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HOUSING MARKET OF AMSTERDAM

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Abstract

In this paper it is shown how households dynamics can be incorporated into operational dynamic urban models.

First, some dynamic model approaches which can be used for modeling urban change are being discussed. It is concluded that in combining these different approaches one needs an "accounting framework" which links the various sub models in a dynamic urban economy in a consistent way.

Next, an "accounting framework" of a dynamic model for Amsterdam is presented. The household sub model is discussed in detail and some preliminary results are given.

1. Introduction

In recent years dynamic spatial models have increasingly come to the fore in urban and regional research (see among others, Andersson and Kuenne, 1986, Bahrenberg et al., 1984, Dendrinis and Mullally, 1985, Fischer and Nijkamp, 1986, Haag and Weidlich, 1984, and Turner, 1980). Such modeling efforts aim at replicating a complex, multidimensional and nested spatial structure, composed of various subsystems (housing, transportation etc.). If such subsystems in a spatial system are intertwined in a non-linear dynamic way (with sometimes even differences in their successive rates of change), a large spectrum of switches in the evolutionary patterns of a spatial system may take place. Clearly, a major problem is then how to model the endogenous dynamics of such complex spatial systems, such that various trajectories of these systems can be generated and replicated by such a model.

Models which have gained a high degree of popularity in this context are inter alia: differential and difference equations, calculus of variation, optimal control theory, dynamic programming, Markov theory, and singularity and bifurcation theory. The choice of a particular model depends on the research objectives and available data. In the present paper the attention will be focussed on an operational dynamic model for the city of Amsterdam, with a particular emphasis on the dynamic behaviour of households in the housing market.

The paper is organized as follows. In section 2 some background remarks on dynamic urban models will be given. Next, section 3 is devoted to the design of a dynamic accounting framework for household on the urban housing market, while section 4 treats the integrated household migration model for the housing market of Amsterdam. Section 5 will present a set of empirical results based on the Amsterdam model. The paper ends with some final remarks in section 6.

2. Dynamic Urban Modeling: Background Remarks

In the field of dynamic urban models, various research directions have been chosen in the recent past. Some examples are:

- dynamic lowry models (cf. Wilson, 1981a, and Harris and Wilson, 1978); these approaches have been applied inter alia in the field of urban shopping and facilities modeling.
- master-equation models (cf. Haag, 1985a, 1985b, 1985c, and Haag and Weidlich, 1984); such modeling efforts can inter alia be found in the area of dynamic migration analysis.
- urban ecology models (cf. Dendrinis and Mullally, 1981, 1983); these models serve to describe inter alia structural changes in urban evolutionary patterns.
- micro simulation (cf. Whithed and Sarley, 1974, Law and Kelton, 1982,

Wegener, 1981, 1983, Wilson, 1981b, and Clarke and Williams, 1985); these modeling efforts have inter alia been used in urban residential choice analysis.

- dynamic discrete choice models (cf. Davies, 1984, Davies and Croughley, 1984, Dunn and Wrigley, 1985, Fischer and Nijkamp, 1986, Heckman, 1981, and Hensher and Wrigley, 1984); such models have often used panel or longitudinal data to analyse the dynamics of disaggregate discrete choices of among others households and firms regarding residential or locational behaviour.

These model approaches have been developed from different viewpoints, by different persons, in different countries. However, they show also remarkable similarities. The resemblance between entropy (and information-theoretic) methods, such as used in the Lowry-models, and discrete choice models is well-known (see e.g. Nijkamp and Reggiani, 1986). Ecological dynamic models can also be rewritten in the form of dynamic Lowry-models, if there are spatial interactions between different points of origin as a result of competition between all zones of the spatial system at hand. In addition, discrete choice models can easily be incorporated in a micro simulation context. Similarities also exist between the master-equation, the Lowry-model and micro simulation. The various models can apparently be used next to or in combination with each other. This offers a large flexibility in building an integrated model with mutually linked submodels. Different subsystems need different submodels, according to the desired aggregation level, the level of detail of the available data, etc. This requires a consistent design of an integrated dynamic urban model. A consistent linkage between the submodels/subsystems of an urban system can be established with the help of an 'accounting framework' (Wilson, 1981b, Clarke and Wilson, 1985). Such an accounting framework will be one of the pivotal elements of our paper. The accounting framework developed for the household and housing market model for Amsterdam will now be described in the next section.

3. An Accounting Framework for Households and the Housing Market

An accounting system is a classification of elements in organized series (vectors, matrices). It is sometimes argued that an accounting system is nothing more or less than double-entry book-keeping (see Stone, 1971). Input-output analysis is one of the earliest examples of such a system. The main advantage of an accounting system is its coherent and consistent representation. In our household and housing market analysis we will also use such an accounting framework. Figure 1 gives the accounting framework for the Household and Housing market submodels of a more comprehensive model for Amsterdam.

The rectangles represent the state matrices at time t , the rhombs represent

the transition matrices, which are calibrated in the various submodels. In previous publications (see Van Wissen et al., 1985) we already described the contents of the various submodels. In this paper attention is focussed on the Household submodels in which changes with regard to demography (ageing, death, birth) and household membership are estimated. In section 4 the construction of the transition matrix of households is described in more detail.

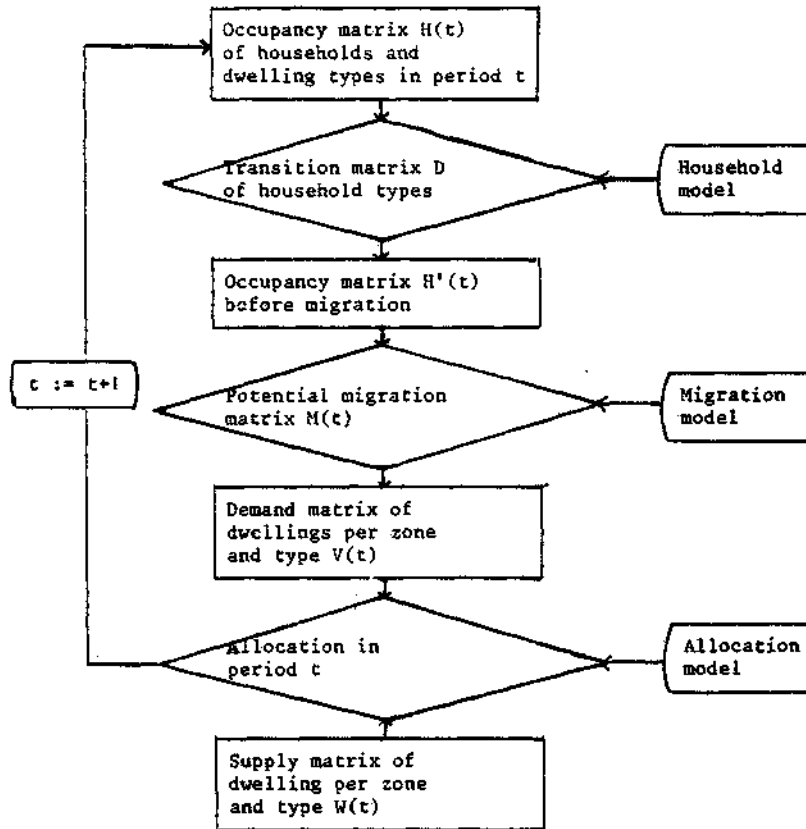


Figure 1. Accounting framework for the household and housing market submodels.

4. The Household Submodel

4.1. Introduction

One of the main modeling efforts in the Household submodel is the estimation of the transition matrix of households. In this respect two directions can be envisaged: (a) the modeling of individual transition probabilities from household membership g in time t to g' in time $t+1$, and (b) the modeling of household transitions. Both approaches appear to have advantages and disadvantages (cf. Keilman, 1984a). Focussing attention on the individual level avoids the definitional problems involved in household modeling. Moreover, in general data are often only available at an individual

level (death, birth, marital status, household membership, etc.). There is however the problem of consistency (Keilman, 1984b): if the model predicts for instance the transition of x heads from 2-person household to 3-person household members. Also, in many applications the individual outcomes have to be evaluated at the household level (e.g., the consequences of the outcomes for the housing needs).

A problem that arises in modeling the transition patterns of households directly is the complexity of the household formation process. Households can split and rejoin in various ways, and the total number of households may change as a result of these processes.

Most household models take for granted the individual level (Harsman and Marskjö, 1977, Gordijn and Heida, 1985, Möller, 1982) with the exception of Webber (1983). If one wants to combine advantages from both sides, it seems a good strategy to model individual transitions and aggregate them next to the household level (see also Willekens, 1984, p. 19). This implies a macro-level orientation together with a detailed break down in household categories (see also Harsman and Snickars, 1983).

In next subsections we will describe a simple method for constructing a household transition table. Central elements in this model are the assessment of individual transitions by using hybrid loglinear models, the household composition matrix that links the individual age distribution to the household composition (cf. Akkerman, 1980, 1982, 1985), and dependence of transitions on basic demographic events. First, the general form of the household transition matrix will be explained in the next subsection.

4.2. The household transition matrix

Consider an equation of the form:

$$\bar{g}(t+1) = D \bar{g}(t)$$

This difference equation describes the development of the household distribution \bar{g} in discrete time. The turnover matrix D describes the rate at which households of a type g' in $(t+1)$ are formed a household of type g in time t . By showing how these rates are being constructed the interpretation of D is rather straightforward.

Let $T = \{t_{g',g}\}$ be a flow matrix of individuals moving from a type g household in time t to a type g' -household in time $t+1$. Let s_g be the average number of persons in a household type g . We now assume that every member of a type g' household at time $t+1$ brings in a proportional part of that household, i.e., a share equal to $1/s_{g'}$. Thus, if we weigh every individual with his share of the new household, we obtain a "household"-flow matrix (although there is no direct micro interpretation for these numbers). To obtain a matrix of rates, we divide each number of households

g^0) in type g at time t .

Summarising this, we obtain:

$$d_{g'g} = t_{g'g} / (s_{g'} g^0_g) \quad (6)$$

It is clear from (6) that summation of d over g' does not add up to 1. This represents the effect of household formation and dissolution on the total number of households. If this household transition matrix is embedded within an overall accounting framework, we can easily keep track of the number of lost and gained households.

Having defined now the transition matrix, we turn to estimation methods in the next subsection.

4.3. The estimation of transition rates

As mentioned before, a good starting point for estimation is the individual flow matrix T . We assume that changes in household status are conditional upon age and current household status. A basic requirement of the model is consistency with basic demographic forecasts on birth's and death's. Thus we are interested in the number of people in age group k and household category g at time t , who are in age group k' and household category g' at time $t+1$: $n_{g'g k'k}$. Summation over k and k' result in the table $\{t_{g'g}\}$, and summation over g and g' gives a matrix that is closely related to the well known Leslie-matrix in demography (see e.g. Keyfitz, 1977). This matrix contains the number of survivors from age category k into $k+1$, and the number of birth's by age of the adults.

A third subtable is the so-called household composition table $A(t) = \{a_{kg}\}$, that is the cross-classification of persons by age and household category. The household composition table is used in household demography for forecasting household distributions (see Akkerman, 1982, 1985). Equivalent, the household composition table at time $t+1$ is $\{a_{k'g'}\}$. Using log-linear terminology, the four-way table $n_{g'g k'k}$ can be decomposed into subtables of lower order.

The saturated log-linear model that describes the table exactly is given in the equation:

$$\begin{aligned} \log(n_{g'g k'k}) = & \mu + \mu^{(1)}_k + \mu^{(2)}_{k'} + \mu^{(3)}_g + \mu^{(4)}_{g'} \\ & + \mu^{(5)}_{k'k} + \mu^{(6)}_{g'g} + \mu^{(7)}_{gk} + \mu^{(8)}_{g'k'} + \mu^{(9)}_{gk'} + \\ & \mu^{(10)}_{g'k} + \mu^{(11)}_{gk'k} + \mu^{(12)}_{g'gk} + \mu^{(13)}_{g'gk'} + \\ & \mu^{(14)}_{g'k'k} + \mu^{(15)}_{g'gk'k} \end{aligned} \quad (1)$$

From standard log-linear theory (see Birch, 1963) it is known that for every parameter μ a sufficient statistic for estimation is the corresponding marginal total. For example, the sufficient statistics for the parameters $\mu^{(7)}_{gk}$ is the household composition table $A(t)$. The argument can be reversed however: given certain (partial) information on marginal frequencies of a multiway table, what log-linear model can be estimated that uses this information?

The saturated model (1) has as many parameters as cells in the table. In general however, the data have a less complex structure, and certain interaction parameters can be fixed at zero.

A closer examination of the model (1) reveals that a number of simplifications can be made:

1. The survival-parameters $\mu^{(5)}_{k'k}$ reduce to $\mu^{(5)}_{k+1,k}$, because survival is only possible to the next age cohort (assuming for the moment that the prediction interval is equal to the age-interval). In addition to the subdiagonal elements $\mu_{k+1,k}$ we have a number of fertility parameters $\mu^{(5)}_{0k}$, corresponding to the number of children born per fertile age cohort k . The remaining parameters $\mu_{k'k}$ are zero by definition.

2. Given the almost linear relation between age distribution at time t and $t+1$, we can assume that any significant interaction involving age (t) and some other term implies conditional independence between age ($t+1$) and this term. This leads to a number of simplifications in the model:

$$\begin{aligned} \mu^{(9)}_{gk'} &= \mu^{(10)}_{g'k} = \mu^{(11)}_{gk'k} = \mu^{(13)}_{g'gk'} = \mu^{(14)}_{g'k'k} \\ &= \mu^{(15)}_{g'gk'k} = 0 \quad (\text{since } \mu^{(7)}_{gk} \neq 0 \text{ and } \mu^{(12)}_{g'gk} \neq 0) \end{aligned}$$

Inspection of the remaining parameters shows that the remaining parameters can be split in a subset with known marginal frequencies, viz:

$$\mu^{(1)}_k, \mu^{(2)}_{k'}, \mu^{(3)}_g, \mu^{(5)}_{k'k} \text{ and } \mu^{(7)}_{gk}, \text{ and a subset of unknown parameters}$$

that relate to the true household transitions for which there is no census information.

In order to estimate these parameters we use a table of prior values from a longitudinal survey on household development, and the so-called incidence matrix, with entries 1 (if a household transition is logically possible) and 0 (if a household transition is logically impossible).

Prior values and structural zero's can be incorporated in loglinear models, leading to so-called hybrid incomplete loglinear models (see e.g. Willekens, 1980, 1985, and Fienberg, 1972).

If we denote the prior values as $m^*_{g'gk'k}$ the resulting model is:

$$\log(n_{g'gk'k}) = \mu + \mu^{(1)}_k + \mu^{(2)}_{k'} + \mu^{(3)}_g + \mu^{(5)}_{k'k} + \mu^{(7)}_{gk} + \log(m^*_{g'gk'k}) \quad (2)$$

It can be shown (Willekens, 1980) that any interaction term in the prior table $\{m^*_{g'gk'k}\}$ that is not estimated using known marginal frequencies (i.e. any interaction term other than $\mu^{(5)}_{k'k}$ or $\mu^{(7)}_{gk}$) is preserved in the resulting table of expected cell frequencies $\{\hat{n}\}$.

So, the model can be interpreted as a multiproportional adjustment problem, and is also consistent with minimum information gain principles (see Snickars and Weibull, 1977).

The final outcome of this model is an individual flow matrix T that can be transformed into a household transition matrix D using formula (6).

The model described is related to the data-availability in the Amsterdam situation, but can easily be adjusted to other situations. For example, some information may be obtainable on the household distribution at time $t+1$, or even the total household composition matrix can be fixed in advance.

Once a reliable estimate of the transition table is available, the consequences for the housing demand can be estimated, as depicted in figure 1.

The final part of this research note is devoted to an illustrative application of the household model in the Amsterdam context.

5. An empirical application of the Household model in the city of Amsterdam

Before presenting some results it should be stressed that the household model developed so far only predicts changes in household structure of the population cohort within the city of Amsterdam in 1971. The predicted household structure in 1981 is not a prediction of the Amsterdam household structure. Many households will have outmigrated since 1971, while others have entered the city. Thus, differences between predicted and actual household structure are both the result of differential migration processes and shortcomings in the model (see below).

The basic data for the model were taken from the 1971 national census data (CBS, 1971). For the survival and fertility rates the Amsterdam statistical office figures were taken (Yearbook Amsterdam, 1971). The 11 household categories can be seen from figure 2. Prior information was available on transitions between households by size. In addition to the assumption made in section 4.3, it was assumed that the relation between age and household composition remained constant, i.e. $\mu_{g'k'} = \mu_{gk}$.

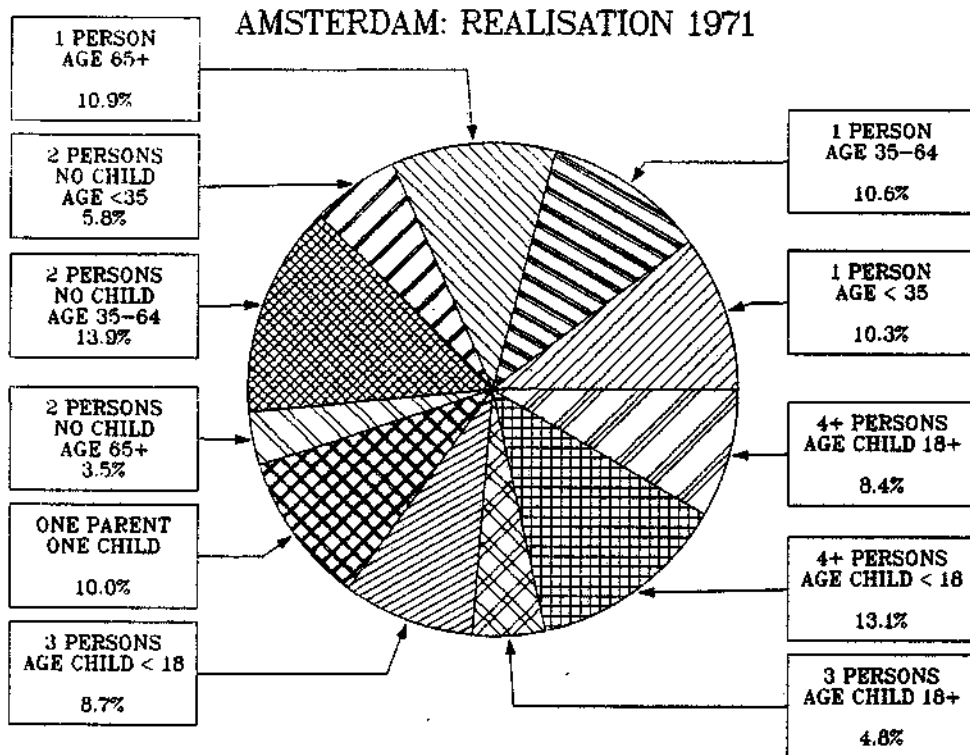


Figure 2. Observed household distribution in 1971 for Amsterdam

In figure 2 the household distribution of 1971 is visualised. The total number of households is 323,500. Two-and more -person households without children with heads in the age group 35-64 form the largest household category, followed by households with four and more children under 18 years old (13.8 and 13.1 percent respectively). Small household categories are one parent-households (3.5 percent), young (under 35) two-and more person households (5.8 percent), and three-person households with one child over 18 years old (4.8 percent).

The projection of households for the year 1981 leads to the pattern depicted in figure 3a.

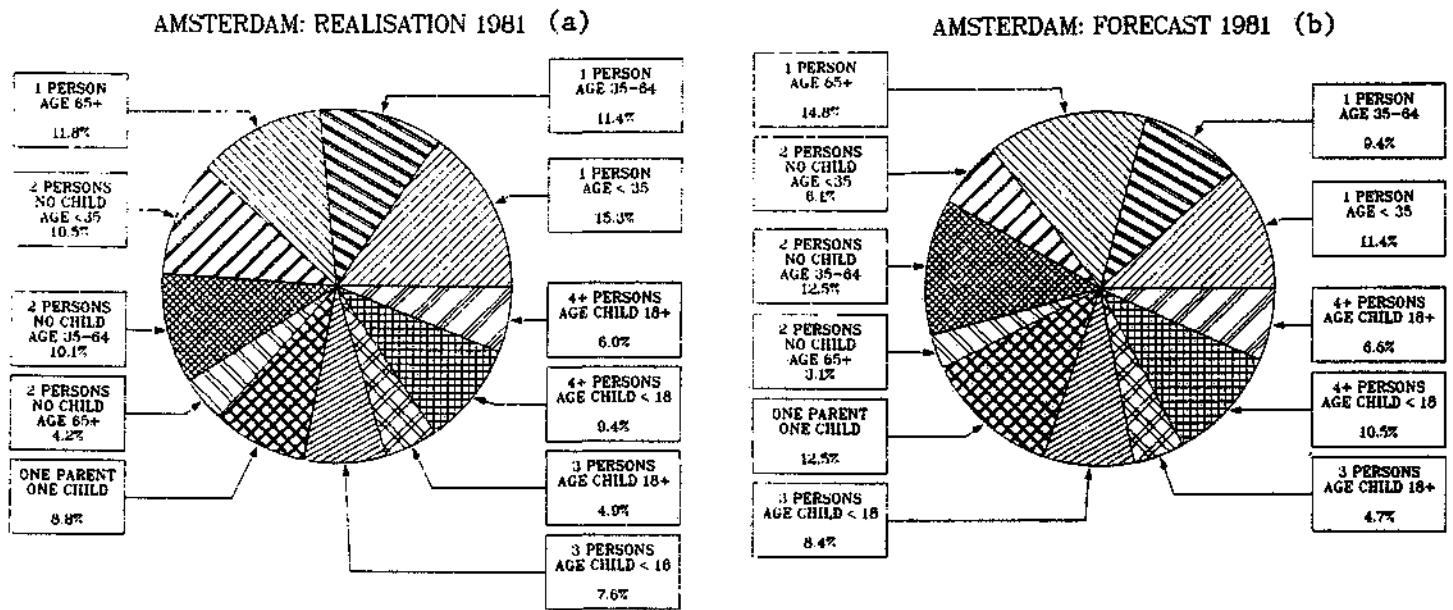


Figure 3. Projected (a) and observed (b) household distribution in 1981 for Amsterdam

Compared with the distribution of 1971 the following trends emerge:

- . sharp increase in the number of retired households, particularly in the number of one person retired households (from 10.9 to 14.9 percent)
- . slight increase in the number of young singles (from 10.3 to 11.4 percent) and young peoples under 35 without children (from 5.8 to 6.1 percent)
- . decrease in the number of large families (four and more persons)

These shifts are similar to national trends towards smaller households.

The observed trend towards retired households can be observed at a national level too, but in absolute terms this is much more important in Amsterdam due to the specific population age distribution.

As pointed out, the present model takes no account of transitions from households to non-households. A part of the predicted retired households will be in homes for aged persons.

If the predicted distribution is compared with the observed figures, the combined impacts of migration processes and structural changes in household composition become clear. The observed household distribution in Amsterdam shows that the number of young households without children (both single and multi-person households) is much higher than predicted on the basis of the 1971 situation. The observed percentage of singles under 35 is 15.3, while the predicted percentage is only 11.4.

The number of retirees is heavily overpredicted, but this is due to the fact that the model assumes there is no transition of persons from house-

holds to non-households.

Other differences are an even lower number of large households than predicted, and more one-parent households. These differences are consistent with the known impact of migration on household distribution (families with children leave the city and settle in the suburbs). Continuation of the predicted trend leads to a stable household distribution (fig. 4).

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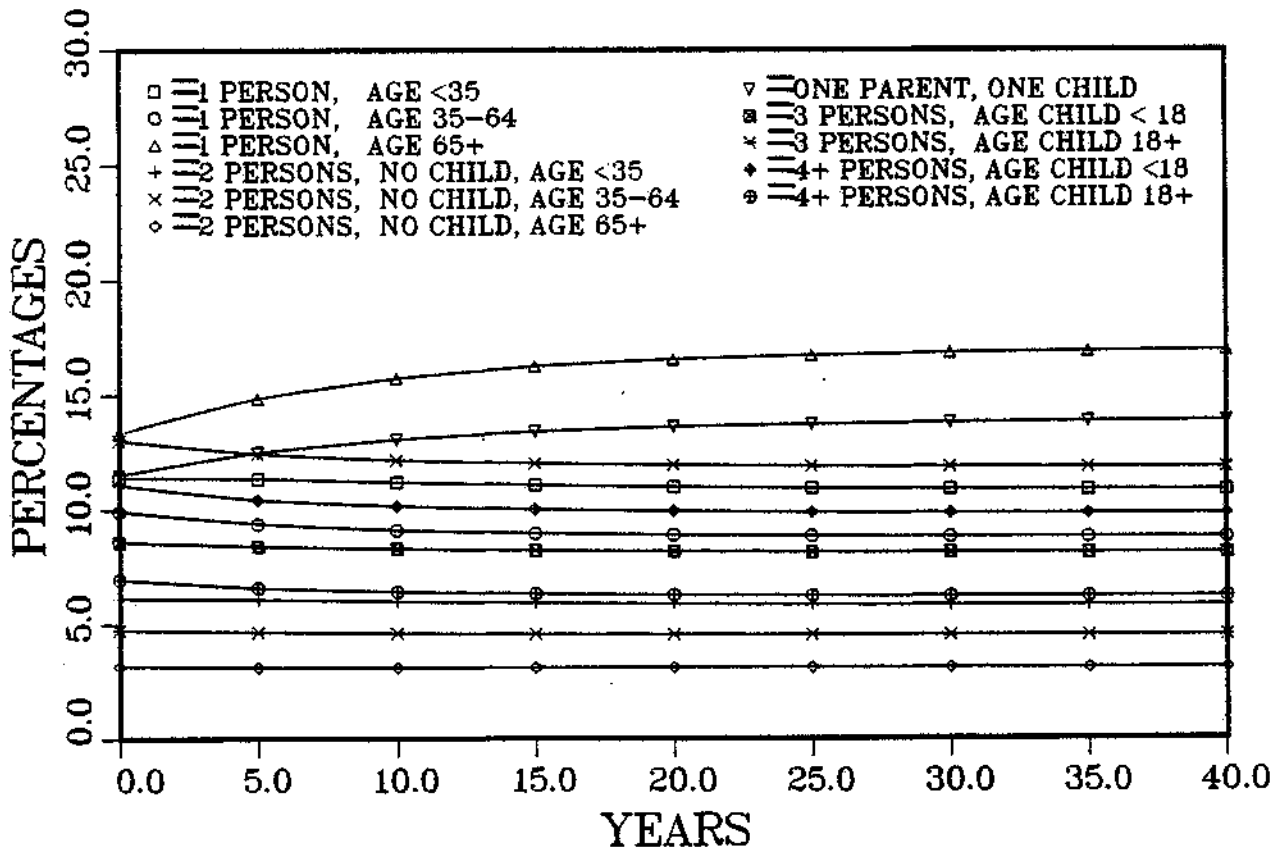


Figure 4. Relative shares of the 11 household types in Amsterdam without migration (year 0 = 1971).

6. Final Remarks

This paper has elaborated the general structure of the household model, and the linkages of life cycle dynamics and migration processes. Furthermore, some results were presented using a simple version of the household model assuming a stable household composition and non-stationary population. The model employed here will in the next stage of an analyses be extended toward a detailed zone-specific household model, so that next we will be able to connect these processes the housing situation. This intermediate stage will then finally allow the estimation of the total demand of housing and the confrontation of demand and supply.

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