Comment on ''Coupling constant and quark-loop expansion for corrections to the valence approximation''

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Lee and Weingarten have recently criticized our calculation of quarkonium and glueball scalars as being ''incomplete'' and ''incorrect.'' Here we explain the relation of our calculations to full QCD.

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Lattice techniques provide an invaluable tool for calculating the properties of hadrons [1]. As a matter of practical necessity, these calculations involve approximations to full QCD. While the spectrum of glueballs has been computed with increasing precision [2–4], this is within quenched QCD. To make contact with experiment requires one to get closer to the full theory by allowing for the creation of $q\overline{q}$ pairs. Different attempts to do this for light scalars, both quarkonium and glueball, have been made by Boglione and Pennington (BP) [5] and by Lee and Weingarten (LW) [6]. In a very recent paper, LW have criticized the former attempt as being *incomplete* and *incorrect*. We believe their extensive discussion is in error in claiming key aspects of QCD have been omitted by BP. Let us explain.

The BP treatment, like that of Tornqvist [7] and others [8], is based on a specific approximation to QCD, in which only hadronic (color singlet) bound states and their interactions occur. One begins with the QCD Lagrangian, for which the only parameters are quark masses and the strength of the quark-gluon interaction. Λ_{QCD} and other scheme-dependent parameters enter on renormalization. One then formally integrates out the quark and gluon degrees of freedom and obtains a Lagrangian involving only hadronic fields with their interactions, in an infinite variety of ways, all of which are determined by the parameters of the underlying theory. We then focus on the ten lightest scalar states. The bare states are realized by switching off all their interactions. Consequently, their propagators are those of bare particles: they are stable. To take this limit, each coupling in the effective Lagrangian of hadronic interactions is multiplied by a parameter λ_i and these λ_i are taken to zero. This does not necessarily correspond to a simple limit of QCD. Nevertheless, we plausibly assume that the ten lightest non-interacting states, which result in this limit, are the nine members of an ideally mixed quarkonium multiplet and an (orthogonal) glueball. Notice that the names *quarkonium* and *glueball* are just a convenient way of referring to the quantum numbers of these states. Individual quark and gluon fields play no role. However, they are, of course, implicit in the formation of hadronic bound states.

The Tornqvist [7] and BP treatment is then to switch on the '' dominant'' interactions of the light scalars by tuning the appropriate parameters λ_i from $0\rightarrow 1$ for the couplings of the bound states to two (or more) pseudoscalars.[1] It is by turning on the interactions that the bare states are ''dressed.'' Figure 1 represents the Dyson summation of such contributions to the inverse propagator. This dressing does not correspond to the creation of a single $q\overline{q}$ pair. Multiple pairs and all the gluons (Fig. 1) needed to generate color singlets and respect the chiral limit are implicitly included. Indeed, it is well known [9], that any picture of pions as simple $q\overline{q}$ systems loses contact with the Goldstone nature of the light pseudoscalars, so crucial for describing the world accessible to experiment. This important (chiral) limit is embodied in our calculation. The resulting hadronic interactions have a dramatic effect on the scalar sector. For instance, the a_0 and an f_0 emerge at 980 MeV with large $K\overline{K}$ components [10], even though their bare states are members of an ideal multiplet 4–500 MeV/c^2 heavier. LW criticize these results as not including the specific gluonic counterterm, Fig. 2, and not explaining why.

The explanation is clear: our analysis only includes color singlet states, both internally and externally as unitarity requires. Colored configurations of whatever kind are implicitly included and not readily dissected. If such counterterms are relevant to the dressing by pseudoscalar (Goldstone) pairs, they have been included.

In spirit, our analysis [5,11] is close to that of Refs. [7,8]. Propagators are dressed by hadron clouds, as in Fig. 1. These determine the right hand cut structure of meson-meson scattering amplitudes. However, in the work of Ref. [7], this *s*-channel dynamics is assumed to control the whole scattering amplitude, with left hand cut effects (and crossed-channel exchanges) neglected, even though this violates crossing symmetry [12,11]. In our treatment [5,11], particularly here where we consider mixing, only propagators are computed and no further assumptions are needed.

Of course, our analysis does have approximations. For instance, the scale of hadronic form factors for a gluish state is assumed to be similar to that of well-established $q\overline{q}$ hadrons. This may not be the case. Moreover, our treatment only

[1]For the glueball, the four pion channel may be particularly important.

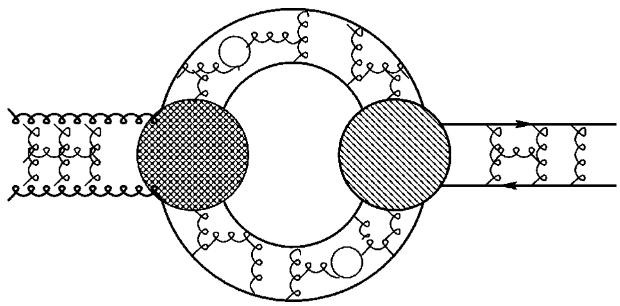

FIG. 1. A pictorial representation of mixing between quarkonium and glueball physical states through common meson-meson channels, included in the Dyson summation of the scalar bound state propagator.

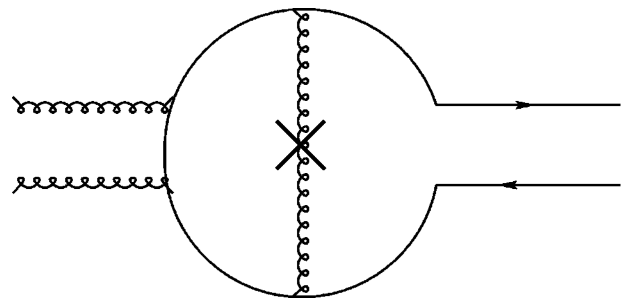

FIG. 2. Feynman diagram representing the counterterm contribution to the glueball-quarkonium mixing amplitude as given by Lee and Weingarten [6].

incorporates interactions with two pseudoscalars and, to a lesser extent, with multipion channels. It is these that determine both the sign and magnitude of the mass shifts generated. For the quarkonium states, the dressing by the light two pseudoscalar channels always produces a downward shift in mass. The size of these shifts of between 100 and 500 MeV (depending on flavor) is set phenomenologically [10] by the $K_0^*(1430)$. A much smaller shift of 10–25 MeV for the precursor glueball is set by the strength of the glueball to two pseudoscalar coupling calculated on the lattice by Sexton *et al.* [13]. The suppression of the couplings of the resulting ''dressed'' hadron to two pseudoscalars happens [11] irrespective of the exact mass of the bare glueball [2–4]. The inclusion of more channels, like $\rho\rho$ and $K^*\overline{K^*}$, may well be important in dressing this state and may alter the rather small mass shifts we found for that sector in both magnitude and sign. Of course, only physically accessible hadronic intermediate states contribute to the imaginary part of the propagator, Fig. 1. Unopen channels contribute only to the real (or dispersive) part and result in renormalizations of the undressed parameters.

By including in our calculation key aspects of the hadron world, in the way described here and in [5], we believe we must have approached closer to full QCD—despite the criticism of Lee and Weingarten.

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