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Co-operative learning and adaptive instruction in a mathematics curriculum

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Is it possible or desirable to keep 12- to 16-year-old students together in heterogeneous classes? This question has received much attention from researchers, policy makers, teachers and parents. In many studies it has been shown that grouping practices like tracking and streaming have negative effects, especially for the lower groups (Oakes 1985). Recently new methodologies and new techniques have made it possible to distinguish between individual effects and group effects. From these analyses we have obtained insights into the effects of the composition of the class (e.g., as reflected in its mean cognitive level) on the learning gains of individual students (Goldstein 1987, Longford 1988). But, quite apart from the discussion about methodological issues, a new discussion is going on and new plans are being made for the middle school level in the USA, especially in California (Middle Grade Task Force 1987).

The recommendations of the California Middle Grade Task Force about grouping practices are clear: heterogeneous grouping practices should be normative in middle grade classrooms. If permanent or semi-permanent 'ability' grouping or tracking occur for all or most of a student's schooldays, substantial harm can result. Researchers are invariably consistent in their conclusion that large numbers of poor and minority students in particular are precluded from realizing the true meaning of equal access when tracking occurs. The Task Force proposes that students should be prepared with a varied repertoire of learning strategies which enable them to engage in independent study and co-operative learning and to give and receive tutorial instruction. This discussion and these proposals have been called 'Detracking or Unstreaming the Middle School'. Many proponents of heterogeneous classes look for co-operative learning strategies in order to find an alternative to tracking and to avoid separation between students (Oakes 1985, Slavin 1987).

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In our view co-operative learning cannot be the solution unless two aspects are taken into consideration: instructional strategies and the content of the curriculum; and the composition of classes and groups. The first aspect deals with the instructional model for co-operative learning and with the content of, for example, mathematics education. These *can be* brought together in a specially designed curriculum. There are good examples of curricula for co-operative learning in which instructional strategies and mathematical content have been designed in such a way that they stimulate co-operation and learning. But learning will result only if there is a fit between the cognitive level of the students and the cognitive demands that are made by the curriculum. Instructional arrangement for co-operative learning must provide *explicit adaptation*, not only to a mean class level, but also to the level of individual students.

In other words, the full potential of co-operative learning can only be attained if the issue of class composition is part of the design and organization of co-operative learning. There is evidence that the percentage of low-aptitude students in a class (as indicated by the class mean-aptitude score at the beginning of a school year) is an important variable for the learning gains of each individual student in the class. The more able students there are in a class, the better the learning. In general, the mean achievement score of a class is a determinant of the quality of the teaching and learning processes in that class and this will be reflected in the learning of individual students (Good and Marshall 1984, Dar and Resh 1986, Terwel and van den Eeden 1992). Webb (1982) has shown the effects of the composition of the small co-operative groups within a class.

Our general point of view is that the learning of an individual student does not take place in a vacuum but in an environment in which variables of different levels are at work. Some variables can be seen at *class level* (implementation of the curriculum, class composition) others are at the *individual* level (aptitude of the individual student) or at *group level* (mean aptitude score of the group). Although interdependency of individuals and groups is often involved in co-operative learning, it can be hypothesized that the composition of a class plays an even more important role under such a model than in traditional, whole-class instruction. Consequently, application of co-operative learning in a school with between-class ability grouping (tracking, streaming or setting) may not have the expected effects for those students who need it most!

Many educators are preoccupied by the phenomenon of heterogeneity (e.g., standard deviation of aptitude scores as measured by a pretest) while paying less attention to the cognitive level of a class (mean aptitude score). There are indications in the literature that class means are more important for learning than standard deviations (Dar and Resh 1986, Dreeben and Barr 1987). In this paper we report a study of the effects of a mathematics curriculum and two context variables, *time* and *mean cognitive level of the class*, on the results of learning. First the goal of the project and the research questions are stated, then a description of the instructional model is given. After a description of the design, the hypotheses, the methods and the model of analysis, the results of the study are presented. The paper ends with some conclusions and a discussion of the outcomes.

The AGO 12 to 16 Project

The AGO 12 to 16 Project (the acronym AGO stands for the Dutch equivalent of 'Adaptive Instruction and Co-operative Learning') seeks to develop and evaluate a mathematics curriculum which is suitable for mixed-ability groups in secondary education. The research questions we will address here are, first, whether this curriculum is feasible and effective, and, second, what effects, if any, the context variables *time* and *mean cognitive level* of the class have on learning.

Many mathematics programmes make insufficient allowance for the differences in intellectual ability that exist in mixed-ability classes. In order to change this situation we developed a mathematics curriculum with adaptive qualities. The evaluation of the experimental curriculum was carried out in two stages. During the first stage the curriculum was used at two schools with the aim of investigating the feasibility of the programme. Experience with the implementation of the programme led to some improvements in the experimental materials. By and large the *AGO model* appeared to be feasible in secondary classrooms. In the second stage, which was on a large scale, the focus was on the effectiveness of the programme. Six hundred students, 13 teachers and six schools were involved in the research. Teachers in the experimental group were trained in AGO methods and in implementing the new AGO curriculum. Teachers in the control groups worked with the existing programme following their usual methods of teaching.

The AGO model

The AGO model tries to create a balance between (1) basic skills and concepts developed through whole-class instruction and (2) problem-solving developed by guided-discovery learning in co-operative groups. The model combines aspects of co-operative learning and adaptive instruction (Terwel 1986, 1990) and the various instructional strategies are designed to develop higher-order thinking. In the model arrangements are made to adapt teaching to differences in aptitude between students.

The AGO model is designed for the middle grades and consists of the following instructional stages:

1. whole-class introduction of a mathematics topic;
2. small-group co-operation in groups of four students;
3. teacher assessments: tests and observations;
4. alternative learning paths depending on assessments. These paths consist of two different modes of activity:
   (a) individual work with the possibility of consulting other students;
   (b) the opportunity of working in a remedial group under direct guidance and supervision of the teacher;
5. individual work in heterogeneous groups with possibilities for students to help each other;
6. whole-class reflection and evaluation of the topic.

This cycle is extended through a series of lessons (units) over, e.g., 3–5 weeks, preferably in extended blocks of uninterrupted instructional time. An existing Dutch mathematics curriculum was reconstructed in accordance with the AGO model.

Each cycle begins with whole-class instruction, for example, in the form of a systematic explanation or a socratic dialogue. The aim of this introduction is to motivate students, to allow them to review the necessary starting knowledge, to give an overview of the learning unit and to introduce the most important concepts and solution procedures. The teacher is free to incorporate whole-class instruction during other components in the cycle.

After the whole-class introduction pupils work in small heterogeneous groups of four. The assignments are designed especially for group work. It is characteristic of AGO that group assignments (where possible or desirable) are presented in real-life contexts. The concepts and solution procedures necessary for the programme's problems are explicitly taught (i.e., in the classroom) before the pupils start their assignments. The assignments are constructed in such a way that collective solutions in, say, groups of four 'make sense'. Problems can be solved in different ways (depending on levels in the learning process). In view of the support that is devoted to the solution process in the learning materials, as well as the supervision that the class or group receives from the teacher, the group process may be described as 'guided rediscovery' (compare Freudenthal's [1973b] concept of 're-invention'). In a whole-class intermezzo pupils report on the solutions they have arrived at in the groups and reflect, under the guidance of a teacher, on the differences in solutions and methods of solution. There then follows a diagnostic test for each individual pupil. This test may be more or less open depending on the aims of the relevant cycle. It is a means of verifying the level that each pupil has attained. The teacher marks or grades the test and discusses the results in class. She or he decides on the basis of the results and his or her personal experiences with the pupils how to continue, i.e., whether there should be multiple learning tracks for weaker and stronger pupils. Pupils who fall behind, and whose knowledge clearly shows gaps, receive specially adapted instruction from the teacher in groups of, for example, six to 10 pupils. Other pupils work independently on individual assignments. Conferring is allowed.

In the next stage, pupils work independently on assignments in the same heterogeneous groups as for the co-operation component, but the method of working differs in that pupils work independently on different assignments. Pupils are again allowed to ask each other for assistance. The teacher supervises individual pupils. Finally, the teacher ends the classroom cycle. Again, pupils are allowed to report. The teacher winds the cycle down with a recapitulation of the most important concepts and solution procedures.

In the research project the AGO cycles finished with a final test. This was a research test, but in addition it was used by the teacher as a means of determining class test marks. The teacher discusses the results after the final test and subsequently introduces a new learning unit.

The AGO model is based on principles of mathematics teaching and elements from cognitive theories and theories of motivation (Freudenthal 1973a, 1973b, Lesh 1981, Terwel 1986). Since working in small groups is of particular importance in the AGO model we shall elaborate the theoretical background to this approach.

According to theories of cognition it is to be expected that working in groups accelerates the learning process. The causes of the positive effects of group work may be found in five factors inherent in this type of learning environment.

1. Pupils in small groups are confronted by their fellow pupils in the group with different solutions and points of view. This may lead to socio-cognitive conflicts which are accompanied by feelings of uncertainty. This may cause a willingness in pupils to reconsider their own solutions from a different perspective. The resulting processes stimulate higher cognitive skills. In principle pupils can also conquer the uncertainty caused by different points of view with other members of the group, particularly where difficult or complicated assignments are concerned.
2. Small groups offer group members the opportunity to profit from the knowledge available in the group as a whole. This may take the form of knowledge, skills and experiences which not every member of the group possesses. Pupils use each other as 'resources' under those circumstances. Leechor (1988) calls this 'resource sharing'.
3. Collaboration in small groups also means that pupils are forced to verbalize their thoughts. Such verbalizations facilitate understanding through cognitive reorganization on the principle that 'Those who teach learn the most'. Offering and receiving explanation enhances the learning process. Group members not only profit from the knowledge and insights transmitted through 'peer tutoring', but they can also internalize effective problem-solving strategies by participating in the collective solution procedures.
4. Positive effects of group work can also be expected on the basis of motivation theory. Co-operation intensifies the learning process. Pupils in the 12 to 16 age-group are strongly oriented towards the peer group and very interested in interaction with their peers.
5. From the point of view of teaching methods in mathematics positive effects may be expected of the kinds of assignments that are used in groups. Varied assignments, which appeal to different cognitions and experiences, offer pupils the possibility of applying their strengths in the search for solutions. (See, for example, Freudenthal's [1973 a, 1973 b] theory of learning-process levels and Cohen, Lotan and Leechor's [1989] concept of multi-ability task.)

Samples of curriculum materials

The AGO model has been developed into a partial curriculum with learning materials for pupils and a manual for teachers. A number of chapters from an existing textbook (*Wiskunde Lijn*) were incorporated into these curriculum materials. In the research project the same mathematical content is used in both the experimental and the control groups.

The most radical adaptation of the existing textbook chapters concerned the group-work stage. The following examples of group assignments illustrate some of the five aspects of co-operative learning mentioned above. The assignments are based on the (mostly) individually oriented *Wiskunde Lijn* problems which will be discussed by way of comparison.

The following is a sample from the AGO materials. It concerns a real-life problem with several possible procedures.

> With the group:
> Annie Zorgvliet weighs her baby Flip every morning. When his weight reaches 4000 grams the baby will be allowed to eat morsels of fruit, in addition to drinking milk. The baby's growth has been satisfactory recently and the scales show a weight of 4100 grams. Annie is ready to go and prepare a banana when she discovers that the weighing was wrong, because Flip has a rattle in each hand. The rattles are identical and without them Flip weighs 3900 grams.
>
> (*a*) Make a drawing of a pair of scales with a drawing of Flip and both his rattles on one side and a weight of 4100 grams on the other. Write Flip's weight in his picture.
> (b) Write down the equation that goes with the scales. This means that you first write down the weight of the things on the left-hand side of the scales, then you write down the equals sign = and then you write down the weight of the things on the right-hand side of the scales.
> (c) What is the weight of one rattle?
> (d) Can you think of other ways of writing down the equation?

Figure 1 represents a worked-out solution to the first question.

Part (c) of the problem can be solved algorithmically by means of a more or less formal equation (e.g., $3900+2x=4100$, or $3900+?+?=4100$, or an equation with pictures of rattles); the solution could also be solved verbally or by trial and error. These various solution procedures are on different levels. In the original *Wiskunde Lijn* text several methods are dealt with in successive exercises as a summary of previous subject-matter. In the AGO materials different solution methods are juxtaposed through the openness of the procedures and extra questions such as (d). (Problems that are capable of several solutions are fruitful for group discussions. The time for a multi-solution assignment to feature in the curriculum can be chosen in such a way that the chance of obtaining more than one solution method is optimal.)

The real-life situation used in the sample problem will be more familiar or attractive to some pupils than to others. To meet this potential difficulty a parallel problem is available, with a story about a boxer. Both cases allow less able pupils to make a useful contribution based on their experiences of the real-life situation.

Figure 1. Partially worked out real-life problem.

The following illustration is a problem which can be solved at several levels and, in addition, has more than one solution.

> *With the group:*
> Draw a figure with a 5th-order rotation centre. You are allowed to use compasses, a ruler, colouring pencils, a protractor and tracing paper. *Think about the problem first and then discuss how it should be done.*

The required rotation angle of 72° can be constructed by gradual approximation or by prior calculation. Many different figures are possible. Two solutions are represented in figure 2.

The juxtaposition of different levels is particularly important in offering possibilities for a deepening of the learning process, since it involves orientation towards the higher level as well as a looking back at the lower level (see Freudenthal 1973 a).

The counterpart from *Wiskunde Lijn* is a set of problems in which the order of the rotation centre and the magnitude of the accompanying rotation angles in a series of figures has to be determined. These problems are more closed in character.

Figure 3 contains a representation of the *Wiskunde Lijn* assignment on the left, with the AGO assignment on the right.

In the AGO method equivalent problems are distributed, solved and discussed in the group. The method of working in parallel on equivalent problems was inspired by Lesh (1981). The presentation of equivalent problems, followed by communal checking, highlights the relationships between the problems. Scrutiny of these relationships contributes to the pupils' discovery of the conceptual model underlying the problems. The AGO assignment contains explicit questions about the relationships in questions.

The above examples often require intensive co-operation. At the same level there were also assignments which required less obligatory forms of co-operation, and which were therefore also suitable for homework. (For a description of the group processes based on qualitative observations [protocols] see Herfs *et al.* [1991].)

Research design and hypothesis

The research design is a pretest–post-test control-group design. Four schools, consisting of 15 classes in all, implemented the experimental AGO curriculum, while two schools with eight classes functioned as the control group. At the start of the experiment there were no significant differences in mean pretest (aptitude) scores of the students in the two conditions. The main hypothesis underlying the project is that there exists a positive

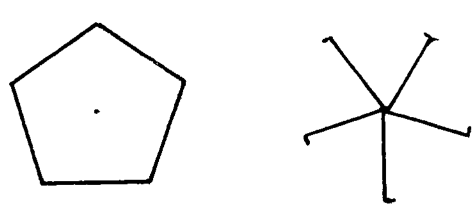
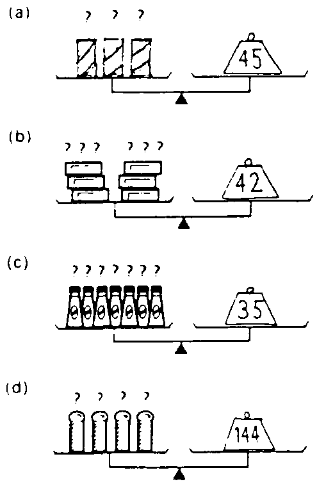


Figure 2. Partially worked out problem with several solutions.

WISKUNDE LIJN

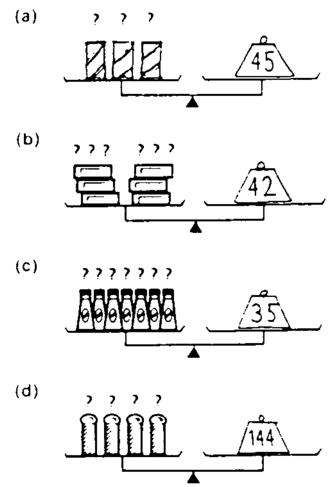
Write down the weighing stories as briefly as possible, and use the symbol ? in your summary. Then solve the equations.



AGO

WITH THE GROUP. Distribute problems (a), (b), (c) and (d).

Write down the weighing stories as briefly as possible. Then solve the equations.



DISCUSS WITH THE GROUP.

(e) What was the same in the equations? What was different?

(f) What had to be done in all the equations to find the solution?

Figure 3. Working in parallel with equivalent problems.

relationship between implementation of the AGO model and learning results. In experimental terms we formulated the hypotheses as follows:

1. Adaptive group education (AGO) will lead to better learning results in mathematics than educational contexts in which AGO characteristics are not implemented.
2. The higher the degree of AGO implementation, the better the learning.

The first hypothesis can be tested by comparing data gathered in AGO classes and non-AGO classes. The second hypothesis is testable in a within-experimental-group comparison (AGO classes only).

Methods, instruments and indices

Several instruments were used in the study, Here we offer only a short description of each instrument. (The instruments are described in more detail in the final report of this study [Herfs *et al.*, 1991].) All instruments proved to be reliable and valid.

Quantitative observations were carried out in all classes in order to investigate the (degree of) implementation. The observations were carried out by means of structured time-sampling observation; the scale was specially designed for this project and is entitled 'Adaptive Instruction Observation Scale' (AIOS). The categories were: *modes of instruction* (e.g., group work; individual work and whole-class instruction), *activities of the teacher*, *activities of individual students*, and *time-on-task*. The lessons selected for observation were chosen in such a way that different phases of the AGO cycle were covered in the experimental group. Observations in the control group were at comparable moments.

A cognitive pretest (mathematical reasoning) was administered to both groups at the start of the AGO period. It consisted of two sub-scales of the *Prüfsystem für Schul- und Bildungsberatung* (PSB), (Horn 1969). Scores on the pretest were used as a covariate and as a measure for determining the mean class aptitude level for each class.

In addition, a cognitive post-test in mathematics was administered. This consisted of two sub-tests (mathematics mappings, mathematics equations). Both sub-tests contained 11 open-ended problems that conformed to the content of the curriculum in both groups.

A teacher questionnaire (TQUEST) was completed by all participating teachers in order to investigate how much time was spent in covering the curriculum and how much time was spent by the students on individual work.

Another questionnaire, the PERCIA Scale (Perception of the Curriculum In Action) was completed by the students. PERCIA measures classroom processes and learning climate (learning environment scale). There were items on teacher whole-class instruction, climate, differentiation, co-operation, task-orientation and mathematics in real-life situations. The format of the instrument is based on Fischer and Fraser (1983) and revised in the AGO project for the specific co-operative learning environment.

A summary of the variables, descriptions and instruments follows:

Variables	*Description*	*Instrument*
CONDITION	treatment: AGO versus non-AGO	CURRICULUM
PRETEST	aptitude: mathematical reasoning	PSB
POST-TEST	achievement in mathematics	POST-TEST
INCLIM	whole class instruction/climate	PERCIA
GROUPW	percentage time in small groups	AIOS
REMIND	percentage time in stage 4 AGO	AIOS
TASK	percentage time on task in class	AIOS
INDWO	percentage time individual work	TQUEST
TIME	time spent in curriculum coverage	TQUEST

Means, standard deviations and correlations were obtained for all measures (Herfs *et al.* 1991). In order to estimate the effects of the AGO model (compared with the control condition) a regression analysis was first carried out at the individual level. In addition a multi-level analysis was carried out in order to decompose the variance at the two levels individual and class. The hypotheses were tested within the random coefficient (RC) model of multi-level analysis. The RC model may be conceived as consisting of two steps, although the steps are executed simultaneously. The first step concerns a within-group regression. In the second step the results of the first step are incorporated in a between-group regression analysis (Leeuw and Kreft 1986). Longford's (1988) VARCL programme was used in the analysis.

Results

Implementation

Systematic observations indicate that the experimental AGO curriculum was implemented according to the criteria we formulated. Comparison between the processes as observed (e.g., group work, assessments and special help) in the experimental and the control group clearly shows the expected differences between the two research conditions. Consequently we conclude that the AGO model has proved to be workable in practice, although we found differences in implementation between classes within the experimental condition, e.g., some teachers spent more time in small groups that others.

Effects

AGO students showed higher achievement than non-AGO students. There was no significant difference in mean pretest score between the experimental and the control group. However, the post-test scores showed a significant difference: the mean score for the control group was 19·37 (SD 6·62) versus 24·97 (SD 9·68) for the experimental group. The effect size (the difference of means divided by the standard deviation of the control group) is 0·85. (This is higher than most of the effect sizes Slavin [1987] reports. If we correct for the (non-significant) differences between the two groups at the pretest we get an effect size of 0·68, which is still larger than most of the findings Slavin reports.) Thus we conclude that there is a substantial difference in effect between the two conditions.

We now present the outcomes of a regression analysis at the individual level. Table 1 gives the results of this regression analysis. In the analysis reported in table 1, the first hypothesis was confirmed. CONDITION accounts for 6% of the variance over that already explained by the pretest. This is a significant and meaningful difference.

To decompose the variance at the individual level and the class level, a multi-level analysis was then carried out. The multi-level analysis confirmed the conclusion of the individual regression analysis regarding the learning effects in mathematics (see table 2). Model 1 is the maximal model. This model contains all the variables, whether significant or not. Model 2 is the reduced model and contains only the significant coefficients. From model 2

Table 1. Regression analysis at the individual level: effect of CONDITION on POST-TEST with PRETEST as covariable (AGO and non-AGO students, $n=572$).

Step	Multiple R	R^2	F(Eqn)	Sign. F	Variable	Beta
1	0·50	0·25	189·86	0·000	Pretest	0·50
2	0·56	0·31	129·00	0·000	Condition	0·25
		0·06				

Table 2. Coefficients and variances as outcomes of the multi-level analysis: the effects of PRETEST, TIME and CONDITION on the results of learning of the AGO and non-AGO students ($n=572$, standard errors in parentheses).*

	Model 1	Model 2
Fixed part		
Individual effect		
A_0 INTERCEPT	−255·9675	−245·1519
B_0 PRETEST	3·7310 (0·8653)	3·5503 (0·8522)
Class effect Intercept explained by		
A_1 PRETEST	3·8218 (0·8113)	3·7880 (0·7746)
A_2 TIME	0·0443 (0·0180)	0·0386 (0·0161)
A_3 CONDITION	7·4020 (5·003)	3·1343 (1·569)
Slope explained by		
B_1 PRETEST	−0·0429 (0·0141)	−0·0427 (0·0139)
B_2 TIME	−0·0009 (0·0003)	−0·0008 (0·003)
B_3 CONDITION	−0·0805 (0·0861)	
Random part		
s^2 individual	31·5348	31·5900
t^2 intercept (class)	12·6063 (0·5403)	9·1203 (0·5084)
v^2 slope (class)	0·0013 (0·0459)	
DEVIANCE	3646·3393	3646·4636
DEV-difference		0·1243
DF-difference		2

* Coefficients in Fixed part and variances in Random part.

we conclude an individual effect of the PRETEST. There is also an effect of the mean PRETEST and the TIME on the intercept and on the slope at the class level. These are interesting results. However, the most important outcome is the effect of CONDITION on the intercept. This analysis clearly shows a positive effect of the AGO model. Thus there is clear evidence for the positive effect of the AGO model on the results of learning in mathematics. The multi-level analyses also show that the two context variables, mean class aptitude as measured by PRETEST and the amount of TIME spent in class covering the curriculum content, had a significant effect on students' learning. A higher mean for mathematical aptitude and a greater amount of time spent on mathematics correlate positively with learning results.

The second hypothesis suggests that the more the AGO characteristics are implemented, the better the learning results will be. In order to test this hypothesis, a multi-level analysis was carried out within the experimental condition (AGO students only). The results are presented in table 3.

From model 2 in table 3 the following conclusions can be drawn. We find a positive effect of PRETEST at the individual level. We also find effects of PRETEST (mean class aptitude) and TIME (this is in line with the outcomes of the analysis in table 2, model 2). In addition table 3 gives information about the effects of specific AGO characteristics. The percentage of time spent working in small groups (GROUPW) is a positive factor in explaining the learning results. Group work has a positive effect on the slope. On the other

Table 3. Coefficients and variances as outcomes of the multi-level analysis: the effects of AGO characteristics, PRETEST and TIME on the results of learning of the AGO students (N=381, standard error in parentheses).*

	Model 1	Model 2
Fixed part		
Individual effect		
A_0 INTERCEPT	−209·2692	−258·7376
B_0 PRETEST	3·7731 (1·4034)	4·1236 (0·9367)
Class effect		
Intercept explained by:		
A_1 INCLIM	−6·6328 (8·0989)	
A_2 GROUPW	0·0816 (0·3171)	
A_3 REMIND	0·0894 (0·3171)	−0·1208 (0·0558)
A_4 INDW	0·2473 (0·2047)	−0·1044 (0·0419)
A_5 TASKO	−0·0553 (0·3004)	
A_6 PRETEST	3·6289 (0·9999)	3·8597 (0·7776)
A_7 TIME	0·0493 (0·0322)	0·0575 (0·0245)
Slope explained by:		
B_1 INCLIM	0·0381 (0·1413)	
B_2 GROUPW	0·0022 (0·0054)	0·0022 (0·0011)
B_3 REMIND	−0·0035 (0·0055)	
B_4 INDW	0·0012 (0·0036)	
B_5 TASKO	0·0013 (0·0052)	
B_6 PRETEST	−0·0486 (0·0184)	−0·0468 (0·0141)
B_7 TIME	−0·0010 (0·0006)	−0·0012 (0·0004)
Random part		
s^2 individual	29·7054	29·6685
t^2 intercept (class)	2·9325 (0·4441)	5·6686 (0·5261)
v^2 slope (class	0·0004 (0·0464)	
DEVIANCE	2392·6033	2399·2404
DEV-difference		6·6371
DF-difference		8

hand, the percentage of time spent on individual work (INDW) has a negative effect on learning results (negative effect on intercept).

An unexpected outcome of this study was that REMIND (working with a remedial group while the rest of the class works individually (Stage 4 of the AGO model) appears to have a negative effect on learning results (negative effect on the intercept). Consequently, this part of the second hypothesis had to be rejected. Finally, no effects were found for the AGO characteristics whole class instruction (INCLIM) and percentage of time on task (TASKO) and, consequently, those parts of the second hypothesis has to be rejected as well.

Summary

The main conclusion of the study is positive. The AGO model as a whole proved to be practical and effective in learning mathematics. The AGO model

has a positive effect on the intercept, which means that the mean scores of AGO classes are higher than the mean scores of non-AGO classes. It may be concluded that, on the average, students benefit from learning in AGO classes as compared with non-AGO classes. The experimental treatment (AGO model) has no effects on the slope. This means that AGO does not increase or decrease the differences between students in the same class.

In addition, and as expected, positive effects of two context variables were found: (1) the total amount of time spent in class covering the mathematical content and (2) class composition as indicated by the mean pretest score (aptitude) of the class.

Two specific conclusions can be drawn about components of the AGO model:

(a) the percentage of time spent working in small groups has a positive effect on the slope of a class: students benefit from co-operation in small groups; but group work increases the differences in achievement between the students in the class. Higher- and lower-achieving students in the same class benefit differentially from working together in small groups.
(b) One part of the AGO model (the alternative learning paths, to adapt instruction to individual differences) did not produce the expected positive effects.

Discussion

Co-operative learning is often proclaimed to be a far-reaching means of drawing out untapped potential. Students are said to be resources for each other in co-operative learning environments. The process of co-operative learning is a resource-sharing process. Students may benefit from the collective supply of concepts and strategies in their group and class. By implication they become more dependent on the qualities of their fellow students and more dependent on their own strengths and weaknesses. Students will benefit more or less from co-operative learning depending on their access to the resources. Access assumes the existence of resources (the untapped potential). The bigger and the more complex the cognitive potential in a class, the more the students will benefit from co-operative learning. However, individual students may differ in many aspects and consequently differ in access to the existing resources in their class.

The findings from our study about the positive, but differential effects of group work indicate that students benefit from group work, but that high-aptitude students benefit more than those with low aptitudes. Our findings about the differential effects of group work for high- and low-aptitude students are in line with the conclusions of several other studies. In a review of the cognitively oriented research literature, Leechor (1988: 43) concludes: 'it is often found that high-achieving students are more active and influential in the interaction process and learn more than low-achieving students'. However, Leechor's own empirical findings to some extent contradict the conclusions from the literature. Webb (1982) found that high-achieving students offer more explanations than low achievers and that giving explanations is positively correlated with learning results.

The findings and explanations in the AGO project have relevance for the practice of teaching and for grouping practices. If high- and low-achieving students benefit differentially from group work it becomes questionable whether any measures are required at all. In the context of the whole instructional organization of the AGO model, additional measures do not seem necessary. AGO as a whole showed no differential effects (see table 2).

But if we reflect on group work as such the question still remains. And if the answer is yes, what can be done? Cohen *et al.* (1989) are convinced that some action must be taken, and make a plea for two approaches: (1) reduction of uncertainty and (2) status treatment. In the AGO project a third approach is used, namely, (3) adapting instruction to individual differences. Each of these approaches is based on different explanations of the phenomena of differential effects. We will consider these explanations and approaches in turn.

1. *Reduction of uncertainty:* Cohen *et al.* (1989) see 'uncertainty of the task from the students' point of view' as one of the key factors in explaining the differential effects of working in small groups. Low-achieving students experience more uncertainty than higher-achieving students. Low-achieving students have less access to the learning task than high achievers. Talking and working together is particularly effective in reducing the uncertainty of the task for students with minimal academic skills. Students can benefit from the assistance of their peers.

2. *Status treatment:* Another problem, related to differential effects, is the problem of status. Some students are more active than others. This is seen by Cohen *et al.* (1989) as an unwanted domination that needs to be treated. They propose two status treatments (multiple-ability tasks and assigning competence) in order to boost the status and the participation of low-status students. Cohen *et al.* (1989) found evidence for the effectiveness of status treatment in weakening the problem of unwanted domination.

3. *Adapting instruction:* Differential effects can also be explained in terms of cognitive theories. If teachers' assessments show that some students do not benefit from discovery learning in small groups, an explanation can be found in differences in aptitude. Those students who do not have the prerequisite declarative and procedural knowledge cannot benefit from working together in small groups. Another explanation can be given on the basis of research in aptitude/treatment interaction: low-aptitude students may need a more structured approach than high-aptitude students (Snow 1989). These explanations are considered in the AGO project, and as a result a third approach has been used as a part of the AGO model. This approach involves adapting instruction by the teacher to differences in aptitudes between students. After whole-class instruction and working in small groups, the teacher makes an assessment of every individual student and gives special instruction to those students who apparently have not benefited from instruction and group work. This help (adaptation/scaffolding) is given in a special-help group of (e.g., 4–8) students under direct supervision of the teacher, while the others work for themselves on more advanced tasks.

Although the AGO model as a whole showed substantial and consistent effects on learning results in mathematics, our specific hypothesis about the effect of Stage 4 was not confirmed. On the contrary, we found a negative effect on the intercept. The more time spent in Stage 4 of the AGO model, the lower the intercept of the class. This means a lower mean score in a class, which does not necessarily imply a lower score for the students who received special help. We cannot totally exclude possible factors such as 'quality of implementation' or 'status problems' in explaining the findings. However, we can also look for other explanations.

One might wonder whether this finding can be attributed to management problems which have lowered the task orientation (time-on-task) of the class. The answer is negative. There exists a rather high, positive correlation at the class level between the percentage of the time devoted to Stage 4 and task orientation. An interesting question remains: is Stage 4 of the AGO model detrimental to learning results or is it the other way round? Stage 4 can also be seen as a reaction from the teacher to learning difficulties in a particular class, and without Stage 4 the results could be even worse. We have some indications that this is the case. In 10 of the 15 AGO classes the teacher gave special help to students according to Stage 4 in the AGO model. These 10 classes had a lower mean pretest score than the five who did not have a special help group. In addition those 10 classes also had a lower mean post-test score than the other five classes (20·8 versus 30·3).

By itself this difference in class means cannot explain the negative effect of Stage 4. Our model controls for differences in class mean. Nevertheless, it is conceivable that a lower class mean indicates a higher proportion of low SES students. In conjunction with this lower SES level of a class, there could be other variables (e.g., verbal ability or social skills) which exert a negative influence on the potential of a class to benefit from the earlier stages of AGO instruction. Thus we interpret the negative effect of Stage 4 as an indicator of a causal process that works in a direction opposite to the one we had assumed in our original model. Stage 4 is now taken to reflect the existence of learning difficulties in a class. The teacher implements Stage 4 as a response to these learning difficulties.

Inspection of the posterior means of random effects (table 3, model 2) sheds some light on this question. The 10 classes in which Stage 4 was implemented have an average of $-5{\cdot}5$ posterior means versus an average of 1·02 in the five classes in which the teacher decided not to implement Stage 4. Thus we conclude that in the classes in which the teacher has given special help according to Stage 4 in the AGO model there is variance left which we cannot explain. Perhaps there is a variable involved which has a negative effect, e.g., SES or specific learning difficulties (not in our model, so we cannot control for these). At any rate it seems that the teachers in the 10 classes made the correct decision to give those classes special (adaptive) instruction, because they contained students with very low achievement, probably due to variables we have not controlled for in our design. Our design does not allow for more specific conclusions. The answer can only be found in an experiment in which the classes that are in need of special help (according to systematic assessments by the teacher), are divided into an experimental group which receives the treatment of Stage 4 in the AGO model, and a control group which is given no special help at all. Thus there still remain some interesting questions about the effects of adapting instruction as a part of the AGO model.

The AGO research provided clear evidence about the positive effects of the model as a whole. The AGO model has a positive effect on the intercept, which means that the mean score of AGO classes is higher than the mean score of non-AGO classes. In general, students benefit from learning in AGO classes as compared with non-AGO classes. However, it is again important to note that the experimental treatment (the AGO model as a whole) has no effects on the slope. This means that AGO has no differential effects on the learning of high- and low-aptitude students in the same class.

We will now consider the practical implications of the effects of the two context variables 'class composition', as indicated by the mean aptitude of a class, and 'allocated time'. Both variables have consistent, positive effects on the intercepts but at the same time have negative effects on the slopes. This means that both variables can be used to influence learning results. A higher mean and/or more time to cover the curriculum in general has a positive effect on learning but decreases the differences between students. Schools and teachers can make a difference at the organization level by manipulating class composition and time. Streaming creates classes with differences in mean aptitude. From our study and many others it is evident that the students in the lowest stream will proceed less well than their fellow students in the higher streams. Furthermore, classes that are given more time to go through the curriculum seem to have better results than classes with less time, but increasing the time has different effects for low- and high-achieving students. High achievers do not benefit from more time. It is important to note that a higher mean and/or more time implies main effect(s) (intercept) and differential effect(s) (slope) on low- and high-aptitude students within a class.

The positive effects of class composition (indicated by the mean pretest score of a class) for all students can also be explained by the 'access theory'. Access to resources merely presupposes the existence of an untapped potential. In classes with a more restricted cognitive potential students will benefit less from co-operative learning than in classes with a rich gold mine as indicated by a high mean pretest score (mean aptitude).

Between-class ability grouping may result in increasing differences in aptitude and status between students. These differences become part of the self-image of the students and the expectations of teachers. This situation may set in motion a downward spiral for the lower streams, which can only be broken by abandoning streaming and tracking in the first years of secondary education. In addition, special measures need to be taken with respect to within-class differences in status and aptitude between students. Applying the AGO model appears to be a good alternative.

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