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QUALITATIVE DISCRETE MULTIPLE CRITERIA CHOICE MODELS IN REGIONAL PLANNING

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1. Introduction

Plan evaluation models have become a meaningful and operational tool in regional policy analysis. The methodology of plan evaluation however, has not demonstrated a static picture in the last decades, but rather a series of drastic changes. In the sixties, cost-benefit and cost-effectiveness analysis have gained a great deal of popularity. The impossibility to put justifiable price tags on various impacts of policy decisions has led in the beginning of the seventies to various adjusted evaluation methods, such as the expected value method, the planning balance sheet analysis and the goals-achievement analysis;

Later in the seventies multiple criteria analysis made a substantial contribution to a more appropriate evaluation methodology, especially because this approach did not need to make the stringent assumptions underlying a market-oriented (price-based) evaluation. One of the evident strong points of multiple criteria analysis was the fact that intangible aspects of decision-making (such as environmental decay, social inequality, etc.) could be taken into consideration. For a survey of these methods we refer to Nijkamp (1979, 1980), Rietveld (1980) and Voogd (1982).

While these methods provided a rigorous progress in operational plan evaluation analysis, it was, at the same time, also realized that in many practical choice and decision situations, various impacts were not only intengible, but also incommensurable, so that these impacts could not be measured by means of the usual cardinal metric system. This awareness has led to a new development in multiple criteria analysis, in which especially qualitative aspects have come to the fore. These qualitative aspects may relate to plan impacts or policy weights measured in an ordinal, binary or nominal sense.

The aim of the present is to provide a brief survey of such qualitative evaluation methods by pointing out some stronger and weaker points. On the basis of this survey, the need for a more justified qualitative evaluation method will be made clear. This new method, the so-called regime method, will be described in more detail, while also an empirical illustration of this method in the field of multiregional conflict analysis will be given.

2. Some Qualitative Multiple Criteria Methods

2.1 Introduction

The uncertainties regarding the assessment and political evaluation of effects of decisions in the area of regional, urban, environmental and physical planning have induced the development of a wide variety of qualitative multiple criteria evaluation methods. The main aim of all these methods is to provide a rational basis for solving discrete choice problems characterized by multiple (incommensurable) evaluation criteria. Usually, multiple criteria analysis is based on two kinds of input data, viz. an impact matrix and a set of political weights attached to the criterion effects.

If we assume the existence of I alternatives ($i=1, \ldots, I$) and J judgement criteria ($j=1, \ldots, J$), we obtain the following $I \times J$ impact matrix P :

$$P = \begin{array}{c} \\ 1 \\ \vdots \\ I \end{array} \overset{\quad 1 \ \cdots\cdots\cdots\cdots \ J}{\begin{bmatrix} & & \\ & p_{ij} & \\ & & \end{bmatrix}} , \qquad (1)$$

where p_{ij} represents the impact of alternative i on criterion j. In case of a qualitative evaluation problem, the elements of p_{ij} may be ordinal, binary or nominal in nature.

The set of weights provides information on the relative importance attached to the outcomes of the successive J criteria; they will be denoted as a vector $\underline{\lambda}$:

$$\underline{\lambda}' = [\lambda_1 \ , \ \ldots\ldots, \ \lambda_J]' \qquad (2)$$

Clearly, a qualitative evaluation problem may also imply that the vector $\underline{\lambda}$ does not contain cardinal tradeoffs, but ordinal or binary weights.

It should be noted that in many choice situations, $\underline{\lambda}$ is not unique. This may be caused by the existence of multiple decision agencies, multiple interest groups, cross-boundary decision problems, or political cautiousness in expressing explicit tradeoffs. In such cases, it may be more meaningful to construct a set of consistent <u>policy scenarios</u> that aim at representing hypothetical but feasible political weights so as to obtain a coherent frame of reference for the evaluation problem at hand (cf. Blair, 1979). Suppose the existence of N scenarios. Then the following scenario matrix Λ may be constructed :

$$\Lambda = \begin{array}{c} \\ 1 \\ \vdots \\ J \end{array} \overset{1 \;\cdots\cdots\cdots\; N}{\begin{bmatrix} & & \\ & \lambda_{jn} & \\ & & \end{bmatrix}} \; , \qquad (3)$$

where λ_{jn} indicates the weight attached to the jth criterion effect in the nth scenario.

It should be added that there are also alternative ways of dealing with uncertainties regarding $\underline{\lambda}$, for instance interactive multiple criteria methods (see Nijkamp and Spronk, 1981, Rietveld, 1980, and Spronk, 1981). If no information at all regarding political preferences exists, the most reasonable way is then to carry out an <u>unweighted</u> evaluation.
Now some qualitative multiple criteria methods will be discussed in more detail.

2.2 Dominance and Frequency Analysis

A dominance and frequency analysis is one of the simplest multiple criteria evaluation methods (see Van Delft and Nijkamp, 1977). A <u>dominance</u> analysis is based on a systematic investigation of the impact matrix P in order to identify the alternatives that provide - in regard to all criteria - better outcomes than remaining alternatives. Consequently, only the dominating alternatives need a further examination, while all others may be eliminated.

A <u>frequency</u> analysis goes one step further. The outcomes of the impact matrix are subdivided into 3 categories, high outcomes (H), intermediate outcomes (M) and low outcomes (L). In a similar way, the weights may be classified into: important (T) or unimportant (O). By systematically classifying all alternatives into the various combined impacts-weights classes, a <u>strength-weakness</u> analysis may provide insight into the 'strong' and 'weak' alternatives. This strength-weakness analysis can be extended towards a probabilistic - oriented approach by calculating the frequency of scores of the alternatives in the combined impacts-weights classes. The following frequency table has to be constructed :

	importance class T			importance class O		
	H	M	L	H	M	L
alternative 1	α					
alternative 2						
⋮						

Table 1. A frequency table for a multiple criteria analysis

Then element α in Table 1 indicates that alternative 1 has α times an outcome that falls into impact category H (high impact), while each of these α outcomes belongs to preference class T (important). In an analogous way, the scores of all alternatives in all combined impacts-weights classes can be interpreted. In this way also, a more refined dominance analysis can be applied, so that inferences can be drawn regarding the conditional probabilities that any of the alternatives scores best.

Though, no unpermitted mathematical operations are carried out in the dominance and frequency analysis, two problems inherent in this method have to be mentioned: 1) the transformation of information from P and $\underline{\lambda}$ into a limited number of size classes and importance classes is not always unambiguous, and 2) there is no guarantee that a single and umambiguous final solution can be identified. An obvious advantage of this method is of course, its simplicity.

2.3 Prioritization Analysis

The prioritization method developed by Saaty (1977) aims at assigning numerical values to weights in a qualitative multiple criteria analysis. The method takes for granted the availability of verbal statements regarding the relative importance of evaluation criteria. In more precise terms : this analysis assumes that for each pair of criteria (j, j') the decision-maker is able to indicate the relative importance of criterion j with respect to criterion j' . The responses of the decision-maker may be categorized as : equally important, slightly more important, etc. These responses may next be assigned a numerical value $b_{jj'}$ on a scale running from 1 to 9.

This leads to a the construction of a J x J paired comparison matrix B. This matrix is defined such that the diagonal elements are equal to 1, while the following symmetry condition holds :

$$b_{jj'} = 1/b_{j'j} \tag{4}$$

Next, this information contained in matrix B is used to assess the quantitative value of the elements λ_j from the weight vectors $\underline{\lambda}$. It should be noted that if the preference information in B were entirely consistent, the vector $\underline{\lambda}$ could be directly derived from any row of B. This would be based on the following consistency condition :

$$B \underline{\lambda} = \underline{\lambda} J \tag{5}$$

where J is the number of criteria.

However, as noticed in the literature on paired comparisons (see Kendall, 1970, e.g.) usually many inconsistencies emerge in paired comparisons experiments due to the inability of the human mind to comprehend completely a complex choice problem. Consequently, usually each row of B would lead to a (slightly) different weight vector $\underline{\lambda}$. Then the prioritization method aims at providing the best approximation of $\underline{\lambda}$ by means of the largest eigenvalue of B. This is evidently a fairly straightforward and simple operation. The approximated cardinal values of $\underline{\lambda}$ can next be used in further steps of an evaluation analysis (for instance, an expected value method) in order to infer final conslusions regarding the best ranking of alternatives.

This method has been applied to several plan evaluation problems (see Blair, 1979, and Johnson, 1980). Some critical comments on the prioritization method may be made, however. In the case of a large number of criteria, a paired comparison method leads to unmanageable amounts of outcomes, so that then serious inconsistencies may arise. Next, the implicit assumption of this method is a linear utility function; this is evidently not always a justifiable assumption. Furthermore, the estimated values of $\underline{\lambda}$ depend somewhat on the length of the scale used in the B matrix. Finally, the method of largest eigenvalues is only one out of many methods to derive an appropriate vector of weights from B. In this regard, a sensitivity analysis would be a meaningful complement to the eigenvalue method (see for an extensive critical review, Johnson, 1980).

2.4 Ordinal Concordance Analysis

Concordance analysis is a popular multiple criteria method that is based on a pairwise comparison of alternatives (see, among others, Van Delft and Nijkamp, 1979, Roy, 1972, and Rietveld, 1980).

The central concept in a concordance analysis is the so-called concordance set $C_{ii'}$; this is the set of all evaluation criteria for which alternative i in the impact matrix P is at least equally attractive as alternative i' . Clearly, this set can be determined irrespective of the specific scale of measurement of the impact matrix.

Another and related important concept is the concordance index $c_{ii'}$. This index represents the extent to which alternative i is better than alternative i' , only in so far as the pertaining criteria are included in the concordance set $C_{ii'}$. Usually, this index is defined as the sum of weights attached to the criteria included in $C_{ii'}$, i.e.

$$c_{ii'} = \sum_{j \in C_{ii'}} \lambda_j \tag{6}$$

Next, one may attempt to identify a dominating alternative by employing graph theory, threshold values or relative dominance indicators.

In an analogous way, one may define a discordance set and a discordance index. The discordance index reflects the extent to which alternative i is worse than i' , if the pertaining criteria belong to the discordance set. Instead of using weights in this index (cf. (6)), the corresponding relative pairwise differences from the impact matrix are then taken into consideration. By combining the results from the concordance and discordance approach, final inferences on the ranking of alternatives may be made.

In the case of ordinal information, the calculation of the concordance index and the discordance index is more difficult, as numerical operations like (6) are not permitted. Sometimes the use of graph theory may be helpful, while also a transformation of qualitative information to size and importance classes (see Section 2.2) may be made. In the latter case, the concordance and discordance indices may be based on the frequencies that - for each importance class - alternatives score with respect to a certain size of class of impacts (see Table 1). (See also Van Delft and Nijkamp, 1977.)

Despite the wide range of applications of the concordance analysis, some limitations have to be mentioned. First, a unique solution is not guaranteed, as the final ranking of alternatives is based on both the concordance and discordance index. Next, in the case of qualitative information it is not easy to develop a variant of the concordance analysis which is not based on unpermitted operations.

2.5 Multidimensional Scaling Analysis

Scaling analysis is a mathematical statistical technique developed mainly in the field of psychometrics. Though various kinds of scaling techniques do exist (such as multidimensional and homogeneous scaling analysis), each scaling technique aims at transforming a qualitative data input (mainly ordinal data) into a cardinal input of lower dimensionality. In a sense, a scaling technique may be regarded as a qualitative principal component analysis. In the recent past, scaling methods have found many applications in the area of planning, regional economics and geography (see also Nijkamp, 1979, and Voogd, 1982).

It is evident that several concepts from scaling analysis may also be applicable to qualitative multiple criteria analysis. Various approaches can be imagined in this case. In the first place, one may use a scaling technique in order to transform a qualitative impact matrix into a cardinal matrix with less dimensions. Then the cardinal configuration of the initial qualitative matrix provides a metric picture of the Euclidean distances both between the alternatives and between the effects.

Secondly, one may also apply a scaling analysis jointly to a qualitative impact matrix and a qualitative weight vector. In that case, both the impacts and the weights have to be transformed into a cardinal metric scale. Though this is mathematically fairly difficult, one may ultimately arrive at cardinal results for both impacts and weights (see also Nijkamp and Voogd, 1979). The final result of this analysis is that one is able to indicate precisely which rank order of alternatives is consistent with a certain rank order of ordinal weights.

The scaling method also deserves a critical evaluation. A major advantage of this method is that no unpermitted operations on ordinal numbers are carried out, but the method itself is fairly complex and not easy to explain to practitioners. In addition, a unique solution is not guaranteed; only a conditional statement, viz. which rankings are in agreement with a certain weight structure, can be made. Nevertheless, this method is methodologically more elegant than the abovementioned techniques.

2.6 Permutation Analysis

The permutation analysis is also suitable for ordinal information in both P and $\underline{\lambda}$ (see Mastenbroek and Paelinck, 1976). This method addresses especially the question: which rank order of alternatives is (after a series of permutations) in harmony with the ordinal information contained in P and $\underline{\lambda}$?

Assume again I alternatives. Then the total number of possible permutations is equal to $I!$ Each permutation will be numbered as $\rho (\rho=1, \ldots, I!)$ In the permutation analysis, each rank order from the permutations is confronted with the ordinal information contained in each of the J columns of the impact matrix. Then Kendall's rank correlation coefficient is used in order to calculate the statistical correlations between the $I!$ rank orders and the J columns of P. This leads altogether to $I! \times J$ rank correlation coefficients denoted by τ_j^ρ. Clearly, a certain permutation ρ (i.e., rank order of alternatives) is more attractive as the value of τ_j^ρ is higher.

However, it should also be taken into account that the weight vector $\underline{\lambda}$ contains additional information. If $\underline{\lambda}$ were cardinal, an expected value method might be applied in order to calculate the maximum weighted value of τ_j^ρ. In the case of ordinal information on $\underline{\lambda}$, the following programming model is specified :

$$\begin{cases} \max\limits_{\rho} ! \sum_{j=1}^{J} \lambda_j \tau_j^\rho \\ \text{subject to} \\ \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_J \\ \sum_{j=1}^{J} \lambda_j = 1 \end{cases} \tag{7}$$

The constraints in (7) reflect the ordinal information about the λ_j and are used as follows. First , all extreme points that are in agreement with the constraints on the weights are generated : (1, 0, ..., 0), $(\frac{1}{2}, \frac{1}{2}, 0, \ldots, 0)$, $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, \ldots, 0)$, ..., $(\frac{1}{J}, \frac{1}{J}, \ldots, \frac{1}{J})$. Next a set of combinations of weights reflecting interior points of the above-mentioned extreme area are generated. Then one may calculate for each of these combinations which permutation ρ yields the maximum value of the objective function in (7). On the basis of this information, one may try to identify a rank order that is in agreement with the ordinal information from P and $\underline{\lambda}$.

Here also some critical remarks are to be made. In the first place, if the number of alternatives is fairly large, the number of permutations will drastically increase (for instance, if I=10, then $I! = 3{,}628{,}800$), so that the method becomes unmanageable. Secondly, the weights λ_j are dealt with in a rather unusual way by relating them to the rank correlation coefficients τ_j^ρ instead of to the criterion impacts. This implies that a decision-maker will have great difficulties in understanding the way in which the weights indicated by him play a role in the whole evaluation. A final disadvantage is that from a mathematical and computational point of view, this method is rather cumbersome to explain, which may of course, affect its acceptability in planning practice.

2.7 Regime Analysis

The regime analysis is a recently developed qualitative multiple criteria analysis. This method has initially been developed in the area of soft econometrics (see Nijkamp and Rietveld, 1982). It was based on a combination of Kendall's paired comparison method for ordinal data and logit analysis. After some successful applications in the field of soft econometric explanatory modelling, it turned out that some ideas from the regime method might also be used in the field of qualitative multiple criteria analysis. This method avoids unpermitted calculations with ordinal numbers, while the various steps to be undertaken are in principle relatively simple. Finally, this method is in principle able to arrive at unambiguous final conclusions regarding the best ranking of alternatives. The details of this method will be given in the next section.

3. Regime Analysis

3.1 Introduction

The reason why ordinal data is difficult to work deal in evaluation studies, is that usual numerical operations such as addition and multiplication cannot be applied. Consider for example, the following decision problem:

$$P = \begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 2 & 1 & 3 \\ 3 & 1 & 2 & 1 \end{bmatrix} \qquad (8)$$

$$\underline{\lambda}' = (1 \quad 4 \quad 2 \quad 3) \qquad (9)$$

The impact matrix P contains the ranking of 3 alternatives according to 4 criteria. A rank number 3 indicates the best outcome, while a rank 1 is assigned to the worst outcome per criterion. The elements of the weights vector $\underline{\lambda}$ indicating the importance of the various criteria can be interpreted in a similar way. When one ignores the ordinal nature of this data, one can simply compute the attractiveness of the various alternatives by adding the products of plan impacts p_{ij} and weights λ_j. This is obviously a rather unsophisticated treatment of ordinal data.

A better way to proceed with ordinal data in evaluation studies is to focus on differences between alternatives by means of pairwise comparisons. The way such pairwise comparisons can be carried out in order to arrive at conclusions about the relative attractiveness of alternatives is the core idea of regime analysis and can be sketched as follows.
Assume (for the ease of presentation) that cardinal values for $\underline{\lambda}$ are known: (.20 .35 .22 .23). When we consider the pair of alternatives (1, 2) we note that alternative 2 is preferred to 1 for criteria 1 and 4. The reverse holds true for the criteria 2 and 3. Therefore we propose to use $\mu_{1,2} = \lambda_2 + \lambda_3 - \lambda_1 - \lambda_4$ as an indicator for the differences in attractiveness between alternatives 1 and 2. In this case $\mu_{1,2}$ is positive viz.(.14), this indicates the the second alternative is less attractive than the first one. Note that - due to the ordinal nature of the information implied by P - in the indicator $\mu_{1,2}$ no attention is paid

to the size of the difference between the impacts of alternatives; it is only the sign of the difference that is taken into account. A final ranking of alternatives can be determined on the basis of the index $\mu_{ii'}$ for all pairs of alternatives i, i' .

A problem which has to be faced in regime analysis is that - in contrast with the assumption made above - only ordinal information on the weights $\underline{\lambda}$ is available. In that case, it is not possible to compute values of the indices $\mu_{ii'}$ as proposed above. To solve this problem, we have developed a method to infer conclusions regarding the values the indices $\mu_{ii'}$ can adopt, given ordinal information about the weights. This method can be considered as the heart of the regime analysis.

After this rather informal introduction to the main idea of regime analysis, we will present a more elaborate description in the next sections.

3.2 The Regime Matrix

In regime analysis we focus on the sign of differences between impacts of alternatives. Consider two alternatives i and i' . The sign of the difference between these alternatives according to criterion j can be described by $r_{ii',j}$, which is defined as follows:

$$\begin{cases} r_{ii',j} = 1 & \text{if} \quad p_{ij} > p_{i'j} \\ \phantom{r_{ii',j}} = -1 & \text{if} \quad p_{ij} < p_{i'j} \end{cases} \tag{10}$$

For example, on the basis of (3.1) we may conclude that $r_{12,1} = -1$. When two alternatives i and i' are compared according to all criteria j=1, ..., J, we may construct a regime vector $\underline{r}_{ii'}$:

$$\underline{r}_{ii'} = (r_{ii',1} \, , \, r_{ii',2} \, , \, \ldots, \, r_{ii',J})' \tag{11}$$

For example, given (3.1), we find for the regime vector pertaining to alternatives 1 and 2: (-1 +1 +1 -1). One arrives at a regime matrix

R by taking together the regime vectors for all pairs of alternatives:

$$R = \begin{bmatrix} \underline{r}'_{12} \\ \vdots \\ \underline{r}'_{1I} \\ \\ \underline{r}'_{21} \\ \vdots \\ \underline{r}'_{2I} \\ \vdots \\ \vdots \\ \underline{r}'_{I1} \\ \vdots \\ \underline{r}'_{I,I-1} \end{bmatrix} \qquad (12)$$

Thus we find for the regime matrix based on the impact matrix (8):

$$R = \begin{bmatrix} -1 & +1 & +1 & -1 \\ -1 & +1 & +1 & +1 \\ +1 & -1 & -1 & +1 \\ -1 & +1 & -1 & +1 \\ +1 & -1 & -1 & -1 \\ +1 & -1 & +1 & -1 \end{bmatrix} \qquad (13)$$

Obviously, the arrays in R are not independent from each other. One half of the number of arrays can be deduced from the other half, because $\underline{r}'_{ii'} = -\underline{r}'_{i'i}$. The regime matrix is simply a transformation of the impact matrix P. It is not difficult to see that no information is lost by this transformation: given R it is possible to determine the underlying matrix P.

3.3 <u>An Index for Differences in Attractiveness of Alternatives</u>

As mentioned in Section 3.1 we propose to use

$$\mu_{ii'} = \sum_j \lambda_j r_{ii',j} \qquad (14)$$

as an index for the difference in attractiveness of alternatives. Since there is only ordinal information available on the λ_j, it is not possible to compute unique values of $\mu_{ii'}$. This does not imply that no conclusions can be inferred about the $\mu_{ii'}$'s, however. Consider the example, the first regime in (13). Suppose that we know that the weights λ_j satisfy (9), so that:

$$\begin{cases} \lambda_2 \geq \lambda_4 \geq \lambda_3 \geq \lambda_1 \geq 0 \\ \sum_j \lambda_j = 1 \end{cases} \tag{15}$$

For these weights we find that $\mu_{1,2} = -\lambda_1 + \lambda_2 + \lambda_3 - \lambda_4$ is non-negative in all cases, which means that on the basis of a pairwise comparison, alternative 1 is preferred to 2. In a similar way it can be shown that given (15), alternative 1 is preferred to alternative 3, and that alternative 2 is preferred to 3. Thus we arrive at a transitive rank order of alternatives.

It is not possible to arrive at such definite conclusions for all rankings of weights, however. If, instead of (15), we would assume that

$$\begin{cases} \lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4 \geq 0 \\ \sum_j \lambda_j = 1 \end{cases} \tag{16}$$

it is not difficult to see that the first regime may give rise to both negative and positive values of μ's. For example, if $\underline{\lambda}' = (.40\ .35\ .20\ .05)$, $\mu_{1,2}$ is positive, whereas for $\underline{\lambda}' = (.45\ .30\ .15\ .10)$, $\mu_{1,2}$ is negative. Therefore, the corresponding regime (+1 -1 -1 +1) is called a critical regime.

Critical regimes give rise to difficulties, since no definite conclusion can be drawn about the sign of the pertaining index μ. The main idea of regime analysis is to circumvent these difficulties by partitioning the set of feasible weights so that for each subset of weights a definite conclusion can be drawn about the sign of the index μ. This is the subject of the following section.

3.4 Partitioning the Set of Feasible Weights

For the ease of presentation we will first consider the case of four criteria. Next, we will indicate how a partitioning can be achieved for an arbitrary number of criteria.

Let us assume that (16) is the ordinal information available about weights. The set of weights satisfying (16) will be denoted as S. This set can be represented as a convex polyhedron with extreme points A, B, C, D :

$$\begin{cases} A : (1, 0, 0, 0) \\ B : (\frac{1}{2}, \frac{1}{2}, 0, 0) \\ C : (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0) \\ D : (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}) \end{cases} \qquad (17)$$

A partitioning of S can be arrived at in three steps.

1. Identify the critical regimes. Then we have to check for all regimes whether μ as defined in (14) may assume both negative and positive values given that $\underline{\lambda}$ is an element of S. The total number of regimes to be examined is $2^4 = 16$.

   The number of critical regimes appears to be equal to four:

$$\begin{bmatrix} +1 & -1 & -1 & +1 \\ -1 & +1 & +1 & -1 \\ +1 & -1 & -1 & -1 \\ -1 & +1 & +1 & +1 \end{bmatrix} \qquad (18)$$

   Note that once we know that $\underline{r}$ is a critical regime, also $-\underline{r}$ is a critical regime. Thus the number of critical regimes is even.

2. Characterize the subsets of S by means of the structure of the critical regimes. The four critical regimes give rise to two <u>critical equations</u> :

$$\begin{cases} f_1 (\underline{\lambda}) = \lambda_1 - \lambda_2 - \lambda_3 + \lambda_4 = 0 \\ f_2 (\underline{\lambda}) = \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4 = 0 \end{cases} \qquad (19)$$

   Note that the coefficients of the critical equations have been derived from the critical regimes in (18).

The following subsets of S can distinguished by means of these equations

$$\begin{cases} S_1 = S \cap \{\underline{\lambda} \mid f_1(\underline{\lambda}) > 0 \text{ and } f_2(\underline{\lambda}) > 0\} \\ S_2 = S \cap \{\underline{\lambda} \mid f_1(\underline{\lambda}) > 0 \text{ and } f_2(\underline{\lambda}) < 0\}. \\ S_3 = S \cap \{\underline{\lambda} \mid f_1(\underline{\lambda}) < 0 \text{ and } f_2(\underline{\lambda}) < 0\} \\ S_4 = S \cap \{\underline{\lambda} \mid f_1(\underline{\lambda}) < 0 \text{ and } f_2(\underline{\lambda}) > 0\} \end{cases} \tag{20}$$

An examination of $S_1, \ldots, S_4$ reveals that S_4 is empty, so that ultimately three relevant subsets remain.

3. Identify the extreme points of the subsets found in the second step. The subsets S_1, S_2 and S_3 are convex polyhedra. The extreme points of the polyhedra can be determined graphically in case of 4 criteria. In addition to the four points of (17) we find:

$$\begin{cases} E : (\frac{1}{2}, \frac{1}{4}, \frac{1}{4}, 0) \\ F : (\frac{1}{2}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}) \end{cases} \tag{21}$$

The characterization of S_1, S_2 and S_3 by means of the extreme points can be found in Table 2.

subset	extreme points	centroid	relative size
S_1	A, B, E, F	(30/48 11/48 5/48 2/48)	.50
S_2	B, D, E, F	(21/48 14/48 8/48 5/48)	.25
S_3	B, C, D, E	(19/48 16/48 10/48 3/48)	.25

Table 2. Subsets of weights in case of four criteria.

In the table we also present the centroid of the polyhedra, computed as the mean of the extreme points. In the last column of the table we have indicated the relative size of the subsets. Each figure in this column is calculated as the absolute value of the determinant of the matrix composed of the coordinates of the extreme points of each subset, divided by the determinant of (17). We note that the subsets are not equally large. If we assume that the weights are uniformly distributed in S, we may conclude that the probability that $\underline{\lambda} \in S_1$, S_2 or S_3 is equal to .50, .25 and .25, respectively.

We will subsequently indicate how the relevant subsets can be found given an arbritrary number of criteria.

1. Determine the critical regimes by examining the following sets of constraints:

$$\begin{cases} \lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_J \geq 0 \\ \sum_j \lambda_j = 1 \\ \sum_j r_j \lambda_j > 0 \end{cases} \tag{22}$$

and

$$\begin{cases} \lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_J \geq 0 \\ \sum_j \lambda_j = 1 \\ \sum_j r_j \lambda_j < 0 \end{cases} \tag{23}$$

where r_j may assume values +1 and -1. If both (22) and (23) contain feasible solutions, the pertaining values for r_j characterize a critical regime.

2. Assume that in the first step 2 L critical regimes have been determined. These regimes give rise to L critical equations:

$$f_l\ (\underline{\lambda}) = \sum_j r_{lj} \lambda_j = 0 \qquad l = 1, \dots, L \tag{24}$$

Hence, 2^L potential subsets of S can be generated:

$$\begin{cases} S_1 = S \cap \{ \underline{\lambda} \mid f_1(\underline{\lambda}) > 0,\ f_2\ (\underline{\lambda}) > 0, \dots, f_L\ (\underline{\lambda}) > 0\} \\ S_2 = S \cap \{ \underline{\lambda} \mid f_1(\underline{\lambda}) > 0,\ f_2\ (\underline{\lambda}) > 0, \dots, f_L\ (\underline{\lambda}) < 0\} \\ \text{etc.} \end{cases} \tag{25}$$

For each of these S_l one has to examine whether it contains feasible solutions. Thus one arrives at $M \leq 2^L$ non-empty subsets.

3. The extreme points of the M polyhedra have to be generated by means of an appropriate algorithm (see for example Zeleny, 1974). Once the extreme points are known, the relative size of the subsets can be found by computing determinants of the pertaining matrices.

The partitioning method may give rise to difficulties when the number of criteria to be determined is rather high. It appears that as long as J does not exceed 7, the desired results can be found by means of analytical methods without making use of computerized algorithms.

We report some results for J = 2, 3 and 5. When J=2, no partitioning is necessary to arrive at definite conclusions concerning the sign of μ.
When J=3, one critical equation is found, giving rise to two subsets of S. These sets have been presented in Figure 1. For the relative size of S_1 and S_2, we find the values $\frac{3}{4}$ and $\frac{1}{4}$, respectively.

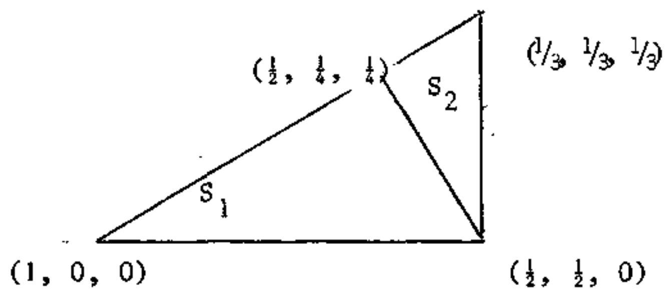

Figure 1. Partitioning the set of feasible weights in case of three criteria.

When J=5, six critical equations can be found, giving rise to seven subsets of S. The extreme points arising in this case have been summarized in Table 3.

A (1 0 0 0 0)	F ($\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{4}$ 0 0)
B ($\frac{1}{2}$ $\frac{1}{2}$ 0 0 0)	G ($\frac{1}{2}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ 0)
C ($\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{3}$ 0 0)	H ($\frac{1}{2}$ $\frac{1}{8}$ $\frac{1}{8}$ $\frac{1}{8}$ $\frac{1}{8}$)
D ($\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ 0)	I ($\frac{1}{3}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$)
E ($\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$)	J ($\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$)
	K ($\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{8}$)

Table 3. Extreme points in case of five criteria.

The seven subsets of weights related to J=5 are described in Table 4. As indicated in the last column, the relative size of the subsets varies considerably.

subset	extreme points	relative size
S_1	A, B, F, G, H	.31
S_2	B, F, G, H, I	.10
S_3	B, F, G, I, D	.21
S_4	B, F, I, D, K	.16
S_5	B, F, C, D, K	.16
S_6	B, I, D, K, J	.05
S_7	C, D, K, J, E	.01

Table 4. Subsets of weights in case of five criteria.

3.5 Determining a Final Ranking of Alternatives

Once the partitioning of the weights set has been achieved, the further application of the regime method is straightforward. For each subset of S it is possible to indicate umambiguously the sign $\mu_{ii'}$ for each pair of alternatives. Let $\mu_{ii'}$ be defined as follows:

$$\begin{cases} \upsilon_{ii'} = +1 & \text{if } \mu_{ii'} > 0 \\ \upsilon_{ii'} = -1 & \text{if } \mu_{ii'} < 0 \end{cases} \qquad (26)$$

Then a pairwise comparison matrix V can be constructed consisting of elements equal to +1 or −1, and with zeros on the main diagonal.

A final ranking of alternatives can be achieved on the basis of V in several ways. A simple way is to use the row totals of V as indicators for the overall attractiveness of alternatives. Thus a final ranking can be achieved on the basis of:

$$s_i = \sum_{i'} \upsilon_{ii'} \tag{27}$$

This approach will be illustrated by means of the impact matrix P as defined in (8) in combination with the ordinal information on weights contained in (16) Consider the subset S_1 as defined in Table 1. Take an interior point of the subset (e.g., the centroid).

Determine the sign of $\mu_{ii'}$ for all regimes occurring in the regime matrix R as defined in (16). Thus we find for the pairwise comparison matrix V_1 :

$$V_1 = \begin{bmatrix} 0 & -1 & -1 \\ +1 & 0 & -1 \\ +1 & +1 & 0 \end{bmatrix} \tag{28}$$

On the basis of V_1 we may conclude that alternative 3 is preferred to 2, which in turn is preferred to 1.

For the two other subsets of weights we find:

$$V_2 = \begin{bmatrix} 0 & -1 & +1 \\ +1 & 0 & -1 \\ -1 & +1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & +1 & +1 \\ -1 & 0 & -1 \\ -1 & +1 & 0 \end{bmatrix} \tag{29}$$

The second pairwise comparison matrix does not give rise to a definite ranking of alternatives. On the basis of the third matrix we may conclude that alternative 1 is preferred to 3, which is again preferred to 2.

Given the probability of occurrence of S_1, S_2 and S_3 (.50 .25 and .25, respectively) the results of the regime analysis can be summarized in a rank order frequency matrix (see Table 5). This matrix contains the probability that alternative i has rank number k for all i and k. From the matrix we may infer, among others, that - given the information that $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4 \geq 0$ - the probability that the third alternative achieves the highest rank is 58%. The probability that this alternative achieves the lowest rank is small (8%), so that we may conclude that alternative 3 is a good candidate for the final selection.

	rank 1	2	3
alternative 1	.58	.08	.33
2	.33	.58	.08
3	.08	.33	.58

Table 5. Rank order frequency matrix.

3.6 Implementation of Regime Analysis

The most intricate part of regime analysis is obviously the determination of a partitioning of the weights set as described in Section 3.4.

It is important to note that it is not necessary to determine a partitioning again and again for each individual evaluation problem. For example, once the information contained in Table 2 is available, it can be used for all decision problems with ordinal data on P and $\underline{\lambda}$, given 4 criteria.

Let us assume that a partitioning has been determined for various numbers of criteria and that the results are stored in a manual. More specifically, we will assume that the manual contains for each J: a) the number of subsets $S_1, \ldots, S_M$, b) an interior point $\underline{\lambda}_m$ of each subset (e.g. the centroid) m=1, ..., M, and c) the relative size q_m of each subset ($\sum_m q_m = 1$).

Given the manual, a regime analysis of an arbitrary ordinal impact matrix P of size (IxJ) and an ordinal weights vector $\underline{\lambda}$ with J elements can be carried out as follows (see also Figure 2).

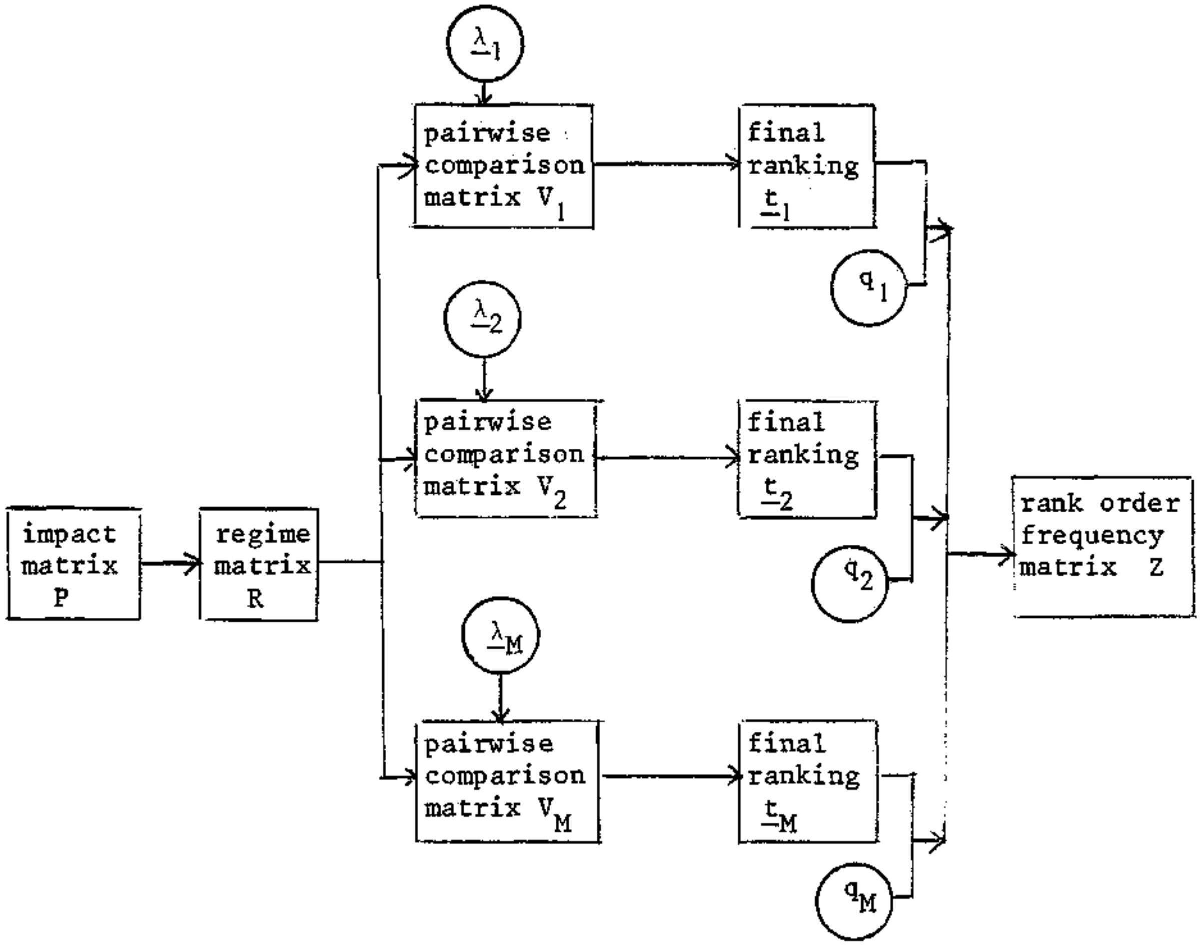

Figure 2. Schematic representation of regime analysis.

1. Given that the number of criteria is equal to J, take from the store the weights vectors $\underline{\lambda}_1, \ldots, \underline{\lambda}_M$ and the relative proportions $q_1, \ldots, q_M$. Note that the weights vectors $\underline{\lambda}_m$ are based on the assumption that $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_J \geq 0$. They should be made in agreement with the ordinal information contained in $\underline{\lambda}$ by the appropriate permutations.

2. Construct the regime matrix R with elements $r_{ii'j}$, on the basis of the impact matrix P by means of (12).

3. Construct for each $\underline{\lambda}_m$ (m=1, ..., M) a pairwise comparison matrix V_m by determining the sign of

$$\mu_{ii'} = \sum_j \lambda_{mj} r_{ii',j} \qquad (30)$$

for all pairs of alternatives i and i' .

4. Determine for each V_m a final ranking of alternatives $\underline{t}_m$ by means of (27)

5. Synthesize the results by constructing a stochastic ranking matrix Z on the basis of $\underline{t}_m$ and the relative proportions q_m (m=1, ..., M).

To what extent can the regime analysis be carried out for relatively large numbers of criteria and alternatives? A large number of criteria (say more than 10) will give rise to difficulties when partitioning the weights set: the potential number of relevant subsets may become excessively large (cf. Section 3.4).

A large number of alternatives (say 30) does not give rise to difficulties, however, since all matrices indicated in Figure 2 remain of moderate size.

3.7 Extensions

The regime method can be extended in several directions. We will subsequently discuss the following cases:

1. The information on weights is less specific than assumed thus far.
2. The information on weights is more specific than assumed thus far.
3. The information on impacts is less specific than assumed thus far.
4. The information on impacts is more specific than assumed thus far.

1. An example of less specific information on weights arises in case of an incomplete ranking of weights. One may know, for example, that $\lambda_1 \geq \lambda_2$ and $\lambda_1 \geq \lambda_3$, but it may be impossible to say whether $\lambda_2 \geq \lambda_3$ or $\lambda_2 \leq \lambda_3$. In such a case the regime analysis can simply be carried out for all possible rankings: $\lambda_1 \geq \lambda_2 \geq \lambda_3$ and $\lambda_1 \geq \lambda_3 \geq \lambda_2$.

2. In addition to a ranking of weights, more information may be available on weights. For example, one may know that $\lambda_1 \geq \lambda_2 + \lambda_4$. Information of this kind can be used to delete some of the subsets of feasible weights.

3. In case of an incomplete ranking of impacts one may proceed in the same way as when the ranking of weights is incomplete. If one knows for example, that $p_{11} \geq (p_{21}, p_{31}) \geq p_{41}$, one can carry out a regime analysis twice: a first time for (4, 3, 2, 1) and a second time for (4, 2, 3, 1) in the impact matrix.

   Another way to deal with incomplete rankings is to redefine the elements of the regime matrix as follows:

$$\begin{cases} r_{ii',j} = 1 & \text{if } p_{ij} > p_{i'j} \\ r_{ii',j} = 0 & \text{if } p_{ij} = p_{i'j} \\ r_{ii',j} = -1 & \text{if } p_{ij} < p_{i'j} \end{cases} \tag{31}$$

   In this case the number of critical equations and hence the number of subsets found in the partitioning of the weights set will be larger than in case of a complete ranking of impacts.

4. A situation which often occurs in evaluation studies is that for some criteria ordinal information is available, and for other criteria cardinal information. In this case, the use of the standard version of regime analyses is less satisfactory since not all available information is used. Then a better way to deal with cardinal impacts for certain criteria j is to redefine the elements $r_{ii',j}$ of the regime matrix for the pertaining criteria, so that the magnitude of the difference between p_{ij} and $p_{i'j}$ can be taken into account. This requires a standardization of the impacts p_{ij}. A possible way to proceed is:

$$r_{ii',j} = (p_{ij} - p_{i'j})\, c_j \tag{32}$$

   where c_j is equal to :

$$c_j = 1 \;/ \sum_{i,i'} | p_{ij} - p_{i'j} | \tag{33}$$

This standardization has the effect that the sum of the absolute values in each colume of the regime matrix is equal.

An important consequence of (32) is that the partitioning of the weights set carried out in Section 3.4 is no longer of use. In principle, it is possible to determine a partitioning in accordance with each arbitrary regime matrix, but this is not attractive since it may give rise to much work which is only useful for one particular case. Therefore, a more appropriate approach is to make use of numerical methods. For example, assume that the weights are uniformly distributed between the constraints ($\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_J \geq 0$). Use a random generator to produce a series of weights vectors $\underline{\lambda}$ which are in agreement with the ordinal information. Compute a pairwise comparison matrix for each $\underline{\lambda}$ and summarize the results by means of a rank order frequency matrix. When the number of weight vectors is large enough, the last matrix is a good basis for the selection of an alternative.

4. An Evaluation of the Antwerp-Rhine Canal

In this section regime analysis will be used for the evaluation of alternative waterways between Antwerp and the river Rhine. The connection between Antwerp and the river Rhine has been a source of conflict between Belgium and the Netherlands for many decades. A good connection with the river Rhine is important for Belgium since this makes large industrial centres in Germany accessible for ships from the port of Antwerp. The problem with the Antwerp-Rhine Canal is that Belgium is the main country to benefit from it, while nearly the whole trajectory is on Dutch territory. Given the conflict of interest between the Dutch ports Amsterdam and Rotterdam on the one hand and Antwerp on the other hand, it is not surprising that it took a very long time before the Dutch and Belgian governments reached an agreement. A large number of alternative trajectories have been proposed during this century but none were acceptable for both parties.

It was only after the formation of the Benelux (a customs union between Belgium, the Netherlands and Luxemburg) in 1944, that the negotiations became fruitful. A final agreement was reached in 1963 and the Antwerp-Rhine Canal was opened in 1975.

In this section, we will analyze the conflicts between Belgium and the Netherlands, given four main alternative trajectories for the Antwerp-Rhine Canal.

1. Zero-option (ZERO) This means a continuation of the existing situation (see Figure 3a.). This gives rise to a considerable detour through the so-called Kanaal door Zuid Beveland (a narrow canal with several locks that can only be used by small vessels).

2. Improvement of the Kanaal door Zuid Beveland (KZB) This improvement obviously does not give rise to a shorter route, but it makes the Antwerp-Rhine route accessible for larger vessels. Besides it gives rise to a smaller number of locks.

3. Schelde-Rhine Canal (SRC) This is a shorter connection than KZB. Existing waterways are used as much as possible (see Figure 3b.).

4. Antwerp-Moerdijk Canal (AMC) This is the shortest connection between Antwerp and the Rhine. Existing waterways are used to a moderate extent so that a considerable part of the connection has to be dug (see Figure 3c.).

The option ultimately chosen was the SRC variant. We assume that for the Dutch government the following criteria were of importance.

1. Costs. Given the Belgian interest in the Antwerp-Rhine connection, Belgium was prepared to pay a main part of the construction costs. The Belgian share varied from approximately 45% for KZB to more than 90% for AMC.

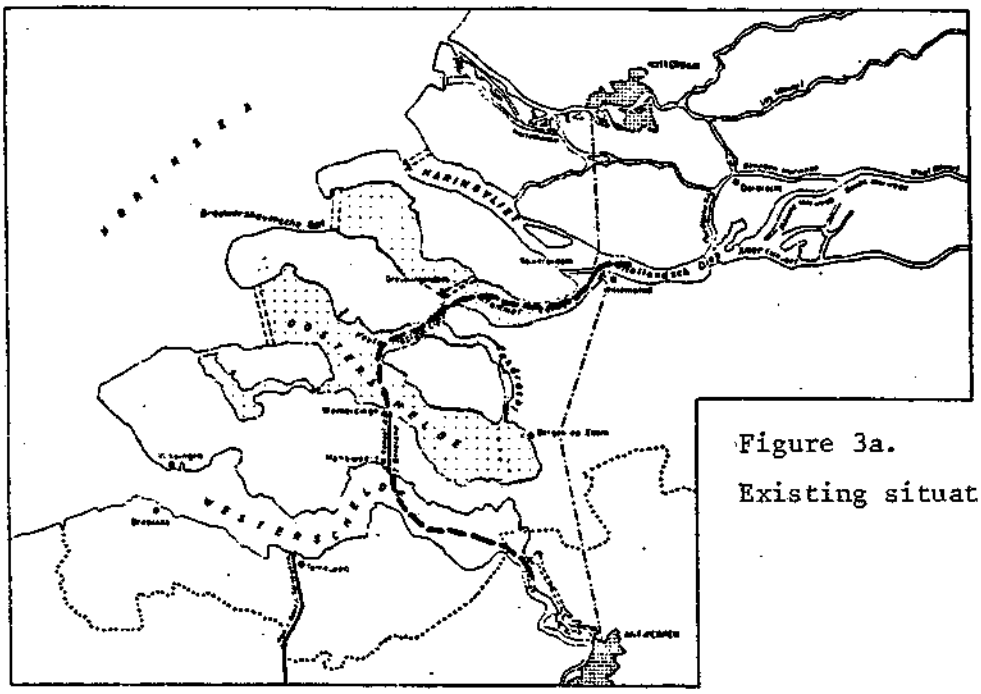

Figure 3a.
Existing situation

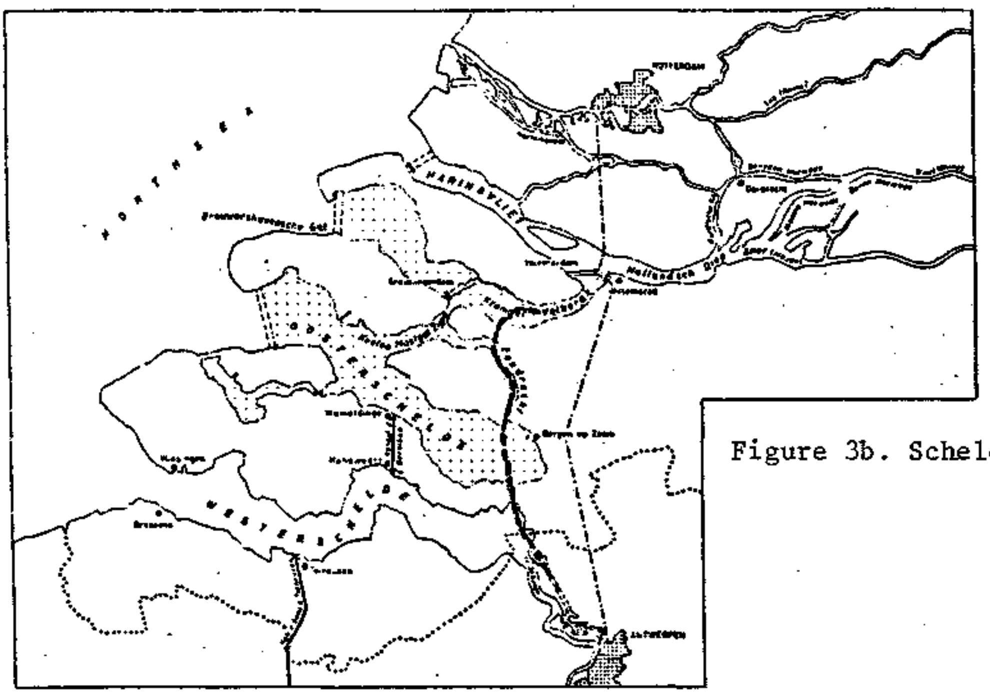

Figure 3b. Schelde-Rhine Canal

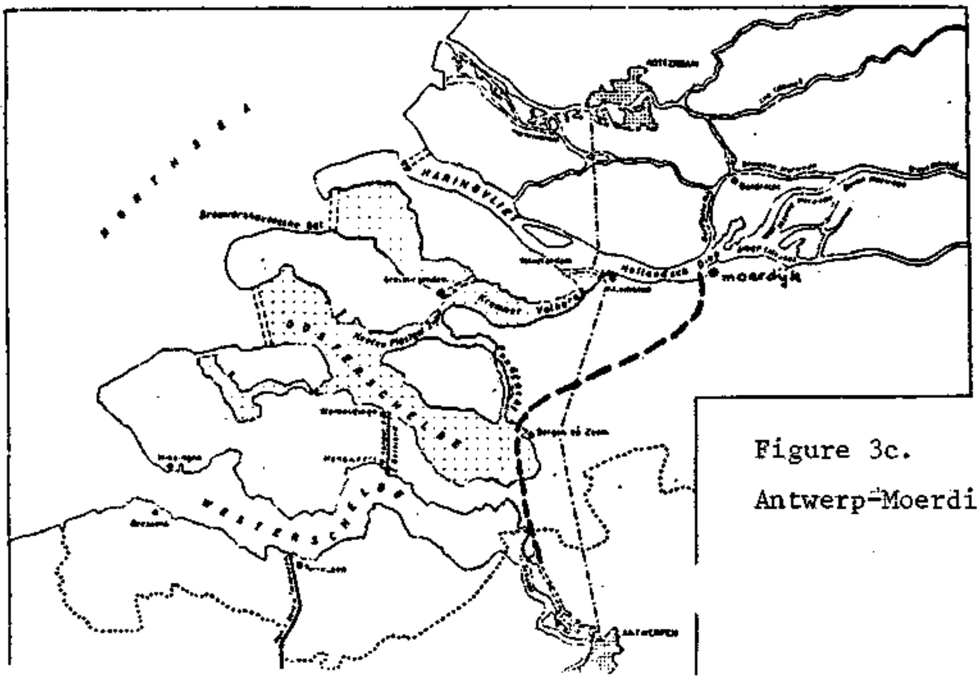

Figure 3c.
Antwerp-Moerdijk Canal

2. Employment. The employment effects of the alternatives are hard to determine. On the one hand, one might argue that the better the water connection, the more the Dutch economy would benefit: it makes Belgium more accessible for the Dutch. On the other hand, one has to recognize that a good connection of Antwerp with the Rhine makes Antwerp more competitive compared to Dutch ports. It is our impression that the last view prevailed in the Dutch approach to the Antwerp-Rhine Canal.

3. Environment. The various trajectories may substantially affect the natural environment.

4. Agricultural Interests. Construction of the AMC would imply that an area of approximately 1200 ha. would be withdrawn from agricultural purposes. For the SRC and KZB smaller areas would be lost.

5. Quality of Waterway. The quality depends, among others, on the length, the number of locks, the maximum capacity of the ships that can make use of it and the influence of tidal movements.

A detailed analysis (see Kutsch Lojenga and Nijkamp, 1977) gives rise to the following impact matrix for the Netherlands (Table 6). The higher the score in this matrix, the more favourable the outcome.

	Costs	Employment	Environment	Agriculture	Quality of Waterway
ZERO	4	4	3	4	1
KZB	1	3	4	3	2
SRC	2	2	2	2	3
AMC	3	1	1	1	4

Table 6. Impact Matrix for the Netherlands

The Belgian impact matrix is of course, related to the Dutch one. We assume that the environmental and agricultural criteria are not taken into account in Belgium given the fact that these primarily pertain to Dutch interests. The Belgian costs of the canal do not run parallel to the Dutch costs given the fact that the total costs are shared differently for each alternative. For the employment criterion we find obviously better outcomes, the better the quality of the waterway.

Thus the Belgian impact matrix reads:

	Costs	Employment	Quality of Waterway
ZERO	4	1	1
KZB	3	2	2
SRC	2	3	3
AMC	1	4	4

Table 7. Impact matrix for Belgium.

In addition to these impact matrices expressing national interests, one may also construct an impact matrix expression the common interest of the countries (BENELUX). For this matrix we will assume that the possible negative employment effects of a canal for the Netherlands will be more than off-set by positive effects on Belgium. Further, given the high share in the costs paid by the Belgians, we will assume that the BENELUX cost profile is in agreement with the Belgian figures. Therefore the BENELUX impact matrix reads:

	Costs	Employment	Environment	Agriculture	Quality of Waterways
ZERO	4	1	3	4	1
KZB	3	2	4	3	2
SRC	2	3	2	2	3
AMC	1	4	1	1	4

Table 8. Impact matrix for the BENELUX.

When we consider the Belgian impact matrix, it is not difficult to draw conclusions on the attractiveness of alternatives. The rank order of alternatives depends on the importance of the cost criterion versus the other criteria (employment and quality of waterway). Given the Belgian efforts to get an Antwerp-Rhine connection, it is reasonable to assume that the cost criterion does not dominate. Thus we arrive at the rank order AMC-SRC-KZB-ZERO.

A ranking of alternatives from the Dutch viewpoint is more difficult to obtain. If we assume that all rankings of weights are equally probable, we arrive at the rank order frequency matrix in Table 9a. This matrix gives rise to a rank order opposite to the Belgian one:

(a)	rank 1	2	3	4
ZERO	.03	.00	.18	.79
KZB	.18	.06	.60	.15
SRC	.03	.94	.03	.00
AMC	.76	.00	.18	.06

(b)	rank 1	2	3	4
ZERO	.00	.00	.00	1.00
KZB	.94	.00	.06	.00
SRC	.00	1.00	.00	.00
AMC	.06	.00	.94	.00

Table 9. Rank order frequency matrices for the Netherlands.

ZERO, KZB, SRC, AMC. Obviously, this rank order depends on the assumption that all rank orders of criteria are equally probably. If more specific assumptions are made, other rank orders of alternatives may be arrived at. For example, if one would assume that $\lambda_1 \geq \lambda_2 \geq \lambda_5 \geq (\lambda_3, \lambda_4)$, one obtains Table 9b. In this table AMC is a good candidate for the second rather than the last position.

We will next consider the BENELUX impact matrix. Table 10a. contains the rank order frequency matrix when all rankings of weights are equally probable. This gives rise to the rank order KZB, ZERO, SRC, AMC. Here again we find that there is no combination of weights for which SRC is the best alternative. In Table 10b. we represent the rank order frequencies based on the assumption that $(\lambda_1, \lambda_2, \lambda_5) \geq (\lambda_3, \lambda_4)$.

(a)	rank 1	2	3	4
ZERO	.24	.00	.51	.24
KZB	.00	.24	.24	.51
SRC	.00	.76	.24	.00
AMC	.76	.00	.00	.24

(b)	rank 1	2	3	4
ZERO	.70	.00	.13	.17
KZB	.00	.70	.17	.13
SRC	.00	.30	.70	.00
AMC	.30	.00	.00	.70

Table 10. Rank order frequency matrices for the BENELUX.

In this case the BENELUX rank order is in agreement with the Belgian preferences. Table 10 clearly shows that AMC is a controversial alternative. Its position (best versus worst) depends highly on the specification of the weights to be attached to the criteria. The same holds true - though to a somewhat lesser extent - for the ZERO alternative. The rankings of KZB and SRC are clearly less sensitive to minor changes in weights.

We arrive at the conclusion that the SRC alternative (the one ultimately selected) is neither for the Netherlands, nor for Belgium, the most preferred one. It has the character of a compromise. Given the fact that SRC received a higher rank in the Belgian view than in the Dutch view, one might conclude that the outcome of the negotiations have been more favourable for Belgium than the Netherlands. It is important to note that the selection of SRC is not in accordance with the ranking arising from the common interest (BENELUX). Thus, compromise decision-making is not necessarily the best way to serve the common interest.

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