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Higher twist and transverse momentum dependent parton distributions: A light-front Hamiltonian approach

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In order to study twist-3 and transverse momentum dependent parton distributions, we use light-front time-ordered PQCD at order α_s to calculate various distribution functions for a dressed quark target. This study enables us to investigate in detail the existing relations between twist-3 and transverse momentum dependent parton distributions. Our calculation shows explicitly that two versions of such relations, considered to be equivalent, occur in the literature which need to be distinguished. Moreover, we examine sum rules for higher twist distributions. While the Burkhardt-Cottingham sum rule for g_2 is satisfied, the corresponding sum rule for h_2 is violated.

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I. INTRODUCTION

In view of the increasing accuracy of recent and planned high energy scattering experiments, more and more attention is paid to the study of parton distributions which are of higher twist and (or) dependent on the transverse momenta of the partons. The twist-3 distribution functions are accessible through the measurement of certain asymmetries in polarized deep inelastic scattering (DIS) [1] and Drell-Yan processes [2]. The transverse momentum dependent (k_T-dependent) structure functions play an important role both in Drell-Yan processes and semi-inclusive DIS [3]. In such reactions, e.g., the transverse momenta and the transverse spin of the partons can get coupled giving rise to azimuthal asymmetries (see e.g. Refs. [4–7]), which are very suitable observables for studying the correlations of quarks and gluons in hadrons. Often, effects due to higher twist and transverse momenta appear simultaneously like in the recent HERMES measurements of the longitudinal single spin asymmetry in semi-inclusive pion production [8]. In this work, we study these higher twist and k_T-dependent structure functions and their interrelations in the framework of light-front Hamiltonian QCD.

As is well known, twist-3 and k_T-dependent parton distributions are related [5,6,9–11] as a consequence of Lorentz invariance. These relations impose important constraints on the distribution functions, which allow one to eliminate unknown structure functions in favor of known ones whenever applicable. Consequently, they have been used frequently in the literature to facilitate matters, for instance in studying the evolution of k_T-dependent distribution functions [9,10,12].

Our motivation here is to investigate the validity of these Lorentz invariance relations by explicit calculation of all the involved distribution functions. There exists a very convenient tool based on the light-front Hamiltonian description of composite systems utilizing many-body wave functions, which enables us to study these relations in the context of perturbative QCD. This tool has already been used successfully in the literature to calculate unpolarized and polarized parton distributions [13] as well as the transversity distribution [14]. The simplicity of this approach has also been exploited to make a critical examination of the Wandzura-Wilczek relation [15]. Here, we use the same approach to calculate the higher twist and k_T-dependent parton distributions perturbatively to order α_s, and then study the Lorentz invariance relations by employing a dressed quark target. We demonstrate that there exist two sets of relations that, although assumed to be the same, are not identical in reality. More precisely, only one set of relations is verified for a dressed quark target, whereas the drawback in the other case can be traced back to the absence of quark-gluon-quark correlators, which seem to be crucial ingredients in the Lorentz invariance relations in a gauge theory.

Our calculation also gives us the opportunity to investigate the sum rules for the twist-3 distributions g_2 and h_2. The Burkhardt-Cottingham (BC) sum rule for g_2 [16] is satisfied for the dressed quark target, but the corresponding sum rule for h_2 [17,18] turns out to be violated. To our best knowledge, the violation of the sum rule for h_2 in a perturbative treatment is a new observation.

The paper is organized as follows. In Sec. II, we outline the definition of the parton distributions relevant for our discussion, and give a detailed account of their relations due to Lorentz invariance. In Sec. III, first the operators for twist-3 distributions are expressed in terms of dynamical fields, and some relevant points regarding the dressed quark target are discussed. Then we present our results for the different parton distributions and a detailed investigation of the two sets of Lorentz invariance relations. In Sec. IV, we study the sum rules for g_2 and h_2 and conclude in Sec. V. Some conventions are summarized in an Appendix.

II. PARTON DISTRIBUTIONS: DEFINITIONS AND THEIR INTERRELATIONS

In this section we recall the definitions of various parton distributions that already exist in the literature and introduce the Lorentz invariance relations among them. We restrict the discussion below to twist-3 structure functions, while in the case of k_T-dependent functions we limit ourselves to the twist-2 level which is sufficient for our purpose. For a complete discussion one should go back to the original references mentioned below.

To begin with, we specify the correlator $\Phi(x)$ of two quark fields on the light-front[1] in terms of which all the structure functions are defined

$$\Phi_{ij}(x)=\frac{1}{\sqrt{2}}\int\frac{d\xi^-}{2\pi}e^{ik\cdot\xi}\langle P,S|\overline{\psi}_j(0)\psi_i(\xi)|P,S\rangle\Big|_{\xi^+=\boldsymbol{\xi}_T=0}, \tag{1}$$

with $k^+=xP^+$. The target state is characterized by its four-momentum P and the covariant spin vector S ($P^2=M^2$, $S^2=-1$, $P\cdot S=0$). Note that $\Phi(x)$ can easily be made gauge invariant by putting an appropriate gauge link between the quark fields. However, since the non-locality in the operator is only in the longitudinal direction ξ^- and we shall be working in the light-front gauge ($A^+=0$), we can always get rid of the gauge link in Eq. (1).

Now, the parton distributions appear in a general decomposition of the correlator $\Phi(x)$ where one finds three functions at twist-2 and three functions at the twist-3 level [2,5],

$$\begin{aligned}\Phi(x)=&\frac{1}{2}\{f_1(x)\not{n}_+ +\lambda\, g_1(x)\gamma_5\not{n}_+ +h_1(x)\gamma_5\not{S}_T\not{n}_+\}\\&+\frac{M}{\sqrt{2}P^+}\Big\{e(x)+g_T(x)\gamma_5\not{S}_T\\&+\lambda\, h_L(x)\gamma_5\frac{[\not{n}_+,\not{n}_-]}{2}\Big\},\end{aligned} \tag{2}$$

with n_+ and n_- being two lightlike vectors satisfying $n_+\cdot n_-=1$. The helicity of the target state is given by λ, while $S_T^\mu\equiv(0,0,\boldsymbol{S}_T)$ represents the transverse spin of the target. Sometimes different notations for twist-2 distributions are used in the literature [$f_1(x)=q(x)$, $g_1(x)=\Delta q(x)$, $h_1(x)=\Delta_T q(x)=\delta q(x)$]. The twist-3 part contains the well-known transversely polarized structure function g_T, and two chiral-odd distributions e and h_L. Note that in Eq. (2) we have not considered the so called T-odd parton distributions.

The structure functions in Eq. (2) are projected out by performing traces of $\Phi(x)$ with suitable Dirac matrices. Using the abbreviation $\Phi^{[\Gamma]}\equiv\mathrm{Tr}(\Phi\Gamma)/2$, we give the explicit expressions for those structure functions that are relevant for our discussion here,

$$\begin{aligned}g_1(x)&=\frac{1}{\sqrt{2}\,\lambda}\Phi^{[\gamma^+\gamma_5]}\\&=\frac{1}{4\,\lambda}\int\frac{d\xi^-}{2\pi}e^{ik\cdot\xi}\langle P,S|\overline{\psi}(0)\gamma^+\gamma_5\psi(\xi)|P,S\rangle,\end{aligned} \tag{3}$$

$$\begin{aligned}h_1(x)&=\frac{1}{\sqrt{2}\,S_T^i}\Phi^{[i\sigma^{i+}\gamma_5]}\\&=\frac{1}{4\,S_T^i}\int\frac{d\xi^-}{2\pi}e^{ik\cdot\xi}\langle P,S|\overline{\psi}(0)i\sigma^{i+}\gamma_5\psi(\xi)|P,S\rangle,\end{aligned} \tag{4}$$

$$\begin{aligned}g_T(x)&=\frac{P^+}{\sqrt{2}M\,S_T^i}\Phi^{[\gamma^i\gamma_5]}\\&=\frac{P^+}{4\,M\,S_T^i}\int\frac{d\xi^-}{2\pi}e^{ik\cdot\xi}\langle P,S|\overline{\psi}(0)\gamma^i\gamma_5\psi(\xi)|P,S\rangle,\end{aligned} \tag{5}$$

$$\begin{aligned}h_L(x)&=\frac{P^+}{2\sqrt{2}\,M\,\lambda}\Phi^{[i\sigma^{+-}\gamma_5]}\\&=\frac{P^+}{8\,M\,\lambda}\int\frac{d\xi^-}{2\pi}e^{ik\cdot\xi}\langle P,S|\overline{\psi}(0)i\sigma^{+-}\gamma_5\psi(\xi)|P,S\rangle,\end{aligned} \tag{6}$$

where, like in Eq. (1), all the correlators are understood to be on the light-front, i.e. $\xi^+=\boldsymbol{\xi}_T=0$.

In a similar way, k_T-dependent parton distributions are defined starting from the following correlation function where the non-locality in its operator structure is not only in ξ^- but in $\boldsymbol{\xi}_T$ as well,

$$\begin{aligned}\Phi_{ij}(x,\boldsymbol{k}_T)=&\frac{1}{\sqrt{2}}\int\frac{d\xi^-d^2\boldsymbol{\xi}_T}{(2\pi)^3}\\&\times e^{ik\cdot\xi}\langle P,S|\overline{\psi}_j(0)\psi_i(\xi)|P,S\rangle\Big|_{\xi^+=0}.\end{aligned} \tag{7}$$

Here we have assumed that in the $A^+=0$ gauge together with antisymmetric boundary conditions for the transverse gluon field, the gauge link can still be omitted as argued, e.g., in Ref. [5]. In the general decomposition of this correlator one naturally finds more distribution functions due to the presence of an extra vector k_T^μ [5],

$$\begin{aligned}\Phi(x,\boldsymbol{k}_T)=&\frac{1}{2}\Big\{f_1(x,\boldsymbol{k}_T^2)\not{n}_+\\&+\Big(\lambda\, g_{1L}(x,\boldsymbol{k}_T^2)+\frac{\boldsymbol{k}_T\cdot\boldsymbol{S}_T}{M}g_{1T}(x,\boldsymbol{k}_T^2)\Big)\gamma_5\not{n}_+\\&-h_{1T}(x,\boldsymbol{k}_T^2)i\sigma_{\mu\nu}\gamma_5 S_T^\mu n_+^\nu\\&-\Big(\lambda\, h_{1L}^\perp(x,\boldsymbol{k}_T^2)+\frac{\boldsymbol{k}_T\cdot\boldsymbol{S}_T}{M}h_{1T}^\perp(x,\boldsymbol{k}_T^2)\Big)\\&\times\frac{i\sigma_{\mu\nu}\gamma_5 k_T^\mu n_+^\nu}{M}\Big\}.\end{aligned} \tag{8}$$

[1]Our definition of light-front components of a generic 4-vector as well as further conventions are summarized in the Appendix.

Here we have shown only the twist-2 part, which is sufficient for our purpose, and we have omitted the *T*-odd functions as before.

Like in the previous case, one projects out the structure functions in Eq. (8) by performing traces of $\Phi(x,\boldsymbol{k}_T)$ with suitable Dirac matrices. Two of the projections necessary for our discussion are

$$\frac{1}{\sqrt{2}}\Phi^{[\gamma^+\gamma_5]}(x,\boldsymbol{k}_T)=\lambda\, g_{1L}(x,\boldsymbol{k}_T^2)+\frac{\boldsymbol{k}_T\cdot\boldsymbol{S}_T}{M}g_{1T}(x,\boldsymbol{k}_T^2), \quad (9)$$

$$\frac{1}{\sqrt{2}}\Phi^{[i\sigma^{i+}\gamma_5]}(x,\boldsymbol{k}_T)=S_T^i h_{1T}(x,\boldsymbol{k}_T^2)+\frac{k_T^i}{M}\Big(\lambda\, h_{1L}^\perp(x,\boldsymbol{k}_T^2)+\frac{\boldsymbol{k}_T\cdot\boldsymbol{S}_T}{M}h_{1T}^\perp(x,\boldsymbol{k}_T^2)\Big). \quad (10)$$

Note that depending on the target polarization the same projection of $\Phi(x,\boldsymbol{k}_T)$ allows one to calculate different structure functions. For example, from Eq. (9) we get $g_{1L}(x,\boldsymbol{k}_T)$ or $g_{1T}(x,\boldsymbol{k}_T)$ for the target being polarized in the longitudinal or transverse direction, respectively. Therefore, Eqs. (7)–(10) give us a well-defined way to calculate $g_{1T}(x,\boldsymbol{k}_T)$ and $h_{1L}^\perp(x,\boldsymbol{k}_T)$ which are necessary for the subsequent discussion. In what follows we need the k_T^2-moments of these two functions, which are defined as [5]

$$g_{1T}^{(1)}(x)=\int d^2\boldsymbol{k}_T\frac{\boldsymbol{k}_T^2}{2M^2}g_{1T}(x,\boldsymbol{k}_T^2),$$

$$h_{1L}^{\perp(1)}(x)=\int d^2\boldsymbol{k}_T\frac{\boldsymbol{k}_T^2}{2M^2}h_{1L}^\perp(x,\boldsymbol{k}_T^2). \quad (11)$$

Now that we have given all the definitions of relevant structure functions, we are in a position to discuss the existing relations among them. These are usually of two kinds—one follows from the QCD equations of motion and the other comes as a consequence of Lorentz invariance. Here we are mainly interested in the latter ones which, according to Refs. [5,6], read as

$$g_T(x)=g_1(x)+\frac{d}{dx}g_{1T}^{(1)}(x), \quad (12)$$

$$h_L(x)=h_1(x)-\frac{d}{dx}h_{1L}^{\perp(1)}(x). \quad (13)$$

These relations have been derived from the general Lorentz covariant decomposition of the correlation function Φ of two quark fields before it is constrained on the light-cone and, hence, they are quite naturally referred to as Lorentz invariance relations. On the other hand, a similar relation for g_T also attributed to Lorentz-invariance has already been proposed in Ref. [9] and extended for h_L in Ref. [10]. A detailed account on these relations can be found in Ref. [11]. Using the notations of Ref. [11], with necessary modifications for the conventions that we follow, the corresponding relations are given by

$$g_T(x)=g_1(x)+\frac{d}{dx}\overline{K}(x)+\int dx'\frac{\overline{D}(x,x')+\overline{D}(x',x)}{x'-x}, \quad (14)$$

$$h_L(x)=h_1(x)+\frac{1}{2}\frac{d}{dx}\widetilde{K}(x)+\frac{1}{2}\int dx'\frac{\widetilde{D}(x,x')+\widetilde{D}(x',x)}{x'-x}. \quad (15)$$

Note that here quark-gluon-quark light-front correlators are involved, which depend on two momentum fractions denoted as $x=k^+/P^+$, $x'=k'^+/P^+$ and the new correlation functions appearing in Eq. (14) are given by

$$\overline{K}(x)=\frac{1}{4\,M\,S_T^i}\int\frac{d\xi^-}{2\pi}e^{ik\cdot\xi}\times\langle P,S|\overline{\psi}(0)\gamma^+ i\partial_T^i\gamma_5\psi(\xi)|P,S\rangle, \quad (16)$$

$$\overline{D}_1(x,x')=-\frac{g_s P^+}{8\,M\,S_T^i}\int\frac{d\xi^-}{2\pi}\frac{d\eta^-}{2\pi}e^{ik\cdot\xi-ik'\cdot\eta}\times\langle P,S|\overline{\psi}(\eta)\gamma^+ \mathbb{A}_T(0)\gamma^i\gamma_5\psi(\xi)|P,S\rangle, \quad (17)$$

$$\overline{D}_2(x',x)=-\frac{g_s P^+}{8\,M\,S_T^i}\int\frac{d\xi^-}{2\pi}\frac{d\eta^-}{2\pi}e^{ik'\cdot\eta-ik\cdot\xi}\times\langle P,S|\overline{\psi}(\xi)\gamma^+\gamma^i \mathbb{A}_T(0)\gamma_5\psi(\eta)|P,S\rangle, \quad (18)$$

while the ones in Eq. (15) are

$$\widetilde{K}(x)=-\frac{1}{4\,M\,\lambda}\int\frac{d\xi^-}{2\pi}e^{ik\cdot\xi}\times\langle P,S|\overline{\psi}(0)\gamma^+ i\boldsymbol{\partial}_T\gamma_5\psi(\xi)|P,S\rangle, \quad (19)$$

$$\widetilde{D}_1(x,x')=-\frac{g_s P^+}{8\,M\,\lambda}\int\frac{d\xi^-}{2\pi}\frac{d\eta^-}{2\pi}e^{ik\cdot\xi-ik'\cdot\eta}\times\langle P,S|\overline{\psi}(\eta)\gamma^+ \mathbb{A}_T(0)\gamma_5\psi(\xi)|P,S\rangle, \quad (20)$$

$$\widetilde{D}_2(x',x)=-\frac{g_s P^+}{8\,M\,\lambda}\int\frac{d\xi^-}{2\pi}\frac{d\eta^-}{2\pi}e^{ik'\cdot\eta-ik\cdot\xi}\times\langle P,S|\overline{\psi}(\xi)\gamma^+ \mathbb{A}_T(0)\gamma_5\psi(\eta)|P,S\rangle, \quad (21)$$

with $\overline{D}(x,x')=\frac{1}{2}[\overline{D}_1(x,x')+\overline{D}_2(x',x)]$ and $\widetilde{D}(x,x')=\frac{1}{2}[\widetilde{D}_1(x,x')+\widetilde{D}_2(x',x)]$. In principle, Eqs. (12), (13) and Eqs. (14), (15) (henceforth, referred to as set A and set B,

respectively) should contain the same information and, in fact, are assumed to be identical. But there exists hardly any proof of that.

These relations are quite remarkable, in particular, since they involve at the same time functions describing longitudinally and transversely polarized targets and therefore will provide us with a consistency check while comparing data for the measured structure functions from different experiments. Moreover, they can be quite useful to predict the evolution of one of the structure functions once the evolutions of others are known, as has been done in Refs. [9,10,12]. Keeping their importance in mind, it is worthwhile to delve more into these relations. We do this in the next section by checking them through explicit calculations for a dressed quark target in the framework of light-front time-ordered perturbative QCD (PQCD).

III. PARTON DISTRIBUTIONS: RESULTS AND DISCUSSION

Before presenting our results, it is useful to disentangle the twist-3 parton distributions into simpler structures which manifest the different aspects of the QCD dynamics contained in them. To achieve this, we re-express the structure functions g_T and h_L in terms of dynamical fields, the so-called good fields, like in the twist-2 case which right from the beginning contains only the good fields. That is, we eliminate the constrained field ψ_- via the constraint equation (A2) in terms of ψ_+ and A_T^i which are the only dynamical fields in the Hamiltonian formulation of light-front QCD. Thus, g_T and h_L defined in the previous section become

$$g_T(x,Q^2)=\frac{P^+}{4\,M\,S_T^i}\int\frac{d\xi^-}{2\pi}e^{ik\cdot\xi}\langle P,S|(\overline{O}_m+\overline{O}_{k_T}+\overline{O}_g)|P,S\rangle,$$

$$=g_T^m(x,Q^2)+g_T^{k_T}(x,Q^2)+g_T^g(x,Q^2), \tag{22}$$

$$h_L(x,Q^2)=\frac{P^+}{8\,M\,\lambda}\int\frac{d\xi^-}{2\pi}e^{ik\cdot\xi}\langle P,S|(\tilde{O}_m+\tilde{O}_{k_T}+\tilde{O}_g)|P,S\rangle,$$

$$=h_L^m(x,Q^2)+h_L^{k_T}(x,Q^2)+h_L^g(x,Q^2), \tag{23}$$

where we have introduced the operators

$$\overline{O}_m=m_q\psi_+^\dagger(0)\,\gamma^i\left(\frac{1}{i\vec{\partial}^+}-\frac{1}{i\overleftarrow{\partial}^+}\right)\gamma_5\psi_+(\xi),$$

$$\overline{O}_{k_T}=-\,\psi_+^\dagger(0)\left(\gamma^i\frac{1}{\vec{\partial}^+}\vec{\boldsymbol{\partial}}_T+\overleftarrow{\boldsymbol{\partial}}_T\frac{1}{\overleftarrow{\partial}^+}\gamma^i\right)\gamma_5\psi_+(\xi),$$

$$\overline{O}_g=g_s\psi_+^\dagger(0)\left(\mathbb{A}_T(0)\frac{1}{i\overleftarrow{\partial}^+}\gamma^i-\gamma^i\frac{1}{i\vec{\partial}^+}\mathbb{A}_T(\xi)\right)\gamma_5\psi_+(\xi), \tag{24}$$

and

$$\tilde{O}_m=2\,m_q\psi_+^\dagger(0)\left(\frac{1}{i\vec{\partial}^+}-\frac{1}{i\overleftarrow{\partial}^+}\right)\gamma_5\psi_+(\xi),$$

$$\tilde{O}_{k_T}=-\,2\,\psi_+^\dagger(0)\left(\frac{1}{\vec{\partial}^+}\vec{\boldsymbol{\partial}}_T+\overleftarrow{\boldsymbol{\partial}}_T\frac{1}{\overleftarrow{\partial}^+}\right)\gamma_5\psi_+(\xi),$$

$$\tilde{O}_g=2\,g_s\psi_+^\dagger(0)\left(\mathbb{A}_T(0)\frac{1}{i\overleftarrow{\partial}^+}-\frac{1}{i\vec{\partial}^+}\mathbb{A}_T(\xi)\right)\gamma_5\psi_+(\xi). \tag{25}$$

Here m_q is the quark mass and $\boldsymbol{A}_T=\Sigma_a T^a\boldsymbol{A}_T^a$ the transverse gauge field, while $1/\partial^+$ is defined in the sense of the principal value prescription as given in Eq. (A3). The above light-front expressions make the physical picture of twist-3 structure functions clear. It explicitly shows the contributions associated with the quark mass, quark transverse momentum and quark-gluon coupling operators. Although one naively expects that the contributions depending explicitly on the quark mass are suppressed, it turns out that each of them is equally important to extract the information contained in twist-3 structure functions. Notice that Eqs. (22),(23) correspond to what in the literature is often referred to as the relations among various light-front correlators coming from the QCD equations of motion (see e.g. Ref. [5]).

Having presented the relevant issues as far as the operator structures involved in the parton distributions are concerned, some comments regarding our calculation and the target state are in order. The calculation is straightforward and we shall avoid giving unnecessary details except mentioning the following points. (For details we refer the reader to Ref. [13].)

Firstly, all the required structure functions are calculated for a dressed quark target given by the following Fock-space expansion truncated at the lowest non-trivial order,

$$|k,\lambda\rangle=\mathcal{N}\Bigg\{b_\lambda^\dagger(k)|0\rangle+\sum_{\lambda_1\lambda_2}\int\frac{dk_1^+d^2\boldsymbol{k}_{T1}}{\sqrt{2(2\pi)^3k_1^+}}\frac{dk_2^+d^2\boldsymbol{k}_{T2}}{\sqrt{2(2\pi)^3k_2^+}}$$
$$\times\sqrt{2(2\pi)^3}\delta^3(k-k_1-k_2)$$
$$\times\Phi_{\lambda_1\lambda_2}^\lambda(x,\boldsymbol{\kappa}_T)b_{\lambda_1}^\dagger(k_1)a_{\lambda_2}^\dagger(k_2)|0\rangle+\cdots\Bigg\}, \tag{26}$$

where $b_\lambda^\dagger(k)$ and $a_\lambda^\dagger(k)$ are the creation operators of quarks and gluons on the light-front which obey the usual commutation relations [see Eqs. (A8), (A9)]. The most important ingredient in the above dressed quark state is the two particle boost-invariant wave-function which can be calculated using light-front time-ordered PQCD and is given by

$$\Phi_{\lambda_1\lambda_2}^\lambda(x,\boldsymbol{\kappa}_T)=-\frac{g_sT^a}{\sqrt{2(2\pi)^3}}x\frac{\sqrt{1-x}}{\boldsymbol{\kappa}_T^2}\chi_{\lambda_1}^\dagger$$
$$\times\left\{2\frac{\kappa_T^i}{1-x}+\frac{1}{x}(\tilde{\boldsymbol{\sigma}}_T\cdot\boldsymbol{\kappa}_T)\tilde{\sigma}_T^i-i\,m_q\tilde{\sigma}_T^i\frac{1-x}{x}\right\}$$
$$\times\chi_\lambda\varepsilon_{T,\lambda_2}^{i*}, \tag{27}$$

with x and $\boldsymbol{\kappa}_T$ being the relative momenta of the quark. Note that the m_q-dependence in the above wave function has its origin in the helicity flip part of the light-front QCD Hamiltonian. This is an essential term in investigating the dynamics of transversely polarized targets and, hence, is also very important as far as our calculation is concerned. The constant $\mathcal{N}$ appearing in Eq. (26) is determined by the normalization condition

$$\langle k',\lambda'|k,\lambda\rangle = 2(2\pi)^3 k^+ \delta(k^+ - k'^+)\,\delta^2(\boldsymbol{k}_T - \boldsymbol{k}'_T)\,\delta_{\lambda,\lambda'}, \tag{28}$$

and to the order α_s given by [13]

$$\mathcal{N} = 1 - \frac{\alpha_s}{2\pi} C_f \ln\frac{Q^2}{\mu^2}\int_0^1 dx\,\frac{1+x^2}{1-x}. \tag{29}$$

Here a hadronic scale μ has been introduced such that $\boldsymbol{\kappa}_T^2 \gg \mu^2 \gg (m_q)^2$, which can be considered as the factorization scale separating the ''hard'' and ''soft'' dynamics of QCD. This scale μ also serves as the lower cutoff of the involved transverse mometum integration, whereas Q^2 is the upper cutoff.

Secondly, in our calculation we also need a transversely polarized target, for example, in the case of g_T. This is obtained by a superposition of two different helicity states. Thus, the one polarized in the x-direction can be expressed by

$$|k,S^1 = \pm 1\rangle = \frac{1}{\sqrt{2}}(|k,\uparrow\rangle \pm |k,\downarrow\rangle). \tag{30}$$

Lastly, the quark mass renormalization enters in the calculation at α_s-order and we use the following expression for the renormalized quark mass m_q^R in terms of its bare mass m_q [19],

$$m_q^R = m_q\left(1 + \frac{3}{4\pi}\alpha_s C_f \ln\frac{Q^2}{\mu^2}\right). \tag{31}$$

We now present the results of our calculation, i.e., all the relevant structure functions for the dressed quark target in Eq. (26) up to order α_s. We first give the twist-3 structure functions g_T and h_L. It turns out that all the three terms in Eq. (22) and Eq. (23) have nonzero contribution to the corresponding twist-3 structure functions and for clarity we provide them separately. For $g_T(x)$ we obtain

$$g_T^m(x,Q^2) = \frac{m_q}{M}\Bigg\{\delta(1-x) + \frac{\alpha_s}{2\pi}C_f \ln\frac{Q^2}{\mu^2} \times\left[\frac{2}{1-x} - \delta(1-x)\int_0^1 dx'\,\frac{1+x'^2}{1-x'}\right]\Bigg\}, \tag{32}$$

$$g_T^{k_T}(x,Q^2) = -\frac{m_q}{M}\frac{\alpha_s}{2\pi}C_f \ln\frac{Q^2}{\mu^2}(1-x), \tag{33}$$

$$g_T^g(x,Q^2) = \frac{m_q}{M}\frac{\alpha_s}{2\pi}C_f \ln\frac{Q^2}{\mu^2}\frac{\delta(1-x)}{2}. \tag{34}$$

A similar calculation for h_L gives

$$h_L^m(x,Q^2) = \frac{m_q}{M}\Bigg\{\delta(1-x) + \frac{\alpha_s}{2\pi}C_f \ln\frac{Q^2}{\mu^2} \times\left[\frac{1}{x}\left(\frac{1+x^2}{1-x}\right) - \delta(1-x)\int_0^1 dx'\,\frac{1+x'^2}{1-x'}\right]\Bigg\}, \tag{35}$$

$$h_L^{k_T}(x,Q^2) = -\frac{m_q}{M}\frac{\alpha_s}{2\pi}C_f \ln\frac{Q^2}{\mu^2}\frac{2(1-x)}{x}, \tag{36}$$

$$h_L^g(x,Q^2) = \frac{m_q}{M}\frac{\alpha_s}{2\pi}C_f \ln\frac{Q^2}{\mu^2}\left[\frac{1-x}{x} + \frac{1}{2}\delta(1-x)\right]. \tag{37}$$

Note that the above results represent purely the PQCD dynamics to the α_s-order relevant for the leading logarithmic approximation, i.e., we only keep the terms proportional to $\ln Q^2/\mu^2$. As is well known, at this order there also appear finite terms which are not considered here. It should be noted that all the individual contributions in g_T as well as h_L in the perturbative calculation are of the same order (namely, proportional to m_q/M), which means that the mass dependent terms g_T^m and h_L^m are not suppressed contrary to the common belief. As mentioned above, m_q is the bare quark mass and up to order α_s it is given by Eq. (31) in terms of the renormalized quark mass m_q^R. On the other hand, M is the renormalized target mass and, therefore, in our case it is identical to m_q^R itself, $M = m_q^R$. Taking this into account, we finally get g_T and h_L as follows:

$$g_T(x,Q^2) = \delta(1-x) + \frac{\alpha_s}{2\pi}C_f \ln\frac{Q^2}{\mu^2} \times\left[\frac{1+2x-x^2}{(1-x)_+} + \frac{1}{2}\delta(1-x)\right], \tag{38}$$

$$h_L(x,Q^2) = \delta(1-x) + \frac{\alpha_s}{2\pi}C_f \ln\frac{Q^2}{\mu^2} \times\left[\frac{2}{(1-x)_+} + \frac{1}{2}\delta(1-x)\right], \tag{39}$$

where we have used the well-known ''plus''-prescription. Equation (38) reproduces[2] the result already obtained in Ref. [15] for a dressed quark target. Also a covariant one-loop calculation with a quark target yields exactly the same ex-

[2]Our result differs from that obtained in Ref. [15] by a factor of $\frac{1}{2}$ which appears in the definition of g_T that we are using. This is not relevant for our purpose as long as we use one consistent set of definitions for all the parton distributions.

pression for g_T [20]. Note that the result for h_L, obtained for the first time here, does not contain any singularity at $x=0$, even though the individual pieces in Eqs. (35)–(37) do.

To investigate the validity of the relations in set A, we need to calculate the structure functions on the right-hand side (RHS) of them which involves explicitly calculating g_1, $g_{1T}^{(1)}$, h_1 and $h_{1L}^{\perp(1)}$ for the dressed quark target. Carrying out the evaluation of $g_{1T}^{(1)}$ and $h_{1L}^{\perp(1)}$ as given in Eq. (11), we get to the α_s-order

$$g_{1T}^{(1)}(x,Q^2)=-\frac{\alpha_s}{2\pi}C_f\ln\frac{Q^2}{\mu^2}x(1-x), \tag{40}$$

$$h_{1L}^{\perp(1)}(x,Q^2)=\frac{\alpha_s}{2\pi}C_f\ln\frac{Q^2}{\mu^2}(1-x). \tag{41}$$

We point out that in the free theory ($\alpha_s=0$) these two k_T-dependent functions vanish since our target carries no net transverse momentum. The same is true for the two functions given earlier in Eqs. (33),(36).

The results for g_1 and h_1 to α_s-order already exist in the literature [13,14] and are given by

$$g_1(x,Q^2)=\delta(1-x)+\frac{\alpha_s}{2\pi}C_f\ln\frac{Q^2}{\mu^2}\times\left[\frac{1+x^2}{(1-x)_+}+\frac{3}{2}\delta(1-x)\right], \tag{42}$$

$$h_1(x,Q^2)=\delta(1-x)+\frac{\alpha_s}{2\pi}C_f\ln\frac{Q^2}{\mu^2}\times\left[\frac{2x}{(1-x)_+}+\frac{3}{2}\delta(1-x)\right]. \tag{43}$$

Note that the α_s-terms for g_1 and h_1 contain the evolution kernels of the corresponding structure functions. Having the explicit results for all the necessary structure functions appearing in the Lorentz invariance relations as given in set A, we can now compare the LHS and RHS of these relations. By doing so, one readily finds that the relations in set A are not satisfied for a dressed quark target. Therefore, the natural conclusion is either that Lorentz invariance is violated in perturbation theory or that the relations in set A do not reflect the complete picture.

It is easy to see that these relations in set A in fact do not reflect the complete picture. To make it evident, we turn our attention now to the relations in set B and first calculate $\overline{K}$ and $\widetilde{K}$ for the dressed quark target to the same order. It turns out that

$$\overline{K}(x)=g_{1T}^{(1)}(x) \quad \text{and} \quad \widetilde{K}(x)=-2\,h_{1L}^{\perp(1)}(x). \tag{44}$$

This immediately leads us to the conclusion that the relations presented in set A are actually different from that in set B unless the contributions coming from the $\overline{D}(x,x')$'s and $\widetilde{D}(x,x')$'s are identically zero, which is unlikely in a general scenario. Carrying out the explicit calculation of these terms, we get

$$\overline{D}_1(x,x')=\overline{D}_2(x',x)=-\frac{\alpha_s}{2\pi}C_f\ln\frac{Q^2}{\mu^2}(x'-x)\,\delta(1-x), \tag{45}$$

$$\widetilde{D}_1(x,x')=\widetilde{D}_2(x',x)=-\frac{\alpha_s}{2\pi}C_f\ln\frac{Q^2}{\mu^2}(x'-x)[\,\delta(1-x)-\delta(1-x')]. \tag{46}$$

We point out that no singularity in $(x'-x)$ shows up in Eqs. (14),(15). From the above results one easily observes that the apparent pole there gets canceled.

Putting the results in Eqs. (42)–(46) back in the RHS of Eqs. (14),(15), we obtain g_T and h_L as given in Eqs. (38), (39) which verifies the relations in set B. Moreover, we see that the discrepancy we found earlier in set A is exactly compensated by taking these $\overline{D}(x,x')$'s and $\widetilde{D}(x,x')$'s properly into account. In other words, from this exercise it turns out that the information contained in these quark-gluon-quark correlators is missing in the relations given in set A, thereby making them incomplete.

Therefore, we finally conclude that the relations in set A and those in set B are not identical—while the first ones are violated, the second ones are satisfied for a dressed quark target up to order α_s. Barring Eqs. (32)–(34), (38) and (42), (43), all the results presented in this section are obtained for the first time here in the context of light-front QCD. It should be noted that in the free theory both sets of relations are satisfied, which is easily verified by setting all the terms proportional to α_s in the above expressions for the structure functions to zero. Since only the quark-gluon-quark correlators seem to be missing in set A, we believe that it is valid and useful in models where no gauge fields are involved. For instance, we have checked explicitly taking the results for the parton distributions as obtained in the spectator model [22] that the relations in set A can be verified. However, in the context of a gauge theory like QCD, one should be careful and always use the relations in set B.

IV. SUM RULES

Our calculation here provides a direct way to investigate the existing sum rules for twist-3 parton distributions in the case of a dressed quark target. Defining the structure functions $g_2\equiv g_T-g_1$ and $h_2\equiv 2(h_L-h_1)$, the following sum rules have been proposed in the literature [16–18]:

$$\int_0^1 dx\, g_2(x)=0, \quad \int_0^1 dx\, h_2(x)=0. \tag{47}$$

From the results presented in the previous section [see Eqs. (38),(39) and (42),(43)] we can immediately write down the expressions for g_2 and h_2 to the order α_s,

$$g_2(x,Q^2)=\frac{\alpha_s}{2\pi}C_f\ln\frac{Q^2}{\mu^2}[2x-\delta(1-x)], \tag{48}$$

$$h_2(x,Q^2)=\frac{\alpha_s}{2\pi}C_f\ln\frac{Q^2}{\mu^2}[4-2\,\delta(1-x)]. \tag{49}$$

The BC sum rule for g_2 follows readily from Eq. (48) as has already been shown in Ref. [15] using the same method. On the other hand, Eq. (49) gives

$$\int_0^1 dx\, h_2(x)=\frac{\alpha_s}{\pi}C_f\ln\frac{Q^2}{\mu^2}, \tag{50}$$

which shows that the h_2 sum rule is violated in perturbation theory. Incidentally, the second moment of h_2 turns out to be zero, $\int_0^1 dx\, x\, h_2(x)=0$, although the significance of this result is not clear.

The violation of the h_2 sum rule in the context of perturbation theory is a new observation. Such a result is unexpected, bearing in mind that the h_2 sum rule has been derived on the same footing of rotational invariance as the BC sum rule [21]. We point out that our observation is different from the findings outlined in Ref. [21], where a possible violation of this sum rule for the experimentally measured structure function has been discussed. There, the origin of such a violation was attributed to quark zero modes giving rise to a δ-function singularity in the parton distribution at $x=0$. Since this kinematical point is usually inaccessible, a significant deviation from the sum rule could occur in the experiment. In contrast, we find the violation already at the level of the parton distribution h_2 calculated to the order α_s. Moreover, in our explicit calculation the final result for h_2 is not inflicted by quark zero modes.

V. CONCLUSIONS

In this work we have calculated higher twist and k_T-dependent parton distributions using the light-front Hamiltonian description of composite systems in terms of multi-parton wave functions. Employing a dressed quark target we have evaluated them to the order α_s in light-front time-ordered PQCD. While we have reproduced the results for g_1, h_1 and g_T, all the other results presented in Sec. III are new.

These calculations, in particular, have given us the opportunity to study the so-called Lorentz invariance relations existing among twist-2, twist-3 and k_T-dependent structure functions. We show explicitly that two distinct sets of such relations exist in the literature. While one set is satisfied [Eqs. (14),(15)] for the dressed quark target, the other one [Eqs. (12),(13)] is not. It turns out that quark-gluon-quark correlators are important for the Lorentz invariance relations, where these pieces are what exactly is missing in Eqs. (12), (13). The implication of our findings on the existing literature is yet to be explored.

Moreover, we have studied the sum rules for the structure functions g_2 and h_2. The BC sum rule for g_2 is fulfilled, whereas the corresponding sum rule for h_2 is violated at the order α_s in perturbation theory. Since both sum rules have been derived on the same basis of rotational invariance, the violation of the h_2 sum rule is surprising and requires further investigation.

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APPENDIX A

In this appendix we summarize our conventions. First, we specify the plus and minus lightcone components of a generic 4-vector a^μ according to $a^\pm\equiv a^0\pm a^3$, and the inner product of two 4-vectors is given by $a\cdot b=\frac{1}{2}a^+b^- +\frac{1}{2}a^-b^+-\boldsymbol{a}_T\cdot\boldsymbol{b}_T$. For the γ matrices we use the light-front representation [15]

$$\gamma^0=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix},\quad \gamma^i=\begin{pmatrix}-i\,\tilde{\sigma}_i & 0\\ 0 & i\,\tilde{\sigma}_i\end{pmatrix},$$

$$\gamma^3=\begin{pmatrix}0 & i\\ i & 0\end{pmatrix},\quad \gamma^5=\begin{pmatrix}\sigma_3 & 0\\ 0 & -\sigma_3\end{pmatrix}, \tag{A1}$$

where $\tilde{\sigma}_1=\sigma_2$ and $\tilde{\sigma}_2=-\sigma_1$. In the usual way we define the dynamical field $\psi_+=\Lambda^+\psi$ and the constrained field $\psi_-=\Lambda^-\psi$, which follows the constraint equation

$$\psi_-=\frac{\gamma^0}{i\partial^+}(i\not{D}_T+m_q)\psi_+, \tag{A2}$$

where $D^\mu=\partial^\mu-ig_sA^\mu$ is the covariant derivative. The operator $1/\partial^+$ is defined as

$$\frac{1}{\partial^+}f(x)=\frac{1}{4}\int_{-\infty}^{\infty}dy\,\epsilon(x-y)f(y), \tag{A3}$$

with $\epsilon(x)$ being the sign-function. In the representation (A1), the projection operators $\Lambda^\pm\equiv\gamma^\mp\gamma^\pm/4$ take the simple form

$$\Lambda^+=\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix},\quad \Lambda^-=\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}. \tag{A4}$$

For the fermion fields we use the two-component notation [23]

$$\psi_+=\begin{pmatrix}\eta\\ 0\end{pmatrix},\quad \psi_-=\begin{pmatrix}0\\ \dfrac{1}{i\partial^+}[\tilde{\boldsymbol{\sigma}}_T\cdot(i\boldsymbol{\partial}_T+g_s\boldsymbol{A}_T)+im_q]\eta\end{pmatrix}, \tag{A5}$$

where the Fourier expansions of the dynamical fields $\eta(x)$ and $\boldsymbol{A}_T(x)$ are given by

$$\eta(x)=\sum_\lambda \chi_\lambda \int \frac{dk^+ d^2\boldsymbol{k}_T}{2(2\pi)^3\sqrt{k^+}}[b_\lambda(k)e^{-ik\cdot x}+d^\dagger_{-\lambda}(k)e^{ik\cdot x}], \tag{A6}$$

$$\boldsymbol{A}_T(x)=\sum_\lambda \int \frac{dk^+ d^2\boldsymbol{k}_T}{2(2\pi)^3 k^+} \times[a_\lambda(k)\boldsymbol{\varepsilon}_{T,\lambda}e^{-ik\cdot x}+a^\dagger_\lambda(k)\boldsymbol{\varepsilon}^*_{T,\lambda}e^{ik\cdot x}]. \tag{A7}$$

Here the creation and annihilation operators for quarks (gluons) obey the anticommutation (commutation) relations

$$\{b_\lambda(k),b^\dagger_{\lambda'}(k')\}=\{d_\lambda(k),d^\dagger_{\lambda'}(k')\} =2(2\pi)^3k^+\delta(k^+-k'^+)\delta(\boldsymbol{k}_T-\boldsymbol{k}'_T)\delta_{\lambda,\lambda'}, \tag{A8}$$

$$[a_\lambda(k),a^\dagger_{\lambda'}(k')]=2(2\pi)^3k^+\delta(k^+-k'^+)\delta(\boldsymbol{k}_T-\boldsymbol{k}'_T)\delta_{\lambda,\lambda'}. \tag{A9}$$

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