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Nonperturbative hyperfine contribution to the b_1 and h_1 meson masses

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Because of the nonperturbative contribution to the hyperfine splitting the mass of the n^1P_1 state is strongly correlated with the center of gravity $M_{\rm cog}(n^3P_J)$ of the n^3P_J multiplet: $M(n^1P_1)$ is less than $M_{\rm cog}(n^3P_J)$ by about 40 MeV (20 MeV) for the $1P(2P)$ state. For $b_1(1235)$ the agreement with experiment is reached only if $a_0(980)$ belongs to the 1^3P_J multiplet. The predicted mass of $b_1(2^1P_1)$ is ≈ 1620 MeV. For the isoscalar meson a correlation between the mass of $h_1(1170)$ $[h_1(1380)]$ and $M_{\rm cog}(1^3P_J)$ composed from light (strange) quarks also takes place.

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I. INTRODUCTION

Since the discovery of the h_c meson [1] the hyperfine (HF) splittings of the *P*-wave states in heavy quarkonia have been investigated in many papers [2–6]. In Refs. [5] and [6] it was clarified why the HF shift of the h_c meson with respect to the center of gravity $M_{\rm cog}(^3P_J)$ of the χ_c mesons turns out to be small, $\Delta_{\rm HF}(h_c)=-0.87\pm0.24$ MeV [7]. It is due to a cancellation of the perturbative and nonperturbative contributions, which are both small and have opposite signs: $\Delta^{\rm P}_{\rm HF}(c\bar{c})\approx-1.7\pm0.3$ MeV and $\Delta^{\rm NP}_{\rm HF}(c\bar{c})\approx 1$ MeV. Here the total HF shift $\Delta_{\rm HF}$ is defined in the following way:

$$\Delta_{\rm HF}=M_{\rm cog}(n^3P_J)-M(n^1P_1). \qquad (1.1)$$

For light mesons the HF splittings of the *P*-wave states are of special interest, since for them the perturbative spin-spin interaction is suppressed as for any $L=1$ state, while the nonperturbative HF interaction is expected to become larger. In our study it will be shown that the nonperturbative contribution $\Delta^{\rm NP}_{\rm HF}$, defined through the vacuum correlators, does dominate and $\Delta_{\rm HF}(1{\rm P})$ is about 30 MeV. Although the magnitude of the splitting depends on such vacuum characteristics as the gluon condensate G_2 and the gluonic correlation length T_g, the total $\Delta_{\rm HF}(nP)$ turns out to be positive in all cases considered.

In our calculations of the HF splittings we shall follow the approach developed in Ref. [8] where the spin-dependent interaction is considered as a perturbation and averaging the spin factors in a meson Green's function is performed without the expansion in inverse powers of quark masses, used in the usual treatment [9]. Therefore the spin-spin potential from Ref. [8] can be used for massless quarks and the HF splittings appear to be proportional to $[\mu_0(nL)]^{-2}$, where $\mu_0(nL)$ is the effective dynamical mass of a light quark, which is defined by the extremum of the Hamiltonian deduced from the QCD Lagrangian. It is essential that $\mu_0(nL)$ depends on the quantum numbers of the state considered and is not small; for the nP meson containing a light quark and antiquark, $\mu_0(1P)\approx0.40$ GeV and $\mu_0(2P)\approx0.52$ GeV and $\mu_0(1P)=454$ MeV and $\mu_0(2P)=566$ MeV for the $nP\, s\bar{s}$ states.

For the isovector $1P$ mesons $[b_1(1235)$ and the ground states of the a_J mesons] the calculated $\Delta_{\rm HF}(1P)$ is 39(19) MeV for two different vacuum gluonic correlation lengths: $T_g=0.3(0.2)$ fm, and with the use of the experimental mass of $b_1(1235)$ we obtain that

$$M_{\rm cog}(1^3P_J,I=1)=1258\pm10\ \ {\rm MeV}, \qquad (1.2)$$

where the theoretical error comes from the uncertainty in the value of the gluonic length T_g. From this result an important consequence follows, namely, the number (1.2) is compatible with the experimental masses of the a_J mesons $(n=1)$ only if $a_0(980)$ [but not $a_0(1450)$] belongs to the isovector 1^3P_J multiplet, i.e., $a_0(980)$ is a usual $q\bar{q}$ state.

For the $b_1(2P)$ meson the mass $M(b_1(2P))\approx1620$ MeV is predicted. The situation with the isoscalar *P*-wave mesons (h_1 and f_J) is also discussed and a correlation between the masses of $h_1(1170)$ and $M_{\rm cog}(1^3P_J)=1245$ MeV for $f_0(980)$, $f_1(1285)$, $f_2(1270)$, as well as between the mass of $h_1(1380)$ and $M_{\rm cog}(1^3P_J)\approx1420$ MeV for $f_0(1370)$, $f_1(1420)$, and $f_2(1430)$ [or $M_{\rm cog}=1470$ MeV if $f_2'(1525)$ belongs to a multiplet composed of a strange quark and antiquark] can also be interpreted as a manifestation of a positive (≈30 MeV) nonperturbative HF splitting.

II. NONPERTURBATIVE HYPERFINE INTERACTION

The HF splitting of the *P*-wave mesons originates both from perturbative and nonperturbative interactions:

$$\Delta_{\rm HF}(nP)=\Delta^{\rm P}_{\rm HF}(nP)+\Delta^{\rm NP}_{\rm HF}(nP), \qquad (2.1)$$

where the perturbative term for $L=1$ exists only in second order of α_s and will be discussed in Sec. V. The quantity $\Delta^{\rm NP}_{\rm HF}$ is defined by the nonperturbative spin-spin potential, which is usually presented in the form

$$V_{\mathrm{HF}}^{\mathrm{NP}}(r)=\frac{1}{3m_q^2}V_4^{\mathrm{NP}}(r). \tag{2.2}$$

As was shown in Ref. [8] the spin-spin potential $V_4^{\mathrm{NP}}(r)$ appears to be the same for heavy and light mesons (if the spin-dependent interaction is considered as a perturbation) and can be expressed through the vacuum correlators $D(x)$ and $D_1(x)$ which were introduced in Ref. [10] and calculated in lattice QCD [11,12]:

$$V_4^{\mathrm{NP}}(r)=2\int_0^\infty d\nu\left[3D(r,\nu)+3D_1(r,\nu)+2r^2\frac{\partial D_1(r,\nu)}{\partial r^2}\right]. \tag{2.3}$$

By definition, at the origin ($x=0$) these correlators are related to the gluon condensate $G_2=\alpha_s/\pi\langle F^a_{\mu\nu}(0)F^a_{\mu\nu}(0)\rangle$:

$$D(0)+D_1(0)=\frac{\pi^2}{18}G_2, \tag{2.4}$$

where the physical value of $G_2=0.04\pm0.02\ \mathrm{GeV}^4$ is usually taken.

In lattice calculations it was found that $D(x)$ and $D_1(x)$ can be parametrized as exponentials at separations $x\gtrsim0.2$ fm [11–13]:

$$D(x)=d\exp\left(-\frac{x}{T_g}\right),$$
$$D_1(x)=d_1\exp\left(-\frac{x}{T_g^{(1)}}\right),$$
$$(x>0.2\ \ \mathrm{fm}), \tag{2.5}$$

with the gluonic correlation lengths T_g and $T_g^{(1)}$, which turn out to be different in the quenched approximation and full QCD. In the general case the parameters d and d_1, obtained in lattice measurements, differ from $D(0)$ and $D_1(0)$.

In full QCD with dynamical fermions ($n_f=4$) the correlation length was found to be relatively large and the D_1 correlator is small and can be neglected in some cases [12]:

$$T_g\approx0.3\ \ \mathrm{fm},\quad d_1\approx\frac{1}{10}d,\quad (n_f=4). \tag{2.6}$$

It was shown in Ref. [12] that in this case the correlator $D(x)$ can be taken as an exponential over all distances, i.e., $d=D(0)$,

$$D(x)=D(0)\exp\left(-\frac{x}{T_g}\right),\quad (T_g\approx0.3\ \ \mathrm{fm}) \tag{2.7}$$

and from Eq. (2.4) in this case

$$D(0)\approx\frac{\pi^2}{18}G_2=0.55\ \ G_2. \tag{2.8}$$

Then from Eq. (2.3) the potential $V_4^{\mathrm{NP}}(r)$ is given by the expression

$$V_4^{\mathrm{NP}}(r)=6d\int_0^\infty\exp\left(-\frac{\sqrt{r^2+\nu^2}}{T_g}\right)d\nu=6drK_1\left(\frac{r}{T_g}\right),$$
$$d=D(0). \tag{2.9}$$

The string tension σ is defined in the general case as

$$\sigma=2\int_0^\infty d\nu\int_0^\infty d\lambda D(\sqrt{\lambda^2+\nu^2}), \tag{2.10}$$

and for $D(x)$ taken as an exponential at all distances it reduces to the relation

$$\sigma=\pi dT_g^2\ \ \text{or}\ \ d=\frac{\sigma}{\pi T_g^2},\ \ G_2\approx\frac{18\sigma}{\pi^3T_g^2}. \tag{2.11}$$

If σ is fixed and not large ($\sigma\approx0.14\ \mathrm{GeV}^2$) then for the gluon condensate a reasonable value $0.036\ \mathrm{GeV}^4$ (for $T_g=0.3$ fm) follows. In this case the nonperturbative HF splitting is

$$\Delta_{\mathrm{HF}}^{\mathrm{NP}}(nP)=\frac{2d}{m_q^2}\langle rK_1(r/T_g)\rangle_{nP}=\frac{2\sigma}{\pi T_g^2m_q^2}\langle rK_1(r/T_g)\rangle_{nP}. \tag{2.12}$$

For light mesons the HF shift in the form of the relation (2.12) gives a dominant contribution also in cases when $D(x)$ cannot be interpolated up to the origin, see below. The matrix elements in Eq. (2.12) will be calculated in our paper with the use of the solutions of the spinless Salpeter equation and the definition of the effective mass m_q of a light quark will be discussed in Sec. III.

Here we would like to notice that the potential $V_4^{\mathrm{NP}}(r)$ in Eq. (2.9), corresponding to the exponential correlator from Ref. [12], has an essential shortcoming. From our calculations it follows that this term gives a rather large nonperturbative shift in charmonium,

$$\Delta_{\mathrm{HF}}^{\mathrm{NP}}(1P,\ c\bar{c})\gtrsim5.0\ \ \mathrm{MeV},\quad (T_g=0.3\ \ \mathrm{fm}), \tag{2.13}$$

so that the total splitting (2.1) turns out to be positive for h_c in contradiction with the experimental negative number. Therefore, to explain the HF splitting of the 1P state in charmonium, one needs to know $D(x)$ in detail at small distances, since the HF splitting in heavy quarkonia appears to be very sensitive to the behavior of the correlators $D(x)$ and $D_1(x)$ at short distances (this problem will be considered in another paper). However, for the light P-wave mesons the behavior of the correlators $D(x)$ and $D_1(x)$ at short distances was found to be inessential, and for them the potential $V_4^{\mathrm{NP}}(r)$ in the form of Eq. (2.9) can be used with 5%–10% accuracy.

Nevertheless, for completeness we give below expressions for the correlator $D(x)$ and for $V_4^{\mathrm{NP}}(r)$, modified such as to make clear that there exists the opportunity to combine a small, "physical" value of the gluonic condensate G_2 and a small correlation length T_g. Otherwise the values fitted in lattice calculations (quenched approximation), $T_g \approx 0.2$ fm in Ref. [11] and $T_g \approx 0.12$ fm in Ref. [13], give rise to very large "unphysical" values of G_2, $\approx 0.14\ \mathrm{GeV}^4$ and $0.23\ \mathrm{GeV}^4$, respectively.

To this end $D(x)$ is supposed to be a constant at $x < x_0$, which differs from the coefficient d in Eq. (2.5) and can be taken as

$$D(x) = \mathrm{const} = d\,\exp\left(-\frac{x_0}{T_g}\right), \quad x \lesssim x_0, \quad x_0 \approx 0.2\ \ \mathrm{fm}, \tag{2.14}$$

while at $x \geqslant x_0$, $D(x)$ is given by the exponential (2.7) as it was observed in lattice measurements. Then even for very small $T_g = 0.6\ \mathrm{GeV}^{-1} = 0.12$ fm, the small value $G_2 \approx 0.02\ \mathrm{GeV}^4$ can be obtained for the gluon condensate. For the modified correlator $D(x)$, Eq. (2.14), the modified nonperturbative spin-spin potential is

$$\begin{aligned}\tilde{V}_4^{\mathrm{NP}}(r) = 6d\Bigg[& e^{-(x_0)/T_g}\sqrt{x_0^2 - r^2} \\ &+ \int_{\sqrt{x_0^2-r^2}}^{\infty} d\nu\,\exp\left(-\frac{\sqrt{r^2+\nu^2}}{T_g}\right)\Bigg]\theta(x_0 - r) \\ &+ 6drK_1\left(\frac{r}{T_g}\right)\theta(r - x_0).\end{aligned} \tag{2.15}$$

For the P-wave light mesons the difference in the nonperturbative HF shift for the potential $V_4^{\mathrm{NP}}(r)$ and $\tilde{V}_4^{\mathrm{NP}}(r)$ does not exceed 10% and therefore the simpler potential $V_4^{\mathrm{NP}}(r)$, defined by Eq. (2.9), can be used. Still for the h_c meson in charmonium such a modification of the spin-spin potential is important.

III. SPECTRUM AND MATRIX ELEMENTS

The fine structure and HF splittings in light mesons, with the exception of π and K, are typically much smaller than the differences between the unperturbed levels [17] and therefore the spin-dependent interaction can be considered as a perturbation. Then the choice of an unperturbed Hamiltonian is of great importance and here the unperturbed approximation is formulated with the help of the spinless Salpeter equation,

$$\{2\sqrt{\mathbf{p}^2 + m^2} + V_0(r)\}\psi_{nL}(r) = E_{nL}\psi_{nL}(r), \tag{3.1}$$

where m is the current mass of a quark and $V_0(r)$ is the static potential. We have chosen this equation since under some assumptions it can be deduced from the QCD Lagrangian. In particular, if in the Feynman-Schwinger representation [13,14] the backward trajectories are neglected, then for $L = 0$ the QCD Hamiltonian for the spinless quark (antiquark) coincides with Eq. (3.1) and for $L = 1$ the correction to the equation (3.1) is not large [15]. Therefore we can use the Salpeter equation for the P-wave states.

For light mesons in Eq. (3.1) the current mass is taken to be zero and the static potential $V_0(r)$ is taken in the form of the Cornell potential,

$$V_0(r) = -\frac{4}{3}\frac{\alpha_{\mathrm{eff}}}{r} + \sigma r + C_0, \tag{3.2}$$

where α_{eff} is an effective Coulomb constant. One can expect that for light mesons, which have the rather large size $R \gtrsim 1$ fm, $(R = \sqrt{\langle r^2 \rangle})$, the value of α_{eff} will probably be close to the so-called freezing value $\alpha_{\mathrm{fr}} = \alpha_{\mathrm{eff}}(r \to \infty)$ which was found in Refs. [16,17], and has the value

$$\alpha_{\mathrm{fr}} = 0.50 \pm 0.05, \tag{3.3}$$

if the screening effects are neglected. However, even for such a large α_{eff}, at long distances, $r \gtrsim 6\ \mathrm{GeV}^{-1}$, the Coulomb interaction is small compared to the linear confining potential and in most cases can be neglected. Therefore, we consider here two variants:

$$\alpha_{\mathrm{eff}} = 0 \quad (\text{case A}), \quad \alpha_{\mathrm{eff}} = 0.45 \quad (\text{case B}). \tag{3.4}$$

To fix the string tension σ in the static potential (3.2) one needs to take into account that although the Salpeter equation with a linear potential σr provides a linear Regge trajectory, however, as shown in Refs. [15], the slope of the Regge trajectory for the Salpeter equation

$$\alpha' = \frac{1}{8\sigma} \tag{3.5}$$

differs from the slope α'_{st} in the string picture where

$$\alpha'_{\mathrm{st}} = \frac{1}{2\pi\sigma_{\mathrm{st}}}, \tag{3.6}$$

with the standard value of $\sigma_{\mathrm{st}} \approx 0.182\ \mathrm{GeV}^2$. Therefore, to provide the experimentally observed slope, the value of σ in the Salpeter equation should be taken smaller than σ_{st}:

$$\sigma = \frac{\pi}{4}\sigma_{\mathrm{st}} = 0.143\ \ \mathrm{GeV}^2. \tag{3.7}$$

In most of our calculations just this number will be taken, but in some cases the value $\sigma \approx \sigma_{\mathrm{st}} \approx 0.18\ \mathrm{GeV}^2$ will be also used for comparison. Thus in case A the static interaction is characterized by the parameter σ only, with its value given by the number (3.7). With this smaller value of σ the masses of the excited states in our calculations will be lower than in Ref. [17] (where the same Salpeter equation was solved with $\sigma_{\mathrm{st}} = 0.18\ \mathrm{GeV}^2$) and closer to the experimental meson masses for the excited states.

IV. DYNAMICAL MASSES OF LIGHT QUARKS

In Refs. [8] a relativistic Hamiltonian H_R was derived from the meson Green's function in the Feynman-Schwinger representation with the use of the auxiliary field (einbein)

TABLE I. The dynamical masses $\mu_0(nL)$ (in MeV) for different light mesons (the current mass $m=0$).

	$1S$	$2S$	$3S$	$4S$	$1P$	$2P$	$1D$
Set A $\sigma=0.143$ GeV^2 $\alpha_{\rm eff}=0$	298	445	557	650	399	516	480
Set B $\sigma=0.143$ GeV^2 $\alpha_{\rm eff}=0.45$	375	513	616	703	436	551	508
Set C $\sigma=0.18$ GeV^2 $\alpha_{\rm eff}=0$	335	500	625	729	448	579	539

approach. For $L=0$ and a spinless quark (antiquark) H_R is given by the operator

$$H_R=\frac{\mathbf{p}^2+m^2}{\mu(\tau)}+\mu(\tau)+\frac{\sigma^2 r^2}{2}\int_0^1\frac{d\beta}{\nu(\beta)}+\frac{1}{2}\int_0^1\nu(\beta)d\beta, \tag{4.1}$$

where $\mu(\tau)$ and $\nu(\beta)$ are the auxiliary operators and $\mu(\tau)$ is defined in the following way:

$$\mu(\tau)=\frac{1}{2}\frac{dt}{d\tau}. \tag{4.2}$$

In the definition (4.2) τ is the proper time and t is the actual time. With the use of the steepest descent method the extremal values $\mu_{\rm ex}(\tau)=\mu_0$ and $\nu_{\rm ex}(\beta)=\nu_0$ can be obtained with the following result:

$$\mu_0=\sqrt{\mathbf{p}^2+m^2},\quad \nu_0=\sigma r. \tag{4.3}$$

Then the relativistic Hamiltonian H_R in Eq. (4.1) reduces to the spinless Salpeter operator

$$\tilde{H}_R=\frac{\mathbf{p}^2+m^2}{\mu_0}+\mu_0+\sigma r\rightarrow 2\sqrt{\mathbf{p}+m^2}+\sigma r. \tag{4.4}$$

In what follows the extremal value μ_0, which is an operator, will be replaced by the average of this operator, which depends on the quantum numbers nL of the state considered, i.e.,

$$\mu_0(nL)=\langle\sqrt{\mathbf{p}^2+m^2}\rangle_{nL}\quad \text{for}\quad m\neq 0,$$

$$\mu_0(nL)=\langle\sqrt{\mathbf{p}^2}\rangle_{nL}\quad \text{if}\quad m=0, \tag{4.5}$$

where m is the current mass of a quark (antiquark) and for light quarks we take $m=0$, while for the strange quark $m_s=170$ MeV will be used.

The definition (4.5) of the effective mass of a light quark was already discussed in Ref. [18] where it was shown that the expectation value of $\tilde{H}_R$ in Eq. (4.4) coincides with that for the nonrelativistic Schrödinger Hamiltonian, if the effective mass is defined as in Eq. (4.5).

As seen from the definition (4.5) the dynamical mass of a light quark $\mu_0(nL)$ appears to coincide with half the average of the kinetic-energy operator:

$$\mu_0(nL)=\tfrac{1}{2}\overline{E}_{\rm kin}(nL). \tag{4.6}$$

In Table I the values of $\mu_0(nL)$ are given for different sets of the parameters of the static potential $V_0(r)$. From Table I one can see that the influence of the Coulomb interaction is rather weak even for an $\alpha_{\rm eff}$ as large as $\alpha_{\rm eff}=0.45$, except for the $1S$ case, where it changes the dynamical mass by roughly 25%. This happens because the sizes of the light mesons are large, e.g., the root-mean-square radii $R(nL)$ for the different states are as follows:

$$R(1S)=0.8\text{–}0.9\ \text{fm};\quad R(2S)=1.3\text{–}1.4\ \text{fm};$$

$$R(3S)=1.6\text{–}1.8\ \text{fm};\quad R(4S)=1.9\text{–}2.1\ \text{fm};$$

$$R(1P)=1.0\text{–}1.2\ \text{fm};\quad R(2P)=1.4\text{–}1.6\ \text{fm};$$

$$R(1D)=1.3\text{–}1.4\ \text{fm};\quad R(2D)=1.6\text{–}1.8\ \text{fm}. \tag{4.7}$$

At such long distances the Coulomb interaction is small, only $\lesssim 10\%$ compared to the linear term σr. Moreover one cannot exclude that at $r\gtrsim 1.2$ fm the screening of the Coulomb interaction may be important and therefore the Coulomb term in the static potential is even smaller and can be neglected, being important only for the $1S$ ground state.

To illustrate our results, the spin-averaged masses of the low-lying mesons are presented in Table II and compared to the experimental values (isovector and isoscalar mesons) and also to the masses from the paper by Godfrey and Isgur [17], where the same Salpeter equation is solved for a different set of parameters:

$$\sigma=0.18\ \text{GeV}^2,\quad \alpha_{\rm GI}(r)\leqslant\alpha_{cr}=0.60,$$

$$C_0=-253\ \text{MeV},\quad m=220\ \text{MeV}. \tag{4.8}$$

As seen from Eq. (4.8) in [17] a rather large value was taken for the current mass m of a light quark, while in our calculations the best fit was obtained with Set A:

TABLE II. The spin-averaged masses $M_{\mathrm{cog}}(nL)$ (in MeV) of the low-lying light mesons.

State	$2S$	$3S$	$1P$	$2P$
This paper				
$\sigma=0.143$ GeV^2	1424	1870	1241	1707
$\alpha_{\mathrm{eff}}=0$				
$C_0=-357$ MeV	fit			
Ref. [17]	1420	1970	1260	1820
Experiment	1424	>1800	1252 [a]	1632 [c]
$(I=1)$	± 44		1306 [b]	1683 [d]

[a]This value of $M_{\mathrm{cog}}(1P)$ is obtained if $a_0(980)$ belongs to the 1^3P_J multiplet.
[b]This value of $M_{\mathrm{cog}}(1P)$ is obtained if $a_0(1450)$ belongs to the 1^3P_J multiplet.
[c]This value of $M_{\mathrm{cog}}(2^3P_J)$ is obtained if $a_2(1660)$ belongs to the 2^3P_J multiplet.
[d]This value of $M_{\mathrm{cog}}(2^3P_J)$ corresponds to the case when $a_2(1750)$ belongs to the 2^3P_J multiplet.

$$\sigma=0.143\ \mathrm{GeV}^2,\quad \alpha_{\mathrm{eff}}=0,\quad m=0,\quad C_0=-357\ \mathrm{MeV}. \tag{4.9}$$

The constant C_0 in Eq. (4.9) was chosen to fit $M_{\mathrm{cog}}(2^3S_J)=1424$ MeV.

In Table II the experimental numbers refer to the isovector mesons, which are not mixed with $s\bar{s}$ and are expected not to have a large hadronic shift. From this table one can see that (i) a better agreement with the experimental masses is obtained if $a_0(980)$ is a member of the 1^3P_J multiplet; (ii) in our calculations the masses of the $3S$ and $2P$ states lie about 100 MeV lower than in [17] and are closer to the experimental numbers for $M_{\mathrm{cog}}(2a_J)$ and $\pi(1800)$.

With the use of the dynamical masses $\mu_0(nL)=m_q$, presented in Table I, the nonperturbative HF splitting can be calculated, since from Eq. (2.3) we obtain

$$\Delta_{\mathrm{HF}}^{\mathrm{NP}}(nL)=\frac{2d}{\mu_0^2(nL)}\left(J_1+\frac{d_1}{d}J_2\right), \tag{4.10}$$

where we have taken into account the second correlator $D_1(x)$ in Eq. (4) to have the opportunity to vary the values of the correlation length T_g. In particular for $T_g=0.2$ fm the ratio $d_1/d\approx 1/3$ was found in Ref. [11].

In Eq. (4.10),

$$J_1=\langle rK_1(r/T_g)\rangle_{nL},$$

$$J_2=\langle rK_1(r/T_g)\rangle-\frac{1}{3T_g}\left\langle r^2K_0\left(\frac{r}{T_g}\right)\right\rangle. \tag{4.11}$$

Here it is assumed that the gluonic correlation lengths T_g and $T_g^{(1)}$ in Eq. (2.3) are equal, as it was observed in lattice measurements of $D(x)$ and $D_1(x)$ for $n_f=0$ [11,13]. We shall also fix the string tension σ and from the definition Eq. (2.10) the parameter d is

$$d=\frac{\sigma}{\pi T_g^2}. \tag{4.12}$$

We estimate the accuracy of the calculated numbers to be about 10%. The nonperturbative HF splittings of the S-wave and P-wave light mesons are given in Table III for two values of the correlation length: $T_g=0.5$ fm and $T_g=0.2$ fm (in both cases $\sigma=0.143$ GeV^2, $\alpha_{\mathrm{eff}}=0$).

As seen from Table III the nonperturbative HF shift is large, ≈ 100 MeV, for the $1S$ ground state; for other states the numbers weakly depend on the value of T_g with the exception of the $1P$ state for which $\Delta_{\mathrm{HF}}^{\mathrm{NP}}$ is different for $T_g\approx 0.3$ fm and $T_g\approx 0.2$ fm, which are taken from the lattice measurements of the gluonic correlators [11,12]. In most cases the magnitude of HF splitting is between 20–50 MeV.

We consider also the P-wave mesons composed of a strange quark and antiquark taking for the current mass of a strange quark $m_s=170$ MeV. Then the dynamical mass of the s quark for different nL states turns out to be about 50 MeV higher than for a light quark (cf. Table I); in particular,

$$\mu_0(2S,s\bar{s})=505\ \mathrm{MeV},\quad \mu_0(1P,s\bar{s})=454\ \mathrm{MeV},$$

$$\mu_0(2P,s\bar{s})=566\ \mathrm{MeV}. \tag{4.13}$$

Correspondingly, the spin-averaged masses of the $s\bar{s}$ mesons appear to be about 170 MeV higher than those for light mesons; e.g., taking the set A of the parameters (3.4) and the constant $C_0=-250$ MeV, defined from a fit to the spin-averaged mass of the $2S$ states [$\phi(1680)$ and $\eta(1440)$], we have obtained that

$$M_{\mathrm{cog}}(1P,s\bar{s})=1424\ \mathrm{MeV},\quad M_{\mathrm{cog}}(2P,s\bar{s})=1885\ \mathrm{MeV}. \tag{4.14}$$

At this point it is of interest to note that $M_{\mathrm{cog}}(1P,s\bar{s})$ coincides with the center of gravity of the multiplet: $f_0(1370)$, $f_1(1420)$, and $f_2(1430)$ which are expected to have a large

TABLE III. The nonperturbative HF splittings $\Delta_{\mathrm{HF}}^{\mathrm{NP}}(nL)$ (in MeV) for light mesons.

State	$1S$	$2S$	$3S$	$1P$	$2P$
$T_g=0.3$ fm	125	56	30	44	27
$T_g=0.2$ fm	96	48	25	24	20

TABLE IV. The hyperfine splittings of the S-wave light mesons (in MeV) with $\alpha_{\overline{MS}}=0.31$.

	$1S$	$2S$	$3S$	$4S$
$\Delta_{\rm HF}^{\rm P}$	194	125	94	75(60)
$\Delta_{\rm HF}$(total), $T_g=0.3$ fm	329	185	144	96
$\Delta_{\rm HF}$(total), $T_g=0.2$ fm	290	173	119	95
Experiment		165 ± 100		

$s\bar{s}$ admixture, but it is 50 MeV smaller if $f_2(2P,s\bar{s})$ is identified with the $f_2'(1525)$ meson.

For the $1P$ $s\bar{s}$ state the nonperturbative HF shift can be calculated from expression (2.12) for $T_g=0.3$ fm and Eq. (4.10) for $T_g=0.2$ fm with the following result:

$$\Delta_{\rm HF}^{\rm NP}(1P,s\bar{s})=\begin{cases}37\ \text{MeV}, & T_g=0.3\ \text{fm},\\ 20\ \text{MeV}, & T_g=0.2\ \text{fm}.\end{cases} \tag{4.15}$$

V. PERTURBATIVE HYPERFINE SPLITTINGS

From experiment it is known that the HF and fine-structure splittings are practically small for all light mesons (with the exception of the π and K mesons) compared to their masses and therefore the spin-dependent effects can be considered as a perturbation. Then, as was shown in Ref. [8], the spin-dependent potentials can be derived by averaging the spin factors, which are present inside the meson Green's function defined in a gauge invariant way. In this approach the expansion in inverse quark masses is not used and in Ref. [8] it was deduced that to order α_s all perturbative spin-dependent potentials $V_i(r)$ $(i=1,2,3,4)$ for light mesons coincide with those in heavy quarkonia, the only difference is that the pole mass of a quark should be replaced by the dynamical mass $\mu_0(nL)$ of a light quark [for a heavy quark $\mu_0(nL)$ coincides with the current mass to order α_s]. In particular, the perturbative spin-spin potential between a light quark and a light antiquark is defined as

$$V_{\rm HF}^{\rm P}(r)=\frac{V_4^{\rm P}(r)}{3\mu_0^2(nL)}. \tag{5.1}$$

Then for the S-wave mesons the perturbative HF splitting is given by the well-known expression:

$$\Delta_{\rm HF}^{\rm P}(nS)=\frac{8}{9}\,\frac{\alpha_s(\mu)}{\mu_0^2(nS)}|R_{n0}(0)|^2, \tag{5.2}$$

where $\alpha_s(\mu)$ is the strong coupling in the modified minimal subtraction ($\overline{\rm MS}$) renormalization scheme. In Ref. [17] the spin-spin interaction was modified with a smearing function with a characteristic momentum scale of about 1.8 GeV. Consequently we can write in Eq. (5.2) for the S-wave mesons

$$\alpha_s(\mu)\approx\alpha_s(1.8\ \text{MeV})=\alpha_s(M_\tau)\approx 0.31\text{–}0.33. \tag{5.3}$$

Since the scale μ coincides with the mass M_τ of the τ lepton we take here $\alpha_s(\mu)=0.31$.

The wave function at the origin entering Eq. (5.2) cannot be precisely defined for the Salpeter equation, since the expansion of the wave function $\psi_{nL}(r)$ (18) in a basis (which is used here for the numerical calculations as suggested in Ref. [19]) is diverging at the point $r=0$. Therefore, we define $R_{n0}(0)\equiv\psi(nS,r=0)$ as in the einbein approach [8] also taking into account the Coulomb interaction that gives a correction of about 10%–20% and the largest one is for the ground state ($\approx 30\%$). Then $R_{n0}(0)$ can be presented in the form

$$R_{n0}(0)=\sqrt{\mu_0(nS)\sigma}\,\xi(nS), \tag{5.4}$$

where the coefficients $\xi(nS)$ are the following: ($\alpha_{\rm eff}=0.39$), $\xi(1S)=1.31$, $\xi(2S)=1.20$, $\xi(3S)=1.16$, and $\xi(4S)=1.14$ and the values of the wave function at the origin are

$$R_{10}(0)=0.294\ \text{GeV}^{3/2},\quad R_{20}(0)=0.30\ \text{GeV}^{3/2},$$
$$R_{30}(0)=0.325\ \text{GeV}^{3/2},\quad R_{40}(0)=0.34\ \text{GeV}^{3/2}. \tag{5.5}$$

From these numbers one can see that the wave function at the origin is almost constant, but slowly growing because of the increase of the dynamical mass $\mu_0(nS)$ with n.

The values of the perturbative splittings for the nS states are given in Table IV ($\alpha_{\overline{MS}}=\alpha_s=0.31$). If one neglects the Coulomb correction in the wave function $R_{n0}(0)$ then $\Delta_{\rm HF}^{\rm P}$ will be about 30%–50% smaller. To check our choice of $R_{n0}(0)$ one can calculate the leptonic width of $\rho(770)$:

$$\Gamma_{e^+e^-}=\frac{2\alpha^2|R_{10}(0)|^2}{M_\rho^2}\left(1-\frac{16}{3\pi}\alpha_s\right), \tag{5.6}$$

which gives the following value for the leptonic width ($\alpha_{\overline{MS}}=0.31;\alpha=1/137$)

$$\Gamma_{e^+e^-}[\rho(770)]=7.36\ \text{keV}, \tag{5.7}$$

that turns out to be in good agreement with the experimental number $\Gamma_{e^+e^-}(\exp)=6.77\pm0.32$ keV [8] (for $\alpha_{\overline{MS}}=0.33$ the leptonic width is $\Gamma_{e^+e^-}=6.8$ keV).

From the number (5.5) for R_{20} one can expect that $\Gamma_{e^+e^-}[\rho(1450)]\approx 1.7$ keV and the fraction $\Gamma_{e^+e^-}/\Gamma_{\rm total}$ for $\rho(1450)$ is seven times smaller than for $\rho(770)$.

From the comparison of the nonperturbative and perturbative spin-spin splittings in Tables III and IV one can see that for all nS states $(n\neq 1)$ the perturbative splitting

TABLE V. The predicted masses of the S-wave mesons in MeV ($T_g = 0.2$ fm).

	$\pi(2S)$	$\rho(2S)$	$\pi(3S)$	$\rho(3S)$	$\pi(4S)$	$\rho(4S)$
Theory	1294	1467	1781	1900	2170	2265
Ref. [17]	1300	1450	1880	2000		
Experiment	1300 ± 100	1465 ± 25	1800 ± 13			2149 ± 17

[a]The mixing of 4^3S_1 and 2^3D_1 states is not taken into account.

$\Delta^{\rm P}_{\rm HF}(nS)$ turns out to be about two times larger than $\Delta^{\rm NP}_{\rm HF}$, while for the $1S$ state the nonperturbative contribution is larger; about 60% of $\Delta^{\rm P}_{\rm HF}(1S)$.

Knowing the HF splittings we can calculate the masses of the isovector mesons (see Table V) neglecting the coupling to the other channels.

We would like to notice here that all our calculations were done for a massless quark (antiquark) with only two parameters: the string tension $\sigma = 0.143\ \mathrm{GeV}^2$ [which defines the dynamical mass of the quark (antiquark) $\mu_0(nS)$ and the spin-averaged spectrum] and the value $\alpha_{\overline{MS}} \approx \alpha_{\overline{MS}}(M_\tau) \approx 0.31$ suggesting that the characteristic "smearing radius" is small as in Ref. [17]. Still, in such a simple picture, the agreement with experiment is reasonably good and our masses for the $3S$ states are about 100 MeV lower than in Ref. [17] and close to the experimental mass of $\pi(1800)$.

To obtain the masses of the $4S$ states one needs to take into account the mixing of these states with the $2D$ states with $M_{\rm cog}(2D) = 1972$ MeV (for the same set of parameters A). The mixing will be done elsewhere.

VI. THE MASSES OF THE b_1 AND h_1 MESONS

For the P-wave state the perturbative HF splitting is of order α_s^2 and is expected to be small. To estimate the perturbative contribution one can use the expression [20]

$$\Delta^{\rm P}_{\rm HF} = \frac{8}{9}\,\frac{\alpha^2_{\overline{MS}}}{\pi m_q^2}\left[\frac{1}{4} - \frac{1}{3}n_f\right]\langle r^{-3}\rangle_{nP}$$
$$\to \frac{2}{3}\,\frac{\alpha^2_{\overline{MS}}}{\pi \mu_0^2(nP)}\langle r^{-3}\rangle_{nP}, \quad (n_f = 3). \tag{6.1}$$

This perturbative HF shift is negative and in Eq. (6.1) m_q is repaced by the dynamical mass of a light quark. This is allowed since the P-wave HF potential $V_4^{\rm P}(r)$ neither depends on the renormalization scale or on the mass of a quark (antiquark). This expression follows from the perturbative spin-spin potential for $L \neq 0$ [21]:

$$V^{\rm P}_{\rm HF}(r) = \frac{1}{3m_q^2}V_4^{\rm P}(r),$$
$$V_4^{\rm P}(r) = \frac{8}{3\pi}\alpha^2_{\overline{MS}}\left(\frac{1}{3}n_f - \frac{1}{4}\right)\boldsymbol{\nabla}^2\frac{\log r}{r}$$
$$= \frac{8}{3\pi}\alpha^2_{\overline{MS}}\left(\frac{1}{4} - \frac{1}{3}n_f\right)\frac{1}{r^3}. \tag{6.2}$$

This short-range spin-spin potential has a characteristic size $R_{\rm HF}$, which can be estimated from the value of the matrix element $\langle r^{-3}\rangle_{nP}$:

$$\langle r^{-3}\rangle_{1P} = 0.019\ \ \mathrm{GeV}^3, \quad \langle r^{-3}\rangle_{2P} = 0.030\ \ \mathrm{GeV}^3. \tag{6.3}$$

If $R_{\rm HF}(nP) = (\langle r^{-3}\rangle_{nP})^{-1/3}$ then $R_{\rm HF}(1P) \approx 0.75$ fm and $R_{\rm HF}(2P) \approx 0.65$ fm are rather large. From these estimates one can conclude that for the P-wave states $R_{\rm HF}(nP) \approx 0.65$ fm appears to be much larger than for the nS states, where in the smearing function $R_{\rm HF}(nS) = (1.8\ \mathrm{GeV})^{-1} \approx 0.11$ fm was taken from Ref. [17] At the distances $R_{\rm HF} \approx 0.65$ fm, the value of $\alpha_{\overline{MS}}$ needs to be taken at the smaller renormalization scale and is very close to the freezing value $\alpha_{\overline{MS}}(q=0)$ which is expected to be $\alpha_{\overline{MS}}(q=0) \approx 0.5$. Therefore, here we take $\alpha_{\overline{MS}}(q=0) \approx 0.45$. The numbers obtained from Eq. (6.1)

$$\Delta^{\rm P}_{\rm HF}(1P) = -5.1\ \ \mathrm{MeV}, \quad \Delta^{\rm P}_{\rm HF}(2P) = -4.8\ \ \mathrm{MeV} \tag{6.4}$$

are much smaller than the nonperturbative shift given in Table III and have opposite signs. Combining both contributions, one obtains the total HF splitting,

$$\Delta_{\rm HF}(1P) = \begin{cases} 39\ \ \mathrm{MeV} & \text{if } T_g = 0.3\ \ \mathrm{fm}, \\ 19\ \ \mathrm{MeV}, & \text{if } T_g = 0.2\ \ \mathrm{fm}, \end{cases} \tag{6.5}$$

or the average number $\Delta_{\rm HF} = 29 \pm 10$ MeV. Knowing the mass of $b_1(1235)$,

$$M[b_1(1P)] = 1229.5 \pm 3.2\ \ \mathrm{MeV}, \tag{6.6}$$

the predicted mass for the center of gravity of the 1^3P_J multiplet ($T_g = 0.3$ fm) is

$$M_{\rm cog}(1^3P_J) = 1258 \pm 3.2(\mathrm{exp}) \pm 10(\mathrm{th})\ \ \mathrm{MeV}. \tag{6.7}$$

The number obtained for $M_{\rm cog}(1^3P_J)$ is in surprisingly good agreement with the experimental mass $M_{\rm cog}(1^3P_J,\mathrm{exp}) = 1252$ MeV, if $a_0(980)$ belongs to the 1^3P_J multiplet, and does not agree with $M_{\rm cog}(1^3P_J) = 1306$ MeV obtained in the case that $a_0(1450)$ belongs to the 1^3P_J multiplet. Thus a strong correlation between the masses of $M_{\rm cog}(1^3P_J)$ and $b_1(1235)$ follows from our analysis and to fit the experimental data one must assume that $a_0(980)$ belongs to the 1^3P_J multiplet and is a $q\bar{q}$ state.

Then $a_0(1450)$ can be considered as a member of the 2^3P_J multiplet with $M_{\rm cog}(2P)=1633$ MeV from Table II and, therefore, with the use of the total HF shift, we predict for the mass of $b_1(2P)$

$$M(b_1(2P))=1610\text{–}1618 \ \text{MeV}, \tag{6.8}$$

since the *total* HF shift from Table III and Eq. (6.4) is

$$\Delta_{\rm HF}(2P)=\begin{cases}22 \ \text{MeV}, & T_g=0.3 \ \text{fm},\\ 15 \ \text{MeV}, & T_g=0.2 \ \text{fm}.\end{cases} \tag{6.9}$$

In the approximation of closed channels used here the HF shift of $h_1(1170)$ and $b_1(1235)$ should be the same, see Eq. (6.9). However, for $h_1(1170)$ the experimental value of the HF shift is larger, 73 ± 19 MeV, and therefore one cannot exclude that $h_1(1170)$ has a small hadronic shift, $\Delta M_{\rm had}=35\pm20$ MeV [note that $h_1(1170)$ has a much larger width, $\Gamma(h_1)\approx360$ MeV, than $b_1(1235)$]. There also exists the state $h_1(1380)$ with $M(^1P_1)=1386\pm19$ MeV. It is assumed that $h_1(1380)$ is mostly composed of a strange quark and antiquark. Then from the calculated $\Delta_{\rm HF}(\text{total})\approx35$ MeV ($T_g=0.3$ fm and $\Delta^{\rm P}_{\rm HF}=4$ MeV) one can obtain the center of gravity of the 1^3P_J multiplet of $s\bar{s}$ mesons:

$$M_{\rm cog}(1^3P_J,s\bar{s})\approx M(1^1P_1)+35 \ \text{MeV}\approx1425\pm19 \ \text{MeV}. \tag{6.10}$$

This number can be compared with $M_{\rm cog}(1^3P_J)$ obtained in the case if $f_0(1370)$, $f_1(1426)$, and $f_2(1430)$ are members of the 1^3P_J multiplet and mostly $s\bar{s}$ states:

$$M^{(1)}_{\rm cog}(1^3P_J)\approx1422 \ \text{MeV} \tag{6.11}$$

and this experimental mass is in good agreement with the predicted mass (6.10). In the other case, when $f_2(1525)$ is a member of the 1^3P_J multiplet, the ''experimental'' value of the center of gravity,

$$M^{(2)}_{\rm cog}(2^3P_J)\approx1474 \ \text{MeV} \tag{6.12}$$

is not correlated with the mass of $h_1(1380)$ and the shift of the mass of $h_1(1380)$ appears to be larger (about 80 MeV) than in our calculations.

VII. CONCLUSIONS

We investigated the nonperturbative spin-spin interaction in light mesons and established the following.

(1) For the $1S$ state the HF shift due to the nonperturbative effects is rather large, because the dynamical mass is relatively small, so that $\Delta^{\rm NP}_{\rm HF}\approx0.4\ \Delta_{\rm HF}(1S,\text{total})$, while for the excited nS states it is only about 15% of the total shift.

(2) Because of the positive sign of the nonperturbative HF splitting, the mass of the n^1P_1 state is strongly correlated with $M_{\rm cog}(n^3P_J)$ being 30 ± 10 MeV smaller than $M_{\rm cog}(n^3P_J)$. The value of this shift depends on the gluonic correlation length adopted.

(3) With the use of the mass of $b_1(1235)$ our predicted mass of $M_{\rm cog}(1^3P_J,I=1)$ is 1258 ± 10 MeV and this number is in agreement with the experimental masses of the $a_J(1P)$ mesons only if $a_0(980)$ belongs to the 1^3P_J multiplet.

(4) For $b_1(2P)$ we predict the mass $M[b_1(2P)]\approx1.62$ GeV.

(5) Our analysis can be applied also to the isoscalar mesons where $h_1(1170)$ and $M_{\rm cog}(1^3P_J)=1245$ MeV lie rather close to each other if $f_0(980)$ is a member of the 1^3P_J multiplet.

(6) In the approximation when $h_1(1380)$, $f_0(1370)$, $f_1(1420)$, and $f_2(1430)$ are considered to be composed mainly of a strange quark and antiquark, the difference $\Delta=M_{\rm cog}(1^3P_J,s\bar{s})-M(h_1(1380))\approx35$ MeV is in full agreement with our estimate of the nonperturbative HF shift, $\Delta^{\rm NP}_{\rm HF}\approx35$ MeV for the correlation length $T_g=0.3$ fm.

(7) The preferable value of the gluonic correlation length $T_g=0.3$ fm was obtained from our analysis of the HF splittings of different mesons in accordance with the lattice data of Ref. [12].

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