chained, and satisfies all of Fishers (1922) tests for price indices. When the definition is extended to allow for changes in the utility function, only one of the six tests fails to be satisfied.

We restrict ourselves to economic consumer indices (for alternative indices, see Samuelson and Swamy (1974), Diewert (1981)). These are defined in the context of a consumer optimization problem. Much attention has been paid to the question of how well one index approximates (or bounds) another. In this paper, we consider the question what we wish to approximate.

There are price, quantity and value indices. The price index is usually defined as the change in the cost of obtaining a reference welfare level. Quantity indices are changes in welfare, where welfare is measured in terms of (Deaton and Muellbauer (1983, p.179)) a utility, a money, or a quantity metric. Value indices are the changes in total expenditure. Quantity and value indices are relatively uncontroversial. We concentrate on price indices.

---

* The research reported in this paper is carried out as part of the project "Demand and Supply in Equilibrium and Disequilibrium", which has been approved by the Dutch Office of Education and Sciences. The author wishes to thank Nol Merkies for valuable comments and suggestions. Reprint order forms should be sent to the author, Department of Econometrics, Free University, P.O. Box 7161, 1007 MC Amsterdam, The Netherlands.

To motivate the new definition, consider the single commodity case. Last year, I bought a textbook for $100. I found it so good that I recommend it this year to all my students. We obtain a *quantity discount*, and pay only $50 per copy. It seems arguable, without further information, that the price of this textbook has effectively dropped by half. The usual price indices, however, may decide upon a different price change, dependent upon additional information on the pricing system of the publisher. In particular, if the pricing system has not changed, the usual price indices do not indicate any price change whatsoever.

2. A new definition

In this section, we propose and interpret a new definition of price indices.

According to the usual definitions, the price index depends on the *arbitrary* choice of a reference utility level, at which prices are evaluated for both periods. An implicit assumption is that the choice of level is irrelevant. The point of the textbook example in the introduction is that prices may vary with quantities (e.g. Hausman (1985)). "The price" is only identified if we specify the quantity purchased. The same point can be made at the "macro-level": the price of an additional unit (and hence the average price) of utility may vary with the level of utility. Thus, if we wish to capture price changes, we may also need to specify the corresponding quantity changes. This is the underlying idea in the following definition.

Define the price index P_{01} from time 0 to time 1 as follows:

$$P_{01} = (c(U^1,p^1)/U(x^1)) \ / \ (c(U^0,p^0)/U(x^0))$$

where p and x are vectors of prices and quantities respectively, welfare U may be measured in a utility, a money, or a quantity metric (consistent with the metric used in the quantity index), and c denotes the cost function. We assume that the utility function (and hence the cost function) remains constant over time, and the superscript i denotes values attained in period i.

We may allow for changes in the utility function (quality changes of a commodity), by making the utility function and the cost function time dependent:

$$P_{01} = (c^1(U^1,p^1)/U^1(x^1)) \;/\; (c^0(U^0,p^0)/U^0(x^0))$$

In this extended definition, there is nothing which the numerator and the denominator have in common. This is consistent with the Arrow and Debreu perspective, in which commodities are differentiated both by place and by time. The change in price of a particular textbook over time is conceptually analogous to the difference in price between an apple and a barrel of oil. However, the empirical application of this extended definition requires us to quantify the changes in the utility function, which is not easy.

To illustrate this index, consider two special cases:
(*) There is only one commodity. Take for instance, the textbook case described above: (with a constant utility function standardized to be homogeneous) the price index is a half. The price index is a ratio of prices at the two periods, *evaluated at the quantities purchased in the two periods* considered.
(*) If the utility function is homothetic, both "reference welfare levels" U^0 and U^1 drop out, the price index depends on prices only, and if the utility function is constant, the proposed price index reduces to the usual price index. However, the usual indices are interpreted somewhat differently.

How to interpret the proposed index? Consider first the interpretation of the quantity index U^1/U^0. This quantity index is a ratio, which has been interpreted both in terms of the numerator and in terms of the denominator. Similar interpretations hold for the value index c^1/c^0.
*) Interpretation of the numerator. Diewert (1976) observes that, by considering the denominator U^0 as a base period normalization, quantity aggregates can be computed from quantity indices. The numerator quantity aggregate in fact *equals* the quantity index. A statement to the same effect is made by Samuelson an Swamy (1974): [the quantity index] "must itself be a cardinal indicator of ordinal utility".
*) Interpretation of the denominator. Samuelson and Swamy (1974) write that (now taking the numerator U^1 as normalization) the quantity index

may also be interpreted as a cardinal indicator of the reciprocal of the quantity aggregate at the base period.

The interpretation of the proposed price index combines the interpretations of the numerator and denominator: it is the ratio of price aggregates, the change in the average cost of utility. We need to be a little careful if we create runs of price indices. If we interpret the price index as a normalized price aggregate, we can develop runs of price indices by using the same normalization throughout. If, alternatively, we interpret the price index explicitly as capturing price changes over time, we divide two price aggregates. In terms of normalizations, we use the same normalization for numerator and denominator, and the normalizations drop out.

Why is this price index meaningful? It summarizes all information about prices needed for budget allocation over time. To demonstrate this, we take the perspective advocated by Pollak (1975), namely that we should consider any price aggregate as a subindex. Following Deaton (1986, p. 1815), but allowing for a more general measure of utility, we can rewrite the separable two-period utility optimization problem

$$\text{Max } U = U\ (u^0,u^1),\ \text{subject to } c = c^0(u^0,p^0) + c^1(u^1,p^1)$$

as follows

$$\text{Max } U = U(u^0,u^1),\ \text{subject to}$$

$$c = u^0 .\ c^0(u^0,p^0)/u^0 + u^1\ .\ c^1(u^1,p^1)/u^1$$

In this relationship, we can interpret (with Deaton (1986)) utilities as quantities, and interpret the ratio of cost function to utilities as prices. In particular, we can interpret the ratio of the two prices (one measured in terms of the other) as the price index.

3. The Fisher tests

Wald (1937) demonstrated that price and quantity indices cannot generally satisfy all Fisher's tests. His counter example hinges on the fact that two different bundles of commodities may be purchased with the same prices. If we rule out this possibility, by requiring that the utility function remains constant over time, we demonstrate in this section that the proposed price indices satisfy all of the Fisher (1922) tests, as presented in in Allen (1975, p. 45). For a similar analysis of the usual price index definition, see Samuelson and Swamy (1974). When the utility function changes over time, it can be verified that the proportionality index will generally not be satisfied.

(i) Identity Test

When one year is compared with itself, the index shows 'no change', i.e. $P_{00}=1$. This test is satisfied:

$$P_{00} = (c(U^0,p^0)/U(x^0)) \;/\; (c(U^0,p^0)/U(x^0)) = 1$$

(ii) Proportionality Test

When all prices move in proportion, so does the index, i.e. $P_{01}=\alpha$ when $p_1=\alpha p_0$ for each item. This test is satisfied, as the optimized utility function is homogeneous of degree zero in prices (ruling out money illusion), and the cost function is (therefore) homogeneous of degree one in prices. Thus:

$$P_{01} = (c(U^0,\alpha p^0)/U(x^0)) \;/\; (c(U^0,p^0)/U(x^0)) = \alpha$$

(iii) Change-of-units Test

P_{01} is invariant under any change in the money or physical units in which individual prices are measured. The assumption that utility is unaffected by the units of measurement is very plausible. As far as changes in the physical units is concerned, this affects the price proportionally, so that this test is also satisfied.

(iv) Time Reversal Test

The joint effect of changing all prices and subsequently changing them back should be indicated by a joint price index showing 'no change': $P_{01}.P_{10} = 1$. This test is closely related to the Circular Test:

(v) Circular Test

The price index between two periods should not be affected by price movements inbetween the two periods: $P_{01}.P_{12}=P_{02}$.

The advantage of the proposed definition is very clear for the time reversal and the circularity tests. As we have defined the numerator and the denominator independently, these tests are automatically satisfied.

(vi) Factor-reversal Test

The price and the quantity index between them account for the value change: $P_{01}.Q_{01}=V_{01}$.

In our definition of the price index, this last test is also satisfied. We have defined price indices (by analogy with quantity and value indices) as the ratio of aggregates, and the factor reversal test can be interpreted as testing whether or not the product of the quantity and the price aggregate (both in the numerator and the denominator) equal the value aggregate.

4. Concluding Comments

In this paper, we define price indices as the change in the average cost of a utility unit. This definition requires no arbitrary reference welfare level, can be consistently chained, and satisfies all of Fishers (1922) tests for price indices.

If the utility function is homothetic, the proposed index reduces to the usual definitions, and many special cases have been found (see e.g. Diewert (1976)). The main practical value of our contribution lies in reinterpreting these standard definitions. The interpretation as the change in the cost of an average utility unit is consistent with the interpretation of the quantity and the value indices (in the sense of the factor reversal test).

It remains to be verified whether the proposed definition results in easily computable price indices exact for a plausible class of *non*-homothetic utility functions.

References

Allen, R.G.D. (1975), *Index Numbers in Theory and Practice*, MacMillan, London.

Deaton, A.S. and Muellbauer, J. (1983), *Economics and Consumer Behavior*, Cambridge University Press, Cambridge.

Deaton, A.S. (1986), "Demand Analysis", in: Z. Griliches and M.D. Intriligator (eds.), *Handbook of Econometrics*, Vol 3, North-Holland, Amsterdam, 1767-1839.

Diewert, W.E. (1976), "Exact and Superlative Price Indices", *Journal of Econometrics* 4, 115-45.

Diewert, W.E. (1981), "The Economic Theory of Index Numbers: A Survey", in: A.S. Deaton (ed.), *Essays in the Theory and Measurement of Consumer Behaviour*, Cambridge University Press, Cambridge, pp. 163-208.

Fisher, I. (1922), *The Making of Index Numbers*, Houghton Mifflin, Boston.

Hausman, J.A. (1985), "The Econometrics of Non-linear Budget Sets," *Econometrica* 53, 1255-1282.

Pollak, R.A. (1975), "Subindexes in the Cost of Living Index", *International Economic Review* 16, 135-50.

Samuelson, P.A. and Swamy, S. (1974), "Invariant Economic Index Numbers and Canonical Duality: Survey and Synthesis", *American Economic Review* 64, 566-93.

Wald, A. (1937), "Zur Theorie der Preisindexziffern", *Zeitschrift fur Nationalokonomie* 8, 179-219.