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ANALYSIS OF DYNAMIC SPATIAL INTERACTION
MODELS BY MEANS OF OPTIMAL CONTROL

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ANALYSIS OF DYNAMIC SPATIAL INTERACTION MODELS BY MEANS OF OPTIMAL CONTROL

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Abstract

This paper deals with the design of general classes of dynamic spatial interaction models. On the basis of a general (well-behaved) multi-period objective function and of a dynamic model representing the evolution of a spatial interaction system, an optimal control model is constructed. Particular attention is given to the equilibrium and stability conditions. It turns out that it is possible to identify steady-state solutions for a dynamic spatial interaction model. Furthermore, it can also be demonstrated that the entropy model is a specific case of the above mentioned spatial interaction system.

1. Introduction

In the seventies a great deal of publications in the field of quantitative geography and regional economics has been devoted to spatial interaction analysis. This macro (or meso) oriented approach has evoked many interesting methodological questions, such as the macro (or meso) behavioural interpretation of spatial interaction models (e.g. a generalized cost interpretation) and the micro behavioural foundation of spatial interaction analysis (e.g. based on a disaggregate choice theory). In a recent paper (see Nijkamp and Reggiani, 1986) it has been demonstrated that spatial interaction models of the Wilson type are compatible with stochastic discrete choice theory, in particular with multinomial logit models based on random utility theory both in a static and dynamic sense.

Only recently, more attention has been devoted to dynamic spatial interaction analysis. Examples can be found in Batten and Boyce (1986), Boyce and Southworth (1979), Clarke and Wilson (1983), Byler and Gale (1978), Coelho (1977), Griffith and Lea (1983), Haynes and Phillips (1982), Kahn (1981), Leonardi (1983), Lombardo and Rabino (1983), Nijkamp and Poot (1986), Nijkamp and Reggiani (1985), Rabino (1985), Sonis (1984), Williams and Wilson (1980) and Wilson (1981). Various specifications have been chosen for such dynamic models, for instance, Markov transition models, synergetic multi-actor models, Volterra-Lotka type of models, and so forth.

In the present paper an attempt will be made at developing an optimal control formulation for a spatial interaction model. Instead of choosing an entropy objective function, we will use here a general well-behaved objective function (of which the entropy function is a specific case). Besides, an additional dynamic equation for push-pull effects will be added. Hence, the results will be more general than those emerging from a standard entropy approach even if it is, from an analytical viewpoint, difficult to establish a formal relationship with the family of discrete choice models.

The paper is organized as follows. In section 2 the general dynamic spatial interaction model will be formulated as an optimal control model. Next, in section 3 special attention will be given to the equilibrium conditions and to the stability of the optimal control solutions. Finally, some concluding remarks will be made.

2. A General Dynamic Spatial Interaction Model

In this section an optimal control representation of a dynamic spatial interaction model will be given. For the ease of presentation, but without loss of generality, we will assume a transport system in which all origin-destination flows are time dependent. These flows will be regarded as control variables in the Pontryagin sense.

It will be assumed that the total volume of flows from origin i, i.e. O_i, may be regarded as a state variable whose evolution is dependent - by means of a linear function - on the net push out - pull in effects of place i (see also Nijkamp and Reggiani, 1986. This leads then to the following dynamic equation:

$$\dot{O}_i = \alpha_i O_i + \delta_i \left(\sum_{j=1}^{J} T_{ji} - \sum_{j=1}^{J} T_{ij} \right) \qquad (2.1)$$

Equation (2.1) can also directly be derived from the well-known dynamic migration model developed by Okabe (1979) and Sikdar and Karmeshu (1982):

$$\dot{P}_i = \alpha_i P_i + \sum_{j=1}^{J} T_{ji} - \sum_{j=1}^{J} T_{ij} \qquad (2.2)$$

where α_i is the natural growth rate of population at the ith place and P_i is the population size. Obviously, in our transportation system we can derive (2.1) from (2.2) by assuming that O_i is linearly dependent (through the parameter δ_i) on the population size in i, as follows:

$$O_i = \delta_i P_i \qquad (2.3)$$

or:

$$\dot{O}_i = \delta_i \dot{P}_i \qquad (2.4)$$

Finally, instead of a conventional entropy function we assume the following more general, well-behaved (i.e. concave) objective function (see also Nijkamp, 1975) reflecting a collective utility function for all points of origin and destination within a given time horizon T:

$$\max \omega^* = \int_0^T \omega\, (T_{ij}, O_i) e^{-rt} dt \qquad (2.5)$$

where T_{ij} stands for the whole set of flow variables $T_{11}, \ldots, T_{IJ}$. A cumulative entropy function (see Sonis, 1986) is a special case of (2.5).

Maximization of (2.5) subject to (2.1) requires the use of first-order conditions for an optimal control model. The relevant Hamiltonian is then:

$$H\ (T_{ij},\ O_i,\ \psi_i, t) = \omega\ (T_{ij},\ O_i)\ e^{-rt} + \sum_{i=1}^{I} \psi_i\ \dot{O}_i \qquad (2.6)$$

where ψ_i represents a co-state variable.

The first-order conditions for a maximum solution of this optimal control model are:

$$\left.\begin{aligned} \frac{\delta H}{\delta T_{ij}} &= 0 && \forall\ i,j \\ \frac{\delta H}{\delta O_i} &= -\dot{\psi}_i && \forall\ i \\ \frac{\delta H}{\delta \psi_i} &= \dot{O}_i && \forall\ i \end{aligned}\right\} \qquad (2.7)$$

All variables in (2.7) are provided with a discount rate. According to Kamien and Schwartz (1981) it is more appropriate to analyze the first-order conditions in terms of current values at each point t than in terms of their equivalent at time 0, as in the first case a set of autonomous (i.e. non time-dependent) differential equations describing the optimal solution is obtained.
Then the following adjusted Hamiltonian may be used:

$$H^*\ (T_{ij},\ O_i,\ \psi_i) = e^{rt}\ H = \omega\ (T_{ij},\ O_i) + \sum_{i=1}^{I} \psi_i^*\ \dot{O}_i \qquad (2.8)$$

with

$$\psi_i^* = e^{rt}\ \psi_i \qquad (2.9)$$

Next, by differentiating (2.9) with respect to time, we obtain:

$$\begin{aligned} \dot{\psi}_i^* &= r\ e^{rt}\ \psi_i + e^{rt}\ \dot{\psi}_i \\ &= r\ \psi_i^* - e^{rt}\ \frac{\delta H}{\delta O_i} \\ &= r\ \psi_i^* - e^{rt}\ \frac{\delta\ (e^{-rt} H^*)}{\delta O_i} \\ &= r\ \psi_i^* - \frac{\delta H^*}{\delta O_i} \end{aligned} \qquad (2.10)$$

If we substitute (2.8) into (2.10), we obtain:

$$\dot{\psi}_i^* = r\ \psi_i^* - \frac{\delta\omega\ (T_{ij},\ O_i)}{\delta\ O_i} + \delta_i\ \psi_i^* \ = \ (r + \delta_i)\ \psi_i^* - \frac{\delta\ \omega\ (T_{ij},\ O_i)}{\delta\ O_i} \qquad (2.11)$$

The remaining first-order conditions from (2.7) are straightforward, so that we arrive at the following system:

$$\left.\begin{array}{ll} \dfrac{\delta\ H^*}{\delta\ T_{ij}} = 0 & \forall\ i,j \\ \dot{\psi}_i^* = (r + \delta_i)\ \psi_i^* - \dfrac{\delta\ \omega}{\delta\ O_i} & \forall\ i \\ \dot{O}_i = \dfrac{\delta\ H^*}{\delta\ \psi_i^*} & \forall\ i \end{array}\right\} \qquad (2.12)$$

where for the ease of presentation the arguments of ω are omitted. By adding also (2.1) to the latter system, an autonomous set of equations is obtained, where time is not an explicit argument.

3. Equilibrium and Stability Solutions

In this section we will study more carefully the solution paths of the above-mentioned system (either in explicit form or in a qualitative sense). The first necessary optimality condition of (2.12) can be written as:

$$\frac{\delta\ H^*}{\delta\ T_{ij}} = \frac{\delta\ \omega}{\delta\ T_{ij}} - \delta_i\ \psi_i^* = 0 \qquad (3.1)$$

This condition states that the marginal value of the collective utility function for the system at hand equals the shadow price of the push of the dynamic state equation.

If the objective function would be a cumulative entropy function, subject to some constraints on the origins and on interaction costs, it can be shown that the optimal values of the control variables can be calculated from the following (generalized) production-constrained spatial interaction model:

$$T_{ij} = A_i^*\ O_i\ D_j\ \ \exp(-\beta^* c_{ij}) \qquad (3.2)$$

where A_i^* is a generalized balancing factor (see for a formal derivation Annex A).

Now it may be interesting to analyze the optimal control solution in the (O_i, ψ_i^*) plane, as in this plane we do not have an explicit solution for T_{ij} (ω is also depending on T_{ij}). In this case we have to analyze more thoroughly equations (2.1) and (2.11), which represent a pair of differential equations in O_i and ψ^*_i. In order to analyze the solution trajectories that are compatible with (2.1) and (2.11), we will first consider the $\dot{O}_i = 0$ locus, i.e.,

$$O_i = -\left(\sum_{j=1}^{J} T_{ji} - \sum_{j=1}^{J} T_{ij}\right)\delta_i/\alpha_i \qquad (3.3)$$

It is noteworthy that $\sum_{j=1}^{J} T_{ij}$ and $\sum_{j=1}^{J} T_{ij}$ are not a function of ψ_i^*.

Next we consider the points for which $\dot{\psi}_i^* = 0$, i.e.,

$$\psi_i^* = \frac{\delta\ \omega}{\delta\ O_i} / (r + \delta_i) \qquad (3.4)$$

Assuming a concave objective function ω, we have the following second-order conditions:

$$\frac{\delta^2\ \omega}{\delta\ T_{ij}^2} < 0 \qquad (3.5)$$

and:

$$\frac{\delta^2\ \omega}{\delta\ O_i^2} < 0 \qquad (3.6)$$

The latter result implies that (3.4) represents a downward sloping curve (see Figure 1).

Now the question arises whether we can infer some conclusions regarding the ultimate state equilibrium. As ω is unspecified so far, it is difficult to provide a precise analytical derivation, but it is possible to approximate the dynamic state and costate equations ((2.1) and (2.11), respectively) in a Taylor series around the steady state solution (for O_i^s and ψ^{*s}_i)of system (3.3) and (3.4).

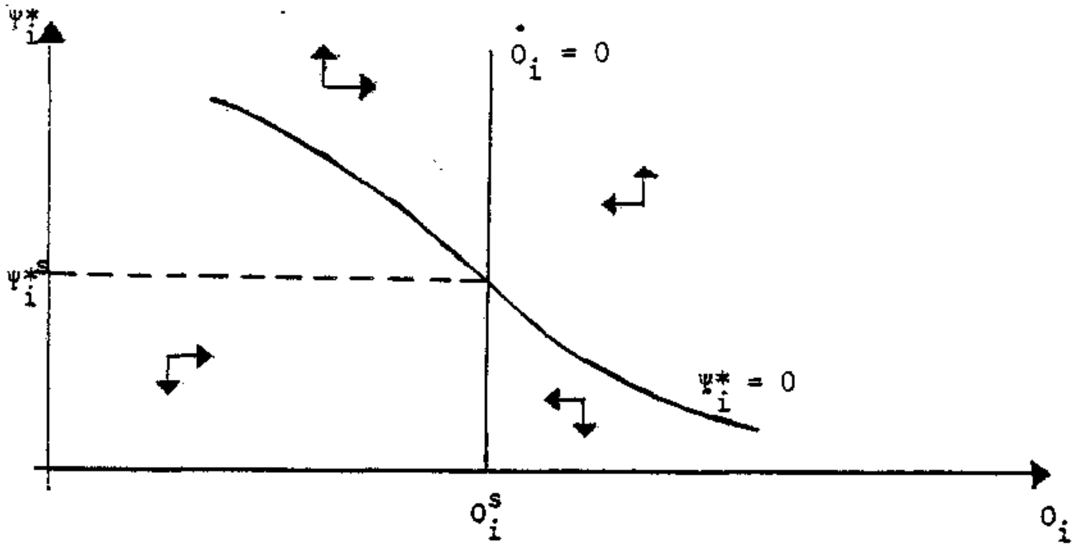

Fig. 1. Solution trajectories and steady state for O^s_i and ψ^{*s}_i

This leads to the following expressions:

$$\dot{O}_i = -\delta_i(O_i - O^s_i) \qquad (3.7)$$

and

$$\dot{\psi}^*_i = -\frac{\delta^2\omega\ (O^s_i)}{\delta\ O^2_i}(O_i - O^s_i) + (r + \delta_i)\ (\psi^*_i - \psi^{*s}_i) \qquad (3.8)$$

Now we have to examine the characteristic roots of (3.7) and (3.8) in order to study the configuration of the equilibrium point. These characteristic roots are:

$$K = r/2 \pm (r + 2\ \delta_i)/2 \qquad (3.9)$$

as is easily seen by writing the following characteristic equation (see Kaplan, 1958):

$$\begin{vmatrix} -\delta_i - k & 0 \\ -\dfrac{\delta^2\ \omega\ (O^s_i)}{\delta\ O^2_i} & r + \delta_i - k \end{vmatrix} = 0 \qquad (3.10)$$

or: $k^2 - pk + q = 0$ (3.11)

$$\left.\begin{aligned} \text{with: } p &= k_1 + k_2 = r > 0 \\ q &= k_1 \cdot k_2 = -\delta_i\ (r + \delta_i) < 0 \end{aligned}\right\} \qquad (3.12)$$

and:

$$\Delta = p^2 - 4q = (r + 2\ \delta_i)^2 > 0 \qquad (3.13)$$

Now it is clear that - given that $q < 0$ - (1) Δ is positive and (2) the roots k_1 and k_2 are real and distinct. However, from (3.9) it can easily be derived that both roots have an opposite sign, i.e.,

$$k_1 < 0 < k_2 \qquad (3.14)$$

Consequently, the steady state reflects a saddle point solution, so that the equilibrium is compatible with a stable state.

This result could also be analyzed in a geometric way. For example, from equation (3.7) and Figure 1, it can be seen that at the right hand side of the locus $\dot{O}_i = 0$, $\dot{O}_i < 0$, while at the left hand side $\dot{O}_i > 0$. Similarly, from equation (3.8) one can easily show that above the locus $\dot{\psi}_i^* = 0$, ψ_i^* is increasing (and hence $\dot{\psi}_i^* > 0$), whereas below the locus $\dot{\psi}_i^* = 0$ the following situation holds: $\dot{\psi}_i^* < 0$.

In Figure 1 the arrows illustrate the areas of increasing and decreasing O_i and ψ^*_i respectively in the phase plane. This solution is consistent with an optimal control approach. In fact, it appears that the pair of differential equations (3.7) and (3.8) arising from system ((2.1), and (2.5)) does not have a totally stable solution, that for <u>all</u> paths converges to a steady state (see Kamien and Schwartz, 1981 and Medio, 1986).

In conclusion, it is clear that by considering a general dynamic spatial interaction model one obtains solution trajectories that approach an equilibrium point as time goes by, reaching the so-called saddlepoint stability.

4. Conclusions

In this paper the stability of a spatial flow system emerging from a general optimal control spatial interaction model has been analyzed. The optimal paths leading to a steady state have been examined. It appears that a steady state solution (reflected inter alia by a saddlepoint) is reached.

The results are, obviously, co-determined by the initial assumptions on state variables and control variables. The discount rate has only the function of changing the marginal value of the costate variable (see (3.4)). For example, from this equation it can easily be seen that an increase in the discount rate decreases the $\dot{\psi}_i^* = 0$ locus, while it leaves the $\dot{O}_i = 0$ locus unaffected in a downward movement of the steady state solution.

The same applies if we increase in the spatial interaction model the parameter δ_i. Thus an increase in the discount rate, in δ_i, moves the equilibrium point downward. In this context, it might also be worth considering the possible movements of the state variables, if the system is not fully deterministic, but subject to stochastic disturbances. This would require the use of a stochastic optimal control model based on e.g. Brownian motion processes), which is still an underdeveloped field which no doubt would warrant further investigation.

ANNEX A. A Production-Constrained Spatial Interaction Model as a Solution of an Optimal Control Entropy Model

In this section, an optimal control problem will be analyzed in which the objective function within a given time horizon T is assumed to be the well-known entropy function which can also be regarded as a specific type of welfare function (see e.g. Wilson, 1970, Coelho, 1977, and Coelho and Williams, 1978):

$$\omega = - \sum_{i=1}^{I} \sum_{j=1}^{J} T_{ij} \left(\ln(T_{ij}/O_i D_j) - 1\right) \qquad (A.1)$$

subject to the standard constraints on origins and costs in a transportation system. Therefore the optimal control problem becomes:

$$\left.\begin{aligned} &\max \omega^* = - \int_0^T e^{-rt} \sum_{i=1}^{I} \sum_{j=1}^{J} T_{ij} \left(\ln \frac{T_{ij}}{O_i D_j} - 1\right) dt \\ &\text{s.t.} \\ &\sum_{j=1}^{J} T_{ij} = O_i \qquad \forall\, i \\ &\sum_{i=1}^{I} \sum_{j=1}^{J} c_{ij} T_{ij} = C \end{aligned}\right\} \qquad (A.2)$$

where c_{ij} is the unit transportation cost between i and j, c is the total cost budget, and D_j a certain given attraction indicator for place j. The constraints in (A.2) are assumed to hold in each time period, and T_{ij} is again assumed to be a control variable. The parameter r reflects a discount rate in order to transform all variables into their present values. In addition, we have the dynamic equation (2.1) for the state variable O_i. This specification bears some resemblance to the cumulative entropy model discussed by Nijkamp and Reggiani (1985) and Sonis (1986).

Owing to the equality constraints from (A.2) the problem becomes a bounded optimal control model, so that the necessary conditions for the optimality can be represented by means of the Hamiltonian and of the Lagrangean function.

The necessary first-order conditions are now (in terms of current values at each point t) (see section 2):

$$\left.\begin{array}{ll} \dfrac{\delta L^*}{\delta T_{ij}} = 0 & \forall\ i,j \\ \dot{\psi}_i^* = r\Psi_i^* - \dfrac{\delta L^*}{\sigma O_i} & \forall\ i \\ \dot{O}_i = \dfrac{\delta H^*}{\delta \psi_i^*} & \forall\ i \end{array}\right\} \qquad (A.3)$$

where:

$$\psi_i^* = e^{rt}\psi_i \qquad (A.4)$$

is the current value multiplier associated with (2.1). The adjusted (current value) Hamiltonian is:

$$H^* = e^{rt} H = -\sum_{i=1}^{I}\sum_{j=1}^{J} T_{ij}\left(\ln \frac{T_{ij}}{O_i D_j} - 1\right) + \sum_{i=1}^{I} \psi_i^* \dot{O}_i \qquad (A.5)$$

while the corresponding adjusted Lagrangean is:

$$L^* = H^* + \sum_{i=1}^{I} \lambda_i^* \left(O_i - \sum_{j=1}^{J} T_{ij}\right) + \beta^* \left(C - \sum_{i=1}^{I}\sum_{j=1}^{J} c_{ij} T_{ij}\right) \qquad (A.6)$$

where λ_i^* and β^* represent the (current) Lagrange multipliers associated with the constraints O_i and C.

The necessary conditions for a constrained maximum with respect to T_{ij} are:

$$\frac{\delta L^*}{\delta T_{ij}} = -\ln \frac{T_{ij}}{O_i D_j} - \lambda_i^* - \beta^* c_{ij} - \delta_i \Psi_i^* = 0 \qquad (A.7)$$

so that:

$$\frac{T_{ij}}{O_i D_j} = \exp(-\lambda_i^* - \delta_i \psi_i^*) \cdot \exp(\beta^* c_{ij}) \qquad (A.8)$$

By defining now:

$$\exp(-\lambda_i^* - \delta_i \Psi_i^*) = A_i^* \qquad (A.9)$$

expression (A.7) becomes:

$$T_{ij} = A_i^* O_i D_j \exp(-\beta^* c_{ij}) \qquad (A.10)$$

This expression is again the usual production-constrained spatial interaction model.

Next if we apply the constraint $\sum_{j=1}^{J} T_{ij} = O_i$ (as defined in A.2) to equation (A.8), we obtain:

$$1 = \exp\,(-\lambda_i^* - \delta_i \Psi_i^*) \sum_{j=1}^{J} D_j \exp\,(-\beta^* c_{ij}) \qquad \text{(A.11)}$$

so that:

$$\exp\,(-\lambda_i^* - \delta_i \psi_i^*) = 1 \;/\; \sum_{j=1}^{J} D_j \exp\,(-\beta^* c_{ij}) = A_i^* \qquad \text{(A.12)}$$

By substituting (A.12) into (A.8), we can easily derive the probability p_j:

$$p_j = \frac{T_{ij}}{O_i} = \frac{D_j \exp\,(-\beta^* c_{ij})}{\sum_{j=1}^{J} D_j \exp\,(-\beta^* c_{ij})} \qquad \text{(A.13)}$$

Expression (A.13) represents a model of the logit type. It is of course also equivalent to the spatial interactive model obtained in (A.10).

Because of the general expression in the term A_i^* defined in (A.12), the spatial interaction model (A.10) or (A.13) is more general than the standard one.

Obviously the same result can also be obtained for a doubly-constrained spatial interaction model. It is clear that in this case we will obtain two balancing factors A_i^* and B_j^* which will be related not only to the Lagrangean multipliers (as in standard spatial interaction model (see Nijkamp and Reggiani, 1985)), but also to the (current value) costate variables.

Next we have to add that the solution (A.10) of our optimal control problem is unique, as we are dealing with a concave integrand. This can also be shown by the following second order conditions:

$$\frac{\delta}{\delta T_{ij}} \left(\frac{\delta H^*}{\delta T_{ij}} \right) = - \frac{O_i D_j}{T_{ij}} < 0 \qquad \text{(A.14)}$$

When we analyze the conditions for the costate variable, we can easily obtain:

$$\dot{\psi}_i^* = (r + \delta_i)\, \psi_i^* - \frac{\delta}{\delta\, O_i} \{ - \sum_{i=1}^{I} \sum_{j=1}^{J} T_{ij} (\ln \frac{T_{ij}}{O_i D_j} - 1) \} - \lambda_i^* \quad (A.15)$$

The final solution of Ψ^*_i cannot be expressed in an analytical sense (as is the usual situation in spatial interaction models), but it can be obtained in a recursive numerical way.

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