NEW DEVELOPMENTS IN MULTIDIMENSIONAL GEOGRAPHICAL DATA AND POLICY ANALYSIS

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Researchmemorandum 1980-11

Paper presented at the
Colloquium on Theoretical and
Quantitative Geography,
Cambridge, September 1980

Abstract

This paper aims at providing a survey of recently developed multidimensional methods in the field of spatial data and policy analysis. After a brief methodological introduction, various multidimensional techniques for data analysis are discussed. Much emphasis is placed on the treatment of soft (ordinal or qualitative) data. Next, several multidimensional methods for policy analysis (multi-objective programming, multicriteria analysis) are discussed. Here again much attention is paid to the treatment of soft information.

1. Introduction*)

Phenomena and problems in modern societies are characterized by complexity, variation and interwoven relationships. This also holds true for spatial patterns and processes. Quantitative geography aims at providing theories and methods which describe such spatial structures and developments in a mathematical and/or statistical way in order to analyse in an operational way the dispersion and coherence of phenomena in regional and urban systems.

The picture of spatial systems is - in general - rather complicated, and hardly any phenomenon in such systems can adequately be described or represented by means of a simple attribute such as a single scalar variable. Normally, such phenomena have a whole set of attributes (aspects, criteria, features) which give a representative mapping of these phenomena. Such a multidimensional representation of phenomena in spatial systems requires adjusted operational methods for an appropriate regional and urban data analysis as well as for a satisfactory policy analysis.

During the seventies a wide variety of multidimensional methods has been developed which are extremely useful for data and decision analysis. Many of them are able to provide an operational framework for the analysis of spatial behaviour and for planning and decision problems. In the present paper the value of multidimensional methods will be set out by providing a selected survey of these methods and of their potential or actual applications.

2. Multidimensional Analysis

The pluriformity of spatial patterns and of changes therein requires very often a multidimensional analytical framework. This is a prequisite for arriving at an operational and comprehensive insight into complex phenomena such as residential location decisions, evaluation of intangibles, the existence of interregional inequalities, decline in environmental quality, spatial interaction and attractiveness, and so forth.

In formal terms, a multidimensional approach implies that a certain variable x is characterized by a vector profile $\underline{v}$ with elements v_i $(i = 1,\ldots,I)$.

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*) Several parts of this paper are based on Nijkamp (1979).

In other words,

$$x \longrightarrow \underline{v} = \begin{bmatrix} v_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ v_I \end{bmatrix} \qquad (1)$$

Usually, the elements of $\underline{v}$ are measured in different dimensions. Sometimes, it is common to standardize the elements of $\underline{v}$. Clearly, an ordinal measurement of the elements of $\underline{v}$ implies already a certain dimensionless standardisation.

Multidimensional data analysis aims at detecting a structure in data presented in vector profiles. To some extent, this modern analysis can be regarded as a straightforward extension of traditional unidimensional methods, although several specific problems may emerge in treating multidimensional profiles. Sometimes, rather cumbersome statistical and mathematical problems may arise. Therefore it is important to employ a set of advanced techniques which may tackle these problems. Examples of such techniques are interdependence analysis, canonical correlation, etc.

A multidimensional approach may also lead to considerable complications in decision and planning problems (cf. the wellknown multi-attribute utility developed by Lancaster (1971)). In this respect, it is extremely important to develop operational methods for policy analysis which are able to take into account conflicts between groups, issues, goals, decision levels etc. Furthermore, uncertainties (lack of reliable information, e.g.) have to be taken into account. The last part of the paper will focus on multidimensional methods for policy analysis.

3.. Pattern and Impact Analysis

Phenomena such as residential quality, environmental pollution, spatial congestion have to be represented by means of a multidimensional pattern. In this way, various units of a spatial systems (districts, regions, e.g.) can also depicted in a comprehensive profile. Suppose, for example, that a residential attractiveness profile $\underline{a}$ is composed of the following elements:

$$\underline{a} = \begin{bmatrix} a_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ a_I \end{bmatrix} = \begin{bmatrix} \text{quantity of dwellings} \\ \text{quality of dwellings} \\ \text{size of recreation areas} \\ \text{availability of shops} \\ \text{cultural facilities} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \qquad (2)$$

Then for a set of regions 1, ..., R (cities, districts etc.) the following multidimensional matrix representation can be constructed:

$$A = \begin{bmatrix} a_{11} & \cdots\cdots\cdots & a_{1R} \\ \vdots & & \vdots \\ \vdots & & \vdots \\ a_{I1} & \cdots\cdots\cdots & a_{IR} \end{bmatrix} \tag{3}$$

A standardisation of the elements of A may be carried out in several ways. A rather easy standardisation is:

$$a^*_{ir} = \frac{a_{ir} - a_i^{min}}{a_i^{max} - a_i^{min}} \tag{4}$$

where a_i^{min} and a_i^{max} are the minimum and maximum values of a_i over all R regions. In (4), the ith indicator is supposed to be a benefit indicator (the higher, the better); otherwise, a reverse standardisation has to be used.

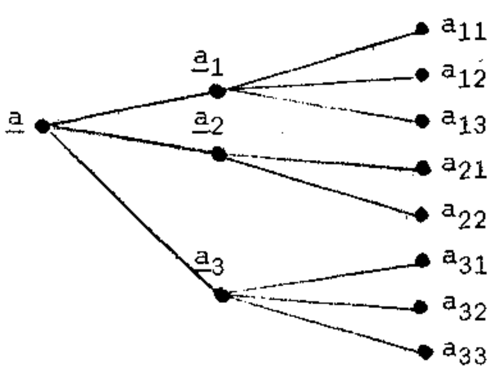
The (unweighted) distance $d_{rr'}$ between the attractiveness profiles of region r and r' can be calculated <u>inter alia</u> via a generalized Minkowski p-metric:

$$d_{rr'} = \{ \sum_{i=1}^{I} (a^*_{ir} - a^*_{ir'})^p \}^{1/p} \quad , p \geq 1 \tag{5}$$

A similarity index between any two profiles can be defined as:

$$s_{rr'} = \frac{1}{1 + d_{rr'}} \tag{6}$$

The above-mentioned pattern analysis of multidimensional phenomena can be extended by making a distinction between main profiles and sub-profiles. In this way, one may divide $\underline{a}$ into main categories (for instance residential quality, recreation, medical care, etc.); next, each main category may be divided into subcategories:



(7)

Such a hierarchical pattern representation is also very relevant in detailed multidimensional impact analyses (among others, in environmental impact statements). In this way, the coherence and variety of the key variables of a spatial system can also be taken account of. The same holds true for shifts in the main categories of a system as a consequence of a change in a main determinant (for example, the decision to build a new road will have a variety of economic, physical, environmental and infrastructural repercussions).

A more comprehensive impact analysis can be based on a stimulus - response approach. The stimuli $\underline{s}$ can be defined on the basis of the elements of a main determinant of a system which exerts a substantial influence on a set of responses $\underline{r}$ (for instance, the elements of the above mentioned attractiveness profiles):

$$\underline{s} \longrightarrow \underline{r} \qquad (8)$$

or:

$$\underline{r} = \underline{f}\ (\underline{s}) \qquad (9)$$

where $\underline{f}$ is a so-called impact function. There are several ways to operationalize $\underline{f}$; for instance, multiple regression analysis, partial least squares, canonical correlation etc. (see later).

4. Interdependence Analysis

A drawback of the multidimensional profile approach is its extensive information pattern, so that a straightforward interpretation is sometimes less easy. Furthermore, many attributes in a profile may contain redundant information. Since a lower number of attributes may facilitate the interpretation of the results, it is worth while to undertake an attempt at reducing the data contained in multidimensional profiles.

A traditional data reduction technique is principal component analysis. This is a transformation from a set of originally mutually correlated variables to a new set of independent variables (based on an orthogonal data transformation in which the original variables are substituted for independent factors). A drawback of these techniques is the fact that new artificial variables are created which can be interpreted on the basis of factor loadings, but which have no clear direct meaning per se.

In respect to this, a more recently developed technique, called interdependence analysis, is more appropriate. Interdependence analysis is an optimal subset selection technique, by means of which a subset of variables which best represents an entire variable set can be chosen [see Boyce et al. (1974)].

The advantage of interdependence analysis is that an optimal subset of original variables is selected, so that a data transformation is not necessary.

Suppose we have matrix A from (4) with R observations (profiles) on I variables. Next, P variables are to be selected from the I variables such that this subset of P variables demonstrates an optimal correspondence with respect to the original data set. Hence, (I-P) variables are to be eliminated.

Now interdependence analysis is based on a series of successive regression analyses between the individual 'dependent' (I-P) variables to be eliminated and the 'independent' variables to be retained. Given (I-P) regressions, the minimum correlation coefficient can be calculated. Next, for all permutations of P in (I-P) variables, a similar operation can be carried out. Then the optimal subset is defined as that subset which maximizes over all permutations the values of the above-mentioned minimum correlation coefficient. This max-min solution bears a correspondence to the equilibrium solution of a game procedure, in which the information contained in a data matrix is reduced such that the selected variables constitute a best representation of the information pattern. See for alternative subset selection criteria Nijkamp (1979).

Applications of interdependence analysis can be found among others in the field of optimal network algorithms, multicriteria evaluation methods, attractiveness analyses of human settlements, spatial complex analyses and spatial inequality analyses.

5. Multidimensional Scaling Analysis

In addition to principal component techniques and interdependence analysis, several other data reduction techniques have been developed during the last decade, such as correspondence analysis [see Benzécri (1973)] and multidimensional scaling (MDS) analysis. Especially MDS techniques have found many applications.

The original rationale behind the use of MDS techniques was to transform ordinal data into cardinal units. Suppose that matrix A from (4) is measured in ordinal units. Then a transformation toward a metric system can be made by assuming that each region r can be represented as a point in a P-dimensional Euclidean space. Since there are R such points, one might interpret the distances between each pair of these R points as a measure for the discrepancy between each pair of profiles. Clearly, the Euclidian co-ordinates are unknown, but they can be gauged by a similarity rule stating that the R points have to be located in the Euclidean space in such a way that their positions correspond to a maximum extent with the ordinal information on the original R profiles. It is clear that the only way to derive metric profiles for each region is to reduce the dimensionality of these profiles. In fact, the degrees of freedom resulting from this reduction in dimensionality are used to transform non-metric data into cardinal units. In other words, the following transition takes place:

$$\hat{A}$$

$$\begin{bmatrix} a_{11} & \cdots\cdots & a_{1R} \\ \vdots & & \vdots \\ a_{I1} & \cdots\cdots & a_{IR} \end{bmatrix} \longrightarrow \begin{bmatrix} \hat{a}_{11} & \cdots & \hat{a}_{1R} \\ \vdots & & \vdots \\ \hat{a}_{P1} & & \hat{a}_{PR} \end{bmatrix} \qquad (10)$$

where A and $\hat{A}$ are measured in non-metric and metric units respectively.

MDS methods can be regarded as extremely powerful tools in spatial data analysis. First of all, they can be used as a data reduction technique as such, but they are especially important in the case of unreliable or soft data (such as ordinal information). MDS techniques allow researchers to draw metric inferences from non-metric input.

There are many geographical applications in the field of MDS techniques: individual perception and preference analyses, mental maps, recreation behaviour, environmental quality analysis, urban renewal projects, and multicriteria analysis.

6. Canonical Correlation

Canonical correlation is especially developed to identify correlations between sets of variables. In contrast to regression techniques which aim at explaining one single variable from an underlying set of variables, canonical correlation attempts to link a profile of variables to another profile of variables.

Canonical correlation attempts to identify the degree of connection between sets of attributes of the same population via a generalized linear correlation analysis. Suppose we have a set of R regions which are characterized by two different profiles; for example, a socio-economic profile composed of I indicators and a spatial-infrastructural profile composed of J indicators. Then one may try to find a correlation between these two multidimensional profiles.

Next, a canonical correlation analysis attempts to identify a relationship between a linear combination of the elements of the first profile and a linear combination of the elements of the second profile, such that the underlying linear model demonstrates a maximum correlation between both linear expressions.

Canonical correlation analysis can be used to test whether or not different profiles characterizing the same phenomenon show a high degree of similarity.

The number of applications of canonical correlation in geographical research is fairly limited, but some applications have been made in the field of regional income analysis, unemployment analysis and spatial pattern analysis.

Sometimes, canonical correlation can also be combined with spectral analysis, especially for time series - cross section problems.

There are two related techniques in the field of canonical correlation analysis, viz. partial least squares [World (1977)] and discriminant analysis [Anderson (1958)].

Partial least squares is a special kind of path model technique which attempts to identify a block structure for latent variables and their indicators as well as between the latent variables themselves (the 'inner' structure) on the basis of iterative regression analysis. To a certain extent, partial least squares can be regarded as an extension of canonical correlation toward more than two profiles.

Discriminant analysis is essentially an assignment method while aims at assigning a certain unit (person, district, e.g.) to a certain class on the basis of a multidimensional profile of attributes of this unit. Stability tests on the results of a discriminant analysis can be carried out via canonical correlation.

7. Spatial Correlation and Econometrics

Spatial (auto)correlation is another phenomenon which frequently occurs in spatial systems. Several test statistics have been developed in order to identify spatial autocorrelation or cross-section correlation (among others by Moran, Geary, Cliff and Ord, and Hordijk). Given a multidimensional profile for a set of regions and given the connectivity structure of the spatial system concerned, several measures for autocorrelation can be defined. These measures can easily be extended for spatio-temporal profiles and for different spatial and temporal lag structures.

A similar approach can be used to detect spatial correlation among the disturbances of a linear spatio-temporal model, so that adjusted econometric techniques can be used to produce consistent parameter estimates [see Nijkamp (1979)]. Some appropriate techniques may be a Zellner generalized least squares method or a Markov scheme method.

In many cases, regional modelling is characterized by the existence of latent (indirectly observed) variables. Such latent variables have usually only a soft or qualitative meaning, but they can be approximated by means of a vector profile of indirect indicators. An appropriate technique for dealing with latent variables is Lisrel; this is based on a maximum likelihood approach and it needs precise information concerning the distribution of the observed variables and the specification of the theoretical model [see for some applications among others Jöreskog (1977) and Folmer (1979)].

A more difficult problem arises, if (parts of) the explanatory variables are only measured in ordinal terms. In that case an MDS approach can be used. Suppose the following model:

$$y = f(\underline{x}, \underline{z}) \qquad (11)$$

where $\underline{x}$ and $\underline{z}$ are profiles with metric and non-metric attributes, respectively. Thus, $\underline{z}$ contains ordinal information. Suppose the number of elements of $\underline{z}$ is K, while the number of observations is R. Then an MDS technique can be applied in order to transform the R x I matrix of ordinal observations into an R x P matrix of metric data (I > P). Next, a normal regression procedure can be applied to the transformed data set [see Nijkamp (1980a)]. Tests on autocorrelation can again be performed via the abovementioned statistics.

8. Ordinal Multidimensional Data

The major part of mathematical and statistical data techniques is based on metric data, although it is surprising that in practical research soft information is very often a rule rather than an exception.

In the past, only a few techniques for ordinal data treatment have been developed. The most well-known examples are the Spearman and Kendall rank correlation coefficients for ordinal data. In regression analysis, dummy variable techniques have become rather popular in order to deal with nominal or qualitative information. In addition, path models (and more recently Partial Least squares and Lisrel techniques) have been developed for latent variables.

It has already been explained in Section 5 that MDS techniques are powerful techniques in geographical research, although some caveats may be: the number of dimensions to be taken into account (just like in factor analysis) and the value of a satisfactory goodness-of - fit. Although MDS methods are in general extremely useful tools for a wide variety of ordinal multidimensional data problems, it may be worth while to examine whether for certain specific problems alternative techniques also may be appropriate.

Beside MDS techniques, it is in some cases useful to make use of order statistics (either in an analytical way or via random generators).

A special problem arises when in the case of model (11) the endogenous variable y is also measured in non-metric units. In that case, it is difficult to apply MDS techniques, since there is only one vector of ordinal data which cannot be reduced to a lower dimension. Then there are two possibilities. First, one may - analogous to a metric regression analysis - write the estimator entirely in terms of (Kendall rank) correlation coefficients. The justification for this analogy is, however, hard to prove.

A second approach may be to make a pair wise comparison of the non-metric data and to assign a zero-one dummy depending on whether or not a certain ordinal number is higher than the other one. In that case, one may apply a probit analysis in order to estimate the probability that a certain outcome of the endogenous variable is higher than another one, given certain zero-one values for the explanatory part of the regression equation.

The latter result also means that ordinal interdependence analyses can be carried out in various ways: (1) via MDS methods, (2) via Kendall rank correlation coefficients, and (3) via soft regression techniques.

The same holds true for canonical correlation, Partial Least Squares and spatial (auto) correlation statistics. Adjusted techniques can also be developed for discriminant analysis and clustering techniques.

9. Multidimensional Preference and Perception Analysis

The multidimensionality principle can also be used to assess individual preference and perception patterns. In regard to this, one may ask individuals to rank their priorities or perceptions (or both) concerning a multidimensional set of items (for example, different shops or recreation areas) by means of ordinal numbers. Next, an MDS approach can be used to draw metric inferences concerning the relative preferences (or perceptions) of the individuals, the discrepancies among individuals and the differences among the items. In this case, a joint MDS procedure may be useful, because such a joint configuration of individuals and items gives a comprehensive representation of the entire preference (or perception) pattern.

Such a preference analysis can also be applied to the supply side of commodities, so that an MDS technique may also provide insight into the relative disequilibrium between the demand and supply side [see for an application to shopping behaviour Blommestein et al. (1980)].

A next step of such a multidimensional analysis may be to construct a behavioural model which tries to explain individual behaviour by means of a multidimensional set of explanatory variables. If the information concerned is nominal or ordinal, a wide variety of disaggregated choice models can be used to assess individual behavioural parameters [see for a survey Van Lierop and Nijkamp (1980a, 1980b)]. Some well-known analytical tools for disaggregated choice models are logit and probit analysis. Both techniques have found many applications in spatial interaction models.

10. Multidimensional Policy Analysis

In the seventies, economists and operations researchers have paid much attention to multidimensional optimization methods as a tool in modern decision-making. The background to this interest in depth in new decision analyses is the lack of operationality of traditional decision techniques. A frequently felt shortcoming of almost all these techniques is the fact that all dimensions of a decision problem have to be translated into a common denominator (like income, profit, efficiency, etc.) or at least have to be made commensurate with the primary objective of a decision problem.

The awareness of a multiplicity of different objectives in decision-making and management has evoked the need for more adequate techniques which take into account the multidimensionality and heterogeneity of individual, social or entrepreneurial behaviour. The need for such adjusted methods is even more apparent due to the mutually conflicting or noncommensurable nature of many objectives. The presence of (partially) incompatible priorities can be considered as an essential characteristic of a wide variety of modern planning and decision problems.

Therefore, recently several attempts have been made to develop more adequate theories and methods which take into account explicitly the existence of multiple criteria in decision-making. The basic feature of these techniques is that a wide variety of relevant decision aspects is included without translating them into monetary units or any other common denominator. These multidimensional optimization methods are also able to integrate intangibles normally failing outside the realm of the traditional price and market system. Expositions of multidimensional optimization theory can be found among others in the following books: Bell et al. (1977), Blair (1979), Boyce et al. (1970), Cochrane and Zeleny (1973), Cohon (1978), van Delft and Nijkamp (1977), Fayette and Nijkamp (1980), Haimes et al. (1975), Hill (1973), Hwang and Masud (1979), Keeney and Raiffa (1976), Nijkamp (1977, 1979, 1980b), Nijkamp and Spronk (1980), Rietveld (1980), Thiriez and Zionts (1976), and Zeleny (1976).

These new approaches are extremely relevant for private and public decision-making in the sphere of production, resources, investment, location, marketing, etc. In all these cases pecuniary elements (like profitability) play an important role, but in addition several other elements are equally important (like social aspects, environmental impacts of production, use of scarce natural resources, risk characteristics, labour conditions, etc.). The multidimensional optimization methods have also a great relevance for regional and urban policy analysis due to the conflicting nature of many goals (either within a region or between a region).

The general format of a multidimensional optimization model is:

$$\max_{\underline{x} \in K} ! \quad \underline{w}(\underline{x}) \tag{12}$$

where $\underline{w}$ is a I x 1 vector of objective functions; $\underline{x}$ a J x 1 vector of decision arguments; and K a feasible area. An example of such a multidimensional programming problem may be: maximize production <u>and</u> employment <u>and</u> environmental quality <u>and</u> energy savings <u>and</u> systems accessibility, subject to the side-conditions set by the economy and technology.

It should be noted that decision-making in a multi-group or multiregional context is fairly complicated, because a part of the one system is under control of another system. Without a master control for the entire system at hand, a compromise choice between conflicting options has to be based on a negotiation or bargaining process between all participating decision-makers. In this respect the notion of interactive decision strategies is very important (see later).

The concept of an efficiency curve plays a central role in multi-dimensional optimization problems, because an efficiency curve precisely reflects the degree of conflict or complementarity between diverging options. The problem of a multidimensional optimization model is to find efficient points $\underline{x}^*$ such that there will exist no other feasible point $\underline{x}$ such that:

$$\begin{aligned} & w_i(\underline{x}) > w_i(\underline{x}^*) \quad , \forall i \\ & \text{and} \\ & w_{i'}(\underline{x}) \neq w_{i'}(\underline{x}^*) \quad , \text{for at least one } i' \ (i' = 1, \ldots, I) \end{aligned} \tag{13}$$

The computationally equivalent problem is:

$$\max \varphi = \sum_{i=1}^{I} \lambda_i w_i (\underline{x}) \tag{14}$$

$$\underline{x} \in K$$

$$\sum_{i=1}^{I} \lambda_i = 1 \quad , \lambda_i \geq 0$$

By means of a parametrisation of the λ's the entire set of Pareto solutions can, in principle, be identified (at least for a convex programming problem), although this may be a rather time-consuming procedure for large problems. Since the λ's are a set of weights (trade-offs) associated with the efficient solutions, any ultimate compromise solution between the diverging objectives can be related ex post to these λ's (note that any optimal solution is efficient).

Another important concept in multidimensional optimization theory is the ideal point. The ideal point $\underline{w}^o$ is a I x 1 vector of maximum values of the successive individual objective functions; in other words, the elements w_i^o of $\underline{w}^o$ are defined as:

$$w_i^o = \max_{\underline{x} \in K} w_i (\underline{x}) \tag{15}$$

It is clear that an ideal point is not a feasible point, but it may serve as an important frame of reference for evaluating points on the efficiency frontier (see later).

Since the aim of this paper is to provide a survey of multiobjective modeling, it may be meaningful to make some classifications. A first typology may be based on a subdivision into continuous and discrete (integer) decision models. Discrete models are characterized by a finite number of feasible alternative choices or strategies (for example, in the case of plan evaluation or project evaluation problems); discrete models are often called multicriteria models. Continuous models are based on an infinite number of possible values for the decision arguments and hence for the objective functions; they are usually called multiobjective optimization models.

Another distinction of multidimensional choice models may be according to the degree of accuracy of information. In this respect one may subdivide such choice models into soft information models (based, for example, on qualitative, fuzzy or ordinal information) and hard information models (based, for example, on deterministic cardinal data input).

Thus, the following figure may be constructed:

	hard information	soft information
multicriteria models	I	III
multiobjective optimization models	II	IV

Fig. 1. A typology of multidimensional optimization models

In the next sections, the categories I-IV of figure 1 will be further discussed.

11. Hard Multicriteria Models

Hard multicriteria models are based on reliable metric information on discrete alternatives (plans, projects or strategies). A first step in all these methods is the construction of an impact matrix which reflects the outcomes of all alternatives with respect to all I relevant decision criteria:

criterion \ alternative	1..............n............N
w_1	
⋮	
w_i	w_{in}
⋮	
w_I	

Table 1. An impact matrix.

The element w_{in} reflect the value of the ith criterion with regard to the nth plan; it is assumed that w_{in} is measured in a normal metric system.

In the past, cost-benefit analysis has been a favourite method to evaluate discrete alternatives. Due to the unpriced nature of several commodities, this method is inappropriate for most urban and regional planning problems [see for an extensive criticism also Nijkamp (1977)]. Some adjusted methods such as the planning-balance-sheet method, the cost-effectiveness analysis and the shadow project approach can be regarded as a significant improvement of traditional cost-benefit analysis, but they do not provide a solution for the problem of judging unpriced and intangible goods.

Instead of using (artificial) prices for unpriced and intangible goods, multicriteria models assign political priorities to certain decision criteria. These weights reflect the relative importance attached by the decision-maker(s) to the outcomes of each criterion. These weights reflect the priority scheme of the decision-maker and may be linear or nonlinear.

It is often a hard task to infer political weighing schemes by means of revealed preferences or questionnaires. When such weights cannot be assessed <u>a priori</u>, two ways are open to proceed with a multicriteria evaluation model:

- to use general alternative scenario's as the basis for deriving alternative sets of weights for future policy choices [see Nijkamp and Voogd (1980)];
- to use an interactive learning procedure during which relative priorities are specified in a stepwise manner [see Van Delft and Nijkamp (1977) and Rietveld (1980); see also section 15].

The following multicriteria methods for discrete decision and evaluation problems may be distinguished:

. trade-off analysis
. expected value method
. correspondence analysis
. entropy analysis
. discrepancy analysis
. concordance analysis
. goals-achievement method

The general feature of these multicriteria methods is that they include a multiplicity of decision criteria, so that they are rather appropriate for modern planning and management problems in which unpriced goods play an important role.

12. Hard Multiobjective Optimization Models

Hard multiobjective optimization models are based on metric information regarding continuous objective functions and constraints. There is also a wide variety of different multiobjective optimization models:

. utility models
. penalty models
. constraint models
. goal programming models
. hierarchical models

. min-max models
. ideal point models

The general conclusion concerning the above-mentioned hard multiobjective optimization models is that especially the last four categories may be appropriate for economic-environmental models, because they use the available information as good as possible without too many arbitrary assumptions or too much additional information.

13. Soft Multicriteria Models

Soft multicriteria models are based on ordinal information or even qualitative information ('good, better, best'). The following soft multicriteria models may be distinguished:

. expected value method
. lexicographic method
. ordinal concordance method
. permutation method
. metagame analysis
. eigenvalue method
. frequency method
. multidimensional scaling method

The final conclusion is that there is a whole series of ordinal evaluation techniques starting from simple but dubious solution methods to complex but satisfactory solution methods. These evaluation techniques are especially useful for regional, urban and environmental management problems, because usually many data on the impacts concerned are uncertain, fuzzy or biased.

14. Soft Multiobjective Optimization Models

Soft multiobjective models are a less developed category of multidimensional choice models. They are characterized by qualitative objective functions (for example, a systems performance measured in ordinal units) and/or qualitative constraints (for example, qualitative impact statements such as 'good, better best').

The number of ways to deal with such choice models is rather limited so far. The following alternative approaches may be distinguished:

. fuzzy set models
. stochastic models
. soft econometric models

The conclusion is that the field of soft multiobjective optimization models is still underdeveloped. For the moment its use for regional and urban decision-making is very limited, although especially the soft econometric models seem to be fairly promising.

15. Interactive Decision Models

The multidimensional choice models discussed so far were based on a certain technique or algorithm to identify a compromise between conflicting objectives. In respect to this, these models can be regarded as a fruitful contribution to environmental, regional and urban policy models.

However, in many planning and decision problems the first (compromise) solution obtained by one of the above-mentioned methods is not considered to be entirely satisfactory. Therefore, instead of regarding the compromise solutions as a final equilibrium point, one may develop a certain interactive learning procedure in order to reach in a series of steps such a satisfactory final compromise solution. This implies that the first compromise solution is only a trial solution which has to be presented to the decision-maker(s) as a frame of reference for judging alternative efficient solutions.
The easiest way to carry out such an interactive procedure is to ask the decision-maker(s) which values of objective functions are satisfactory and which ones are unsatisfactory (and hence have to be improved).

This can easily be done by using a checklist encompassing all first compromise values of the I objective functions (Table 2).

values of first compromise slutions	satisfactory (+ or -)
$\hat{w}_1$	
.	
.	
.	
.	
.	
$\hat{w}_I$	

Table 2. A checklist for the interactive learning process.

Let S represent the set of objective functions which are to be increased in value. Then the decision-maker's judgement concerning the trial compromise solution can be taken into account by specifying the following constraint:

$$w_i \geq \hat{w}_i \quad , \quad \forall \, i \in S \qquad (16)$$

In consequence, the following model has to be solved for the next stage of of analysis (cf. 12):

$$\begin{aligned} &\max! \; w_i\,(\underline{x}) \\ &\underline{x} \in K \\ &w_i\,(\underline{x}) \geq \hat{w}_i \quad \forall\, i \in S \end{aligned} \qquad (17)$$

By specifying (17), a new multidimensional choice problem emerges. This can again be dealt with by means of one of the methods set out in the previous sections. After the calculation of the outcomes of this model, a new (trial) compromise solution arises which can again be checked with the decision-maker(s) by means of Table 2, and so forth. The procedure has to be repeated, until a final satisfactory compromise solution has been identified by the decision-maker(s). [See for applications among others Van Delft and Nijkamp (1977), Fayette and Nijkamp (1980), and Rietveld (1980)].

The steps of an interactive procedure are briefly summarized in figure 2:

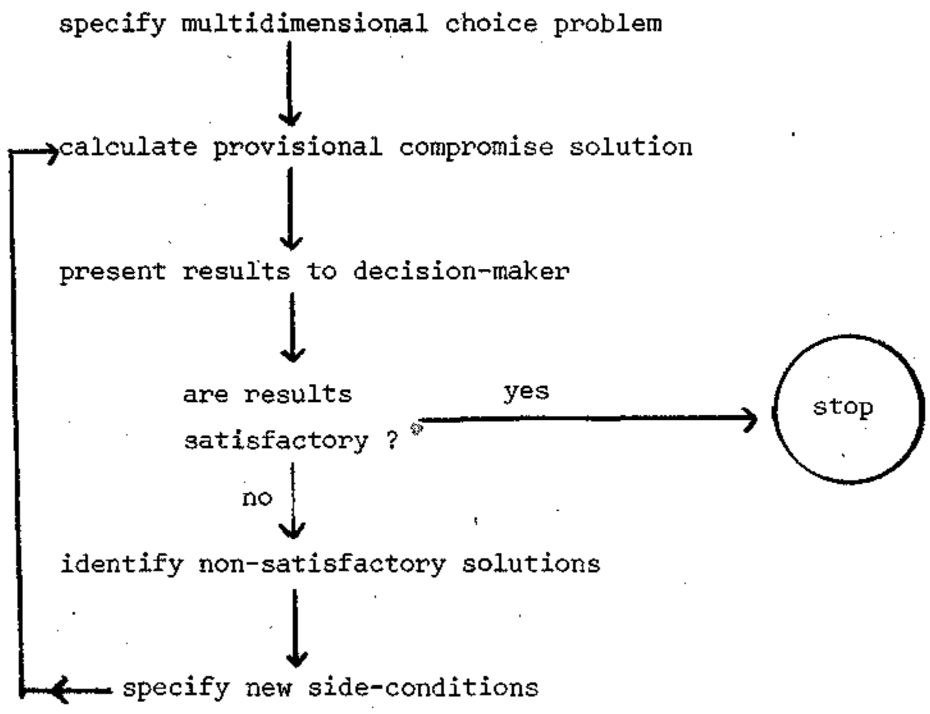


Fig. 2 Steps of an interactive multidimensional choice problem.

The advantages of such interactive procedures are evident: they provide information to the decision-maker(s) in a stepwise manner, they lead to an active role of the decision-maker(s), and they avoid the prior specification of trade-offs (although they can be inferred ex post).

In our opinion, the use of interactive multidimensional choice models is extremely important for environmental, urban and regional decision-making and management, because it enables decision-maker(s) to assign a clear role to intangibles and incommensurables in evaluation and decision problems.

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