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A M S T E R D A M

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Abstract

We describe an economy in which farmers become rationed after a bad harvest and the balance of payments deteriorates progressively as a result. Necessary and sufficient conditions are derived for a 'perverse' (positive) effect of rationing on money stocks held by farmers. In this model the presence of money balances related to the stochastic nature of rationing may frustrate liberalisation attempts.

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SUPPLY RESPONSE AND MONEY DEMAND IN A PEASANT ECONOMY WITH RATIONING AND RISK.

1. Introduction

We consider a developing country which is heavily dependent upon agricultural exports and which is characterized by two institutional rigidities. First, while export crops are privately produced, the government is the sole agent who can buy crops from farmers and export them. The producer price which farmers receive is fixed in real terms. Secondly, the government controls the allocation of imports between farmers and the rest of the economy, the urban sector. The government's own demand (direct and indirect) for imports and that of urban firms and consumers must be satisfied to a minimum extent: the allocation of imports to the urban sector can exceed this minimum level (the 'urban claim') but a lower allocation is politically not feasible. Elsewhere we have applied the model to Tanzania (Bevan et al., forthcoming), but the description characterizes many developing countries.

Using a barter model, we show that in such an economy the market clearing equilibrium is not globally stable. Once goods markets get rationed, the economy gets caught in a downward spiral; the micro-economic reaction of farmers to rationing is magnified by its macro-economic repercussions, via the balance of payments. Further, in non-market clearing conditions, agents' response to price changes are shown to be the opposite of that likely under market clearing. Hence, appropriate changes in pricing policy in an economy subject to rationing cannot be inferred from knowledge about behaviour under market clearing conditions.

In the fix-price equilibrium literature, it is normally assumed, e.g. Malinvaud (1977), that agents know with certainty how much they will be able to buy of the rationed goods. This may apply when, as in Europe during wartime rationing, a coupon system is used. If an economy arrives in a rationing regime by accident, there is however no formal rationing system. In that case availability of consumption goods to farmers is likely to become highly uncertain, both over time and over individuals, as indeed it is in Tanzania. This source of risk has important implications for asset demand.

This point has, to the best of our knowledge, been ignored in the literature. In Malinvaud's well-known model consumers hold an asset ('money') but risk and its consequences for intertemporal behaviour are ignored: the consumer's money balance is an argument of the utility function for the current period.

Benassy (1975, pp. 515-18) has shown how that specification can be derived from a two-period formulation in which availability in the second period is risky. He considers the implications of this formulation for the existence of equilibrium, but not for changes in (mean) availability on optimal money stocks, stating only in passing (p. 516) that rationing leads to a 'flight from money'. We show in section 4 (where the barter model is extended by the introduction of money and stochastic rationing) that this is, in general, not true: under stochastic rationing a decrease in availability may lead to an increase in money demand.
Stochastic rationing in such an economy implies the possibility of a 'honeymoon' for the government, a period during which exports are temporarily higher than the amount consistent with reduced availability of consumption goods in rural areas: farmers work harder in the short run in order to adjust their money stocks. We show that there are three types of equilibria in this model and derive necessary and sufficient conditions for each. These conditions involve the degree of risk aversion and the probability distribution of the amount which can be bought, i.e. both the mean and the riskiness of the ration.

The monetary consequences of stochastic rationing have important policy implications. A liberalization programme in which the government restored farmers' access to consumption goods to the level of the unrationed equilibrium (through an aid-financed increase in imports) might fail as farmers consumed more without raising their production in order to run down their excess money balances. In this situation the government must either give up its policy, as reflected in the two institutional rigidities, or default on part of the monetary claims held by farmers.

2. Rationing of farmers in a barter economy

Consider a farmer who can produce two crops: a cash crop and a food crop. Production is denoted by q_c and q_f respectively. The cash crop is not consumed by the farmer and can be sold at a price p. The food crop is consumed and can be bought and sold at prices p_b, p_s respectively ($p_b > p_s$). In addition to the two crops there are two other goods in the model: leisure (ℓ) and a consumption good (c) which can be acquired only through trade. Production requires only labour and q_i (i=f,c) denotes (by choice of units) both production and labour input.[1] Writing t for the total time available for farm work, leisure is defined by:

$$(1) \qquad q_f + q_c = t - \ell.$$

The budget constraint may be written as

$$(2) \qquad c \leq pq_c + \pi(q_f - f)$$

where f denotes consumption of the food crop and

$$(3) \qquad \pi = \begin{cases} p_s & \text{if } q_f > f \\ p_b & \text{if } q_f < f \end{cases}$$

The farmer's utility function $u(c,f,\ell)$ satisfies $u_{ii} < 0 < u_i$. We assume that the market for the food crop is very imperfect: there is a wide range between the buying and selling price. In particular, we assume that the producer price of the cash crop lies in this range:

$$(4) \qquad p_b > p > p_s.$$

If the farmer were to maximize his utility function subject to (1), (2) and (3) he would choose the vector (c^*, f^*, ℓ^*). This (unrationed) equilibrium he can, however, not attain: demand for the consumption good is rationed:

$$(5) \qquad c \leq \bar{x}$$

where the ration $\bar{x}$ is known with certainty to the farmer and is strictly less than c^*.

Note that if the food crop could be bought and sold at the same price then the farmer would be fully specialized: he would produce only the cash crop or only the food crop, depending upon whether p was greater or less than $p_b = p_s$. Assumption (4), however, implies that he will produce both crops and, in particular, that he will be selfsufficient in food. This follows from (2). The budget constraint must be binding (both in rationed and in unrationed equilibrium), because otherwise (keeping c and f constant) leisure (ℓ) could be increased and (since u_3 is positive) this would increase utility. But if the budget constraint is binding then q_f must be equal to f. For suppose on the contrary, that q_f exceeds f. Then a combination of an increase in cash crop production and a decrease in food crop production such that the budget constraint remains unaffected ($p\Delta q_c + p_s \Delta q_f = 0$) would increase leisure and hence utility: $\Delta\ell = -(\Delta q_f + \Delta q_c) = \Delta q_c\ (-1+p/p_s) > 0$. Similarly, at an optimum f cannot exceed q_f. Because if it did then an increase in q_f (and an offsetting decrease in q_c) would increase leisure: $\Delta\ell = \Delta q_f\ (-1+p_b/p) > 0$. Hence $f = q_f$ and the problem reduces to:

$$\max_{c,\ell,q} u(c, t-\ell-q, \ell)$$

subject to

(6) $\quad c \leq pq$

(7) $\quad c \leq \overline{x}$

where we have dropped the subscript of q_c : q denotes cash crop production. We are interested in the effect on q of changes in the two variables controlled by the government: the producer price of the cash crop and the consumption good ration $(p, \overline{x})$.

Consider first a change in price. If the rationing constraint is effective, then, since (6) must also hold as an equality[2]:

(8) $\quad q = \overline{x}/p.$

Hence the supply response is perverse: cash crop production is decreasing in the producer price. While this conclusion is almost trivial as a theoretical result, it deserves to be emphasized, since those who advocate 'getting prices right' in policy discussions seem to have given little thought to what that phrase might mean in the context of an economy in which farmers are rationed. For example, in Tanzania the IMF has advocated large increases in producer prices for cash crops in order to raise exports. Opponents of the IMF have countered that price increases are unlikely to do much good if farmers cannot spend the extra money; that the policy is likely to be actually harmful if farmers value leisure positively seems to have gone unnoticed.

The effect of an increase in the ration (keeping the producer price fixed) is, of course, positive: from (8), output of the cash crop increases proportionately with the ration.

In unrationed equilibrium, c^* and q^* are functions of p only. If these functions are monotonic, we can write:

$$(9) \qquad c^{-1}(c^*) = p = q^{-1}(q^*).$$

This defines the farmers' offer curve: the locus of values of q (cash crop 'exports' from rural areas to the rest of the economy) and c ('imports' of consumption goods into rural areas) such that utility is maximized, subject to (6). Under this trade-theoretic interpretation we consider farmers and the rest of the economy as two separate countries. Note that for points on the offer curve the 'balance of payments' is in equilibrium (since $c^* = pq^*$).

In Figure 1 we show this offer curve as an increasing, concave function $q^*(c^*)$.[3] Under rationing the offer curve becomes a straight line $q = c/p$ (where c equals $\bar{x}$). If, e.g., the producer price p would be equal to the slope AB/OA then in unrationed equilibrium farmers would choose point B.

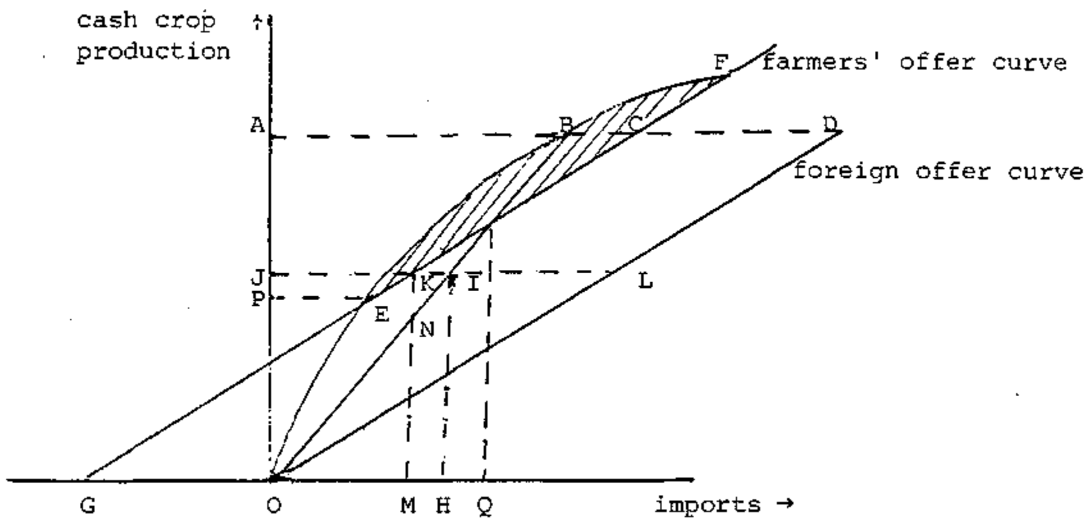

Figure 1. Macro-economic Effects of Rural Rationing.

In rationed equilibrium they would choose a point on the line OB; e.g. for $\bar{x}$ equal to OH, they would choose point I.

3. Macro-economic effects of rationing

So far the analysis has been partial since $\bar{x}$ is treated as exogenous: while changes in availability of consumption goods to farmers affect output of the cash crop, no account has been taken of feedback. To analyse the macro-economic effects of rationing we introduce an urban sector. This is rudimentary, consisting only of the government and of urban consumers.[4] The government imports consumption goods and decides on the allocation of these imports between rural and urban consumers. There are no other imports and the cash crop (which the government procures from farmers) is the only export product.

The world price of the cash crop is given, hence the foreign offer curve is shown in Figure 1 as a straight line. The government is constrained in two ways: first, it is politically committed to price control and, secondly, it must satisfy urban consumption demand (a fixed quantity, equal to CD in the diagram). This urban claim is represented as a shift to the left (OG) of the foreign offer curve. If farmers choose point B, exports would be OA, and AD could be imported, more than sufficient to satisfy both farmers and urban consumers. Note that only points in the shaded area, enclosed by the farmers' offer curve and the displaced foreign offer curve are feasible in this sense.

As drawn, the farmers' offer curve is at B parallel to the foreign offer curve. This is relevant if the government's objective is to maximize urban consumption. It would then set the domestic price of the cash crop at AB/OA. Point B would then be an optimum and the difference between the world price and the lower domestic price would be the revenue-maximizing tax. However, it is not essential to our argument that the domestic price is set at this revenue-maximizing level, but rather that the initial point B lies on the farmers' offer curve, between E and F.

At B no agents are rationed. However, now consider a disturbance to this equilibrium. Suppose that, as a result of a random shock such as a bad harvest, output falls from OA to OJ. Farmers then expect to get a quantity JI of consumption goods but receive only JK since imports fall to JL and KL is allocated to urban consumers. A point such as K cannot be a stationary equilibrium, since if farmers expected to receive only JK they would not be willing to produce as much as MK but only MN. If farmers expect rationing to continue and are sufficiently pessimistic about the ration $\bar{x}$[5] then the

economy gets caught in a downward spiral: cash crop production decreases, exports fall, less can be imported, farmers are more severely rationed, they revise their expectations of $\bar{x}$ downwards and produce even less. This process of cumulative contraction does not converge: it stops when production falls below OP and the urban claim can no longer be honoured.
How can such a decline be arrested? Once the economy has arrived at a point such as K, changing the producer price is not advisable for two reasons. First, we have already noted that the supply response under rationing is perverse. This price would have to be lowered, but the government has no way of knowing by how much to reduce p in order to reach the point on the farmers' offer curve directly above K. There is a danger of overshooting: p might be lowered too much in which case cash crop production would be further reduced. Secondly, if (as is likely) the producer price was sub-optimal to begin with, then farmers would be given conflicting signals: the price would first be reduced (to break out of the rationing regime), but would later, once availability started to improve, have to be raised in order to move along the farmers' offer curve to a point beyond B. The only alternative to a price policy is an increase in the ration $\bar{x}$. As long as the urban claim must be satisfied this is feasible only if aid can be obtained to finance a temporary excess of imports over exports. In the model of this section such an aid-financed recovery would succeed provided the amount of aid was at least MQ.

4. Stochastic rationing in a two-period model with money

We have, so far, discussed a barter economy. Introducing money into that model would not add anything of interest since transaction demand would be the only reason for holding money. However, if we drop the assumption that the farmer knows the amount of the ration with certainty, then there is an additional reason for holding money. In this section we consider the implications of stochastic rationing for the demand for money and the extent to which the holding of money balances affects our previous analysis.

We simplify the model by dropping the food crop: the utility function now has two arguments, consumption of the urban good (c) and the number of hours worked on the cash crop (h). We assume that production requires only labour and that the prices of the cash crop and of the consumption good are fixed at unity in terms of money. Hence, h measures not only effort but also production, the value of sales and the volume of consumption which, in the absence of rationing, can be financed with the proceeds. We assume that the utility function $u(c,h)$ satisfies:

(10a) $u_1 > 0 > u_2;\ u_{12} = u_{21} \leq 0$

(10b) the Hessian of u is negative definite

(10c) $u_1(0,0) + u_2(0,0) > 0$

(10d) $u_2(c,h)$ is convex in c.

Note that u_2 is assumed to be negative, since h is negatively related to leisure. Assumption (10c) ensures that in the absence of risk consumption is positive. Assumption (10d) gives a sufficient (but not necessary) condition for at least one of the constraints (12), (13) to be binding at an optimum.[6]

A period starts just before harvest time. The amount of work involved in harvesting is largely determined by past decisions (e.g. weeding, spraying). We ignore the labour tasks other than harvesting, but we do assume that the amount of work in the current period (h_t) is predetermined. We measure money balances (m_t) just after the harvest has been sold, hence the amount which can be spent in period t+1 is

(11) $m_{t+1} = m_t + h_{t+1} - c_t.$

The ration (x_t) becomes known just after the harvest. The farmer then decides how much to consume, taking into account the constraints

(12) $\quad c_t \leq x_t$

(13) $\quad c_t \leq m_t$

and the effect, via (11), on his future consumption possibilities. If he maximizes expected utility in a two-period model then the problem just after the harvest in the first period is:

$$\max_{c_t, h_{t+1}} \quad W = u(c_t, h_t) + Eu(c_{t+1}, h_{t+1})$$

subject to (11), (12), (13) and m_t, h_t given.

The farmer does not know the ration for the next period (x_{t+1}) but he does know its density function $f(x)$. We assume that $f(x)$ is continuous and positive for all positive x and we write $\bar{x}$ for the mean ration:

(14) $\quad \bar{x} = \int_0^\infty x\, f(x)\, dx.$

After the harvest in the second period the problem is simply:

$$\max_{c_{t+1}} \; u(c_{t+1}, h_{t+1})$$

subject to $c_{t+1} \leq x_{t+1}$; $c_t \leq m_{t+1}$; m_{t+1}, h_{t+1} given

which gives:

(15) $\quad c^*_{t+1} = \min(x_{t+1}, m_{t+1}).$

Hence the two-period problem may be rewritten as:

$$\max_{c_t, h_{t+1}} \quad W = u(c_t, h_t) + \int_0^{m_{t+1}} u(x, h_{t+1})\, f(x)\, dx + u(m_{t+1}, h_{t+1}) \int_{m_{t+1}}^\infty f(x) dx$$

subject to (11), (12), (13) and m_t, h_t given.

Note that in a stationary equilibrium without rationing, consumption, effort and money balances would satisfy:

$$(16) \qquad c_t = h_t = m_t = c^*$$

where c* solves the first-order condition

$$(17) \qquad u_1(c^*,c^*) + u_2(c^*,c^*) = 0$$

which simply states that in equilibrium the marginal benefit of working harder (the utility of extra consumption) must be equal to the marginal cost (the utility of foregone leisure)[7]. We assume that the farmer is in unrationed equilibrium for t = 0 and that rationing starts, unexpectedly, in period 1 [8]:

$$(18) \qquad c^* = m_1 = h_1.$$

We are interested in the decisions taken by the farmer in the average case, i.e. when

$$(19) \qquad x_t = \bar{x}.$$

Differentiation of the objective function with respect to h_{t+1} gives:

$$(20) \qquad \frac{\partial W}{\partial h_{t+1}} = [u_1(m_{t+1},h_{t+1}) + u_2(m_{t+1},h_{t+1})]\int_{m_{t+1}}^{\infty} f(x)\,dx + \int_0^{m_{t+1}} u_2(x,h_{t+1})\,f(x)\,dx$$

$$= \phi(m_{t+1}, h_{t+1})$$

It is convenient to define the function

$$(21) \qquad \xi(m_t,h_{t+1}) = \phi(m_t + h_{t+1} - \bar{x},\ h_{t+1})$$

which is decreasing in both arguments, since (suppressing the arguments of the utility function):

$$(22) \qquad \xi_1 = (u_{11}+u_{21}) \int_z^\infty f(x)\,dx - u_1\, f(z) < 0$$

$$(23) \qquad \xi_2 = \xi_1 + (u_{12}+u_{22}) \int_z^\infty f(x)\,dx + \int_0^z u_{22}\, f(x)\,dx < \xi_1$$

where $z = m_t + h_{t+1} - \bar{x}$.
Consider the function $h_{t+1} = \mu(m_t)$ defined by $\xi(m_t, h_{t+1}) = 0$. Total differentiation of the defining relation gives:

$$(24) \qquad \xi_1 dm_t + \xi_2\, dh_{t+1} = 0$$

hence, since $\xi_2 < \xi_1 < 0$, the slope of the function μ satisfies:

$$(25) \qquad -1 < \frac{d\mu}{dm_t} = -\xi_1/\xi_2 < 0.$$

It immediately follows that the slope of the function

$$(26) \qquad g(m_t) = m_t + \mu(m_t) - \bar{x}$$

satisfies

$$(27) \qquad 0 < \frac{dg}{dm_t} < 1.$$

The function g is relevant because of the following results:

<u>Lemma 1.</u> The solution to the two-period problem satisfies $m_{t+1} = g[\max(\bar{x}, m_t)]$.

<u>Proof.</u> In the appendix it is shown that the solution satisfies $c_t = \min(\bar{x}, m_t)$: at least one of the constraints (12), (13) is binding. The first-order condition for h_{t+1} implies, from (20): $\phi(m_{t+1}, h_{t+1}) = 0$.
Suppose $c_t = \bar{x}$. Then, by construction, $h_{t+1} = \mu(m_t)$ satisfies the first-order condition and

$$(28) \qquad m_{t+1} = m_t + h_{t+1} - c_t = m_t + \mu(m_t) - \bar{x} = g(m_t).$$

If, however, $c_t = m_t$ then $\phi(m_t + h_{t+1} - m_t, h_{t+1}) = 0$ implies $h_{t+1} = \mu(\bar{x})$. Hence in that case

$$(29) \qquad m_{t+1} = m_t + h_{t+1} - c_t = m_t + \mu(\bar{x}) - m_t = g(\bar{x}).$$

Therefore $m_{t+1} = g(m_t)$ if $\bar{x} \leq m_t$ and $m_{t+1} = g(\bar{x})$ otherwise.

<u>Corollary</u> $0 < g(\overline{x}) < c^*$.

<u>Proof.</u> If $h = g(\overline{x})$ then h solves $\phi(h,h) = 0$. From (20) it follows that $u_1(h,h) + u_2(h,h) > 0$. Because of (10), this is consistent with (17) only if $h = g(\overline{x}) < c^*$. To prove that $g(\overline{x})$ is positive, note that (10c) implies $\phi(0,0) > 0$. Substitution in (21) gives $\xi(\overline{x},0) > 0$. Since $\xi(\overline{x}, \mu(\overline{x})) = 0$ and ξ_2 is negative this implies $\mu(\overline{x}) > 0$. Hence, from (26), $g(\overline{x}) > 0$.

<u>Lemma 2.</u> If $\overline{x} < c^*$ then $g(0) > 0$.

<u>Proof.</u> First note that, since $\overline{x} < c^*$ and $c^* > 0$:

$$(30) \qquad \xi(o,\overline{x}) = \phi(o,\overline{x}) = u_1(o,\overline{x}) + u_2(o,\overline{x})$$

$$> u_1(c^*,c^*) + u_2(c^*, c^*) = 0$$

Secondly, from the definition of μ:

$$(31) \qquad \xi[o,\mu(o)] = 0$$

and since ξ is decreasing in its second argument ($\xi_2<0$), (30) and (31) imply that $\mu(o) > \overline{x}$. Substitution in (26) gives the desired result:

$$(32) \qquad g(o) > 0.$$

The function g is shown in Figure 2. Here all parameters of the density function f(x) and of the utility function are kept constant. These parameters determine c^*, the mean $\overline{x}$ and the riskiness of the ration and the function g itself. What remains is the dependence of h_{t+1} (and hence of m_{t+1}) on the initial money stock m_t.[9] In Figure 2, m_t is measured along the horizontal axis. If it exceeds $\overline{x}$ then $m_{t+1} = g(m_t)$, otherwise $m_{t+1} = g(\overline{x})$ (lemma 1).
Since $c_t = \min(\overline{x}, m_t)$ and $m_{t+1} = m_t + h_{t+1} - c_t$, the optimal values of c_t and h_{t+1} follow immediately.
That only the four cases shown are possible follows from the corollary.

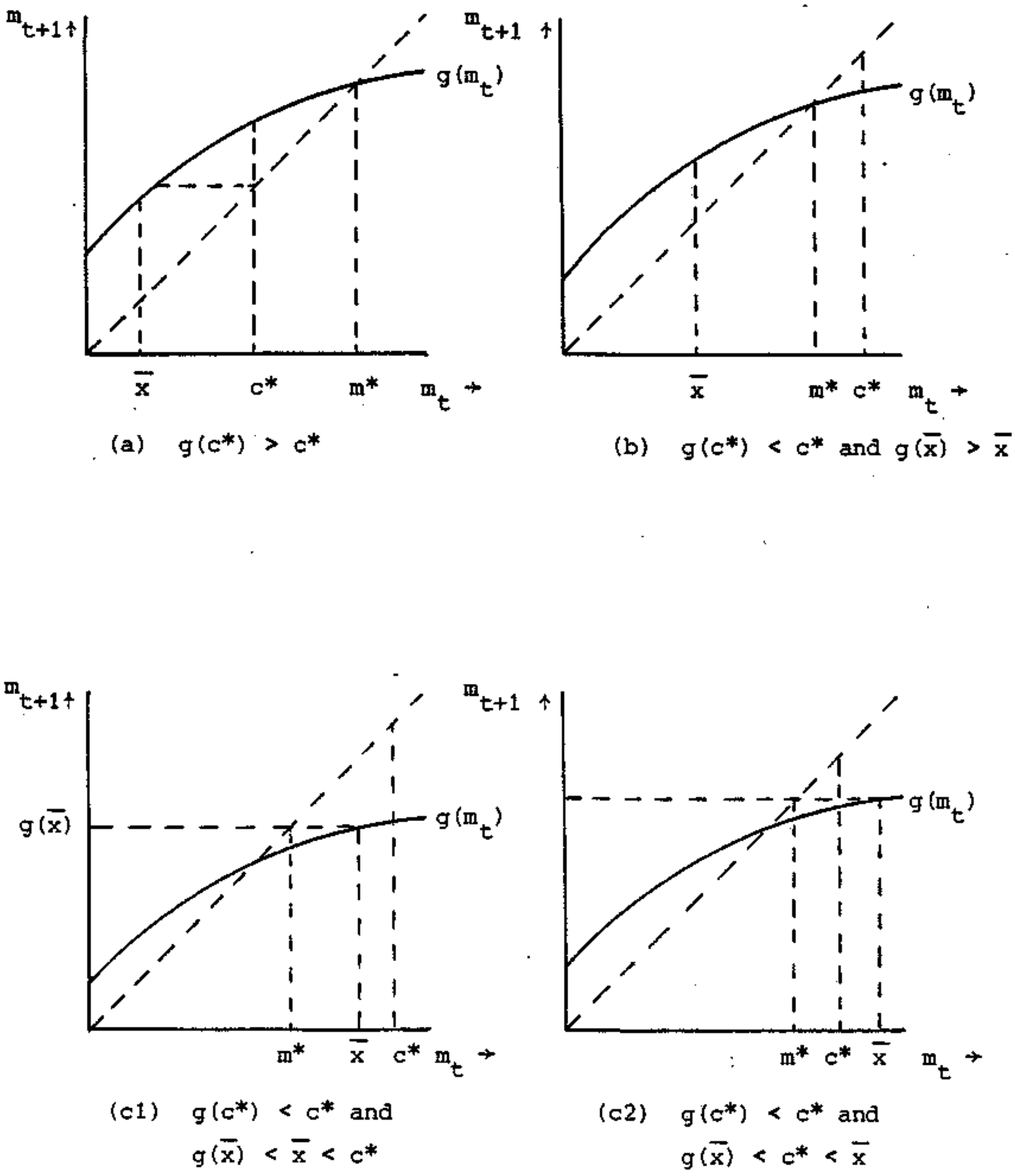

Figure 2. Optimal money balance ($\underline{m}^*$) as fixed point of the mapping $m_{t+1} = g\,[\max\,(\bar{x}, m_t)]$.

Provided the parameters of the problem remain constant, the diagram may also be used to trace changes in the optimal solution over time, by interpreting m_{t+1} as the initial money stock in the next two-period problem. In cases (a) and (b) it is clear that in such a sequence of two-period problems m will converge to m*. Since the sequence starts at $m_1 = c^*$ and c* lies to the left of the fixed point m* in diagram (a) but to the right in diagram (b), the two cases differ in the direction from which the long run equilibrium is approached. Money balances rise in case (a) and fall in case (b).

In case (c2) the mapping $m \to g(m)$ is not relevant. Since $\bar{x} > c^* = m_1$, $m_2 = g(\bar{x}) = m^* < c^*$ (lemma 1 and corollary). Hence the money stock falls from c* to m* and then remains constant. Case (c1) combines features of diagrams (b) and (c2). Since $\bar{x} < c^* = m_1$, initially the mapping $m \to g(m)$ applies. The money stock will fall and there must be a time t* such that $m_{t*} < \bar{x}$; m then falls to m* and remains constant.

In all four cases m* is the fixed point of the mapping $m \to g\,[\max\,(\bar{x},m)]$. The existence of this fixed point follows trivially from (27) and lemma 2 in the first three cases. In case (c2), however, lemma 2 does not apply since then $\bar{x} > c^*$. However since $g(\bar{x})$ is positive (corollary), the line $m_{t+1} = g(\bar{x})$ must intersect the 45° line, hence m* exists (and is positive).

Figure 2 suggests that in case (a), in a sequence of overlapping two-period models, it is optimal for farmers to react to the imposition of rationing by a process of adjustment during which money balances increase (converging to $m^* > c^*$) and effort decreases (converging to $h^* = \bar{x}$); that in case (b) money balances fall over time (converging to $m^* < c^*$) and effort increases (converging to $h^* = \bar{x}$); and, finally, that in case (c) money balances fall and effort increases until, in long-run equilibrium, $m^* = h^* < \bar{x}$. This result we now state and prove.

<u>Theorem 1.</u>

Consider a sequence of two-period problems ($t = 1,2,..$):

$$\max_{c_t,h_{t+1}} \quad W = u(c_t,h_t) + \int_0^{m_{t+1}} u(x,h_{t+1})\, f(x)\, dx + \int_{m_{t+1}}^{\infty} u(m_{t+1},h_{t+1})\, f(x)\, dx$$

$$\text{subject to } c_t \le \bar{x};\ c_t \le m_t;\ m_{t+1} = h_{t+1} + m_t - c_t;\ \text{and } m_1 = h_1 = c^*$$

Then the optimal sequences $m_2, m_3, ..$ and $h_2, h_3, ..$ satisfy:

(a) iff $\xi(c^*,\bar{x}) > 0$:

$$c^* > h_{t+1} > h_{t+2} > \lim_{t\to\infty} h_t = \bar{x};\ \lim_{t\to\infty} m_t = m^* > m_{t+2} > m_{t+1} > c^*$$

(b) iff $\xi(c^*,\overline{x}) < 0 < \xi(\overline{x},\overline{x})$:

$\lim_{t\to\infty} h_t = \overline{x} > h_{t+2} > h_{t+1}$; $c^* > m_{t+1} > m_{t+2} > \lim_{t\to\infty} m_t = m^*$

(c) iff $\xi(\overline{x},\overline{x}) < 0$:

$\overline{x} > h_{t+2} \geq h_{t+1} > 0$; $c^* > m_{t+1} \geq m_{t+2} \geq h_{t+2}$

with the weak inequalities strict only for $t < t^*$.

Proof

1. Case (a) is defined (see fig. 2) by $g(c^*) > c^*$. From (26) this is equivalent to $\mu(c^*) > \overline{x}$. But $\xi(c^*,z)$ is decreasing in z and equal to zero for $z = \mu(c^*)$, hence $\mu(c^*) > \overline{x}$ if and only if $\xi(c^*,\overline{x})$ is positive. Similarly, case (c) is defined by $g(\overline{x}) < \overline{x}$ and this is equivalent to $\mu(\overline{x}) < \overline{x}$ hence $\xi(\overline{x},\overline{x}) < 0$ is both necessary and sufficient for case (c).
2. In case (a), since $m_1 = c^*$ and $m_t = g(m_t)$, the sequence $m_1, m_2, \ldots$ converges monotonically from below to m^*. But from (27), for all $m_t < m^*$:

$$(33) \quad m_{t+2} - m_{t+1} = g(m_{t+1}) - g(m_t) < m_{t+1} - m_t$$

which, since $h_{t+1} = m_{t+1} - m_t - \overline{x}$, implies

$$(34) \quad h_{t+2} < h_{t+1}.$$

Hence effort decreases monotonically and converges to $\overline{x}$.
3. In case (b) the sequence $m_1, m_2, \ldots$ starts to the right rather than the left of the fixed point. Hence the same argument applies, but the inequalities in (33) and (34) are reversed: m_t converges monotonically from above to m^*, and h_t converges monotonically from below to $\overline{x}$.
4. We have already noted that $\xi(c^*,z)$ is decreasing in z. If $\overline{x} < c^*$ then $\xi(c^*,c^*) = \phi(2c^*-\overline{x}, c^*) < \phi(c^*,c^*) = \int_0^{c^*} u_2(x,c^*)\, f(x)\, dx < 0$. Hence $h_2 = \mu(m_1) = \mu(c^*) < c^*$: in case (a) effort is, throughout the adjustment process, lower than in the absence of rationing.
5. In case (c1) $m_1 = c^* > \overline{x}$ and m initially decreases monotonically as in case (b) (Figure 2). But since $g(\overline{x}) < \overline{x}$, there must be a time t^* such that $m_{t^*} < \overline{x}$. Hence (lemma 1) $m_t = c_t = h_{t+1} = g(\overline{x}) = m^* < \overline{x}$ for all $t > t^*$, from which the result in the theorem follows trivially. In case (c2), the first phase, with $m_{t+1} = g(m_t)$, does not apply: $t^* = 1$ hence the money stock is immediately reduced to m^*.

This completes the proof.

We now restrict our attention to the case where mean availability is reduced ($\bar{x} < c^*$). It is useful to decompose the initial effect of rationing on the money balance $(m_2 - m_1)$ into a planned and an unplanned change:

$$(35) \qquad m_2 - m_1 = h_2^* - \bar{x} = (h_2 - c^*) + (c^* - \bar{x}).$$

Since the onset of rationing was unexpected, the farmer had planned to consume a quantity c^* but he can consume only $\bar{x}$. Hence the second term in brackets in (35) is positive and measures an unplanned increase in money holding. This is offset by the first term, which, as we have just seen, is negative. If this first term dominates then $h_2^* < \bar{x}$ and this, as we now know, is the case iff $\xi(c^*, \bar{x}) < 0$. Hence the initial, positive, unplanned effect on money balances is reinforced for $\xi(c^*, \bar{x}) > 0$ and reversed for $\xi(c^*, \bar{x}) < 0$.

Theorem 1 establishes the existence and uniqueness of a long-run (stationary) equilibrium and the convergence and monotonicity of the adjustment process. More importantly, the theorem gives a very simple condition for the sign of the effect of stochastic rationing on effort and on money stocks, a condition which involves only the sign of $\xi(c^*, \bar{x})$. We now turn to the effects of changes in the models parameters on the money stock m^*.

Consider first the situation where the farmer overestimates the mean $\bar{x}$. This may e.g. be the case when the severity of rationing increases: it will take time before the farmer realises that a lower ration x_t does not represent bad luck ($x_t < \bar{x}$: the case of an unlucky draw from an unchanged probability distribution) but an unfavourable shift in the probability distribution ($x_t = \bar{x}$ but $\bar{x}$ has fallen). We model this, slightly artificially, by assuming that the farmer uses in his calculations not f(x) but

$$\dot{f}(x) = \begin{cases} f(x-\varepsilon) & \text{for } x \geq \varepsilon \\ 0 & \text{otherwise} \end{cases}$$

where ε, a positive number, is the amount by which the mean is overestimated. In cases (a) and (b), $h = \bar{x}$ and $m = m^*$ in long-run equilibrium, hence

$$(36) \qquad \xi(m^*, \bar{x}) = [u_1(m^*, \bar{x}) + u_2(m^*, \bar{x})] \int_{m^*}^{\infty} f(x)\, dx + \int_0^{m^*} u_2(x, \bar{x})\, f(x)\, dx = 0.$$

When the mean is overestimated the integral boundary m* must be replaced by m*-ε. Hence:

$$(37) \qquad \xi_1 dm^* + u_1(m^*,\bar{x})\ f(m^*)\ d\varepsilon = 0$$

and, since ξ_1 is negative and u_1 is positive, this implies that the effect of the overestimation of the mean $\bar{x}$ on the money stock m* is positive.

Next consider the case where availability improves: f(x) shifts a distance ε to the right (and the farmer perceives this correctly). In this case $\bar{x}$ must be replaced by $\bar{x} + \varepsilon$ in (36), in addition to the change in the integral boundary. Then:

$$(38) \qquad \xi_1 dm^* + [(u_{12}+u_{22}) \int_{m^*}^{\infty} f(x)\ dx + \int_0^{m^*} u_{22}\ f(x) + u_1\ f(m^*)]\ d\varepsilon = 0$$

The sign of the terms in square brackets is ambiguous, hence without further restriction on the functions u and f, we cannot say whether an increase in availability $\bar{x}$ leads to higher or lower money balances m*.

However, for many reasonable functional forms (e.g. in the numerical example presented below) the effect can be negative. In that case, if rationing becomes more severe, farmers respond, paradoxically, by accumulating <u>larger</u> money balances (e.g. in Figure 3 a move from B to A).
In case (c), $\bar{x}$ must be replaced by m* in (36). It follows that if availability improves:

$$(39) \qquad \xi_1 dm^* + u_1(m^*, m^*)\ f(m^*)\ d\varepsilon = 0$$

hence in this case the effect of $\bar{x}$ on m* is unambiguously positive.

Figure 3 illustrates the effect of availability ($\bar{x}$) on money stocks in long-run equilibrium (m*). Here the parameters of the utility function are kept constant and only the mean of the density function f(x) is allowed to change (the riskiness of the ration is kept constant).
As drawn, Figure 3 shows examples of all four types of equilibria: for $\bar{x}_1 < \bar{x} < \bar{x}_2$ we have case (a) of Figure 2; for $\bar{x} < \bar{x}_1$ or $\bar{x}_2 < \bar{x} < \bar{x}_3$, case (b); for $\bar{x}_3 < \bar{x} < c^*$, case (c1); and for $\bar{x} > c^*$ case (c2)[10]. This is not necessarily

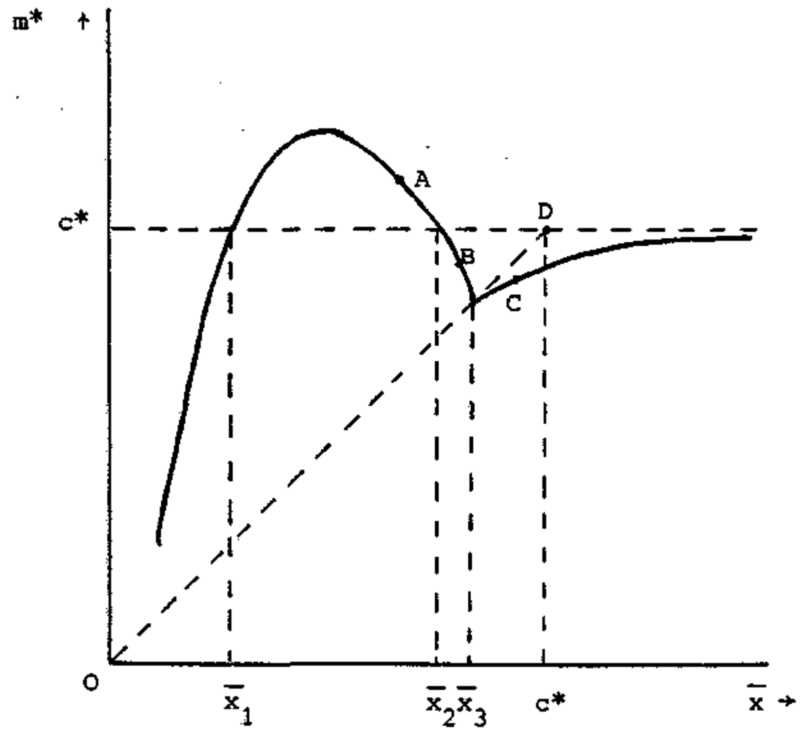

Figure 3. Money balances (m*) and availability ($\bar{x}$) under stochastic rationing.

true; e.g. for a lower degree of riskiness, case (a) may not occur (cf. Table 1). In the absence of rationing the farmer would choose point D. Money stocks are then independent of rationing. Under deterministic rationing the relation between m* and $\bar{x}$ is proportional: the farmer then chooses a point on OD.

Note that the curve does not pass through D: if $\bar{x}$ equals c* the farmer does not behave as in the absence of rationing. Because the ration is risky, $\bar{x}$ is not action-equivalent to c*, even if $\bar{x}$ = c* (the distance between the curve and point D is similar to a risk premium). Under stochastic rationing, the farmer will choose to hold more money than in the deterministic case if rationing is sufficiently severe ($\bar{x} < \bar{x}_3$). We have seen that case (a) arises iff $\xi(c^*,\bar{x})$ is positive, i.e. if $\bar{x}_1 < \bar{x} < \bar{x}_2$ in Figure 3. Money stocks are then in the long run (at points such as A) higher than in the absence of rationing. This implies a policy problem in two senses. First, when availability deteriorates, the negative effect on output is mitigated in the short run by the need to increase money stocks: the balance of payments position will appear to be better than it is in the long run. Conversely, if the government improves availability in order to boost exports output increases less than proportionately because it is optimal to decumulate money. Note that at A, $m^* > c^*$: farmers will reduce their money balances even if full availability were to be restored. The monetary overhang may make it very difficult for a government to break out of the rationing regime.

While the effect of improved availability on m^* is ambiguous, the effect on cash crop production is unambiguously positive. For the long run this result is trivial: since $h^* = \bar{x}$, production increases with availability. But, using (21) we can show that this is also true in the short run:

$$(40) \qquad \lambda_1 dh_2 + \lambda_2 d\varepsilon = 0$$

where

$$(41) \qquad \lambda_1 = (u_{11}+u_{12}+u_{21}+u_{22}) \int_{m_{t+1}}^{\infty} f(x)\ dx + \int_0^{m_{t+1}} u_{22}\ f(x)\ dx - u_1 f(m_{t+1}) < 0$$

$$(42) \qquad \lambda_2 = -(u_{11}+u_{21}) \int_{m_{t+1}}^{\infty} f(x)\ dx + 2u_1\, f(m_{t+1}) > 0$$

hence $\frac{dh_2}{d\varepsilon}$ is positive.

In this sense the analysis of section 3 is not affected by the introduction of money: it remains true that an initial fall in cash crop production is reinforced by its feedback effect, via the balance of payments, on availability.

As an example, consider the separable utility function:

$$(43) \qquad u = c^{1-R}/(1-R) + (t-h)^{1-\gamma}/(1-\gamma) \qquad (R,\gamma > 0;\ R,\gamma \neq 1)$$

which has constant relative risk aversion (of degree R) with respect to consumption. Since $u_1 = c^{-R}$ and $u_2 = -(t-h)^{-\gamma}$, c^* solves $c^R = (t-c)^{\gamma}$. Assume that ln x is normally distributed with mean μ and variance σ^2. Then $\bar{x} \doteq \exp(\mu+\sigma^2/2)$ and

$$(44) \qquad \int_m^{\infty} f(x)dx = F(\frac{\ln m-\mu}{\sigma}) = F(\frac{\ln\ (m/\bar{x})}{\sigma} + \frac{\sigma}{2})$$

where F is the complement of the normal distribution function:

$$(45) \qquad F(z) = \int_z^{\infty} \exp\ (-y^2/2)\ dy.$$

Substitution in (20) gives the relation between $\bar{x}$ and m in long-run equilibrium:

$$F\left(\frac{\ln (m^*/\bar{x})}{\sigma} + \frac{\sigma}{2}\right) = \frac{(m^*)^R}{[t-\min(\bar{x},m^*)]^\gamma}$$

For $R = 2$, $\gamma = 4$, $t = 10 + \sqrt{10}$ (and hence $c^* = 10$), the results are shown in Table 1, for three different values of σ. Column (2) depicts a case of moderate riskiness: $\sigma = 0.5$ implies that x exceeds 0.5 $\bar{x}$, $\bar{x}$ and 1.5 $\bar{x}$ with probability 0.87, 0.40 and 0.14 respectively.[11] In this case the relation between m* and $\bar{x}$ is as shown in Figure 3, with $\bar{x}_1 = 3.7$, $\bar{x}_2 = 9.3$ and $\bar{x}_3 = 9.4$. Hence if $\bar{x}$ falls more than 7 % short of c* then money stocks are higher than in the absence of risk ($m^* > \bar{x}$). In particular, if $\bar{x}$ is less than 93 % but more than 37 % of c*, we get the honeymoon result ($m^* > c^*$). E.g. for $\bar{x} = 6.2$, the optimal money stock is 20 % higher (and velocity almost 50 % lower) than in the absence of rationing. Finally, if rationing is very severe ($\bar{x} < 3.7$) then m* is again less than c*.
For a greater degree of riskiness[12] (e.g. $\sigma = 0.8$, as in column (3)), the peak in Fig. 3 becomes steeper (the same is true for an increase in the degree of risk aversion, R). Conversely, if riskiness decreases the function becomes flatter and, for sufficiently low σ, m* no longer exceeds c* for any value of $\bar{x}$: there is then no honeymoon (column (1)). As σ approaches zero, the function approaches the 45° line.

Table 1. Optimal money stock (m*), mean availability ($\bar{x}$) and riskiness (σ).

		m*	
$\bar{x}$	(1)	(2)	(3)
2.0	4.0	6.7	10.0
3.7	6.1	10.0	13.3
5.0	7.7	11.3	14.1
6.2	8.8	11.9	13.8
8.0	9.8	11.2	11.7
9.0	9.7	10.0	9.7
9.3	9.5	9.3	9.3
10.0	9.6	9.4	9.3
15.0	10.0	9.8	9.6
σ	0.2	0.5	0.8

The present model has two important implications for our analysis. First, adjustment to a regime in which farmers are rationed is not instantaneous. Secondly, the effect of rationing on cash crop production may be perverse during this adjustment process: it may be optimal to work harder during this process (and hence to produce more) than in long-run equilibrium.

It is instructive to determine whether these two results are due to the introduction of stochastic rationing or to the second way in which we changed the model in this section: the introduction of money.

To begin, let us note that, while we have remarked in passing that money is also kept for transaction purposes, in the analysis money has only one role: it is a store of value. Clearly, if the cash crop could be stored costlessly after the harvest then all of the preceding analysis for a monetary economy would apply to a barter economy as well, m now being interpreted as stored output rather than a money balance.[13] In this sense our results are not due to the introduction of money into the model but to the stochastic nature of rationing (given the existence of an asset which would be perfect in the absence of rationing). The role money (or a similar asset) plays in our results becomes clear if we introduce stochastic rationing into a model without assets (i.e. there is no money and neither good is storable). In that case, there is no dynamic adjustment left. The farmer reaches the new, stationary equilibrium instantaneously; $c_t^* = c^* = \bar{x}$ and $h_t^* = h^*$ is the solution to

$$\max_h \int_0^h u(x,h)\ f(x)\ dx + u(h,h) \int_h^\infty f(x)\ dx.$$

The first-order condition is:

$$(46) \qquad \int_0^h u_2(x,h)\ f(x)\ dx + [u_1\ (h,h) + u_2\ (h,h)] \int_h^\infty f(x)\ dx = 0.$$

In this model the farmer faces the risk that some of his work on the cash crop will be wasted. The crop is exchanged directly for consumption goods, but the labour input decision which determines the size of the crop (h) has to be taken before the amount of consumption goods available (x) is known. If h turns out to exceed x then the difference is simply wasted: it cannot be stored (either directly or indirectly as money).

Since the first term in (46) is negative, we have

$$(47) \qquad u_1\ (h^*,h^*) + u_2\ (h^*,h^*) > 0 = u_1\ (c^*,c^*) + u_2\ (c^*,c^*)$$

hence output is lower than in unrationed equilibrium

(48) $h^* < c^*$.

Note that the first-order condition implies

(49) $\phi(h^*,h^*) = 0$.

Assume that the honeymoon condition $\xi(c^*,\bar{x}) > 0$ is satisfied. This implies

(50) $\phi(c^*,\bar{x}) > 0$.

But ϕ is decreasing in both its arguments hence $h^* \leq \bar{x} < c^*$ is not possible, therefore

(51) $\bar{x} < h^* < c^*$.

Hence if the honeymoon condition is satisfied so that in the monetary model production converges to its long-run equilibrium value ($\bar{x}$) from above then in the barter model output is constant at a level above $\bar{x}$. This does not mean that the honeymoon is permanent. On the contrary: the government can, obviously, procure only as much of the crop as farmers are able to exchange for consumption goods so that (while production is higher) exports are permanently equal to $\bar{x}$. Hence in this case rationing is sufficiently severe, the ration is sufficiently risky and/or the farmer is sufficiently risk averse for the wastage of effort implied by $h^* > \bar{x}$ to be optimal. The role of money is now clear: it enables the farmer to adjust to an equilibrium in which this wastage is eliminated. We summarize our results as follows.

Theorem 2.

For $\bar{x} < c^*$, cash crop production (h^*) satisfies (for $t = 2,3,..$)

(a) in the absence of rationing: $h^*_t = c^*$

(b) in the case of deterministic rationing: $h^*_t = \bar{x} < c^*$ (all t)

(c) in the case of stochastic rationing (no money):
$\bar{x} < h^*_{t+1} = h^*_t < c^*$, for $\xi(c^*,\bar{x}) > 0$

(d) in the case of stochastic rationing (with money):
$\bar{x} < h^*_{t+1} < h^*_t < c^*$, for $\xi(c^*,\bar{x}) > 0$.

Exports are equal to $\bar{x}$ in case (c) and to h^*_t in the three other cases.

Hence the point about the model of this section compared with the earlier case of deterministic rationing in a barter model is not that it introduces the possibility of effort exceeding the mean value of the ration ($h^* > \bar{x}$): this would be true a fortiori in case (c). The point is rather that if farmers hold money then exports also exceed $\bar{x}$, but the difference disappears over time. The government enjoys a honeymoon in the sense that it effectively obtains the crop at a lower price during the adjustment process: the farmers' terms of trade deteriorate temporarily.
At the end of the previous section we remarked that the money balances built up by farmers in response to rationing, might make it very difficult to break out of the rationing regime. Figure 3 indicates that this could be true even if the government managed to restore full availability (and remove uncertainty) at a stroke. If the initial situation involves a money stock greater than c^* and farmers are convinced that the new one is at D, they will want to eliminate the excess money balance (m^*-c^*) and this affects output negatively. Unless aid donors are willing to finance the claim against the government which this monetary overhang represents fully, some sort of default is unavoidable. This might take the form of raising the price of the cash crop and the price of the consumption good in proportion. The producer price then remains constant in real terms, but desired money stocks rise since m^* is linear homogeneous in prices. If prices are raised sufficiently, farmers will want to continue to hold the accumulated money balances, even in unrationed equilibrium at point D. A currency reconstruction is an alternative form of default, which is formally equivalent[14].

5. Conclusion

Our first conclusion is that such an economy may, when exposed to a shock, end up in a regime in which farmers are rationed. The economy will not return to an unrationed equilibrium and, indeed, the balance of payments deficit will get larger after the initial shock. Secondly, raising the producer price for export crops will make matters worse. Thirdly, our theoretical analysis in section 4 established that stochastic rationing may have unexpected effects on money demand. The behaviour of farmers in response to rationing differs qualitatively, depending on the sign of $\xi(c^*,\bar{x})$ and of $\xi(\bar{x},\bar{x})$. For sufficiently severe rationing a liberalization attempt which ignores the monetary overhang caused by risk is doomed to fail.

Appendix

1. Existence. Consider the two-period problem:

$$\max_{c_t, h_{t+1}} W = u(c_t, h_{t+1}) + u(m_{t+1}, h_{t+1}) \int_{m_{t+1}}^{\infty} f(x)dx + \int_{o}^{m_{t+1}} u(x, h_{t+1}) f(x)dx$$

subject to:

(A1) $c_t \leq \bar{x} \quad (\lambda)$

(A2) $c_t \leq m_t \quad (\mu)$

(A3) $h_{t+1} \leq c^*(\nu)$

(A4) $c_t, h_{t+1} \geq 0$

(A5) $m_{t+1} = m_t + h_{t+1} - c_t$

(A6) m_t, h_t predetermined, positive and finite.

The objective function W is strictly concave, because of assumption (10); the constraints (A1), (A2), (A3) are convex, and the feasible set is bounded and non-empty (since $c_t = h_{t+1} = 0$ is feasible). Hence the Kuhn-Tucker conditions are sufficient for a unique optimum. It remains to show that the solution used in section 4 satisfies the Kuhn-Tucker conditions. From Figure 2 and theorem 1, m_t is positive for all t and (since $\bar{x}$ is positive) so is $c_t = \min(\bar{x}, m_t)$. In the paper, h_{t+1} was determined as the solution to $\phi(m_{t+1}, h_{t+1}) = 0$ hence

(A7) $W_{h_{t+1}} = \phi(m_{t+1}, h_{t+1}) - \nu = 0$

is satisfied with ν equal to zero. Also, condition (A3), which was not imposed in section 4 but which does not affect the solution, is satisfied. Since c_t is positive, the remaining Kuhn-Tucker condition is

(A8) $u_1(c_t, h_t) - u_1(m_{t+1}, h_{t+1}) \int_{m_{t+1}}^{\infty} f(x)\, dx = \lambda + \mu \geq 0.$

If the inequality in (A8) is strict then either (A1) or (A2) must be binding. Hence if $\lambda + \mu > 0$ then the assumption in section 4 that $c_t = \min(\bar{x}, m_t)$ is justified.

2. Proof of $c_t = \min(\bar{x}, m_t)$

Hence we wish to show that

$$\text{(A9)}\quad u_1(c_t,h_t) > u_1(m_{t+1},h_{t+1}) \int_{m_{t+1}}^{\infty} f(x)\,dx.$$

Substituting $\phi(m_{t+1},h_{t+1}) = 0$ from (20), this is equivalent to:

$$\text{(A10)}\quad u_1(c_t,h_t) + u_2(m_{t+1},h_{t+1}) \int_{m_{t+1}}^{\infty} f(x)\,dx + \int_{o}^{m_{t+1}} u_2(x,h_{t+1})\,f(x)\,dx > 0$$

Since $u_{21} \leq 0$:

$$\text{(A11)}\quad u_2(m_{t+1},h_{t+1}) \int_{m_{t+1}}^{\infty} f(x)\,dx \geq \int_{m_{t+1}}^{\infty} u_2(x,h_{t+1})\,f(x)\,dx.$$

Hence, if

$$\text{(A12)}\quad u_1(c_t,h_t) + \int_{o}^{\infty} u_2(x,h_{t+1})\,f(x)\,dx > 0$$

then (A9) is satisfied. In cases (a), (b) and (c1) we have: c_t, h_t, $h_{t+1} \leq c^*$ and $\bar{x} < c^*$. Hence in those cases:

$$\text{(A13)}\quad u_1(c_t,h_t) + u_2(\bar{x},h_{t+1}) > 0.$$

Since u_2 is assumed to be convex in x, Jensen's inequality gives:

$$\text{(A14)}\quad \int_{o}^{\infty} u_2(x,h_{t+1})\,f(x)\,dx \geq u_2(\bar{x},h_{t+1})$$

and (A12) follows from substitution of (A14) in (A13).
In case (c2) this result does not apply since then $\bar{x} > c^*$. But $u_{21} \leq 0$ implies:

$$\text{(A15)}\quad \int_{o}^{m_{t+1}} u_2(x,h_{t+1})\,f(x)\,dx \geq u_2(m_{t+1},h_{t+1}) \int_{o}^{m_{t+1}} f(x)\,dx.$$

In case (c2) either $m_{t+1} < m_t \leq c^*$ or $m_{t+1} = m_t = g(\bar{x}) < c^*$ (corollary): in either case $m_{t+1} < c^*$. In addition: m_t, c_t, h_t, $h_{t+1} \leq c^*$, hence:

$$\text{(A16)}\quad u_1(c_t,h_t) + u_2(m_{t+1},h_{t+1}) = u_1(c_t,h_t) + u_2(m_{t+1},h_{t+1}) \int_{m_{t+1}}^{\infty} f(x)\,dx + u_2(m_{t+1},h_{t+1}) \int_{o}^{m_{t+1}} f(x)\,dx \geq u_1(c^*,c^*) + u_2(c^*,c^*) > 0.$$

and (A10) follows from substitution of (A15) in (A16).
Hence (A9) is always satisfied: $\lambda + \mu$ is positive and hence $c_t = \min(\bar{x}, m_t)$.

Notes

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1 This linearity assumption is not essential to our argument, but it simplifies the algebra considerably.

2 The price change must be small in the sense that c^* (which will change with p) continues to be larger than p.

3 Our assumptions on the utility function do not guarantee this shape of the offer curve. It is, however, sufficient to assume that the cash crop supply function $q^*(p)$ is upward sloping. E.g. if we approximate this function locally by the iso-elastic function $q^* = \overline{q}\, p^{\eta}$ $(\eta > o)$, then, since $c^* = pq^*$,

$$\frac{dc}{dq} = p + q/q_p > 0$$

and

$$\frac{d^2c}{dq^2} = 1/q_p + (q_p^2 - qq_{pp})/q_p^3 = (1+1/\eta^2)/q_p > o.$$

Hence, for all p for which output is increasing in the producer price, the offer curve $q^*(c^*)$ is increasing and concave. For the special case of a Stone-Geary utility function $u = (c-\overline{c})^{\alpha}\ (f-\overline{f})^{\beta}\ (\ell-\overline{\ell})^{\gamma}$, it may be shown that this sufficient condition (q_p^* positive) is satisfied iff demand for the consumption good is income elastic.

4 Elsewhere, Bevan et al. (forthcoming), we have considered the urban economy in more detail, distinguishing five agents: the government, entrepreneurs in the formal and in the informal sector, wage earners and black marketeers.

5 If farmers consider the bad harvest as a transient phenomenon and realise that the decline in coffee production was the only reason for rationing then they would not change their behaviour at all and the economy would return to point B. More generally, if they expect to be able to buy at least OQ of the consumption good then they will be rationed in the next period, but the equilibrium will be stable: farmers will be able to buy as much as they expected. Hence a necessary condition for the contractionary adjustment process is that farmers' expectations are sufficiently pessimistic in the sense that $E(\overline{x}) < OQ$. Note that in a pure barter model the government would have to abandon its policy immediately after the bad harvest, either by exchanging less than JI for the crop (i.e. by lowering p) or by giving less than KL to urban consumers. This is not the case described in the text.

We implicitly assume that farmers hold money but only enough to finance their expected purchases of consumption goods until the next harvest. If these purchases are spread over time then they will find out that they are being rationed only in the course of the year, after they have exchanged their crop for money. The case where money is not just held for current transaction demand is considered in the next section.

6 An appeal to restrictions on third-order derivatives is common in the economics of risk. A well-known example is the effect of risk on savings: if the first-order derivative of a one-argument utility function is convex, then savings increase with riskiness.
If the utility function $u(c,h)$ is separable ($u_{21} = 0$), as in the numerical example below, then (10d) is satisfied. Separability is sufficient, but not necessary; (10d) is, e.g., also satisfied for a Stone-Geary specification $u = c^{\alpha}(t-h)^{1-\alpha}$ $(0<\alpha<1)$.

7 Note that, since $u_{11}<0$, $u_{22}<0$, $u_{12}\leq 0$, (10c) implies that c^* is positive.

8 Note that we do not assume $\bar{x}<c^*$. In the riskless case this assumption would be natural, but in the presence of risk the farmer's decisions will be affected by rationing even if $\bar{x}\geq c^*$.

9 The other predetermined variable (h_t) does not affect the solution since utility in the two periods is additive.

10 Note that the curve is drawn with a kink for $\bar{x}=\bar{x}_3$. At that point the first derivative is indeed not continuous. This is because the mapping changes from $m\rightarrow g(m)$ to $m\rightarrow g(\bar{x})$.

11 For $\sigma = 0.2$ these probabilities are 0.9996, 0.46, 0.02 and for $\sigma = 0.8$ they are 0.68, 0.34, 0.18.

12 We interpret riskiness in the sense of Rothschild and Stiglitz (1970, 1971). Hence an increase in riskiness is modelled as a mean-preserving spread: an increase of σ and a decrease of μ, keeping $\mu + \frac{1}{2}\sigma^2$ and hence $\bar{x}$ constant.

13 Note that, while the nominal prices of the cash crop and of the consumption good are fixed, neither money nor the cash crop is a perfect asset: rationing limits the convertibility of the asset into consumption goods.

14 The distributional consequences in the urban sector of various methods of sterilizing the monetary overhang (including the effects of a devaluation) are considered in Bevan et al. (forthcoming).

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