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Single transverse-spin asymmetry in the Drell-Yan lepton angular distribution

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We calculate a single transverse-spin asymmetry for the Drell-Yan lepton-pair's angular distribution in perturbative QCD. At leading order in the strong coupling constant, the asymmetry is expressed in terms of a twist-3 quark-gluon correlation function $T_F^{(V)}(x_1,x_2)$. In our calculation, the same result was obtained in both the light-cone and covariant gauges in QCD, while keeping explicit electromagnetic current conservation for the virtual photon that decays into the lepton pair. We also present a numerical estimate of the asymmetry and compare the result to another existing prediction.

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I. INTRODUCTION

It has been known for some time that cross sections involving a single transverse hadronic spin show significant asymmetries relative to the spin direction [1,2]. Because of parity and time-reversal invariance, single longitudinal-spin asymmetries vanish for most inclusive observables in high energy collisions; and single transverse-spin asymmetries open up a new domain of QCD dynamics: physics at twist-3, which is sensitive to the correlations between quark and gluon fields. With the active spin program at Brookhaven National Laboratory's Relativistic Heavy Ion Collider (RHIC), many single transverse-spin asymmetries have been proposed and could be observed in the near future [3–9].

In this paper, we reexamine the single transverse-spin asymmetry in the Drell-Yan lepton angular distribution. Although the asymmetry was studied in Refs. [7–9], two different analytical expressions were derived. In Ref. [9] a formal argument, based on gauge vector independence, was given in favor of the result of Ref. [8], however, uncertainty about the result remained [10]. This motivates us to calculate the asymmetry in a more transparent way, such that color and electromagnetic gauge invariance are manifest.

For the polarized Drell-Yan process $A(p,s_T)+B(p')\rightarrow\gamma^*(Q)[\rightarrow l\bar{l}]+X$ we define the lepton-pair's angular distribution in the virtual photon's rest frame ($\vec{Q}=0$). As shown in Fig. 1, we choose the z axis along the direction of polarized incoming hadron, x axis along the direction of polarization vector $\vec{s}_T$, and the observed lepton momenta as

$$l^\mu=\frac{Q}{2}(1,\sin\theta\sin\phi,\sin\theta\cos\phi,\cos\theta),$$

$$\bar{l}^\mu=\frac{Q}{2}(1,-\sin\theta\sin\phi,-\sin\theta\cos\phi,-\cos\theta). \tag{1}$$

The Q is the invariant mass of the virtual photon, and its four-momentum $Q^\mu=l^\mu+\bar{l}^\mu$.

The Drell-Yan single transverse-spin asymmetry is defined to be the difference of two spin-dependent cross sections, $d\sigma(s_T)/dQ^2d\Omega$ with opposite directions of polarization, divided by their sum,

$$A_N\equiv\frac{1}{2}\left(\frac{d\sigma(s_T)}{dQ^2d\Omega}-\frac{d\sigma(-s_T)}{dQ^2d\Omega}\right)\Bigg/\frac{1}{2}\left(\frac{d\sigma(s_T)}{dQ^2d\Omega}+\frac{d\sigma(-s_T)}{dQ^2d\Omega}\right)$$
$$\equiv\frac{d\Delta\sigma(s_T)}{dQ^2d\Omega}\Bigg/\frac{d\sigma}{dQ^2d\Omega}, \tag{2}$$

where the phase space $d\Omega=d\cos\theta d\phi$.

In Ref. [7], Hammon, Teryaev, and Schäfer (HTS) calculated the Drell-Yan single transverse-spin asymmetry at the tree level in perturbative QCD with the light-cone gauge, and obtained

$$A_N^{(HTS)}=g\left[\frac{\sin 2\theta\sin\phi}{1+\cos^2\theta}\right]\frac{1}{Q}\frac{\left[T(x,x)-x\dfrac{dT(x,x)}{dx}\right]}{q(x)}, \tag{3}$$

where $g=\sqrt{4\pi\alpha_s}$ is the coupling constant for strong interaction, and the summation over quark flavor was suppressed. The $q(x)$ is the normal unpolarized quark distribution. The $T(x,x)=-T_F^{(V)}(x,x)$ was called a twist-3 soft gluon pole function, with the $T_F^{(V)}(x_1,x_2)$ defined as [3]

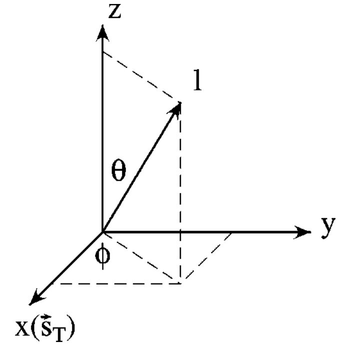

FIG. 1. Coordinate choice for the virtual photon's rest frame.

$$T_F^{(V)}(x_1,x_2)=\int\frac{dy_1^- dy_2^-}{4\pi}e^{ix_1p^+y_1^-+i(x_2-x_1)p^+y_2^-}\times\langle p,\vec{s}_T|\overline{\psi}(0)\gamma^+[\epsilon^{s_T\sigma n\bar{n}}F_\sigma{}^+(y_2^-)]\times\psi(y_1^-)|p,\vec{s}_T\rangle. \tag{4}$$

The $\pm$ components of any vector k are defined in terms of two lightlike vectors n and $\bar{n}$ as $k^+=k\cdot n$ and $k^-=k\cdot\bar{n}$. These lightlike vectors are chosen such that $\bar{n}\cdot n=1$. In our choice of frame, the momentum of the polarized hadron p is up to a mass term proportional to $\bar{n}$.

In Ref. [8], Boer, Mulders, and Teryaev (BMT) also investigated the Drell-Yan single transverse-spin asymmetry. A similar but different result was derived. This result was reexpressed in the notation of Ref. [7] and was found to be [9]

$$A_N^{(BMT)}=g\left[\frac{\sin 2\theta\sin\phi}{1+\cos^2\theta}\right]\frac{1}{Q}\frac{T(x,x)}{q(x)}, \tag{5}$$

which differs from $A_N^{(\mathrm{HTS})}$ in Eq. (3) by the term proportional to $xdT(x,x)/dx$.

In this paper, we calculate the leading order contributions to the Drell-Yan single transverse-spin asymmetry in terms of a generalized QCD factorization theorem [3,11]. In order to test the gauge invariance of our result, we derive the asymmetry in light-cone gauge as well as in covariant gauge of QCD while keeping the electromagnetic current conservation of the virtual photon explicit. The latter also makes the calculation much more transparent than the one of Ref. [8], which uses so-called effective T-odd distribution functions. In our present calculation we find that the results are independent of the gauge choices and we confirm the result of Refs. [8,9].

The derivative term in HTS's result in Eq. (3) has been found in single transverse-spin asymmetries in other processes, like prompt photon production [3] and pion production in hadron-hadron scattering [6], where the derivative term arises from collinear expansions of the partonic parts while calculations were performed in covariant gauge. However, we argue below that in terms of the generalized factorization theorem [11], there is no derivative term for the Drell-Yan asymmetry at the tree-level while it can appear in higher order corrections.

The rest of this paper is organized as follows. In the next section, we derive the single transverse-spin asymmetry in the Drell-Yan lepton angular distribution in perturbative QCD with a choice of light-cone gauge. In our calculation, we monitor the electromagnetic current conservation of the virtual photon by calculating the strong interaction part and the leptonic part separately. In addition, we show explicitly why there is no derivative term. As a cross check, we use the method developed in Ref. [3] to calculate the same Drell-Yan single transverse-spin asymmetry in a covariant gauge in Sec. III. An identical result is obtained. Finally, we present a numerical estimate of the asymmetry and our conclusions in Sec. IV.

II. ASYMMETRY CALCULATED IN QCD LIGHT-CONE GAUGE

In order to calculate the Drell-Yan single transverse-spin asymmetry, A_N in Eq. (2), we need to calculate both the unpolarized cross section, $d\sigma/dQ^2d\Omega$, and the polarized cross section, $d\Delta\sigma(s_T)/dQ^2d\Omega$.

The unpolarized Drell-Yan cross section can be factorized according to the QCD factorization theorem [12,13],

$$\frac{d\sigma_{AB\to l\bar{l}(Q)X}}{dQ^2d\Omega}=\sum_{a,b}\int dx\,\phi_{a/A}(x)dx'\,\phi_{b/B}(x')\frac{d\hat{\sigma}_{ab\to l\bar{l}(Q)X}}{dQ^2d\Omega}, \tag{6}$$

where $\Sigma_{a,b}$ run over all possible parton flavors, the ϕ's are normal parton distributions, and $d\hat{\sigma}/dQ^2d\Omega$ are perturbatively calculable partonic hard parts. At the lowest order, the partonic hard part is given by the $q\bar{q}\to\gamma^*(Q)\to l\bar{l}$ tree-level diagram, and can be expressed as

$$\frac{d\hat{\sigma}_{q\bar{q}\to l\bar{l}(Q)X}}{dQ^2d\Omega}=e_q^2\left(\frac{\alpha_{em}^2}{4\hat{s}}\right)H^{\mu\nu}(xp,x'p')L_{\mu\nu}(l,Q-l)\times\left(\frac{1}{Q^2}\right)^2\delta(Q^2-(xp+x'p')^2), \tag{7}$$

where $\hat{s}=(xp+x'p')^2\approx xx'S$ with total c.m. energy $S=2p\cdot p'$. The parton level hadronic tensor $H^{\mu\nu}$ is given by

$$H^{\mu\nu}(xp,x'p')=\left(\frac{1}{3}\right)[(xp)^\mu(x'p')^\nu+(xp)^\nu(x'p')^\mu-(xp)\cdot(x'p')g^{\mu\nu}] \tag{8}$$

with the explicit color factor (1/3). The corresponding leptonic tensor is

$$L_{\mu\nu}(l,Q-l)=4[l_\mu(Q-l)_\nu+(Q-l)_\mu l_\nu-l\cdot Qg_{\mu\nu}]. \tag{9}$$

Both hadronic and leptonic tensors show explicit electromagnetic current conservation: $Q_\mu H^{\mu\nu}=0$ and $Q^\mu L_{\mu\nu}=0$, which will be an issue below when we calculate the twist-3 contributions to the asymmetry.

By contracting the hadronic and leptonic tensors, we obtain the lowest order unpolarized Drell-Yan cross section in the virtual photon's rest frame,

$$\frac{d\sigma_{AB\to l\bar{l}(Q)X}}{dQ^2d\Omega}=\frac{4\pi\alpha_{em}^2}{9Q^2}\sum_q e_q^2\int dxq(x)dx'\bar{q}(x')\times\left[\frac{1}{4\pi}\left(\frac{3}{4}\right)[1+\cos^2\theta]\delta(Q^2-xx'S)\right], \tag{10}$$

where Σ_q runs over all quark and antiquark flavors and the factorization scale dependence of the parton distributions is suppressed.

The single transverse-spin asymmetry is a twist-3 effect. In contrast to Eq. (6), the Drell-Yan single transverse-spin asymmetry has the following generalized factorization formula [3]

$$\frac{d\Delta\sigma_{AB\to l\bar{l}(Q)X}(s_T)}{dQ^2 d\Omega} = \sum_{a,b} \int dx_1 dx_2 T^{(3)}_{a/A}(x_1,x_2;s_T) \times \int dx' \phi_{b/B}(x') \times \frac{1}{Q} \frac{d\Delta\hat{\sigma}_{ab\to l\bar{l}(Q)X}(s_T)}{dQ^2 d\Omega}, \tag{11}$$

where $T^{(3)}_{a/A}(x_1,x_2;s_T)$ are twist-3 three-parton correlation functions with a dimension of mass and $d\Delta\hat{\sigma}(s_T)/dQ^2 d\Omega$ are perturbatively calculable partonic parts. The explicit $1/Q$ factor in Eq. (11) is a signal of a twist-3 effect and takes care of the mass dimension of the twist-3 correlation functions. In order to extract the lowest order contribution to the Drell-Yan single transverse-spin asymmetry, we need to start with lowest order diagrams with two-fermion and one gluon lines from the polarized hadron, as shown in Fig. 2 [3,11,14]. Because of the virtual photon of the Drell-Yan process, contributions from Fig. 2 can be written in general as

$$I \equiv \left(\frac{1}{Q^2}\right)^2 W^{\mu\nu}(Q) L_{\mu\nu}(l,Q-l) \tag{12}$$

with the leptonic tensor $L_{\mu\nu}$ given in Eq. (9). After collinear expansion of the parton's momenta, and separation of fermion traces [3,11,14], the leading contributions to the hadronic tensor $W^{\mu\nu}$ from the diagram in Fig. 2 can be expressed in the following factorized form:

$$W^{\mu\nu}(Q) = \sum_q e_q^2 \int dx_1 dx_2 T^{\sigma}_q(x_1,x_2;s_T) \times \int dx' \bar{q}(x') H^{\mu\nu}_{\sigma}(x_1,x_2,x';Q), \tag{13}$$

where Σ_q runs over all quark and antiquark flavors. The hadronic matrix element $T^{\sigma}_q(x_1,x_2;s_T)$ is given by

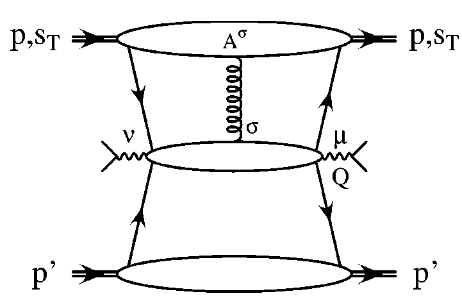

FIG. 2. General diagram contributing to the Drell-Yan single transverse-spin asymmetry.

$$T^{\sigma}_q(x_1,x_2;s_T) = \int \frac{p^+ dy_1^-}{2\pi} \frac{p^+ dy_2^-}{2\pi} e^{ix_1 p^+ y_1^-} e^{i(x_2-x_1)p^+ y_2^-} \times \langle p,\vec{s}_T | \bar{\psi}_q(0) \times \left(\frac{\gamma^+}{2p^+}\right) A^{\sigma}(y_2^-) \psi_q(y_1^-) | p,\vec{s}_T\rangle, \tag{14}$$

where A^{σ} is the gluon field. The partonic part $H^{\mu\nu}_{\sigma}$ in Eq. (13) is given by the four Feynman diagrams in Fig. 3 with the quark lines contracted by $1/2\gamma\cdot p$, and antiquark lines contracted by $1/2\gamma\cdot p'$ [3,11,14]. As we will demonstrate below, the inclusion of all four diagrams are very important for preserving electromagnetic current conservation. The diagrams 3(c) and 3(d) play the same role as the twist-3 contribution of diagram Fig. 1 of Ref. [15] in the calculation of Ref. [8].

It is important to note that the $T^{\sigma}_q(x_1,x_2;s_T)$ in Eq. (14) is not necessarily a twist-3 matrix element, and the hadronic tensor $W^{\mu\nu}$ in Eq. (13) could have twist-2, twist-3, and even higher twist contributions [14]. In the rest of this section, we show how to extract the twist-3 contributions to the single transverse-spin asymmetry from the $W^{\mu\nu}$ in Eq. (13) in light-cone gauge. We will present a similar derivation in a covariant gauge in the next section.

In $n\cdot A(y_2^-)=0$ light-cone gauge, the leading contribution of the gluon field A^{σ} comes from its transverse components (i.e., $\sigma=1,2$) [16], and the corresponding gluon field strength is given by

$$F^{+\sigma}(y_2^-) = n^{\rho}\partial_{\rho} A^{\sigma}(y_2^-). \tag{15}$$

Therefore, in the light-cone gauge, we need to make the following replacement:

$$A^{\sigma}(y_2^-) \to \frac{i}{(x_2-x_1)p^+} F^{+\sigma}(y_2^-) \tag{16}$$

in the matrix element $T^{\sigma}(x_1,x_2;s_T)$ to convert the gluon field $A^{\sigma}(y_2^-)$ into the gluon field strength $F^{+\sigma}(y_2^-)$, the same as that in the twist-3 matrix element $T^{(V)}_F(x_1,x_2)$ in Eq. (4).

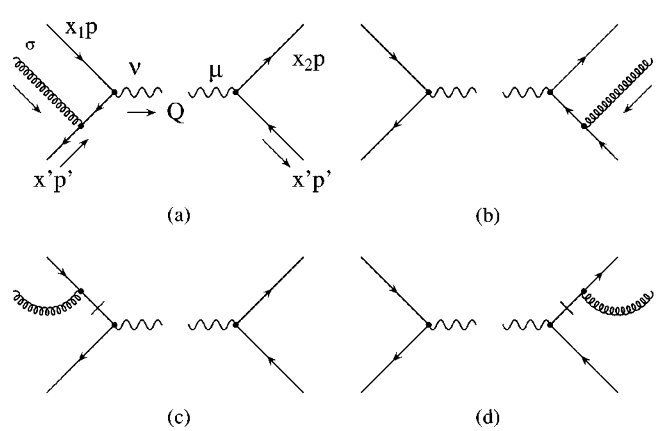

FIG. 3. Lowest order Feynman diagrams contributing to the Drell-Yan single transverse-spin asymmetry.

Because of different behavior of the fermion propagators, we derive the partonic tensor $H^{\mu\nu}_{\sigma}$ in Eq. (13) in two steps. We first calculate the contributions from the two diagrams in Figs. 3(a) and 3(b). With the Lorentz index σ in the transverse direction, we obtain

$$H^{\mu\nu}_{\sigma}(a+b)=g\left(\frac{x_2-x_1}{x_2-x_1+i\epsilon}\right)\left[g^{\mu}_{\sigma}\left(\frac{p}{x'}\right)^{\nu}\delta(Q^2-x_2x'S)+g^{\nu}_{\sigma}\left(\frac{p}{x'}\right)^{\mu}\delta(Q^2-x_1x'S)\right]. \tag{17}$$

Clearly, the partonic part from the two diagrams violates electromagnetic current conservation.

After the collinear expansion, the fermion propagators in the other two diagrams in Fig. 3 are on mass-shell and contributions from these two diagrams are divergent. As shown in Refs. [11,14], the divergent part of these propagators corresponds to the long-distance contributions that should be included into the twist-2 quark distribution; and these propagators also have a finite contact contribution, which is one twist higher. The concept of special propagator was introduced in Ref. [14] to identify the high twist piece of each propagator. Graphically, the special propagator is represented by a normal propagator line plus a bar as shown in Figs. 3(c) and 3(d). The Feynman rule for a special propagator depends on the gauge choice because of the nature of high twist contributions. In light-cone gauge, for example, the rule for a special quark propagator, merged from incoming quark and gluon lines in Fig. 3(c), is given by [14]

$$\frac{i\gamma\cdot(x_2p)}{(x_2p)^2+i\epsilon}\rightarrow\frac{i\gamma\cdot n}{2x_2p\cdot n}\left[\frac{x_2-x_1}{x_2-x_1+i\epsilon}\right], \tag{18}$$

where the "$i\epsilon$" can be either derived following the example given in the next section, or obtained directly from the prescriptions given in the Appendix of Ref. [17]. With the special quark propagator in Eq. (18), we derive the twist-3 contributions from the diagrams in Figs. 3(c) and 3(d),

$$H^{\mu\nu}_{\sigma}(c+d)=g\left(\frac{x_2-x_1}{x_2-x_1+i\epsilon}\right)\left[g^{\mu}_{\sigma}\left(-\frac{p'}{x_2}\right)^{\nu}\delta(Q^2-x_2x'S)+g^{\nu}_{\sigma}\left(-\frac{p'}{x_1}\right)^{\mu}\delta(Q^2-x_1x'S)\right]. \tag{19}$$

Combining the short-distance twist-3 contributions from all four diagrams in Fig. 3, we obtain

$$H^{\mu\nu}_{\sigma}=g\left(\frac{x_2-x_1}{x_2-x_1+i\epsilon}\right)\left[g^{\mu}_{\sigma}\left(\frac{p}{x'}-\frac{p'}{x_2}\right)^{\nu}\delta(Q^2-x_2x'S)+g^{\nu}_{\sigma}\left(\frac{p}{x'}-\frac{p'}{x_1}\right)^{\mu}\delta(Q^2-x_1x'S)\right]. \tag{20}$$

The partonic part $H^{\mu\nu}_{\sigma}$ with complete twist-3 contributions from all four diagrams in Fig. 3 clearly respects the electromagnetic current conservation.

Having the partonic part $H^{\mu\nu}_{\sigma}$ in Eq. (20) and the matrix element T^{σ} in Eq. (14) with its gluon field replaced by the corresponding field strength as in Eq. (16), we derive the twist-3 contributions to the hadronic tensor $W^{\mu\nu}(Q)$ in Eq. (13),

$$W^{\mu\nu}(Q)=\sum_q e_q^2\int dx T^{(V)}_{F_q}(x,x)\int dx'\bar{q}(x')\times\left(\delta(Q^2-xx'S)\frac{g}{2}\epsilon^{s_T\sigma n\bar{n}}\right)\times\left[g^{\mu}_{\sigma}\left(\frac{p}{x'}-\frac{p'}{x}\right)^{\nu}+g^{\nu}_{\sigma}\left(\frac{p}{x'}-\frac{p'}{x}\right)^{\mu}\right]. \tag{21}$$

In deriving the above result, we took the imaginary part of the single pole,

$$\frac{1}{x_2-x_1+i\epsilon}\rightarrow-\pi i\,\delta(x_2-x_1), \tag{22}$$

such that the i from the imaginary part cancels the i in Eq. (16) to ensure a real contribution to the single transverse-spin asymmetry [3]. The apparent factor x_2-x_1 in numerator of Eq. (20) cancels the $1/(x_2-x_1)$ in Eq. (16), which is the result of converting the matrix element T^{σ} to the twist-3 correlation function $T^{(V)}_{F}$. It is clear that the leading order twist-3 hadronic tensor $W^{\mu\nu}(Q)$ in Eq. (21) respects electromagnetic current conservation,

$$W^{\mu\nu}(Q)Q_{\mu}=W^{\mu\nu}(Q)Q_{\nu}=0, \tag{23}$$

with the virtual photon's momentum $Q^{\mu}=(xp)^{\mu}+(x'p')^{\mu}$.

By contracting the hadronic tensor $W^{\mu\nu}(Q)$ with the leptonic tensor $L_{\mu\nu}$ in Eq. (9), and using Eq. (12), we derive the leading order twist-3 contributions from the general diagram in Fig. 2,

$$I^{(3)}=\left(\frac{1}{Q^2}\right)^2\sum_q e_q^2\int dx T^{(V)}_{F_q}(x,x)\int dx'\bar{q}(x')\times\left[-\frac{g}{Q}SQ^2(\sin 2\theta\sin\phi)\,\delta(Q^2-xx'S)\right], \tag{24}$$

where Σ_q runs over all quark and antiquark flavors. Multiplying the twist-3 contributions in Eq. (24) by the overall flux, coupling, and phase space factor, $(\alpha^2_{em}/4S)=1/(2S)e^4(1/32\pi^2)$, as well as a color factor (1/3), we obtain the leading order polarized cross section,

$$\frac{d\Delta\sigma_{AB\rightarrow l\bar{l}(Q)X}(s_T)}{dQ^2d\Omega}=\frac{4\pi\alpha^2_{em}}{9Q^2}\sum_q e_q^2\int dx T^{(V)}_{F_q}(x,x)dx'\bar{q}(x')\times\left[\frac{1}{4\pi}\left(\frac{3}{4}\right)\left[-\frac{g}{Q}\sin 2\theta\sin\phi\right]\times\delta(Q^2-xx'S)\right]. \tag{25}$$

Use $T_q(x,x)=-T^{(V)}_{F_q}(x,x)$, and divide the polarized cross section in Eq. (25) by the unpolarized cross section in Eq.

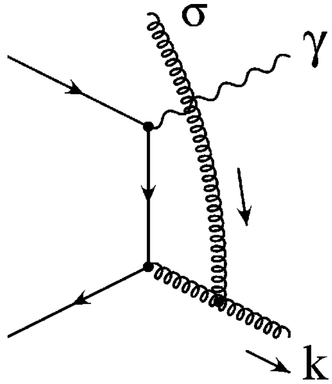

FIG. 4. Sample Feynman diagram generating a double soft-gluon pole.

(10), we derive our result for the single transverse-spin asymmetry for Drell-Yan lepton angular distribution,

$$A_N = \sqrt{4\pi\alpha_s}\left[\frac{\sin 2\theta \sin\phi}{1+\cos^2\theta}\right]\frac{1}{Q}\,\frac{\Sigma_q e_q^2 \int dx T_q(x,x)\bar{q}(Q^2/xS)}{\Sigma_q e_q^2 \int dx q(x)\bar{q}(Q^2/xS)}, \tag{26}$$

where Σ_q runs over all quark and antiquark flavors. If we suppress the sum over quark flavors, as what was done in Refs. [7,8], the asymmetry A_N in Eq. (26) is reduced to the $A_N^{(BMT)}$ in Eq. (5).

With three partons from the polarized hadron, the general Feynman diagram in Fig. 2 can have both soft-gluon as well as soft-quark poles [3]. Because of the infrared behavior of soft gluons, the soft-gluon poles from the general diagram in Fig. 2 can be either a single pole, like the $1/(x_2-x_1+i\epsilon)$ in Eq. (20), or a double pole, such as $1/(x_2-x_1+i\epsilon)^2$. The single pole leads to a dependence on $T(x,x)$ while the double pole results into the derivative term $x dT(x,x)/dx$. For example, consider a diagram in Fig. 4, which gives a higher order correction to the Drell-Yan single transverse-spin asymmetry if the photon is virtual. If the photon is real, it gives a leading order contribution to the single transverse-spin asymmetry in prompt photon production [3]. In light-cone gauge, this diagram potentially has a double soft-gluon pole: one from the gluon propagator and the other from the gluon attached to the polarized hadron. However, in general, one of the two potential poles might be canceled by a vanishing numerator. But, if the outgoing gluon in Fig. 4 has nonvanish transverse momentum, both soft poles could survive and lead to a double soft-gluon pole [3].

However, the potential double poles never survive at the lowest order single transverse-spin asymmetry in the Drell-Yan angular distribution calculated in this paper. Consider the diagram in Fig. 3(a), we have potential poles from the quark propagator,

$$i\,\frac{(x_1-x_2)\gamma\cdot p - x'\gamma\cdot p'}{-(x_1-x_2)x'S+i\epsilon},$$

and the $i/(x_2-x_1)p^+$ in Eq. (16) from the gluon attached to the polarized hadron in light-cone gauge. In terms of the generalized factorization theorem [3,11], momenta of all partons entering the partonic short-distance hard parts have only components collinear to their respective hadron momenta. For the leading order Drell-Yan process, the incoming quark and antiquark lines in Fig. 3 have to be contracted with $1/2\gamma\cdot p$ and $1/2\gamma\cdot p'$ to ensure the twist-3 contributions and color gauge invariance at this twist. Any other combination will result into power suppressed corrections [14]. Because the index σ in Fig. 3 is transverse in light-cone gauge, the $1/2\gamma\cdot p'$ from the twist-2 antiquark line eliminates the $\gamma\cdot p'$ term of the quark propagator, and leaves the partonic contribution proportional to $(x_2-x_1)/(x_2-x_1+i\epsilon)$. The vanishing numerator factor (x_2-x_1) cancels the potential $1/(x_2-x_1)$ pole from the incoming gluon, and leaves a single soft-gluon pole. The derivative term in Eq. (3) was obtained because the authors did not contract the incoming quark and antiquark lines with the $1/2\gamma\cdot p$ and $1/2\gamma\cdot p'$. For a detailed discussion on this issue from the perspective of gauge vector (n) independence, see Ref. [9].

III. ASYMMETRY DERIVED IN A COVARIANT GAUGE

In deriving the single transverse-spin asymmetry in the Drell-Yan angular distribution in the last section, we kept explicit electromagnetic current conservation. In order to test the color gauge invariance of our result, we present a derivation of the same asymmetry in a color covariant gauge in this section.

Similar to our calculation in light-cone gauge, we start with the general decomposition in Eq. (12), and a complete set of partonic Feynman diagrams at the order of $O(g\alpha_{em}^2)$ in Fig. 3. Due to the change of the gauge, one difference is that the gluon field $A^\sigma(y_2^-)$ in the matrix element $T_q^\sigma(x_1,x_2;s_T)$ is no longer dominated by its transverse components. Instead, the leading contribution of the gluon field is from the component parallel to the direction of its momentum, $A^\sigma \propto p^\sigma$ [16]. Therefore, we can get the leading contribution from the gluon field in a covariant gauge by the following replacement:

$$A^\sigma(y_2^-) \rightarrow \frac{1}{p^+}A^+(y_2^-)p^\sigma. \tag{27}$$

In order to convert the $A^+(y_2^-)$ to the corresponding field strength $F^{+\rho}(y_2^-)$, we need to give the gluon field a small transverse momentum k_T, and then convert $k_T^\rho A^+(y_2^-)$ into $F^{+\rho}(y_2^-)$ [3].

With the gluon carrying a small transverse momentum k_T, the hadronic tensor $W^{\mu\nu}(Q)$ in Eq. (12) has a slightly different expression,

$$W^{\mu\nu}(Q)=\sum_q e_q^2\int dx_1dx_2\int d^2k_T T_q(x_1,x_2,k_T;s_T)$$
$$\times\int dx'\overline{q}(x')H^{\mu\nu}(x_1,x_2,k_T,x';Q), \tag{28}$$

where Σ_q runs over all quark and antiquark flavors. The hadronic matrix element $T_q(x_1,x_2,k_T;s_T)$ is defined as

$$T_q(x_1,x_2,k;s_T)=\int\frac{p^+dy_1^-}{2\pi}\frac{p^+dy_2^-}{2\pi}\frac{d^2y_T}{(2\pi)^2}$$
$$\times e^{ix_1p^+y_1^-}e^{i(x_2-x_1)p^+y_2^-}e^{i\vec{k}_T\cdot\vec{y}_T}\langle p,\vec{s}_T|\overline{\psi}_q(0,0_T)\left(\frac{\gamma^+}{2p^+}\right)$$
$$\times\left[\frac{1}{p^+}A^+(y_2^-,y_T)\right]\psi_q(y_1^-)|p,\vec{s}_T\rangle. \tag{29}$$

The partonic part $H^{\mu\nu}(x_1,x_2,k_T,x';Q)$ is again given by the four Feynman diagrams in Fig. 3, except the incoming gluon momentum is now $(x_2-x_1)p+k_T$, and the gluon polarization σ is contracted with p^σ.

In order to extract the twist-3 contributions from these diagrams, we need to get one power of k_T from the partonic part to convert the gluon field A^+ in the matrix element into corresponding gluon field strength [3]. Expand the partonic part at $k_T=0$,

$$H^{\mu\nu}\approx H^{\mu\nu}(x_1,x_2,k_T=0,x';s_T)$$
$$+\frac{\partial}{\partial k_T^\rho}H^{\mu\nu}(x_1,x_2,k_T=0,x';s_T)k_T^\rho+O(k_T^2). \tag{30}$$

The first term in the above expansion corresponds to the eikonal line contribution to the twist-2 quark distribution. The second term in Eq. (30) is responsible for the twist-3 contribution, while the remaining terms correspond to even higher twist contributions.

Substituting the second term of the above expansion into Eq. (28), and integrating over $\int d^2k_T$, we obtain the twist-3 contributions to the hadronic tensor in a covariant gauge,

$$W^{\mu\nu}(Q)=\sum_q e_q^2\int dx_1dx_2T_{F_q}^{(V)}(x_1,x_2)\int dx'\overline{q}(x')$$
$$\times\left[\frac{i}{2\pi}\epsilon^{\rho s_T n\bar{n}}\frac{\partial}{\partial k_T^\rho}H^{\mu\nu}\bigg|_{k_T=0}\right], \tag{31}$$

where the twist-3 correlation function $T_{F_q}^{(V)}(x_1,x_2)$ is given in Eq. (4). Therefore, calculating the twist-3 short-distance contribution is equivalent to extracting the terms linear in k_T from the Feynman diagrams in Fig. 3 with incoming gluon momentum $(x_2-x_1)p+k_T$.

As in the previous section, we calculate the short-distance partonic contributions to the asymmetry in two steps. We first evaluate the contributions from the diagrams in Figs. 3(a) and 3(b), and find

$$\frac{\partial}{\partial k_T^\rho}H^{\mu\nu}\bigg|_{k_T=0}(a+b)$$
$$=-g\left(\frac{1}{x_2-x_1+i\epsilon}\right)\left[g_\rho^\mu\left(\frac{p}{x'}\right)^\nu\delta(Q^2-x_2x'S)\right.$$
$$\left.+g_\rho^\nu\left(\frac{p}{x'}\right)^\mu\delta(Q^2-x_1x'S)\right]. \tag{32}$$

Again, the contributions from these two diagrams do not respect electromagnetic current conservation, and are exactly the same as what we calculated in the light-cone gauge. Therefore, we need to find the other half of the contributions from the diagrams in Figs. 3(c) and 3(d).

In a covariant gauge, the gluon polarization index σ in Fig. 3 is contracted with p^σ. For the two diagrams in Figs. 3(c) and 3(d), the gluon interaction vertices are directly connected to the incoming quark lines, which are to be contracted with $1/2\gamma\cdot p$. Naively, we would conclude that contributions from these two diagrams vanish because $p^\sigma[\gamma_\sigma 1/2\gamma\cdot p]\propto p^2=0$. But, due to the vanishing denominator of the quark propagators of these two diagrams, we actually have a "0/0" situation for the contributions of these two diagrams.

Same as in the light-cone gauge, these two diagrams have not only twist-3 contributions but also long-distance twist-2 contributions. In order to extract the correct twist-3 contributions from these two diagrams in covariant gauge, consider a simple and generic diagram in Fig. 5, where the incoming

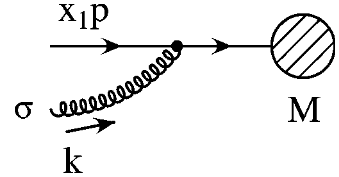

FIG. 5. A generic diagram for extracting the special quark propagator.

quark of momentum $x_1 p$ is on mass-shell, and the gluon's momentum $k \to (x_2 - x_1)p + k_T$. The contribution from this generic diagram can be written as

$$H \equiv \mathrm{Tr}\left[M(x_1 p + k) \frac{i\gamma\cdot(x_1 p + k)}{(x_1 p + k)^2 + i\epsilon} (-i\gamma_\sigma)\gamma\cdot(x_1 p) \right] p^\sigma, \tag{33}$$

where M represents the blob in Fig. 5. Since we are interested in the twist-3 contribution in a covariant gauge, we need only the terms linearly proportional to k_T,

$$H \to H^{(3)} = \mathrm{Tr}[M(x_2 p)\gamma\cdot k_T] \frac{x_1 p^2}{(x_2 - x_1)(2x_1 p^2) + i\epsilon}$$

$$= \mathrm{Tr}[M(x_2 p)\gamma\cdot k_T] \frac{1}{2} \frac{1}{x_2 - x_1 + i\epsilon}. \tag{34}$$

In deriving the first line in Eq. (34), we dropped a k_T^2 in the denominator and the k_T dependence in M, because they correspond to twist-4 or higher contributions.

Following this approach, we derive the twist-3 contributions from the two diagrams in Figs. 3(c) and 3(d),

$$\frac{\partial}{\partial k_T^\rho} H^{\mu\nu}\bigg|_{k_T=0}(c+d) = -g\left(\frac{1}{x_2 - x_1 + i\epsilon}\right)\left[g_\rho^\mu \left(-\frac{p'}{x_2}\right)^\nu \times \delta(Q^2 - x_2 x' S) + g_\rho^\nu \left(-\frac{p'}{x_1}\right)^\mu \times \delta(Q^2 - x_1 x' S)\right]. \tag{35}$$

By adding all twist-3 contributions of the four diagrams in Fig. 3, we obtain in a covariant gauge

$$\frac{\partial}{\partial k_T^\rho} H^{\mu\nu}\bigg|_{k_T=0} = -g\left(\frac{1}{x_2 - x_1 + i\epsilon}\right)\left[g_\rho^\mu \left(\frac{p}{x'} - \frac{p'}{x_2}\right)^\nu \times \delta(Q^2 - x_2 x' S) + g_\rho^\nu \left(\frac{p}{x'} - \frac{p'}{x_1}\right)^\mu \times \delta(Q^2 - x_1 x' S)\right], \tag{36}$$

which clearly respects electromagnetic current conservation.

By substituting Eq. (36) into Eq. (31), and contracting with the leptonic tensor $L_{\mu\nu}$, we derive the same polarized Drell-Yan cross section in Eq. (25) and the same asymmetry in Eq. (26) in a color covariant gauge. Therefore, we can conclude that our result in Eq. (26) for the single transverse-spin asymmetry in the Drell-Yan lepton angular distribution not only respects electromagnetic gauge invariance, but also is the same in both color light-cone and color covariant gauges.

IV. NUMERICAL ESTIMATE AND CONCLUSIONS

In order to estimate the numerical size of the single transverse-spin asymmetry in the Drell-Yan lepton angular distribution in Eq. (26), we need to know the size and sign of the twist-3 quark-gluon correlation functions $T_{F_q}^{(V)}(x,x)$.

Fortunately, the same twist-3 quark-gluon correlation functions $T_{F_q}^{(V)}(x,x)$ appear in a number of other single transverse-spin asymmetries [3,6]. In principle, self-consistency of the twist-3 correlation functions in different physical observables is a test of QCD and its factorization theorems beyond the leading twist physics.

In Ref. [6], single transverse-spin asymmetries in hadronic pion production were calculated in perturbative QCD, in terms of the same generalized factorization theorem that was used in this paper. The same twist-3 quark-gluon correlation functions $T_{F_q}^{(V)}(x,x)$ dominate the leading contributions to the asymmetries. Although the existing data on pion asymmetries have relatively low pion transverse momenta, the general features of the data are naturally explained by the theoretical formulas [6]. By comparing the theoretical calculations with the existing data on the difference between π^+ and π^- asymmetries, it seems that the twist-3 correlation function $T_{F_u}^{(V)}(x,x)$ for valence up quark has an opposite sign to $T_{F_d}^{(V)}(x,x)$ for valence down quark. In addition, a positive $T_{F_u}^{(V)}(x,x)$ is favored.

If we adopt the model introduced in Ref. [6] for the twist-3 quark-gluon correlation functions,

$$T_{F_q}^{(V)}(x,x) \approx \kappa_q \lambda\ q(x) \tag{37}$$

with $\kappa_u = +1 = -\kappa_d$, $\kappa_s = 0$, and $\lambda \sim 100\,\mathrm{MeV}$, we estimate the single transverse-spin asymmetry in the Drell-Yan lepton angular distribution as follows. Taking the angle $\phi \sim 90°$, we have the angular dependence,

$$\left| \frac{\sin 2\theta \sin\phi}{1+\cos^2\theta} \right| \leqslant 0.7. \tag{38}$$

Because the valence up quark is about twice the valence down quark, and the fractional charge square $e_u^2 = 4/9$ is a factor of 4 larger than $e_d^2 = 1/9$, we approximate

$$\frac{\Sigma_q e_q^2 \int dx T_q(x,x)\overline{q}(Q^2/xS)}{\Sigma_q e_q^2 \int dx q(x)\overline{q}(Q^2/xS)} \sim -\lambda \frac{\int dx u(x)\overline{u}(Q^2/xS)}{\int dx[u(x)\overline{u}(Q^2/xS) + \overline{u}(x)u(Q^2/xS)]} \sim -\frac{\lambda}{2}, \tag{39}$$

where the minus sign is due to the definition $T_q(x,x) = -T_{F_q}^{(V)}(x,x)$. Because of the assumption, $k_{\overline{u}} = 0$ [6], we have only one term in the numerator in Eq. (39) in comparison with the two terms in the denominator. The 2 in the second line of Eq. (39) is due to the fact that the two terms in the denominator are equal for proton-proton collision. Taking the maximum value from the angular dependence in Eq. (38), we estimate that the magnitude of the asymmetry in Eq. (26) can be as large as (for $\sqrt{4\pi\alpha_s}\sim 2$)

$$|A_N|\sim 0.7\frac{\lambda}{Q}. \tag{40}$$

For $\lambda\sim 100\,\mathrm{MeV}$, we have A_N of about 3.5% at $Q=2$ GeV, and 1.75% at $Q=4$ GeV, where we chose the Q values to be just below and above the J/ψ resonance. Considering that RHIC's sensitivity to double transverse-spin asymmetries in Drell-Yan is at the percent level [18], these values for the single transverse-spin asymmetry might be detectable.

The numerical values for the single transverse-spin asymmetry in Drell-Yan lepton angular distribution are small, which is consistent with the fact that the single transverse-spin asymmetries are a twist-3 effect. For any inclusive observables, like the Drell-Yan cross section discussed here, the asymmetries should be of the order of Λ_{QCD}/Q with Q being the energy exchange of the collisions. However, the single transverse-spin asymmetries can be large for observables at certain phase space where the twist-2 contributions are steeply falling, and the twist-3 contributions get a kinematic enhancement due to the derivative term [6].

We would like to contrast our small asymmetry estimates with those of Ref. [4], which find large asymmetry values based on their model and its parameters fitted to the pion production asymmetries. A future measurement of the size of the asymmetry can therefore clearly distinguish between the model predictions of [4] (asymmetries between 20% and 40% for the whole x_F range) and the estimates presented here.

In conclusion, we have calculated the single transverse-spin asymmetry for the Drell-Yan lepton-pair's angular distribution in perturbative QCD. We found that our result at leading order in the strong coupling constant, given in Eq. (26), is consistent with what was derived in Ref. [8], but different from what was found in Ref. [7]. We derived our result in both color light-cone and covariant gauges while keeping explicit electromagnetic current conservation. With a model of the twist-3 correlation function, our calculated asymmetry predicts both the sign and magnitude of the asymmetry.

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[1] G. Bunce *et al.*, Phys. Rev. Lett. **36**, 1113 (1976); A. D. Panagiotou, Int. J. Mod. Phys. A **5**, 1197 (1990).
[2] D. L. Adams *et al.*, Phys. Lett. B **264**, 462 (1991); A. Bravar *et al.*, Phys. Rev. Lett. **77**, 2626 (1996).
[3] J.-W. Qiu and G. Sterman, Phys. Rev. Lett. **67**, 2264 (1991); Nucl. Phys. **B378**, 52 (1992).
[4] C. Boros, Liang Zuo-tang, and Meng Ta-chung, Phys. Rev. Lett. **70**, 1751 (1993).
[5] M. Anselmino, M. Boglione, and F. Murgia, Phys. Lett. B **362**, 164 (1995); A. V. Efremov, V. M. Korotkiyan, and O. Teryaev, *ibid.* **348**, 577 (1995); M. Anselmino and F. Murgia, *ibid.* **442**, 470 (1998); J. Tang, hep-ph/9807560; D. Boer, Phys. Rev. D **60**, 014012 (1999).
[6] J.-W. Qiu and G. Sterman, Phys. Rev. D **59**, 014004 (1999).
[7] N. Hammon, O. Teryaev, and A. Schäfer, Phys. Lett. B **390**, 409 (1997).
[8] D. Boer, P. J. Mulders, and O. V. Teryaev, Phys. Rev. D **57**, 3057 (1998).
[9] D. Boer and P. J. Mulders, Nucl. Phys. **B569**, 505 (2000).
[10] O. V. Teryaev, RIKEN Rev. **28**, 101 (2000).
[11] J.-W. Qiu and G. Sterman, Nucl. Phys. **B353**, 105 (1991); **B353**, 137 (1991).
[12] J. C. Collins, D. E. Soper, and G. Sterman, Nucl. Phys. **B261**, 104 (1985); **B308**, 833 (1988); in *Perturbative Quantum Chromodynamics*, edited by A. H. Mueller (World Scientific, Singapore, 1989), p. 1.
[13] G. T. Bodwin, Phys. Rev. D **31**, 2616 (1985); **34**, 3932(E) (1986).
[14] J.-W. Qiu, Phys. Rev. D **42**, 30 (1990).
[15] D. Boer, P. J. Mulders, and O. V. Teryaev, hep-ph/9710525.
[16] M. Luo, J.-W. Qiu, and G. Sterman, Phys. Rev. D **50**, 1951 (1994).
[17] A. H. Mueller and J.-W. Qiu, Nucl. Phys. **B268**, 427 (1986).
[18] O. Martin *et al.*, Phys. Rev. D **60**, 117502 (1999).