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AS A TOOL IN POLICY MODELING

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MIXED QUALITATIVE CALCULUS AS A TOOL IN POLICY MODELING.

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ABSTRACT

Qualitative calculus of policy models is an appropriate method for policy impact analysis in case of imprecise information concerning the structural model parameters. The conditions of the so called sign-solvability analysis of a linear equation system - with information represented by a positive (+), negative (-) or zero (0) impact - are strict, however.
In this paper, the relevance of qualitative calculus for urban policy modelling is discussed.
An extension of the sign-solvability approach (with purely qualitative information) will be dealt with in case of a mixture of qualitative and quantitative information. We will introduce the use of matrix decomposition methods, of theoretically plausible parameter restrictions and of a top-down/ bottom-up approach for sign-solvability. The sign-solvability approach is applied to a dynamic policy simulation model of urban decline in the Netherlands, developed for the city of The Hague. Given the insufficiently reliable data base for estimating the model in a conventional econometric way, qualitative calculus was used in order to infer conclusions regarding the direction of impacts of policy variables.

1. INTRODUCTION

In recent years qualitative calculus has become an increasingly important tool in policy impact analysis. The analysis of qualitative relations in economic models has been originated by Samuelson in 1947 in order to examine - in a comparative static context - the effect on an equilibrium situation due to changes in one or more of the exogenous variables (policy controls, e.g.). This analysis, usually called qualitative calculus, deals with policy impact models in which the relations between variables are analysed in a qualitative way, i.e. when information on the sign of the impact on some response variable is obtained from prior knowledge concerning the signs of the structural parameters in a model. In other words, only qualitative information about the directional relationships between variables in a model - represented by a positive (+), negative (-) or zero (0) impact - is used without quantitative information about the values of the variables. A zero impact denotes then absence of a prior theoretical relationship between a pair of variables. There are at least three main reasons for the development of qualitative calculus and its treatment of qualitative information in economic policy analysis.

- "Ordinarily, the economist is not in possession of exact quantitative knowledge of the partial derivatives of his equilibrium conditions" (Samuelson, 1947, p. 26), because of the limited amount of suitable quantitative data.
- The qualitative information about the various impacts may have a more solid empirical basis than the functional model structure (or model specification) (see Lancaster, 1962).
- There may be difficulties in empirical practice to obtain precise or exactly quantified information because of measurement problems to get the high-level information, lack of time or simply lack of money to collect data (see also Nijkamp et al., 1985).

The main developments in the field of qualitative calculus took place in mathematics (see among others, Greenberg and Maybee, 1981; Maybee, 1980; Maybee and Quirk, 1969), economics (see among others, Lady, 1983; Lancaster, 1962; Ritschard, 1983) and ecology (Jeffries, 1974).

In many situations, the data base for policy impact analysis is unsatisfactory in order to estimate the impact parameters in an adequate way (for instance, due to lack of appropriate time-series). Especially in case of dynamic behaviour of a complex system various fluctuations may occur, which reflect sometimes asymmetric behaviour during different time phases (for instance, upswing and downswing phases in situations with structural changes). This implies that conventional econometric techniques are not always appropriate tools, as they usually assume stable behaviour of parameters. In order to deal with lack of reliable time series, often simulation models are used in order to analyse various trajectories of a system (for instance, as a response to a policy stimulus). An evident disadvantage of this approach is the lack of falsifiability of the parameter values.

An alternative way of dealing with a weak database is qualitative calculus. Qualitative methodological tools may be used to 'quantify' the impact of policy instruments in terms of positive, negative or zero impacts. For example, the impacts variables on like demand for consumption goods and entrepreneurial atractiveness caused by a change in the supply of intermediate services and in the supply of infrastructural facilities may be denoted in qualitative terms by means of positive, negative or zero changes.

The first aim of the present paper is to provide a concise introduction to sign-solvability analysis of a model which is represented by a structural relationship $h(x,y)= 0$ (see section 2). The structural relations encompass a vector y of endogenous variables and a vector x of exogenous variables. The information about the model (called the qualitative structure of the model) consists of two elements, viz.:

- the causal structure determined by the first order derivatives of $h(x,y)= 0$ with respect to variables x and y.
- the sign of the non-zero elements of the matrix of first order derivatives.

Consider now a set of linear equations $Ax+b = 0$. Both matrix A and vector b contain qualitative information with cell-elements +,- or 0. The set of equations can be solved for the unknown vector x, given matrix A and vector b. This system is called full sign-solvable when the cell-entries of the solution $x = -A^{-1}b$ (with cell-elements +, - or 0) are defined in a unique way. In the past years necessary and sufficient conditions for full sign-solvability have been formulated in the literature (see, for example, Bassett et al., 1968).

These conditions can also be interpreted in a graph theoretical way by means of signed directed graphs (or, shortly digraphs) (see for example, Maybee and Greenberg, 1969). The reason to use a graph representation of the impacts between variables is that the conditions of sign-solvability make use of graph-theoretic tools.

The sign solvability approach can be interpreted as a kind of overall sensitivity analysis as follows. If it is possible that the system can be solved for x in a unique way with a vector of signs as the solution, this solution will hold for all possible cardinal values of the matrix A and vector b up to their signs.

One of the major problems in practical applications with purely qualitative information is caused by the severe restrictions inherent in identifying unambiguous solutions for the full sign-solvability problem. In this regard, additional information measured on an ordinal or cardial scale may lead to partial sign-solvable systems which are otherwise not solvable in a purely qualitative sense. Therefore, the second aim of the paper is an analysis of sign-solvability with mixed levels of information. This approach makes use of a mixture of qualitative and quantitative information. Various matrix decomposition and permutation methods have recently been developed in the context of large and complex economic systems (see for a discussion, Greenberg and Maybee, 1981). It will be shown that the approach with matrix permutations is also fruitful for problems with mixed information. An extension of the sign-solvability analysis with purely qualitative information is presented in section 3 and is based on four points:

(i) a matrix décomposition and permutation procedure for sign-solvability analysis.

(ii) the use of a mixture of qualitative and quantitative information by means of a so called top-down and bottom-up approach.

(iii) the use of plausible parameter restrictions for sign-solvability analysis in policy modelling.

(iv) some recently developed computer programs for sign-solvability analysis.

An application of sign-solvability analysis is presented in section 4 for a dynamic urban planning model developed for the city of The Hague in the Netherlands.

2. SIGN-SOLVABILITY ANALYSIS

Sign-solvability analysis may be regarded as a specific form of a qualitative impactanalysis in policy modelling.

Consider a general economic impact model represented by n structural relations $h(x,y)=0$ with y a vector of endogenous variables and x a vector of exogenous variables (including policy instruments). The structural relations may be static or dynamic in nature with either linear or non-linear components. The analysis of sign-solvability means the identification of changes in endogenous variables in a qualitative way (denoted by +, -, 0) due to changes in exogenous variables. This can in principle be obtained by totally differentiating the above mentioned system:

$$\frac{\partial h}{\partial x}\, dx + \frac{\partial h}{\partial y}\, dy = 0 \qquad (1)$$

or, equivalently:

$$\frac{dy}{dx} = - \frac{\partial h/\partial x}{\partial h/\partial y} \qquad (2)$$

The right-hand side of formula (2) consists of two parts, viz. the inverse of $\partial h/\partial y$ and this inverse multiplied with $\partial h/\partial x$. When both parts are uniquely determined in a qualitative way, formula (2) is called sign-solvable.

A set of linear equations, as a specific example of the above mentioned general form of structural relations, can be written in matrix notation as:

$$Ax = -b \qquad (3)$$

with A a matrix of order nxn with elements a_{ij} (i,j=1,..,n) while both x and b are vectors of order nx1. The solution of (3) is given by:

$$x = -A^{-1}b, \qquad (4)$$

provided matrix A is non-singular.

Assume matrix A and vector b contain qualitative information concerning the signs of the cell-entries. Equation (3) is called full sign-solvable if the signs of the elements of vector x in (4) can be identified in a unique way, and it may be interpreted in a way analogous to (2).

Full sign-solvability holds if and only if both the inverse of matrix A and the inverse multiplied with vector b are determined uniquely. There is a number of matrix operations which do not affect the analysis of full sign-solvability (see also Lancaster, 1962), viz.:

(i) permutation of any two rows of both matrix A and vector b. This operation only changes the order in which the equations are written.

(ii) permutation of any two columns of either matrix A or vector b. This operation only changes the order of the variables.

(iii) reversement of all signs in any row of both A and b. This operation multiplies both sides of an equation by a factor -1.

(iv) reversement of all signs in any column of either A or b. This operation multiplies a variable by -1.

The row and column manipulations (i), (ii) and (iii) can be carried out without affecting the solution vector, while the final operation implies the sign-reversement of a particular variable.

Necessary and sufficient conditions for full sign-solvability of (3), with A non-singular, have been formulated by Bassett et al., 1968. The conditions make use of graph theoretic methods, with impacts between variables represented by signed digraphs (directed graphs with either a positive or a negative sign). System (3) is full sign-solvable if and only if the following conditions hold.

(a) the diagonal elements of A be all negative, i.e. $a_{ii} < 0$ for all i.

(b) if $i_1 \neq i_2 \neq \ldots \neq i_k$ $(k>1)$ then

$a_{ii_1} a_{i_1i_2} \cdot \ldots \cdot a_{i_kj} < 0$.

This condition means that all cycles in the matrix A of length at least two need to be non-positive.

(c) $b \leq 0$: the elements of vector b need to be non-positive.

(d) if $b_k < 0$, then every path from i to k is non-negative for $i \neq k$.

The row and column operations (i) to (iv) are useful for a discussion of the conditions of sign-solvability. The first condition of sign-solvability may hold after one or more row or column permutations and sign reversements. Conditions (a) and (b) are necessary and sufficient to determine the inverse of the matrix A in a unique way, while conditions (c) and (d) guarantee a unique sign of the cell-entries from A^{-1} b when conditions (a) and (b) hold. Consider for example the following analytical representation from a set of three linear equations with qualitative information about the impacts between variables:

$$\begin{bmatrix} - & + & - \\ - & - & - \\ + & 0 & - \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ - \\ 0 \end{bmatrix}$$

The impacts of this system are presented in an equivalent way in Figure 1 by means of graphs with elements (A, -b).

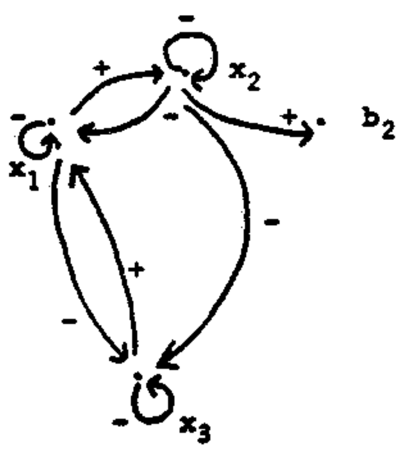

Figure 1. A Graph Representation of a Qualitative Model

All conditions for full sign-solvability hold in this example and the solution (4) becomes:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} - & - & + \\ + & - & 0 \\ - & - & - \end{bmatrix} \begin{bmatrix} 0 \\ - \\ 0 \end{bmatrix} = \begin{bmatrix} + \\ + \\ + \end{bmatrix}$$

The change in the variables x_1,x_2 and x_3 become positive, given the model structure represented by matrix A and the signs of the exogenous variable in vector b.

Other applications of sign-solvability (in the field of, for example, regional economic and macro-economic modeling) can be found in Brouwer and Nijkamp, 1985; Lady, 1983; Lancaster, 1962; Maybee and Quirk, 1969; Ritschard, 1980; Voogd, 1983.

The next section deals with some recent advances in the area of sign- solvability with different levels of information (e.g. a mixture of purely qualitative and quantitative information about the impacts between variables).

3. EXTENSION OF THE SIGN-SOLVABILITY APPROACH FOR PURE QUALITATIVE INFORMATION

The conditions for sign-solvability discussed in the previous section originated from mathematics. They have - in the framework of economic modelling - been further examined among others for Klein's model for the USA (see also Brouwer et al., 1985). Sign-solvability did even not hold for such a small dynamic national economic model for the USA with six equations. We will discuss therefore some adjusted tools and research directions of the sign-solvability approach for policy modelling, viz.:

(i) the use of matrix decomposition and matrix permutation procedures;

(ii) the use of plausible parameter restrictions which may be inferred on theoretical grounds;

(iii) the use of a stepwise procedure to include parameter values from one or more equations which are based on prior information. This stepwise procedure makes a distinction between a so-called top-down and bottom-up approach;

(iv) the use of recently developed computer algorithms for qualitative linear systems.

A number of matrix decomposition and matrix permutation procedures are developed for the analysis of sign-solvability (see also Maybee, 1981). A matrix A is called reducible if a permutation matrix P exists, to reverse rows and columns of the matrix A, such that A will be transformed in to A* with

$$A^* = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix} \qquad (5)$$

with both matrices A_{11} and A_{22} are square matrices and 0 is a zero-matrix. The transformation of A into A* is obtained by

$$A^* = P\ A\ P^T \qquad (6)$$

A matrix decomposition procedure makes a decomposition of matrix A into submatrices, just like for example in Formula (5).

When the matrix A is reducible, the sign-solvability approach can be dealt with in two steps, because:

$$\begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \qquad (7)$$

or:

$$A_{11}\, x_1 \;+\; A_{12}\, x_2 \;=\; b_1$$
$$A_{22}\, x_2 \;=\; b_2 \qquad (8)$$

The necessary and sufficient conditions for sign-solvability of vector x_2 can be analyzed independently from vector x_1. Vector x_2 may be sign-solvable irrespective of whether vector x_1 is sign-solvable (see also Gilly, 1984).
The conditions of sign-solvability make use of only qualitative information. However, in empirical modelling situations additional cardinal information may often be available which can be used as well. Such plausible information may be based on either logical, empirical or theoretical evidence. An example of plausible information is the share of consumption in national income which is not only positive but it should be also smaller than one. As the sign-solvability conditions make use of the signs of minors in the coefficient matrix, this information may be highly relevant.
When such additional quantitative information will be used, it may be possible that an originally not sign-solvable system of equations becomes at least partially sign-solvable (see also Brouwer et al., 1985).
When limited information - in quantitative terms - is available about the impacts between variables a so-called top-down or a bottom-up aproach may lead to interpretable modelling results. Both approaches are stepwise procedures so as to assure that a qualitative system is sign-solvable in a number of steps.
The top-down (or forward selection) approach implies that all equations are assumed to be represented in qualitative terms, so that we then may identify which and how many equations have to be estimated in a quantitative way in order to make the system sign-solvable.
The bottum-up (or backward elimination) approach starts with a complete estimated model and attempts then to identify which and how many equations may be specified in qualitative terms in order to still guarantee sign-solvability.
A main advantage of both the top-down and bottom-up approach is that policy models may become sign-solvable when a mixture of qualitative and quantitative information is used.
A FORTRAN computer algorithm to solve qualitative linear systems by means of a block recursive decomposition procedure has been developed and operationalized at the university of Geneva by Ritschard (1980). The procedure however may become problematic for computational reasons, especially in case of complex dynamic economic models.
The qualitative analysis discussed in this section will be now applied in the next section to a dynamic model of urban developments.

4. SIGN-SOLVABILITY ANALYSIS OF A DYNAMIC SIMULATION MODEL

The use of sign-solvability analysis will be illustrated in this section by means of a dynamic simulation model of urban decline for the city of The Hague in the Netherlands. Like many other cities in industrialized countries, this city is exhibiting a process of decline, in terms of population and jobs. It is however a major problem to select the appropriate (packages of) policy instruments needed to steer the urban evolution into a desired stable direction. Insight into the effects of policies for urban renewal is also hampered due to the lack of an operational urban model, mainly caused by the underdeveloped state-of-the-art in the area of urban econometrics (see also Hutchinson et al., 1985).

The fluctuating pattern of this urban system since World War II makes it unfortunately almost impossible to develop and estimate a satisfactory econometric model, as data on relevant key variables are not available and as there is no guarantee for a symmetric behaviour during the upswing and downswing phases of urban development. This explains also the limited use of integrated urban models in policy practice; urban evolution is still a poorly understood phenomenon.

Consequently, many urban policy models are not based on solid econometric procedures but on simulation experiments. Forrester (1971), for example, developed a simulation model with five major state variables, viz. non-agricultural investment, population, natural resources, pollution and agricultural investment. The policy impact model which will be discussed here is also a simulation model for urban dynamics. This model will be used as the basis for a qualitative calculus approach to urban dynamics.

Parameter and model validation in a conventional econometric way is problematic because of the a-symmetric pattern of urban evolution and the lack of sufficiently quantitative information.

In this context, sign-solvability analysis may be extremely relevant, as this method may be able to predict the qualitative (sign) impact of a policy variable, even if reliable cardinal values of impact coefficients are not available.

The model presented in this section has been used to assess the impact of public policy measures in the long-term evolution of the city. Urban development in time is mainly governed by attractiveness and disattractiveness factors wich play essentially the role of pseudo-prices in the model.

The structure of the model is presented in Figure 2.

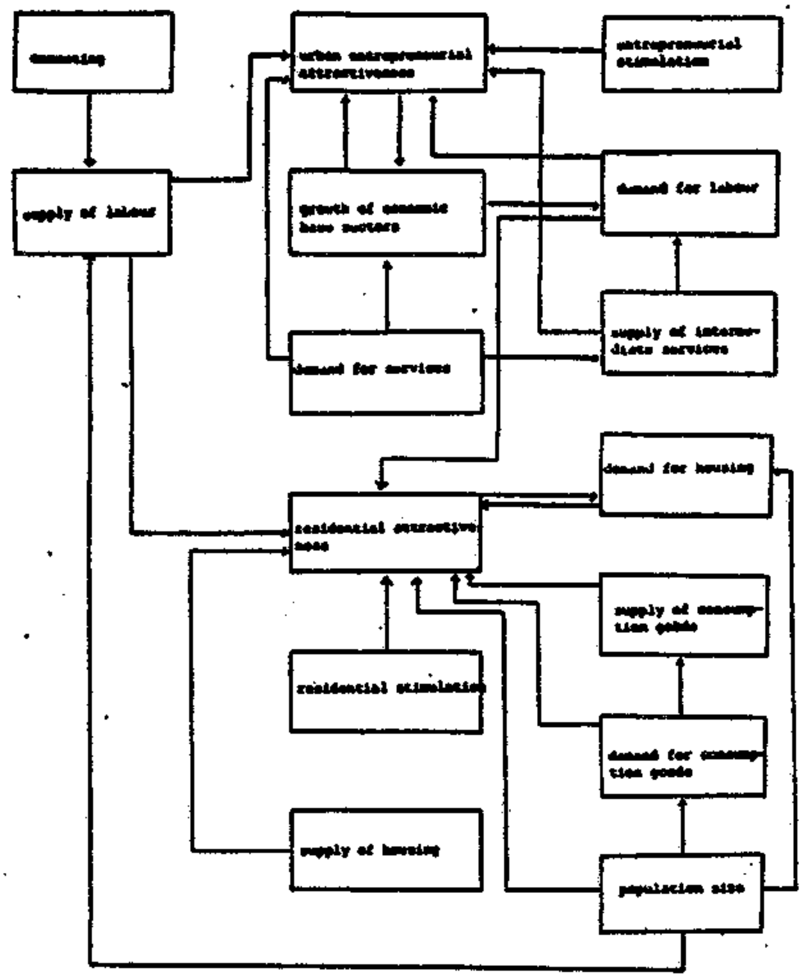

Figure 2. Structural Representation of an Urban Development Process

This model, with first-order time lags which is linear in nature, is denoted in matrix terms as follows:

$$\begin{array}{r} 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11 \end{array}\begin{bmatrix} 1 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\beta_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\gamma_1 & -\gamma_2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -\delta_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -\varepsilon_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\eta_1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\chi_2 & 0 & -\chi_1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\nu_1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -\xi_1 & 0 & 0 & 0 & 0 & -\xi_3 & -\xi_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} A^b \\ V^d \\ T \\ A^w \\ C^d \\ P \\ W^d \\ C^s \\ V^s \\ L^s \\ L^d \end{bmatrix}_t =$$

$$\begin{bmatrix} 1 & -\alpha_3 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 & \alpha_3 & \alpha_1 & -\alpha_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\gamma_1 & -\gamma_2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\delta_3 & -\delta_4 & -\delta_1 & \delta_3 & 0 & -\delta_2 & \delta_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\eta_1 & 0 & 1+\eta_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\chi_2 & 0 & \chi_3-\chi_1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_1 & 0 & 0 & 1-\lambda_1 & 0 & 0 & 0 \\ 0 & \mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 1-\mu_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\nu_1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -\xi_1 & 0 & 0 & 0 & 0 & -\xi_3 & -\xi_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} A^b \\ V^d \\ T \\ A^w \\ C^d \\ P \\ W^d \\ C^s \\ V^s \\ L^s \\ L^d \end{bmatrix}_{t-1} \qquad (9)$$

The model consists of 11 equations with endogenous variables characterized in the following way (see also Nijkamp and Soffer-Heitman, 1979).

1 = urban entrepreneurial attractiveness (A^b);
2 = demand for services (V^d);
3 = growth of economic base sectors (T);
4 = residential attractiveness (A^w);
5 = total demand for consumption goods (C^d);
6 = growth of the population size (migration included) (P);
7 = demand for housing (W^d);
8 = total supply of consumption goods (C^s);
9 = supply of intermediate services (V^s);
10 = supply of labour (L^s);
11 = demand for labour (L^d).

The 11 equations represented in matrix form in (9) are explained briefly below.

The development of urban entrepreneurial attractiveness will be stimulated by a rise in the supply of labour and intermediate services, the growth of economic base sectors as well as entrepreneurial stimulation measures (equation 1 in (9)). The development of economic base sectors (equation 3) is determined by a growth of urban entrepreneurial attractiveness and the demand for services. The simulation model also describes the development of residential attractiveness which is related to the situation of the housing market, the urban labour market, the balance between demand and supply of consumption goods and the growth of population (equation 4). Total demand for consumption goods in equation 5 is related to the population size. The growth of population size (equation 6) is related to the supply of housing and is determined exogenously. The supply of consumption goods will rise when the discrepancy between demand and supply becomes larger. The supply of intermediate services is a function of the difference between demand and supply of such services with a lag of one period (equation 8). The supply of labour is determined by the growth of population and the increase in the number of commuters. Finally, the demand for labour in equation 11, is a function of the development of the growth of economic base sectors, the supply of intermediate services and the supply of consumption goods.

The parameter values used in the simulation stage of urban modelling for the city at hand are presented in Table 1.

Table 1. Parameter Values of an Urban Simulation Model

	α	β	γ	δ	ε	η	χ	λ	μ	ν	ξ
1	1.0	0.9	0.4	1.0	0.9	0.2	1.1	0.5	0.5	0.75	0.2
2	1.0		0.5	1.0		0.9	0.5				0.2
3	0.75			0.75		0.02	0.02				0.5
4	2.0			2.0							

We will now analyze sign-solvability of the linear simulation model, viz. whether modelling results will be obtained which can be interpreted when only qualitative information or a mixture of qualitative and quantitative information is available. The columns of the matrix A are multiplied by -1, so that all main diagonal elements of the matrix sign (A) are negative, and the first condition of sign-solvability holds. This is one of the matrix operations mentioned in section 2 which do not affect the analysis of sign-solvability. The equations of the model represented above can be denoted by a matrix decomposition into submatrices, viz.:

$$\begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_t = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{t-1} \qquad (10)$$

The matrices are denoted in qualitative terms by

$$\text{sign}\ (A_{11}) = \begin{bmatrix} - & 0 & + & 0 & 0 & 0 \\ 0 & - & + & 0 & 0 & 0 \\ + & + & - & 0 & 0 & 0 \\ 0 & 0 & 0 & - & 0 & + \\ 0 & 0 & 0 & 0 & - & + \\ 0 & 0 & 0 & + & 0 & - \end{bmatrix}, \qquad \text{sign}\ (A_{21}) = \begin{bmatrix} 0 & 0 & 0 & + & 0 & + \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ [illegible] & [illegible] & [illegible] & [illegible] & [illegible] & + \\ 0 & 0 & 0 & 0 & 0 & + \\ 0 & 0 & + & 0 & 0 & 0 \end{bmatrix}$$

$$\text{and} \quad \text{sign}(A_{22}) = \begin{bmatrix} - & 0 & 0 & 0 & 0 \\ 0 & - & 0 & 0 & 0 \\ 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & - & 0 \\ 0 & + & + & 0 & - \end{bmatrix}$$

Sign-solvability of the linear system of equations in (9) can be analysed in two steps, because of the matrix decomposition procedure represented in (10), i.e. the variables 1 to 6 in the first step and the other variables in the second step.

A graph representation of the impacts between the variables 1 to 6 is given in Figure 2. The variables y_1 to y_6 denote the endogenous variables for period t and b_1 to b_6 denote these variables with a lag of one period.

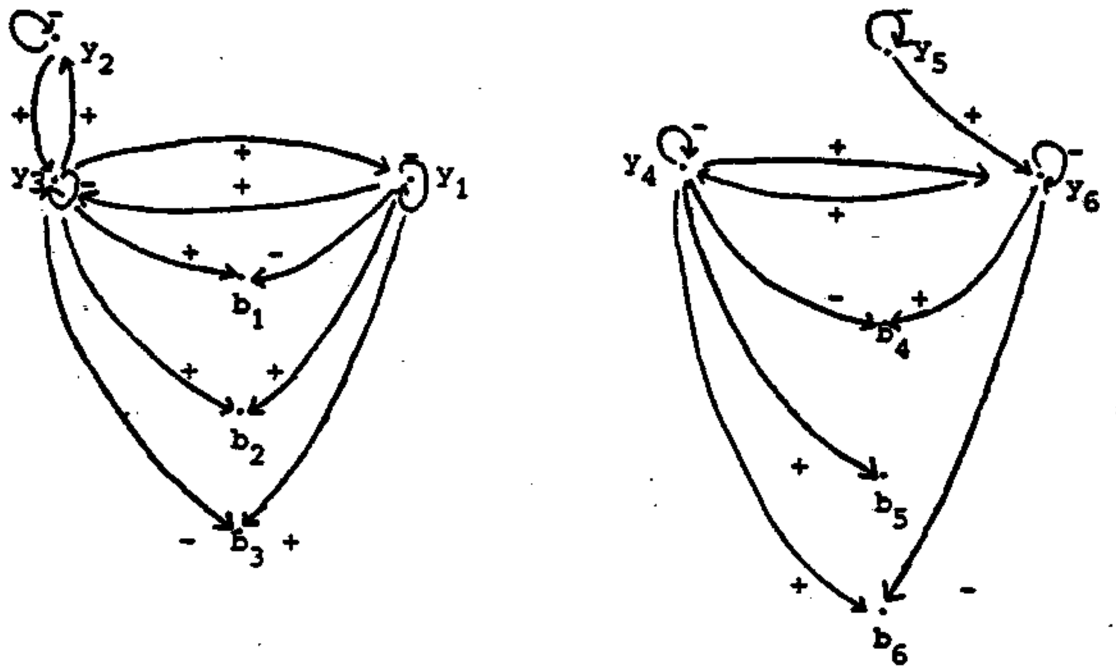

Figure 3. Graph Representation of Qualitative Impacts between Variables.

The matrix A_{11} can be partitioned into two independent sets of variables, viz. a set with the variables (A^b, V^d, T) and a set with the variables (A^w, C^d, P). We will discuss now the four conditions of sign-solvability. The first condition holds for all variables in Figure 3, viz. all main diagonal elements of the matrix A_{11} are negative.

The second condition, viz. that all cycles of length at least two are nonpositive, does not hold for anyone of the two graphs.

We may conclude from these two conditions of sign-solvability (one of them does not hold), that the inverse of the matrix A_{11} with only qualitative information, is not uniquely defined. The conditions three and four do not hold for the graph representation and we may conclude from this that the linear system which corresponds to Figure 2 with only qualitative information is <u>not</u> sign-solvable.

In the above we discussed that the dynamic simulation model is not sign-solvable. However, a system which is not sign-solvable with pure qualitative information, may become sign-solvable when a mixture of qualitative and quantitative information is available. The quantitative information may be obtained from theoretical evidence, prior knowledge or an estimation procedure. Consider for example the parameter values of the equations 1 to 6, presented in Table 1. The sign-values of the reduced form for these variables then becomes:

$$\begin{bmatrix} A^b \\ V^d \\ T \\ A^w \\ C^d \\ P \end{bmatrix}_t = \begin{bmatrix} + & - & + & 0 & 0 & 0 \\ 0 & - & + & 0 & 0 & 0 \\ 0 & - & + & 0 & 0 & 0 \\ 0 & 0 & 0 & + & - & + \\ 0 & 0 & 0 & + & - & - \\ 0 & 0 & 0 & 0 & - & + \end{bmatrix} \begin{bmatrix} A^b \\ V^d \\ T \\ A^w \\ C^d \\ P \end{bmatrix}_{t-1} \qquad (11)$$

The signs are defined uniquely because of the quantitative information about the parameters. The second part of formula (10) then becomes:

$$A_{21}\, x_{1t} + A_{22}\, x_{2t} = B_{21}\, x_{1,t-1} + B_{22}\, x_{2,t-1} \qquad (12)$$

and

$$x_{2t} = -A_{22}^{-1} A_{21}\, x_{1t} + A_{22}^{-1} B_{21}\, x_{1,t-1} + A_{22}^{-1} B_{22}\, x_{2,t-1} \qquad (13)$$

with x_{1t} and x_{2t} the variables 1 to 6 and 7 to 11 successively, for period t. The right-hand side of formula (13) consists of three parts and will be solved with the help of the reduced form representation in (11) and the qualitative impact values for the other variables. The sign-values for these variables then become:

$$\begin{bmatrix} W^d \\ C^s \\ V^s \\ L^s \\ L^d \end{bmatrix}_t = \begin{bmatrix} 0 & 0 & 0 & + & 0 & + \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & + & 0 & + \\ 0 & - & + & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} A^b \\ V^d \\ T \\ A^w \\ C^d \\ P \end{bmatrix}_{t-1} + \begin{bmatrix} 0 & 0 & 0 & + & 0 & + \\ 0 & 0 & 0 & 0 & - & 0 \\ 0 & - & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & + \\ 0 & - & + & 0 & - & 0 \end{bmatrix} \begin{bmatrix} A^b \\ V^d \\ T \\ A^w \\ C^d \\ P \end{bmatrix}_{t-1} +$$

$$+ \begin{bmatrix} - & 0 & 0 & 0 & 0 \\ 0 & - & 0 & 0 & 0 \\ 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & - & 0 \\ 0 & + & + & 0 & - \end{bmatrix} \begin{bmatrix} W^d \\ C^s \\ V^s \\ L^s \\ L^d \end{bmatrix}_{t-1} = \begin{bmatrix} 0 & 0 & 0 & + & - & + \\ 0 & 0 & 0 & 0 & - & 0 \\ 0 & - & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & + & 0 & + \\ 0 & - & + & 0 & - & 0 \end{bmatrix} \begin{bmatrix} A^b \\ V^d \\ T \\ A^w \\ C^d \\ P \end{bmatrix}_{t-1} + \begin{bmatrix} - & 0 & 0 & 0 & 0 \\ 0 & - & 0 & 0 & 0 \\ 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & - & 0 \\ 0 & + & + & 0 & - \end{bmatrix} \begin{bmatrix} W^d \\ C^s \\ V^s \\ L^s \\ L^d \end{bmatrix}_{t-1} \qquad (14)$$

The results from equations (11) and (14) show, for example, that a positive sign for the demand for consumption goods (C^d) during period t-1 will lead to a negative change in the next period for the variables: residential attractiveness (A^w), demand for consumption goods (C^d), growth of population size (P), supply of consumption goods (C^s), and the demand for labour (L^d) and it has no effect on the other variables.

We can conclude from the above that the system of linear equations is not fully sign-solvable when quantitative information about the equations 1 to 6 is available: a number of cell-entries in (14) are still undetermined with regard to their sign, viz. the cell-entries (1,6) with regard to the matrix coresponding to variables 1 to 6 and the cell-entries (5,2) and (5,3) with regard to the matrix corresponding to variables 7 to 11. The use of a top-down/bottom-up approach will therefore now be discussed to deal with a mixture of qualitative and quantitative information. The inverse of the matrix A_{11} is (10) can be partitioned into

$$A_{11} = \begin{bmatrix} A_{111} & 0 \\ 0 & A_{112} \end{bmatrix} \qquad (15)$$

with

$$A_{111} = \frac{1}{1-\beta_1\gamma_2-\gamma_1\alpha_2} \begin{bmatrix} 1-\beta_1\gamma_2 & \alpha_2\gamma_2 & \alpha_2 \\ \beta_1\gamma_1 & 1-\alpha_2\gamma_1 & \beta_1 \\ \gamma_1 & \gamma_2 & 1 \end{bmatrix} \quad (16)$$

and

$$A_{112} = \frac{1}{1-\eta_1\delta_4} \begin{bmatrix} 1 & 0 & \delta_4 \\ \eta_1\epsilon_1 & 1 & \epsilon_1 \\ \eta_1 & 0 & 1 \end{bmatrix} \quad (17)$$

The inverse of the matrix A_{22} in (10) with qualitative information is determined uniquely and becomes:

$$A_{22}^{-1} = \begin{bmatrix} - & 0 & 0 & 0 & 0 \\ 0 & - & 0 & 0 & 0 \\ 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & - & 0 \\ 0 & - & - & 0 & - \end{bmatrix} \quad (18)$$

The decomposition of the matrix A_{11} in (15) shows that the matrices A_{111} and A_{112} can be analyzed separately. This conclusion shows that in the framework of this paper sign-solvability with a mixture of qualitative and quantitative information for the two sets of variables (A^b, V^d, T) and (A^w, C^d, P) can be analyzed separately.

The results for the top-down/bottom-up approach are presented in Table 2. We made use of the reduced form model representation as follows:

$$x_{1,t} = M_1 \, x_{1,t-1} \quad (19\text{-a})$$

and

$$x_{2,t} = M_2 \, x_{1,t-1} + M_3 \, x_{2,t-1} \quad (19\text{-b})$$

The undetermined signs of the cell-entries from the matrices M_1, M_2 and M_3 are presented in Table 2 in columns 2, 3 and 4 successively. Column 1 in Table 2 presents the parameters for which quantitative information is used. Plausible parameter information has been used as well, viz. $1 - \lambda_1 < 1$ and $1 - \mu_1 < 1$, or $0 < \lambda_1 < 1$ and $0 < \mu_1 < 1$.

The cell-entries of the matrices M_1, M_2 and M_3 which are defined for their signs are the ones presented in equations (11) and (14).

Table 2. Mixed Levels of Information and Sign-solvability

Analysis	Quantitative information	Unknown signs of cell-entries: Matrix M_1	Matrix M_2	Matrix M_3
1	$\alpha_1,\alpha_2,\alpha_3,\alpha_4,\beta_1,\gamma_1,\gamma_2$	(4,4),(4,5),(4,6) (5,4),(5,5),(5,6) (6,4),(6,5),(6,6)	(1,4),(1,5),(1,6) (4,4),(4,5),(4,6)	(5,2),(5,3)
2	$\alpha_1,\alpha_2,\alpha_3,\alpha_4,\beta_1,\gamma_1,\gamma_2,\xi_1,\xi_2,\xi_3$	(4,4),(4,5),(4,6) (5,4),(5,5),(5,6) (6,4),(6,5),(6,6)	(1,4),(1,5),(1,6) (4,4),(4,5),(4,6)	
3	$\alpha_1,\alpha_2,\alpha_3,\alpha_4$ $\beta_1,\gamma_1,\gamma_2,\nu_1$	(4,4),(4,5),(4,6) (5,4),(5,5),(5,6) (6,4),(6,5),(6,6)	(1,4),(1,5),(1,6) (4,4),(4,5),(4,6)	(5,2),(5,3)
4	$\alpha_1,\alpha_2,\alpha_3,\alpha_4,\beta_1,\gamma_1,\gamma_2,\chi_1,\chi_2,\chi_3$	(4,4),(4,5),(4,6) (5,4),(5,5),(5,6) (6,4),(6,5),(6,6)	(1,4),(1,5),(1,6) (4,4),(4,5),(4,6)	(5,2),(5,3)
5	$\eta_1,\eta_2,\eta_3,\delta_1,\delta_2,\delta_3,\delta_4$	(1,1),(1,2),(1,3) (2,1),(2,2),(2,3) (3,1),(3,2),(3,3)	(1,6) (5,1),(5,2),(5,3)	(5,2),(5,3)
6	$\eta_1,\eta_2,\eta_3,\delta_1,\delta_2,\delta_3,\delta_4,\varepsilon_1$	(1,1),(1,2),(1,3) (2,1),(2,2),(2,3) (3,1),(3,2),(3,3)	(1,6) (5,1),(5,2),(5,3)	(5,2),(5,3)
7	$\eta_1,\eta_2,\eta_3,\delta_1,\delta_2,\delta_3,\delta_4,\varepsilon_1,\chi_1,\chi_2,\chi_3$	(1,1),(1,2),(1,3) (2,1),(2,2),(2,3) (3,1),(3,2),(3,3)	(5,2),(5,3)	
8	$\eta_1,\eta_2,\eta_3,\delta_1,\delta_2,\delta_3,\delta_4,\varepsilon_1,\xi_1,\xi_2,\xi_3$	(1,1),(1,2),(1,3) (2,1),(2,2),(2,3) (3,1),(3,2),(3,3)	(1,6) (5,1),(5,2),(5,3)	
9	$\eta_1,\eta_2,\eta_3,\delta_1,\delta_2,\delta_3,\delta_4$ ε_1,ν_1	(1,1),(1,2),(1,3) (2,1),(2,2),(2,3) (3,1),(3,2),(3,3)	(1,6) (5,1),(5,2),(5,3)	(5,2),(5,3)
10	$\alpha_1,\alpha_2,\alpha_3,\alpha_4,\beta_1,\gamma_1,\gamma_2,\eta_1,\eta_2,\eta_3,\delta_1,\delta_2,\delta_3,\delta_4,\varepsilon_1$		(1,6)	(5,2),(5,3)
11	$\alpha_1,\alpha_2,\alpha_3,\alpha_4,\beta_1,\gamma_1,\gamma_2,\eta_1,\eta_2,\eta_3,\delta_1,\delta_2,\delta_3,\delta_4,\varepsilon_1$ χ_1,χ_2,χ_3			(5,2),(5,3)
12	$\alpha_1,\alpha_2,\alpha_3,\alpha_4,\beta_1,\gamma_1,\gamma_2,\eta_1,\eta_2,\eta_3,\delta_1,\delta_2,\delta_3,\delta_4$ $\chi_1,\chi_2,\chi_3,\xi_1,\xi_2,\xi_3$			
13	$\alpha_1,\alpha_2,\alpha_3,\alpha_4,\beta_1,\gamma_1,\gamma_2,\eta_1,\eta_2,\eta_3,\delta_1,\delta_2,\delta_3,\delta_4,\varepsilon_1$ $\chi_1,\chi_2,\chi_3,\xi_1,\xi_2,\xi_3$			

An empty block in Table 2 means that the corresponding matrix has no cell-elements which are undetermined for their signs.
The first row in Table 2, for example, presents the cell-entries of the matrices M_1, M_2 and M_3 which are undetermined by sign when quantitative information for the three first equations from equation (9) is used. Additional quantitative information is necessary to determine the signs of these cell-elements. When the analyses 1 and 2 are compared to each other, and quantitative information about the parameters ξ_1, ξ_2 and ξ_3 is introduced, all cell-entries of the matrix M_3 are determined for their signs. This is a result of the so-called top-down approach.
An originally not sign-solvable simulation model, like the analyses 1 to 11 in Table 2, becomes sign-solvable in analysis 12 when quantitative information about the above mentioned parameters is used.
In such a case the qualitative information about the equations for the variables C^d, P, C^s, V^s and L^s is used, as well as theoretically plausible parameter values, viz. ϵ_1, μ_1, $\nu_1 > 0$, $0 < \lambda_1 < 1$ and $0 < \mu_1 < 1$.
The opposite approach to a stepwise selection for sign-solvability based on a backward elimination (the top-down approach) takes the cardinally estimated model as the starting point of the analysis. This is presented in analyses 12 and 13 in Table 2. Both analyses are sign-solvable. The quantitative information about the parameter ϵ_1 does not add any information for the sign-solvability analysis.
For that reason we prefer the top-down selection approach with a stepwise selection procedure and with all parameters in the basic model known up to their sign.

5. CONCLUSION

Some new research directions in the field of mixed qualitative calculus, (especially sign-solvability of an urban planning model) have been discussed in the paper.

Qualitative calculus can be regarded as a method to solve (either static or dynamic) models with qualitative information about the impacts between variables. Sign-solvability is a major issue in policy modelling in case of non-metric information regarding parameters. Mixed sign-solvability analysis and matrix decomposition methods may provide new tools for a qualitative policy impact analysis. A serious disadvantage of a matrix decomposition and matrix permutation approach, discussed in section 3, is the necessity of matrices A_{11} and A_{22} to be squared; the sign-solvability approach becomes problematic when such matrices are not square. In this regard, the potential of a generalized inverse procedure has to be further examined.

However, if a mixture of qualitative and quantitative information about the impact between variables is available, a stepwise selection procedure can be employed in order to obtain solutions for the sign-solvability approach in dynamic and static policy models.

In relation to efficient decomposition procedures, the use of theoretically plausible quantitative information on parameter values and of non-square matrices, A_{11} and A_{22} is useful as an operational tool in sign-solvability analysis of policy models with mixed qualitative-quantitative information.

The previous analyses have demonstrated that qualitative calculus is an extremely useful tool in policy modelling and may be an appropriate complement to conventional econometric techniques and simulation procedures.

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