

1987

# SERIE RESEARCH MEMORANDA

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A STATE OF THE ART REVIEW

M.M. Fischer  
P. Nijkamp

Researchmemorandum 1987-61

december 1987



VRIJE UNIVERSITEIT  
FACULTEIT DER ECONOMISCHE WETENSCHAPPEN  
EN ECONOMETRIE  
AMSTERDAM



FROM STATIC TOWARDS DYNAMIC DISCRETE CHOICE MODELLING:

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Manfred M. Fischer  
Department of Geography  
University of Vienna

Peter Nijkamp  
Department of Economics  
Free University Amsterdam



## Abstract

The main aim of the present paper is to survey some major trends in current research in the field of discrete choice modelling, with particular emphasis on dynamic approaches. The paper is organized as follows. Section 2 provides a brief overview of static disaggregate choice modelling and random utility maximization, based inter alia on multinomial logit and/or probit models, generalized extreme value models, and nested logit models. Particular attention is given here to model representation issues, sampling and estimation issues and model performance issues. Next, section 3 is devoted to some recent developments in the rapidly growing new field of dynamic discrete choice modelling. In contrast to stochastic panel data models of buying behaviour, dynamic discrete choice models incorporate explanatory variables and take adaptive behaviour explicitly into account (i.e. the effect of past experience on choice behaviour). Several dynamic discrete choice model approaches are summarized. Special attention is paid to the seminal work of Heckman. In the final section, complementary and alternative approaches to dynamic choice modelling are discussed, such as the human activity constraint approach, the computational process modelling approach and the master equation approach. It is concluded that contextual effects, multi-actor or synergetic interactions and shifting individual preferences based on learning principles are of primary importance in dynamic discrete choice modelling.



## 1. INTRODUCTION

Spatial systems are never static, but always in a state of flux, in both an absolute and a relative sense. The dynamics of spatial systems may be the result of three types of different forces (see also Jansen et al., 1985, Nijkamp and Reichman, 1986, and Williams, 1981):

- . external influences (i.e. influences from the environment of the system such as e.g. the rise of oil prices on the world market or the global depression affecting international and/or interregional trade volumes),
- . internal dynamics caused by reaction patterns of actors (households, firms, landlords, investors and others), and
- . public policy instruments aiming at influencing the state or structure of a spatial system in order to achieve a set of policy objectives.

Up to now, empirical research on spatial systems is usually underpinned by models and theories of a static or comparative static nature. This perspective assumes the existence of unique equilibrium combinations of spatial system characteristics which change smoothly in response to changes in exogenous variables. Discontinuities in the development of spatial systems are attributed to unexpected external influences rather than to the internal dynamics of spatial systems (see Varaija and Wiseman, 1981).

In recent years however, the interest in fully dynamic spatial models has grown considerably. This increased attention is due to methodological advances in the area of (in)stability and (dis)equilibrium analysis (e.g., Nicolis and Prigogine, 1977, Thom, 1975 and Weidlich and Haag, 1983), to the progress made in the statistical, mathematical and computerized handling of non-linear dynamic models, and to the general awareness that the economies of most countries are going through a stage of structural (i.e., non-linear dynamic) change.

In regional and urban economics and in human geography various approaches have been developed that aim at replicating the evolution of a complex spatial system by means of dynamic models. Examples of such contributions are:

- . dynamic Lowry-models, based on iterative adjustment processes towards a new macro equilibrium state after an initial exogenous impulse (cf. Harris and Wilson, 1978),
- . urban ecology models, based on simplified aggregate models for describing by means of Volterra-Lotka dynamics the evolution of urban systems (cf. Dendrinos and Mullally, 1985),
- . self-organising models, based on evolutionary assumptions regarding the behaviour of dynamic spatial systems (cf. Allen and Sanglier, 1981),
- . micro simulation models, based on a probabilistic approach to the analysis of changes (events) in the state of a complex dynamic system (cf. Wegener, 1983, and Clark and Wilson, 1985),

- event history analysis, based on the statistical analysis of a sample-path observation plan which records relevant information on all changes in the state of a phenomenon within some observation periods (cf. Hannan and Tuma, 1985).

Furthermore, catastrophe and bifurcation theory models have been adopted for analysing the dynamics of spatial systems (see e.g. Wilson, 1981). Such models describe spatial systems characterized by multiple equilibria in which shifts from one equilibrium to another may involve discontinuities. Since these discontinuities are considered as characteristic of the system rather than as the result of external shocks, catastrophe theory based models may provide important new insights into the nature of economic and social processes within spatial systems. Up to now, however, the applications of both the 'Zeeman' and 'Thom' approaches to catastrophe theory based model building has been disappointing (for more details on this issue, see e.g. Varaija and Wiseman, 1981). A major shortcoming is generally the lack of explicit assumptions about micro behaviour or about environmental and technological constraints, while catastrophic events are considered as the result of independent decision-making by a large number of choice-makers. Advances in understanding the process of internal dynamics can only be achieved if it is possible to show how discontinuities are the logical consequence of specified behaviour and circumstances.

In the context of dynamic modelling, Prigogine (1981) has made a very interesting typology of models:

- macro-phenomenological models (in such models the variables are (weighted) average values of micro variables underlying the dynamics of a system),
- micro-stochastic models (these models aim at explicitly replicating the behaviour of micro variables, based on a stochastic approach),
- models based on dynamic laws at a basic level (such models aim at identifying the trajectory of system variables assuming that these are governed by fundamental laws of motion).

Applying this classification to dynamic spatial models Barentsen and Nijkamp (1986) conclude that models of the first type include inter alia dynamic Lowry-models and urban ecology models, while models of the second type can be found in the family of dynamic discrete choice models and of micro simulation models. Finally, models of the third type comprise inter alia models based on self-organizing paradigms allowing for bifurcation and disequilibrium states.

In the past years, the use of disaggregate choice models has been strongly advocated, as such models are in general able to capture stochastic and behavioural aspects of spatial decision-making. Starting from the observation that modelling at the empirical level of the individual actor in the spatial system (consumers or suppliers of activities, such as e.g.



migrants, travellers, real estate developers, or local government decision-makers) offers the promise of new insights into decision-making and choice behaviour processes. Various researchers have devoted considerable efforts to the development of behavioural spatial choice models capable of considering individual choices from a set of discrete alternatives at a point in time. Such discrete choice models have mainly focused on cross-section analysis. Of course, choice models for static analysis may be extended to provide a basis for dynamic analysis. But such extensions are neither as obvious nor as simple as it may seem at first glance. Some progress towards these directions has already taken place during the last few years (see for a survey also Nijkamp et al., 1985, Pitfield, 1984, and Timmermans, 1985). The shift from static towards dynamic modelling efforts is placing new demands on the discrete choice methodology. The present paper aims at providing a survey of some major trends in current research on discrete choice modelling, with particular emphasis in dynamic approaches. Section 2 will give a brief discussion of static discrete choice models, followed by a treatment of important research issues in this context. Next, section 3 presents some developments in dynamic discrete choice modelling. Finally, in section 4 complementary and alternative approaches are discussed.

## 2. DISCRETE CHOICE MODELLING AND RANDOM UTILITY MAXIMIZATION

### 2.1 Introductory Remarks

Many important decisions an individual is facing in his life involve choice from a constrained set of alternatives, such as e.g. residential mobility and housing choice, choice of occupation and workplace location, choice of a car, the mode and route of travel in work trips and shopping trips. Such choices are discrete in nature. However, in many choice contexts, conventional marginalist micro-economic consumer theory takes for granted that the decision variable of a consumer is continuous, which is evidently a less valid assumption.

In the 1970's significant progress has been made in developing and applying random utility based choice models in different spatial choice contexts, mostly in travel demand analysis (see e.g. Domencich and McFadden, 1975, Horowitz, 1979, 1980, Halperin and Gale, 1984, and Fischer, 1986) and more recently also in the area of residential mobility-housing choice analysis (see McFadden, 1978, Anas, 1982, Onaka and Clark, 1983, Van Lierop and Nijkamp, 1984, Clark and Onaka, 1985, Van Lierop and Rima, 1985, Quigley, 1985, Aufhauser et al, 1986, Fischer and Aufhauser, 1986 and Van Lierop, 1986), as well as in labour supply mobility analysis (see Evers and van der Veen, 1983, Maier and Fischer, 1985, Fischer and Maier, 1986). Before discussing dynamic discrete choice models (in section 3), we will give a brief overview of static discrete choice modelling in the present section.

## 2.2 Basic Concepts and Classical Models

Detailed presentations of the assumptions and derivations of discrete choice models are given in Domencich and McFadden (1975), McFadden (1981), Ben-Akiva and Lerman (1985), Fischer and Nijkamp (1985a, 1985b) and others. For the purpose of this paper it is useful to summarize the major assumptions underlying discrete choice models as follows:

- . Each decision-maker  $i$  (individual, household or another decision-making unit) in the population faces a set  $A$  of mutually exclusive choice options  $a = 1, \dots, A'$ .
- . The population of decision-makers is partitioned into population segments  $s = 1, \dots, S'$ . The decision-makers in each segment have the same socio-economic characteristics. Moreover, it is usually assumed that the decision-makers in each segment have identical choice sets. This, however, is not strictly necessary (see e.g. Manski, 1981).
- . The decision-maker  $i \in s$  assigns to each alternative a value  $u_{ia}$  of an objective function, termed utility, and chooses that alternative which yields the maximum utility, i.e.

$$u_{ia} > u_{ia'} \quad \text{for} \quad a' \neq a; \quad a' = 1, \dots, A' \quad (1)$$

- . It is usually assumed that random utility represents variations among decision-makers within the same segment. According to this interpersonal interpretation of random utility, all decision-makers have completely deterministic preferences but these cannot be fully observed by the analyst because certain choice-relevant attributes are unobserved or because the valuation of observed attributes may vary from one decision-maker to the other.

The preferences of a decision-maker  $i$  who belongs to population segment  $s$  are represented through a utility function of the form

$$u_{ia} = U(f_1(x_{ia}), f_2(x_{ia}), f_3(\epsilon_{ia})) \quad (2)$$

where  $U(\cdot)$  is the utility function for the  $s$ -th segment,  $f_1(x_{ia})$  the function containing the observed characteristics of decision-maker  $i \in s$  and alternative  $a$ ,  $f_2(x_{ia})$  a random function representing the idiosyncratic tastes of decision-maker  $i$  (i.e., the difference between the tastes of  $i$  and the average tastes of decision-makers within  $s$ ) and  $f_3(\epsilon_{ia})$  a random disturbance term capturing the effects of unobserved choice-relevant attributes of both the decision-makers and the alternatives.  $x_{ia}$  is a  $K$ -dimensional vector of observed characteristics of decision-maker  $i$  and alternative  $a$ .

The specification of the functional form of  $f_1$ ,  $f_2$ ,  $f_3$  and  $U$  is

the starting point for defining a particular model specification. With only very few exceptions random utility based choice models assume that these functions are linear in the parameters and additive in the variables:

$$u_{ia} = x_{ia} \beta + (x_{ia} \delta_i + \xi_{ia}) = v_{ia} + \epsilon_{ia} \quad (3)$$

where the first term,  $v_{ia}$ , at the right-hand side of (3) is referred to as the systematic (deterministic or representative) component of utility, while the second term,  $\epsilon_{ia}$ , denotes the random component. This component consists of two parts:  $\xi_{ia}$  is a random disturbance term capturing the effects of unobserved attributes of the decision-maker and the choice alternatives, while  $x_{ia} \delta_i$  represents the idiosyncratic tastes of  $i$ .  $\beta$  is a vector of parameters of the representative component of utility and  $\delta_i$  the taste variation parameters vector.

A probabilistic choice model aims at forecasting the probability  $P_{ia}$  that decision-maker  $i$  selects alternative  $a$ :

$$P_{ia} = \text{prob} (u_{ia} > u_{ia'}, \text{ for } a' \in A, a' \neq a), \quad (4)$$

conditional on  $x_i$  and  $\theta$ , where  $x_i = (x_{ia}, a \in A)$  and  $\theta$  is a vector including the  $\beta$ - and  $\delta_i$ - parameters of the choice model concerned. Given a stratification of the population of decision-makers, a specification of the set of alternatives among which a decision-maker can choose and a specification of the utility function of type (3), the form of the choice probabilities (4) depends on the distribution  $F$  chosen for the random components.

In the multinomial logit model widely used in a variety of choice contexts, it is assumed that  $F$  is the independent and identically distributed type I extreme-value distribution

$$F_{\epsilon_i} | x_i, \theta = \prod_{a \in A} \exp(-\exp(-\mu(\epsilon_{ia} - \eta))) \quad (5)$$

and that there is no random taste variation across decision-makers within the same population segment (i.e., the taste variation parameters in (3) are equal to zero).  $\eta$  is a location parameter and  $\mu$  a positive scale parameter. Usually it is assumed that  $\eta = 0$  and  $\mu = 1$ . Under these assumptions the choice probabilities have the form

$$P_{ia} = \exp(x_{ia} \beta) / \sum_{a' \in A} \exp(x_{ia'} \beta) \quad (6)$$

It has been widely recognized that the independence of irrelevant alternatives (IIA) property - a property which implies that the relative choice probability of any two alternatives depends exclusively on their systematic components - can give rise to somewhat odd and erraneous predictions when

the alternatives are close substitutes for each other (this situation often occurs in spatial choice contexts). One of the most widely cited anomalies is the red bus/blue bus paradox. The core of the problem lies in the assumption that the disturbances are mutually independent. This assumption requires that the sources of errors contributing to the disturbances must do so in a way such that the total disturbances are independent. In the case of the blue bus/red bus example this is implausible because the red and blue bus modes share all the unobserved characteristics of buses. Thus, the search for alternatives to the IIA-based multinomial logit model has been a major concern in discrete choice analysis.

Many approaches have been suggested in recent years to accommodate varying degrees of similarity between alternatives. The most general one is the multinomial probit model which can handle arbitrary correlations expressed in the form of a general variance-covariance matrix (see Hausman and Wise, 1978). In this model it is assumed that the vector  $\varepsilon_{i.}$  of random components has an  $A'$ -dimensional multivariate normal distribution with mean vector zero and a general  $(A', A')$  variance-covariance matrix  $\Sigma$ . Then the choice probabilities have the form

$$p_{ia} = \int_{-\infty}^{\infty} \left[ \prod_{a' \neq a} \int_{-\infty}^{\infty} N(\varepsilon_{i.} | 0, \Sigma) d\varepsilon_{ia'} \right] d\varepsilon_{ia} \quad (7)$$

where  $N(\cdot)$  denotes a multivariate normal density function with mean vector zero and a variance-covariance matrix  $\Sigma$ . In contrast to (6) this multinomial probit model allows random taste variations across individuals. The taste variation parameters  $\delta_i$  (see equation (3)) are drawn from a multivariate normal distribution with zero mean and a  $(K, K)$ -variance-covariance matrix  $\Sigma_{\delta}$ , whereas  $\Sigma_{\delta}$  is drawn from a multivariate normal distribution with mean vector zero and a  $(A', A')$ -variance-covariance-matrix  $\Sigma_K$ . In contrast to the multinomial logit model the probit version is, however, computationally rather intractable despite recent progress made in developing more efficient and accurate procedures such as direct numerical integration methods (see Hausman and Wise, 1978), the simulated frequency method (see Lerman and Manski, 1981) and iterative approximation procedures (see Daganzo et al., 1977).

In the generalized extreme value model the joint cumulative distribution function  $F$  is the multivariate extreme value distribution, i.e.

$$F(\varepsilon_{i.} | x_{i.}, \theta) = \exp \left[ -G((\exp(-\varepsilon_{ia}), a \in A), x_{i.}) \right] \quad (8)$$

where  $G$  is a non-negative, homogeneous-of-degree-one function that satisfies certain regularity conditions (see McFadden, 1981). Model (8) implies that the random terms  $\varepsilon_{ia}$  may be correlated across choice alternatives, though they must have equal variances for all choice options. Furthermore, random taste variations are not allowed. Thus, the generalized extreme

value model is more general than the multinomial logit model, but less than the multinomial probit model. The generalized extreme value choice probabilities which are consistent with utility maximization under (8) are given by McFadden (1981, p.228) as

$$P_{1a} = \frac{\theta}{\sum_{i \in A} \theta} \ln G(\exp x_{1j}\beta, \dots, \exp x_{1A}\beta) \quad (9)$$

which follows by direct integration from (8).

The most practical special case of this generalized extreme value model is the nested or hierarchical multinomial logit model. The nested logit model takes for granted a recursive sequential decision structure (such as e.g. a nested structure in a travel choice context marked by the trip frequency choice, the destination choice, the modal choice and finally the route choice as the lowest level, or a nested structure in a residential mobility-housing choice context marked by the mobility decision, the neighbourhood choice, the dwelling type choice and the dwelling unit decision). It consists of a sequential, recursive set of probability choice equations. This model is sequential because the choice of an alternative at a given level of the choice hierarchy is conditional upon the outcomes of higher level choices and it is recursive because the decision also depends upon the composite utilities of choice options available at lower-level choices. Inclusive values representing expectations of the outcomes of lower level choices serve as the feedback linking mechanisms. This implies that the systematic component of utility has to be additively separable. A sufficient condition for nested multinomial logit models to be consistent with random utility maximization is that the inclusive value parameters fall in the unit interval (see McFadden, 1979). Since nested models are able to provide good approximations to true simultaneous decision structures where no choice hierarchy exists, they may also be used to greatly simplify the magnitude and computations of the estimation task in a simultaneous situation, especially when the set of choice options becomes unmanageably large (see Fischer and Maier, 1986). Thus, nested logit modelling provides a convenient methodological framework for treating subsets of choices within a general choice model by relaxing the IIA assumption. Instructive examples of modelling the joint residential mobility-housing choice system can be found in Onaka and Clark (1983), and Clark and Onaka (1985).

Estimation of the parameters of the choice models mentioned above is carried out by the maximum likelihood approach where the log-likelihood function,  $L$ , is given as

$$L = \sum_{i \in A} \theta_{1a} \ln P_{1a} \quad (10)$$

where  $k_{ia}$  is equal to one if decision-maker  $i$  selects choice option  $a$ , and equal to zero otherwise.

## 2.3 Specific Problems and Recent Advances

In this subsection it is attempted to discuss some specific problems and to summarize some major advances which have been made in the following three research areas (see Fischer 1985, 1986): model representation, sampling and estimation, and model assessment.

### (1) Model representation issues

Substantial research efforts have been devoted to issues such as the definition of choice sets, the incorporation of aspects of complex and constrained choice behaviour and alternative decision structures.

Spatial choice problems evidently differ from non-spatial choice problems in the identifiability and the number of available choice options. Spatial alternatives such as alternative shopping destinations can be identified in several ways ranging from specific shops to fairly large spatial aggregates. Moreover, the usual assumption of a universal choice set faced by all decision-makers is very likely to break down; especially in the case of destination sets for non-work trips or in the case of housing choice alternatives (which may be very large in number), the size of the choice set is severely limited in practice by a variety of perceptual constraints on choice behaviour. Decision-makers act under restricted knowledge of alternatives in accordance with individual search and information spaces. In order to accommodate such aspects of constrained behaviour it is necessary to divide the set of potential alternatives into feasible and non-feasible subsets. Then choices can be modelled within these restricted subsets in accordance with the random utility methodology (see Fischer and Aufhauser, 1986).

Current discrete choice models generally assume that information about choice alternatives available to the decision-maker is exogenous and subject to systematic inaccuracies. This classical assumption, however, is unrealistic. The decision-makers' information about choice options is not only imperfect, but also depends on observations made in the course of experience and on information-gathering activities conducted in order to learn more about the choice context (see Manski, 1981). The integration of the dynamic relation between information and decision-making implies a transition from the current cross-section to a dynamic framework where effects of experience, time-discounted preferences, learning processes, habit persistence and so on are becoming central issues. In recent years, there has been an increased interest in developing panel data discrete choice models to examine dynamic aspects of choice behaviour (see section 3).

In current practice, choice models embody compensating decision rules. In such models it is assumed that decision-makers trade off attributes of

the choice options in the decision process. Only little research, however, has been conducted so far, to verify the correspondence between choice processes and the linear additive utility formulations of these compensatory models. Recent research has pointed out the notion that decision-makers do not make judgements according to strictly additive and multiplicative rules (see, e.g., Norman and Louvière, 1974). Payne (1976) presents results which indicate that when decision-makers who are faced with selecting one of many complex alternatives (such as dwelling units available in the housing market) tend to employ simple non-compensating decision rules such as elimination by aspects. Discontinuous evaluation and choice processes may be captured basically by non-compensating choice models. Such models are based on dominance, conjunctive, lexicographic, satisflex, minmax regret, elimination by aspect or related decision rules in which changes in one attribute cannot be compensated by opposite changes in other attributes (see Timmermans, 1984). Future research has to focus on information processing and evaluation mechanisms involved in choice behaviour in order to make models in a behavioural sense more realistic.

(ii) Sampling and estimation issues

The traditional sampling process in discrete choice modelling is exogenously stratified sampling. In exogenous sampling the population of decision-makers is classified on the basis of stratification criteria exogenous to the selected choice options, while next a random sample is drawn from each stratum where different strata may have different sample sizes. Under certain regularity conditions, maximum likelihood estimation from exogenous samples does not present any new problems in comparison with an estimation from random samples.

Quite recently, choice-based sampling has been suggested as an important alternative to exogenous sampling and significant progress has been achieved in developing appropriate maximum likelihood and related statistically sound estimators. In choice-based or endogenous stratified sampling the classification of the decision-makers' population into subsets is based on the observed chosen alternatives, while for each - and within each - subset the required number of decision-makers is drawn at random. Conventional maximum likelihood estimators will be inconsistent and, thus, asymptotically biased in choice-based sampling. This fact has not seldomly been overlooked in empirical applications. But in the recent past significant progress has been made in developing a variety of computationally tractable and statistically appealing choice-based sampling estimation procedures (see Manski and Lerman, 1977, Manski and McFadden, 1981, Cosslett, 1981). Manski (1981) describes three alternative approaches for obtaining statistically sound estimators. The first approach assumes that the attribute density function can be a priori restricted to a parametric family of density functions. This approach has only seldomly been applied in practice because computation is rather costly and theory does not give strong guid-

ance concerning the parametric restriction on the attribute density function. The second approach which does not involve the attribute density, leads inter alia to the 'weighted exogenous sampling maximum likelihood' estimator (see also Manski and McFadden, 1981, 17-18). Use of this estimator assumes that the proportions of the population choosing each choice alternative are known. This assumption is quite often satisfied in applications. The third approach suggested by Cosslett (1981) involves the use of joint maximum likelihood estimation of model parameters and the attribute density function.

It is worthwhile to mention that a properly designed choice-based sample may provide more precise estimators at lower costs than a random sample of the same total size. Due to the lack of information about the choices and independent variables in the population, one may be forced to consider hybrid sampling procedures in which endogenous sampling is linked with additional survey data or statistics taken from a random sample of the entire population. Maximum likelihood estimators for a series of hybrid sampling procedures are provided by Cosslett (1981).

(iii) Model performance issues

Probabilistic choice models are highly sensitive to a large number of specification errors such as misspecification of the choice set, incorrect specification of the probability distribution of the random component and incorrect functional form of the deterministic component of the utility function. Models with specification errors can cause large errors in the choice probabilities. Thus, the identification of specification errors is of central importance in spatial choice modelling.

Three types of specification tests are available. The first type includes informal specification tests for the utility functions, such as the examination of the signs, t-statistics and ratios for the estimated parameters. These procedures are routinely used to arrive at an acceptable specification of the utility functions. They, however, lack power because models with specification errors causing large errors in the choice probabilities may have parameters with the right sign, satisfactory t-statistics and ratios.

The second type of specification tests consists of formal statistical comparisons of models with different specifications and includes likelihood ratio tests, Lagrangian multiplier tests and tests of non-nested hypotheses suggested inter alia by Horowitz (1982, 1983). By means of these statistical procedures it is possible to detect violations with respect to the basic assumptions of the model itself (for example to test for violation of the IIA assumption, for the presence of taste variation in the population, for heteroscedasticity in the utility functions). These test procedures give information on specific causes of specification errors which, however, is reliable only if certain a priori alternative hypotheses with respect to the correct model are true.



The third type of procedures is based on testing the statistical significance of the differences between predictions and observations. For example, Horowitz (1984) suggests formal statistical tests for comparing predicted and observed aggregate shares in population strata and uses the fact that these differences are normally distributed in large samples in order to develop chi-square test statistics, one for the case in which the test and estimation data sets are the same and one for the case in which they are independent. Up to now, the small sample properties of the statistical specification tests, however, are largely unknown.

The static choice model representatives discussed in this section have often been criticized for their temporal stationarity assumptions which are obviously too unrealistic, especially in the case of recurrent discrete choice situations such as short-run destination choices like shopping travel (cf. Clarke et al., 1982, Wrigley and Dunn, 1984a, 1984b, Koppelman and Pas, 1985). Nevertheless they have been successful in gaining a deeper understanding of several aspects of choice behaviour. Following Ben-Akiva and De Palma (1981) a static choice model may be considered as a valid approach to analyse choice behaviour if the following two conditions are met:

- . the dynamic adjustment process has to be sufficiently fast in relation to typical time scales of changes in exogenous choice variables, and
- . the psychological and monetary transfer costs (associated with the transition from one choice alternative to another) are negligible.

### 3. DYNAMIC MODELS OF DISCRETE CHOICE

#### 3.1 Introductory Remarks: Longitudinal Data and Different Survey Designs

It has been increasingly recognized that choice behaviour is very difficult to analyse with only cross-sectional data. In the last few years human geographers and regional scientists have developed an increasing interest in longitudinal survey data. Such data provide the information base for dynamic models of discrete choice.

There are many possible longitudinal survey designs which might be used to collect information on choice behaviour over time. In particular, the panel designs provide the potential to measure different components of change in choice behaviour at the individual level. Following Wrigley (1986) four different longitudinal survey designs which are most frequently used may be distinguished:

##### (1) Repeated cross-sectional surveys

Such surveys draw an independent sample of individuals at different points in time from the same population. As a consequence they provide a representative cross-section of the population at each point in time. A major limitation of this type of surveys in the context of dynamic modell-

ing of discrete choice is the fact that the sample units are not retained from one time period to the next. There is no possibility to decompose observed change in behaviour over time into the two components: changes in population composition and changes in sample unit behaviour. Thus, dynamic models of discrete choice have to be based on panel data. The essence of panel data is information on a (more or less) fixed sample of decision-makers across time such that statements can be made about behavioural response at the individual level. The panel survey designs briefly characterised below may be used for this purpose.

(ii) Classical panel surveys

Classical panel surveys involve repeated measurements on the same individuals at different points in time. That is, in contrast to repeated cross-sectional surveys the sample units are kept in the panel. A major drawback of this type, however, is that the size of the panel is reduced over time by the process of 'panel attrition'. Especially, in the case of long-term panel surveys the panel may become unrepresentative as time proceeds.

(iii) Rotating panel surveys

Rotational panel surveys are characterised by a process of planned 'retirement' of sample units and systematic 'refreshment' by new representative sample units. In this way the problem of 'panel attrition' is circumvented, but at the price of a reduction in measuring components of change in behaviour at the individual level.

(iv) Mixed panel surveys

This type of surveys is a hybrid of the classical panel survey on the one hand and the rotating panel survey or the repeated cross-sectional survey on the other hand. The classical panel survey component is used to measure change at the individual sample unit level. The rotating panel or the cross-sectional survey component is used to check on possible biases from differential rates of attrition among subgroups in the panel.

The great potential of panel data for dynamic modelling stems from both the temporal nature of the data and the data linkage for each decision-maker. Panel data enable one to explicitly recognize the intertemporal nature of choice outcomes, especially the effect of experience on decisions. Moreover, it is expected that the use of panel data results in greater efficiency, in both statistical and behavioural terms, than the estimation of separate relationships in the case of a repeated cross-sectional sample (see Johnson and Hensher, 1982, and Coleman, 1981).

Stochastic models of buying behaviour such as brand choice models and purchase incidence models have been very successful in analysing panel

data. Brand choice models predict which choice alternative will be chosen, given that a decision is made at a particular point in time. Purchase incidence models predict how many choice alternatives will be chosen in a specified time period or when an alternative will be chosen. Most of these models have been originally developed in marketing research and have been brought to the attention of regional scientists and human geographers by Wrigley and Dunn (1984a, 1984b), Davies (1984), Halperin (1985) and Timmermans (1985). Wrigley and Dunn (1984b) successfully apply the Dirichlet model of heterogeneous buyer behaviour in the context of multistore purchasing in Cardiff. Although stochastic panel data models of buying behaviour provide a suitable framework for analysing several aspects of dynamic choice behaviour, in their current form they do not incorporate explanatory variables. Moreover, it is often claimed that a dynamic approach to analysing panel data should explicitly take into account adaptive behaviour (i.e. the effect of past experience on choice behaviour).

In contrast to the stochastic panel data models of buying behaviour, dynamic discrete choice models incorporate explanatory variables and explicitly account for dynamic effects of choice behaviour.

### 3.2 Some Fundamental Issues in Dynamic Discrete Choice Modelling

The extension of discrete choice modelling to incorporate choice behaviour over time raises several important methodological issues, such as (see also Kessler and Greenberg, 1981):

- . the question how to take structural state dependence (i.e. the dependence of current on past behaviour and of future on current behaviour) into account and
- . the question how to deal with serial correlation or spurious state dependence in the omitted (unmeasured or unmeasurable) variables which generate the choice outcome.

To disentangle the influences of structural and spurious state dependence is a difficult, but a key issue in dynamic modelling activities. There are several sources of structural state dependence. Choice outcomes may depend on previous choices (Markovian effects), on the length of time the current state has been occupied (duration-dependence effects), on previous interchoice time (lagged duration-dependence effects) and on the number of times different states have been occupied (occurrence-dependence effects) (Wrigley, 1986). Information is often not available to take all these different structural state dependence effects into account.

Omitted unmeasured or unmeasurable influences on choice behaviour, especially those which result from the censoring of the data base, are likely to introduce a serial correlation effect and a bias in the parameters of the observed variables (see Wrigley, 1986). If the degree of serial correlation in the data is unknown, previous experience may appear to influence

future experience only because it is a proxy for temporally persistent unobservables which determine choices (see Heckman, 1981a).

Some recent developments in the rapidly growing new field of dynamic discrete choice modelling will be discussed in the sequel.

### 3.3 Tardiff's Dynamic Discrete Choice Model

Tardiff (1980) was one of the first who made an attempt to extend discrete choice methodology by introducing structural state dependence effects and serial correlation in the utility functions. He regards recurrent choice as a sequence of static utility maximizing choices by decision-makers whose utility functions may have certain individual, structural and spurious state dependence effects.

Let  $t = 1, \dots, T$  denote an exogenously given sequence of time periods and assume that a decision-maker  $i$  has to choose an alternative  $a(t)$  from the set of choice options in the choice set in period  $t$ . The choice set may vary from decision-maker to decision-maker as well as over time. But then the complex issue of forecasting choice sets arises. Tardiff (1980) suggests that a useful replacement for utility function (3) in such circumstances is:

$$u_{iat} = x_{iat} \beta + \sum_a \gamma_{aa'} C_{ia'}(t-1) + \tilde{\epsilon}_{ia} + \epsilon_{iat}^* \quad (11)$$

Evidently, this utility function explicitly takes the intertemporal nature of choice processes into account.  $u_{iat}$  is the utility of alternative  $a$  for decision-maker  $i$  at time  $t$ ;  $x_{iat}$  is a vector of observed functions of (the decision-maker's and alternative) characteristics which may vary in time;  $C_{ia'}(t-1)$  is a variable with  $C_{ia'}(t-1) = 1$ , if  $i$  chooses  $a'$  in the previous period  $t-1$  and 0 otherwise. The random term is now disaggregated into two components:  $\tilde{\epsilon}_{ia}$  represents unobserved time-invariant effects (fixed effects of unobserved variables) whereas  $\epsilon_{iat}^*$  varies among decision-makers and time periods. The inclusion of the second term at the right-hand side of (11) makes it possible that the choice in one period may influence choices in the following period and thus accounts for first-order Markov effects. If the estimate of  $\gamma$  is positive (negative), it indicates an increased (decreased) choice probability in the subsequent period.

The specific form of a panel data discrete choice model depends on whether structural state dependence and/or spurious state dependence effects are present. By putting various terms in (11) equal to zero, Tardiff considers three special cases of the general data discrete choice model: models with temporal independence, structural state dependence models and spurious state dependence models. These classes will briefly be discussed now.

(i) Models with temporal independence

In these models it is assumed that  $\gamma_{aa'} = 0$  for all  $a$  and  $a'$  (i.e., no structural state dependence exists) and  $\bar{\epsilon}_{ia} = 0$  for all  $i$  and  $a$  (i.e., no spurious state dependence exists). If these assumptions are valid, then the probability of a sequence of choices is disaggregated into a product of static choice probabilities. The observations for decision-makers over time are treated as independent. The time series of choices made by a decision-maker cannot be distinguished in this case from a set of choices made by a cross-section of decision-makers at a single point in time. Thus, the static discrete choice models described in section 2 can be directly applied to this dynamic choice problem. The standard static discrete choice model is a special case when  $T = 1$  (i.e., only one time period exists).

(ii) Structural state dependence models

Case 2 assumes that  $\bar{\epsilon}_{ia} = 0$  for all  $i$  and  $a$ , but that structural state dependence effects are present. Thus, the effects of previous upon current choices are explicitly considered. Since error terms are assumed to be independent across time periods and the second term of the right-hand side of (11) is statistically predetermined, the usual discrete choice models can be directly applied. A special case results when  $x_{iat} \beta$  is zero (i.e., when the current choice is only a function of previous choices). This special case leads to transition probabilities of a first-order Markov model of spatial choice.

(iii) Spurious state dependence models

The key assumption in case 3 is the presence of (non-trivial) spurious state dependence effects and the absence of structural state dependence effects (i.e.,  $\gamma_{aa'} = 0$  for all  $a, a'$ ). Because the choices depend upon observed spurious state dependence effects, they are statistically dependent. The usual estimation procedures are no longer valid. Tardiff (1980) suggests to treat the  $\bar{\epsilon}$ -effects as fixed rather than random (the so-called fixed coefficients model approach). By adopting this approach the  $\bar{\epsilon}_{ia}$  terms are explicitly specified as alternative-specific constants for each decision-maker. Future research is necessary to investigate the statistical reliability of the fixed-effects approach in small samples.

3.4 Heckman's General Model of Dynamic Choice

In contrast to Tardiff (1980), Heckman (1981a) derives a general dynamic model for the analysis of discrete panel data which can be used to analyse the structure of discrete choices made over time from a direct consideration of the complex error variable structure (random-effects approach). The model which may be considered as a generalization of the models dis-

cussed above in several directions is based on the notion that discrete outcomes are generated by continuous variables with cross-thresholds. In applications, these continuous variables are related to well defined economic concepts. For example, in Domencich and McFadden (1975) the continuous variables producing discrete choices are differences in utilities of possible choices. The model based upon the multinomial probit formulation is sufficiently flexible to take into account time dependent explanatory variables, general spurious state dependence patterns for unmeasured attributes as well as complex structural state dependence interrelationships among decisions taken in different time periods. This model will now briefly be described for the sake of illustration.

It is assumed that from a random sample of decision-makers information on the presence or absence of an event (i.e., a discrete choice in our context) in each of  $T$  equi-spaced time periods is assembled. The key assumption of Heckman's general dynamic model is that an event for decision-maker  $i$  in time period  $t$  occurs, if and only if a continuous latent random variable  $y_{it}$  crosses a threshold. Only for convenience this threshold may be assumed to be zero. The random variable  $y_{it}$  may be disaggregated into a purely random disturbance component  $\varepsilon_{it}$  and a deterministic component  $v_{it}$ , i.e.,

$$y_{it} = v_{it} + \varepsilon_{it} \quad (12)$$

with

$$y_{it} \geq 0 \quad \text{if and only if} \quad d_{it} = 1 \quad (13)$$

and

$$y_{it} < 0 \quad \text{if and only if} \quad d_{it} = 0, \quad (14)$$

where  $d_{it}$  is a dummy variable denoting the occurrence of the event under consideration. The distribution of the  $d_{it}$ 's is generated by the distributions of the  $\varepsilon_{it}$ 's and  $v_{it}$ 's where it is assumed that  $\varepsilon_{it}$  is normally distributed with mean zero and a  $(T, T)$ -positive definite covariance matrix. This normality assumption generates a general model which is able to account for a wide variety of error structures for serially correlated unobserved variables.

Assuming that the latent variable  $y_{it}$  is a linear function of observed choice-relevant attributes  $x_{it}$ , of lagged values  $y_{it}$  and of past outcomes  $d_{it'}$  with  $t' \leq t$ , Heckman's general dynamic model may be written as

$$v_{it} = x_{it}\beta + \sum_{j=1}^{\infty} \gamma_{t-jt} d_{it-j} + \sum_{j=1}^{\infty} \lambda_{jt-j} \prod_{\ell=1}^j d_{it-\ell} + G(L) y_{it} \quad (15)$$

where  $G(0) = 0$  and  $G(L) = g_1 L + g_2 L^2 + \dots + g_K L^K$  is a general lag operator,  $L^K y_{it} = y_{it-K}$ . The initial conditions  $d_{it'}$  and  $y_{it'}$  for  $t' = 0, -1, \dots$  (in other words, the relevant presample history

of the process) are assumed to be predetermined or exogenous. This assumption, however, is only valid if the unobserved choice relevant characteristics generating the process are serially independent.

The first term at the right-hand side of (15) may incorporate past and current information and future expectations on exogenous choice-relevant attributes affecting current choices. The second term represents structural state dependence effects. In contrast to (11) the effect of the entire past history of the process on current choice is taken into account and not only the past time period. This term is assumed to be finite. The coefficients for past events (i.e.,  $\gamma_{t-jt}$ ) are considered to be functions of the current time period  $t$  and the time period  $t-j$  in which the event occurred. The third term denotes the cumulative effect on current choices of the most recent continuous experience in a state. It is assumed to be finite. The  $\lambda$ 's denote parameters. Finally, the last term in (15) representing the effect of previous relative evaluations of the two states on current choices captures the action of habit persistence.

Heckman (1981a) shows that (15) is sufficiently flexible to accommodate time-varying exogenous explanatory variables, unobserved variables with a general serial correlation structure (i.e., heterogeneity in the population which has an unmeasurable influence on the choices made) and complex structural interrelationships among decisions taken at different times. Imposing various restrictions on the parameters of the general model, a variety of models such as Markov models, renewal processes, Bernoulli models and Polya schemes emerge as special cases.

### 3.5 Other Dynamic Discrete Choice Approaches

Similar approaches to dynamic discrete choice modelling have been suggested by other researchers. Daganzo and Sheffi (1982) analyse the use of the multinomial probit model approach to panel data and show that the choice of a structural state dependence model, a serial correlation model or any hybrid thereof is simply a specification issue that should be decided by the modeller. The computational complexity of the estimation process increases with the product of the number of alternatives and time periods which can be handled. They also discuss the initial conditions problem that arises in estimating a discrete time-discrete data stochastic process in a situation where serial correlation in the unobservables and structural state dependence in the process are in evidence (see Heckman 1981b). In Daganzo and Sheffi's approach the tricky initial conditions problem is circumvented because choices do not enter explicitly the structural state dependence specification. An application of Daganzo and Sheffi's approach to a binary two-period choice situation can be found in Johnson and Hensher (1982). Computational restrictions on multinomial probit estimation limit

the application of the approach to large-scale discrete choice problems.

Krishnan and Beckman (1979) have developed a dynamic model as an extension of a static logit model for binary choices which is able to capture also preference indifference. The dynamic version of the logit model suggested by Sonis (1984) replaces the principle of utility maximization by a somewhat more realistic choice principle. A decision-maker does not choose the alternative on the basis of a comparison of utilities, but on the basis of a comparison of the temporal marginal utilities which may be interpreted as the expectations of a gain in the future. Thus, his dynamic version is based on accounting for dynamic marginal utilities and, moreover, on the incorporation of a joint influence of interaction and imitation processes between adopters of different types of alternatives as well as on the introduction of the intervention of an active environment which changes the accessibility to choice options for the adopters.

Discrete choice models usually deal with the choices of a single decision-maker defined as a household or individual. There are many household decisions which result from interactions among household members. De Palma and Lefevre (1983) have developed a dynamic extension of the multinomial logit model which allows for such interactions. The model is formulated as an interactive continuous-time Markov process. Similar in spirit is Leonardi's (1981, 1983) work. His time-nested random utility approach introduces a new way of looking at the dynamics of the evaluation process, relating them to the formation of expectations over future. Alternative and partly complementary contributions to simultaneous multi-actor choice problems can be found in Margolis (1980) as well as in Miyao and Shapiro (1981).

Much progress has been made in the field of dynamic discrete choice modelling in the last few years. But unquestionably, there are several problems which are not yet satisfactorily solved up to now, such as, e.g., the problem of attrition bias effects as well as the problem of initial conditions. Future research activities should be also directed to relax the unrealistic assumption that the choice set is fixed over time. A relaxation of this assumption implies the complex issue of forecasting choice sets.

#### 4. OUTLOOK: ALTERNATIVE APPROACHES TO MODELLING THE DYNAMICS OF SPATIAL CHOICE

Recently, it has been shown (Nijkamp and Reggiani, 1986) that discrete choice models can consistently be interpreted in the context of spatial interaction models and Alonso's general theory of movement, in both a static and a dynamic sense: (dynamic) discrete choice models have strong roots in (dynamic) generalized spatial interaction analysis (see also Anas, 1983).

Recently, the random utility maximizing principle which is used in



spatial discrete choice modelling as a principle of spatial human behaviour has increasingly come under attack. Burnett and Hanson (1982), for example, argue that the assumption that intra-urban travel is the outcome of a rational decision-making process, even with limited information, seems to be dubious since increasing evidence indicates that travel is a stable daily routine, also a constrained choice for most, but most likely a deep-seated avoidance behaviour for many, too. The human activity constraint time budget approach to understanding spatial choice behaviour emphasizes especially the need to view choice decisions within a broader context and to adopt a more realistic and more complex conceptualization. Recent advances in measuring activity-travel patterns and in exploring constraints and their effects on the set of choice options within which travel decisions are made are discussed in Burnett and Hanson (1982). This conceptually appealing approach is useful to provide deeper insights into the derived nature of travel and the structure of multipurpose and multistop trips. It emphasizes the way individuals form and reinforce behavioural patterns of travel and other activities in space and time. Spatial and temporal constraints as well as interactions between household members deserve an explicit and more comprehensive treatment. This approach, however, is essentially descriptive rather than explanatory and predictive. Nevertheless, it might be fruitful to integrate basic ideas of this approach into random utility discrete choice models. An earlier interesting attempt in this direction was undertaken by Hensher (1976).

In contrast to the dynamic discrete choice models, the so called heuristic choice modelling approach explicitly attempts to replicate individual decision-making processes. Heuristic choice modelling introduced in geography and regional science by Smith and his colleagues (see Smith et al., 1982, Smith and Lundberg, 1984) adopts the viewpoint that decision-making is a concurrent and heuristically-guided search of a physical space and its mental representation. Quite recently, advances in computer science and cognitive psychology have provided the potential to construct computational process models in order to represent complex choice behaviour of decision-makers with constrained computational capacity (see Smith, et al., 1982). Such models are important in the case of complex choice problems in which exhaustive research is infeasible. Consequently, the decision-maker's memory and his perception of the choice context are major determinants for individual heuristic choice analysis. Computational process models of decision-making behaviour may be constructed in such a way that (a) they incorporate the view that choice behaviour presupposes a joint, heuristically-guided search of a physical space and its mental representation, (b) they enable to take into account individual variability in both mental representation and the related decision making behaviour, (c) they provide potential predictions of individual choices in relatively complex choice contexts, and (d) they are able to generate macro choice

models from individual choice models. The computational process modelling approach may imply a radical departure from models of rational decision-making, as they are based on pattern-matching methods, similarity analysis and context-dependent modular design.

A third interesting development in the area of dynamic discrete choice modelling can be found in the so-called master-equation approach advocated among others by Weidlich and Haag (1983). A master equation describes the evolution of the probability distribution function, representing the transition probabilities for well defined states of a dynamic micro-based spatial system of actors. By using, for instance, a mean value approach an elegant link can be established between micro levels and macro levels of a system, so that structural changes in dynamic systems can be analyzed in a statistically satisfactory way.

The use of master equations has two important advantages. In the first place, it allows to take account of synergetic effects in the behaviour of different individuals (social adaptation processes, congestion, learning effects etc). The socio-configuration includes then the individual transition probabilities based on joint interaction effects. A second major advantage of the master equation approach is that it allows in principle to include micro utility elements (based, e.g., on dynamic discrete choice models of, for instance, the logit type) in the probability distribution for individual choice. Consequently, feedback elements (state dependence, e.g.), heterogeneity (variation between individuals) and non-stationarity (variation over time) can, thus, be taken care of. In this way, one can integrate a solid use of statistics with the use of dynamic discrete choice models.

In conclusion, the area of modelling the dynamics of spatial choice offers a rich potential for path-finding analysis in the field of individual dynamic spatial behaviour. Random utility based discrete choice models have now indeed reached a stage of development in which they offer a flexible tool for analysing a wide range of spatial static and dynamic choice problems. In particular, dynamic disaggregate models of choice appear to gain a great deal of interest, although various severe problems (such as the problem of stationarity, the problem of initial conditions, the problem of attrition bias effects, the problem of transferability of the results over space and time etc.) are not yet satisfactorily solved.

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