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Explicit and Latent Authority in Hierarchical Organizations*

René van den Brink† Robert P. Gilles‡

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Abstract

In this paper we consider the problem of the control of access to a firm's productive asset, embedding the relevant decision makers into a general structure of formal authority relations. Within such an authority structure, each decision maker acts as a principal to some decision makers, while she acts as an agent in relation to certain other decision makers. We study under which conditions decision makers decide to exercise their own authority and to accept their superiors' authority.

We distinguish two types of behavior within such an authority situation. First, we investigate a non-cooperative equilibrium concept describing the explicit, myopic exercise of authority. We find that if monitoring costs are sufficiently small, such explicit authority is exercised fully.

Second, we consider the possibility of subordinates to submit themselves to authority even though such authority is not enforced explicitly. Again for sufficiently small monitoring costs such latent authority can be supported as an equilibrium.

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1 Introduction: Exercising authority

In this paper we develop an alternative approach to modelling the use and enforcement of formal authority within a given hierarchical production organization. We do not intend to develop an alternative theory on the nature of the firm, but rather limit our investigations to the nature of authority in a given formal authority structure that regulates the access of agents to a productive asset.

In contemporary literature on the firm, the nature of authority in (hierarchical) production organizations is a major field of investigation. Since the seminal contributions of Coase [10], Simon [39], Williamson [41, 42], Grossman and Hart [16], and Hart and Moore [18] the literature has mainly developed towards a theory of incomplete contracting which tries to explain the formation of firms from the *ownership* over residual rights, i.e., rights that are not contractible.[1] One of the main limitations of this theory is that it mostly studies situations with a rather limited number of authority relationships. Another problem with this approach is the focus on ownership. As Rajan and Zingales [32] put it: "The property rights view does not consider employees' part of the firm because, given that employees cannot be owned, there is no sense in which they are any different from agents who contract with the firm at arm's length".

Following Rajan and Zingales [32] we place the *control of access to a productive asset* at the center of our investigations and, thus, of our model of enforcing formal authority within a production organization. We pursue an alternative approach, explicitly allowing arbitrarily complex structures of formal authority relations using deterministic concepts from noncooperative as well as cooperative game theory and the theory of social situations (Greenberg [15]). We explicitly assume a given environment consisting of a fixed set of agents[2], a productive asset, and a structure of formal authority relationships between these agents regulating the control of the access to the productive asset. We view such a formal authority relationship as between a "superior" and a "subordinate". The superior is assumed to have the power to control the access of the subordinate to the productive asset. Our formal theory is now based on three primitives:

(1) a description of the productive values that can be generated by the different teams of agents that are generated through accessing the productive asset,

(2) a structure of formal authority relationships which represents the distribution of the power to regulate the access of individual agents to the productive asset, and

[1] For recent developments regarding the theory of incomplete contracting and its foundations we refer to Maskin and Tirole [27] and Hart and Moore [19].

[2] Throughout this paper we use the term "agent" synonymously with the standard notion of an "economic actor". Hence, unless stated explicitly, an agent does not refer to an agent as in a principal-agent relationship.

(3) a utility structure describing the preferences of the agents over the different production situations.

We give a short description of each of these primitives.

First, following the seminal work of Alchian and Demsetz [2], we assume that production is in principle a collective effort. Teams of agents access the productive asset and generate a collective production value.[3] Formally, the potential collective *output values* of the different teams are represented by a cooperative game with transferable utility. This is also the modelling principle of the literature quoted. We assume that these productive capacities are completely independent of the regulation of a team's access to the productive asset of the firm. In that respect these output values only have a *potential* nature.

Second, we introduce an arbitrarily complex structure of formal authority relationships. Our main hypothesis is that one has to distinguish "authority" itself from the deliberate enforcement of authority, or "enforced authority". Following Aghion and Tirole [1] we define *formal authority* of an individual as the formal contracted right of that individual to control the access of certain other individual agents to the firm's asset. Hence, within a formal authority relation we distinguish one superior and one subordinate such that the superior has the right to control the access to the productive asset by the subordinate. An agent is usually a superior to one or more subordinates, but is herself possibly also a subordinate to one or more superiors. In this regard individual agents within an authority structure are "relative principals" as well as "relative agents" in the sense of a regular principal-agent relation.

This implies that a team has to obtain some form of permission from the superiors of the members of the team before it has access to the firm's productive asset. We assume that such permission is only required if formal authority relationships are "enforced" by the various superiors of members of the team. If authority is not enforced, in principle such authority has not to be granted.

Here, we define authority to be *enforced* when costs are incurred to monitor certain subordinates with the aim to actually regulate or control their access to the firm's asset. When an individual agent — as a relative principal or superior within the formal authority structure — decides to enforce her formal authority over some of her subordinates, she engages in monitoring to detect whether a subordinate pursues unauthorized access to the firm's asset. This implies that in principle enforcing authority is costly. If a subordinate does not assume the objectives of the superior, the superior can ultimately sanction that

[3]To support the hypothesis that team production is collective, we quote Alchian and Demsetz [2], page 779: "With team production it is difficult, solely by observing total output, to either define or determine *each* individual's contribution to this output of the cooperating units." For a more elaborate discussion we also refer to Hart and Moore [18] and Ichiishi [21].

subordinate by firing him, i.e., the superior can deny that subordinate access to the firm's productive asset.

Throughout this paper we assume that the incurred costs of monitoring are uniform. Furthermore, we do not complicate the model by assuming that monitoring is imperfect, i.e., we assume that monitoring is perfect. This allows us to handle the enforcement of authority in a completely deterministic fashion and to analyze situations with an arbitrarily complex authority structure. Extension to imperfect monitoring are left for future research, which requires the application of game theoretic models of incomplete and imperfect information.

Third, we introduce a *utility structure* describing the motivations of the agents within the firm. Our main hypothesis is that the individual utilities are completely determined by the output values that are realized by the various teams of agents within the firm. Each individual agent is assumed to participate voluntarily in these value-generating teams and shares in these values. Now each individual agent assesses her position in such a situation only on the realized output values of the various teams. Hence, we assume that the authority structure itself has no direct externalities. It only has indirect consequences on the utility levels generated through the enforcement of authority and the denial of certain agents to access the firm's asset.

Above we introduced the notions of *formal* and *enforced* authority. At the heart of our study is the game theoretic analysis of the strategic decision making processes whether to enforce the assigned formal authority or not. We recall that the concept of formal authority is represented by the given structure of formal authority relations between the agents. For each formal authority relationship it can now be decided whether it should be enforced or not. In our framework the strategic enforcement of authority is developed into two fundamentally different fashions: the *explicit* and *latent* enforcement of authority.

Explicit enforcement of authority is the willful or strategic decision to enforce the formal authority to control the access of a subordinate to the firm's productive asset. As indicated, this is done by monitoring the subordinate, possibly incurring monitoring costs.

Our model of explicit authority is developed as a non-cooperative strategic or normal form authority game. Each individual agent selects which subordinates to monitor within the formal authority structure. This leads to a certain structure of enforced formal authority relations. Monitoring costs are taken into account and determine together with the properties of the utility function of the individual whether enforcing formal authority is profitable for an individual or not. The resulting Nash equilibria describe the resulting individually stable structures of explicitly enforced (formal) authority relationships. Under mild conditions we show that complete exercise of formal authority is warranted under low enough monitoring costs. This is as one would expect.

Latent authority is exercised if the (rational) subordinate voluntarily behaves as if his access to the firm's productive asset were monitored explicitly by his superior, even though there is no actual monitoring taking place, and, thus, formal authority is not explicitly enforced by his superior. Latent authority comes about in situations where rational subordinates take into account the abilities of a superior to exercise their formal authority explicitly by engaging in monitoring. Obviously, under latent authority, social gains are generated and, therefore, is socially preferable over explicit authority. The model of latent authority can be considered to be a formal construction to explain the elusive concept of "loyalty".

The setup of our analysis is as follows. Within the formal structure of an authority situation consisting of the three concepts described above, we develop two models of enforcing authority. The first model is founded on a very straightforward description of explicit monitoring and leads to an understanding when the explicit enforcement of formal authority takes place. The second model describes a more advanced standard of (boundedly) rational behavior that results into latent authority. Our main results are valid under rather mild conditions on the authority and utility structure.

The analysis of latent authority leads to some surprising insights. In case some formal authority is not enforced explicitly, subordinates may act as if such authority is enacted fully. This approach is based on the insight that superiors can induce states in which certain subordinates are monitored. Sufficiently rational subordinates now correctly anticipate under which conditions monitoring will be induced by their superiors. Given these correct beliefs, all subordinates may voluntarily act as if they are fully monitored even though that might not be the case. We show that if monitoring costs are sufficiently low, in the equilibrium state subordinates will voluntarily submit to full authority, i.e., a state of full latent authority emerges. Hence, this approach provides an alternative foundation for the phenomenon that formal authority need not be exercised explicitly in order to be effective, confirming the main insight from standard principal-agent theory which is based on the analysis of much simpler authority situations.

These main insights for these two fundamentally different models of "real" authority — in the sense of Aghion and Tirole [1] — are established under a single condition on the utility structure denoted as *dual monotonicity*. This is a relatively mild condition on the utility structure that is satisfied by most known solution concepts in cooperative game theory. We provide a comparison of this condition to the well known monotonicity requirements from the literature on cooperative games with transferable utility in Appendix B of this paper.

The paper is organized as follows. In Section 2 we develop the constituting elements of our theory. In the third section we introduce our analysis of the explicit exercise of authority through the concept of a normal form authority game. Section 4 is devoted to the analysis of the latent exercise of authority. In Section 5 we give a comparison of both models. Section 6 discusses some concluding remarks as well as the relationship of this approach to the existing literature on authority or power in hierarchical organizations. Throughout this paper the proofs of the main results are relegated to Appendix A.

2 Foundations of the theory

In this section we introduce the three primitives of our theory, discussed in the introduction. These three primitive elements are collected into a so-called authority situation, which gives a complete description of the agents' output values, the formal authority relations between the participating agents, as well as their preferences.

Our formal theory is founded on the theory of cooperative games with an authority structure developed in Gilles, Owen, and van den Brink [14], Derks and Gilles [11], Gilles and Owen [13], van den Brink [7], and van den Brink and Gilles [8]. In this theory a standard cooperative game with transferable utility is extended to incorporate hierarchical authority relationships between the agents. Here we limit ourselves to the formal theory of the so-called conjunctive approach introduced by Gilles, Owen and van den Brink [14].

Throughout the paper we let $N = \{1, \ldots, n\}$ be a given finite set of agents, who engage in productive activities and are collectively endowed with some given, formal hierarchical authority structure.

2.1 Team production

First we introduce a description of the productive capabilities of teams of agents in the given set N seeking access to the firm's asset. We base ourselves on the theory of Alchian and Demsetz [2] on team production. As usual, we use the concept of a cooperative game with transferable utility on N to describe the potential output values that the various teams can generate by accessing the firm's productive asset. The hypothesis that these potential output values can be represented through a cooperative game is also one of the principles underlying Hart and Moore [18] and Ichiishi [21].

Formally, a *cooperative game with transferable utility* — or simply a *game* — on N is a function $v\colon 2^N \to \mathbb{R}$ such that $v(\varnothing) = 0$. A game assigns to every team of agents $E \subset N$ some potential output value $v(E) \in \mathbb{R}$ that can be attained collectively by that team through accessing the firm's productive asset. The collection of all games on agent set N is denoted by $\mathscr{G}^N$.

A game $v \in \mathscr{G}^N$ is *monotone* if for all $E \subset F \subset N$ we have $v(E) \leqslant v(F)$. Note that this implies that $v(E) \geqslant 0$ for all $E \subset N$. A game $v \in \mathscr{G}^N$ is *strictly monotone* if v is monotone and for all $E \subset F \subset N$ with $E \neq F$ we have $v(E) < v(F)$.

Since we consider these games to be descriptions of potential output values rather than realized output values, it is natural to suppose that the agents have preferences over which production situation they participate in. We assume that these preferences are completely based on the (potential) output values that the various teams can attain, and do not depend on the authority relationships between the agents.

Formally, each agent $i \in N$ is assumed to be endowed with a von Neumann-Morgenstern utility function $u_i \colon \mathscr{G}^N \to \mathbb{R}$ over all possible games. Now the composite function $u \colon \mathscr{G}^N \to \mathbb{R}^N$ defines a *utility structure* over $\mathscr{G}^N$. A utility structure describes the desires of the agents in N. In the literature certain utility structures have a prominent place. (We refer to the seminal work of Herstein and Milnor [20].) Roth [34, 36] has shown that the adoption of certain risk-neutrality assumptions leads to the Shapley value (Shapley [37]) as the only feasible vNM utility structure[4]. Here the *Shapley value* $\varphi^S \colon \mathscr{G}^N \to \mathbb{R}^N$ is defined for every agent $i \in N$ and every game $v \in \mathscr{G}^N$ by

$$\varphi_i^S(v) \equiv \sum_{\{E \subset N \,|\, i \in E\}} \frac{(|E|-1)!\,(n-|E|)!}{n!}\,(v(E) - v(E \setminus \{i\})) \tag{1}$$

In case of simple games, Roth [35] shows that the utility structure defined by this type of conditions includes the Banzhaf value (Banzhaf [5]).[5] The following properties of utility structures are important in our analysis.

Definition 2.1 *The utility structure $u \colon \mathscr{G}^N \to \mathbb{R}^N$ on $\mathscr{G}^N$ satisfies*

(i) ***additivity*** *if for all $v, w \in \mathscr{G}^N$ it holds that $u(v+w) = u(v) + u(w)$, where $(v+w)(E) = v(E) + w(E)$ for every $E \subset N$.*

(ii) *the **null player property** if for every $v \in \mathscr{G}^N$ and $i \in N$ with $v(E \cup \{i\}) = v(E)$ for every $E \subset N$, it holds that $u_i(v) = 0$.*

(iii) ***dual monotonicity*** *if for every $v, w \in \mathscr{G}^N$ such that there is an $F \subset N$ for which $v(F) \leqslant w(F)$, and $v(E) = w(E)$ for all $E \in 2^N \setminus \{F\}$, it holds that $u_i(v) \geqslant u_i(w)$ for all $i \in N \setminus F$.*

[4] In the related literature on incomplete contracts the Shapley value has also been used in e.g., Hart and Moore [18]. An implementation of the Shapley value is given by, e.g., Pérez-Castillo and Wettstein [29].

[5] We refer to van den Brink and van der Laan [9] for a complete discussion of the properties of the normalized Banzhaf value.

(iv) ***strong dual monotonicity*** *if for every* $v,w \in \mathscr{G}^N$ *such that there is an* $F \subset N$ *for which* $v(F) < w(F)$*, and* $v(E) = w(E)$ *for all* $E \in 2^N \setminus \{F\}$*, it holds that* $u_i(v) > u_i(w)$ *for all* $i \in N \setminus F$*.*

We remark that both the Shapley value and the Banzhaf value satisfy all properties given in Definition 2.1. Additivity and the null player property are familiar concepts in cooperative game theory. Dual monotonicity and strong dual monotonicity impose that agents are envious of potential payoffs to teams of which they are *not* a member. We study these two properties more exhaustively in Appendix B. We restate strong dual monotonicity in a form that we use throughout this paper.

Lemma 2.2 *A utility structure* $u\colon \mathscr{G}^N \to \mathbb{R}^N$ *satisfies strong dual monotonicity if and only if for all* $v,w \in \mathscr{G}^N$ *and* $i \in N$ *such that*

(i) *there is a team* $F \subset N \setminus \{i\}$ *for which* $v(F) < w(F)$*,*

(ii) $v(E) \leqslant w(E)$ *for all* $E \subset N \setminus \{i\}$*, and*

(iii) $v(E) = w(E)$ *for all* $E \subset N$ *with* $i \in E$*,*

it holds that $u_i(v) > u_i(w)$*.*

Proof. The "only if" part is straightforward. The "if" part follows from repeated application of the definition of strong dual monotonicity given in Definition 2.1. □

2.2 Authority structures

Next we consider the description of formal authority relations between the participants in the production organization. An *authority structure* on N is a map $S\colon N \to 2^N$ that assigns to every agent $i \in N$ a set $S(i) \subset N$ of *direct subordinates*. The class of all authority structures on N is denoted by $\mathscr{S}^N$.

Here we interpret an authority structure $S \in \mathscr{S}^N$ as that an agent $j \in S(i)$ has to obtain "permission" from agent i for any productive activity that he intends to undertake by himself or with other agents in a team, through accessing the firm's productive asset. Therefore, the set $S^{-1}(i) = \{j \in N \mid i \in S(j)\}$ consists of all *direct superiors* of i.

There are several interpretations of what the concept of "permission" might entail. We limit ourselves to the *conjunctive approach*, developed in Gilles, Owen and van den Brink

[14], van den Brink and Gilles [8], and Derks and Gilles [11], in which the induced authority structure establishes complete control of the superior over her direct subordinates.[6] Below we develop a description of this interpretation.

First we introduce some auxiliary concepts.

Let $S \in \mathscr{S}^N$ and $E \subset N$. We define $S(E) = \cup_{i \in E} S(i)$ as the set of direct subordinates of the agents in the team E. Similarly, we define $S^{-1}(E) = \{i \in N \mid S(i) \cap E \neq \varnothing\}$ as the set of direct superiors of the agents in E.

The *transitive closure* of $S \in \mathscr{S}^N$ is the mapping $\widehat{S} \colon N \to 2^N$ which for every agent $i \in N$ is defined by $j \in \widehat{S}(i)$ if and only if there is a finite sequence $h_1, \ldots, h_k \in N$ with $h_1 = i$, $h_k = j$, and $h_{t+1} \in S(h_t)$ for all $1 \leqslant t \leqslant k-1$. The agents in $\widehat{S}(i)$ are called the (direct and indirect) *subordinates* of i in S. Similarly, the agents in $\widehat{S}^{-1}(i) := \{j \in N \mid i \in \widehat{S}(j)\}$ are called the (direct and indirect) *superiors* of i in authority structure S.

Finally, we define $B_S = \{i \in N \mid S^{-1}(i) = \varnothing\}$ and $W_S = \{i \in N \mid S(i) = \varnothing\}$. Here, B_S is the set of position in S that are undominated. They can be interpreted as the "executive officers" within the authority structure S. Similarly, the set W_S consists of all powerless positions in the authority structure S. These positions can be interpreted as "non-management positions", and the agents occupying these positions can simply be indicated as "workers".

Two basic properties of authority structures are used throughout this paper:

Definition 2.3 *An authority structure $S \in \mathscr{S}^N$ is called*

(i) ***acyclic*** *if $i \notin \widehat{S}(i)$ for every agent $i \in N$, and*

(ii) ***transparent*** *if for every $i \in N$ it holds that $S(i) \cap \widehat{S}(S(i)) = \varnothing$.*

Acyclicity requires that there are no formal authority cycles, which is a rather mild requirement. Essentially it implies that the organization structure is "top-down".

The transparency condition implies that within the authority structure an agent is never a direct superior of one of the subordinates of her subordinates, i.e., indirect authority relations never coincide with direct authority relations. This condition therefore imposes that the organization is "lean" and is not burdened with unnecessary authority relations.

We emphasize that neither acyclicity nor transparency imply that the authority structure is *hierarchical* in the sense that there is a unique position at the top of the structure, i.e., the property that $|B_S| = 1$. Hence, throughout this paper we work with very general authority structures, possibly with multiple "executive officers".[7]

[6] Alternatively, in the *disjunctive approach*, developed in Gilles and Owen [13] and van den Brink [7], the imposed authority structure consists of partial control in the sense that only the collective of *all* direct superiors can veto an action of a direct subordinate.

[7] Usually, one might have even in mind an authority structure that is *strictly hierarchical* in the sense that it is acyclic as well as hierarchical. We use such structures to illustrate properties in some of our examples.

2.3 Authority situations

Next we combine the three primitive elements introduced previously. These are the productive abilities — described by a game $v \in \mathscr{G}^N$ — with a formal authority structure — described by some $S \in \mathscr{S}^N$. Furthermore, all agents are endowed with an objective, described by the utility structure u on $\mathscr{G}^N$. The combination of these elements is denoted as an authority situation. Formally, a pair $(v,S) \in \mathscr{G}^N \times \mathscr{S}^N$ is called a *game with an authority structure* on N. A triple (u,v,S) with $u \colon \mathscr{G}^N \to \mathbb{R}^N$ a utility structure and (v,S) a game with an authority structure, is called an *authority situation* on N. We emphasize that the three primitive elements making up an authority structure are assumed to be independent from each other.

Definition 2.4 *Let (v,S) be a game with an authority structure. An agent $i \in N$ is* ***inessential*** *in (v,S) if $i \in W_S$ and $v(E \cup \{i\}) = v(E)$ for every $E \subset N$. Furthermore, an agent $i \in N$ is inessential in an authority situation (u,v,S) if i is inessential in (v,S).*

An inessential agent is a null player in the game v as well as an irrelevant member of the authority structure S in the sense that he has no authority over any other agents.

Next we address how an authority situation can be evaluated. As mentioned we assume throughout that each superior is in principle able to exercise full authority over her subordinates within (v,S). If such full authority is exercised, a team $E \subset N$ cannot form without the appropriate authority from all its superiors $\widehat{S}^{-1}(E)$. Formally, a team $E \subset N$ is *autonomous* in S if $\widehat{S}^{-1}(E) \subset E$. We denote by Φ_S the collection of all autonomous teams in the authority structure S.

If the team E is not autonomous, it cannot freely access the firm's productive asset and attain its potential productive output value. However, we can identify the largest sub-team that can freely access the firm's asset. Formally, E's *autonomous part* in S is given by $\sigma_S(E) = E \setminus \widehat{S}(N \setminus E)$. So, E is autonomous if and only if $\sigma_S(E) = E$.

Definition 2.5 *Let $(v,S) \in \mathscr{G}^N \times \mathscr{S}^N$ be a game with an authority structure on N. Its* ***restriction*** *$\mathscr{R}(v,S) \in \mathscr{G}^N$ is defined by $\mathscr{R}(v,S)(E) = v(\sigma_S(E))$ for every $E \subset N$.*

The induced mapping $\mathscr{R}(\cdot,S) \colon \mathscr{G}^N \to \mathscr{G}^N$ is linear and incorporates the effects of exercising authority over the positions of the agents in the authority relationships S.[8] We illustrate the introduced concepts with an example.

[8] Some of the properties of this mapping are investigated in Gilles, Owen and van den Brink [14]. We remark that similar approaches have been developed to analyze other restrictions on team formation. In particular we refer to the seminal contribution by Aumann and Drèze [3] for situations with coalitional partitions and to the seminal work of Myerson [28] for restrictions induced by communication networks.

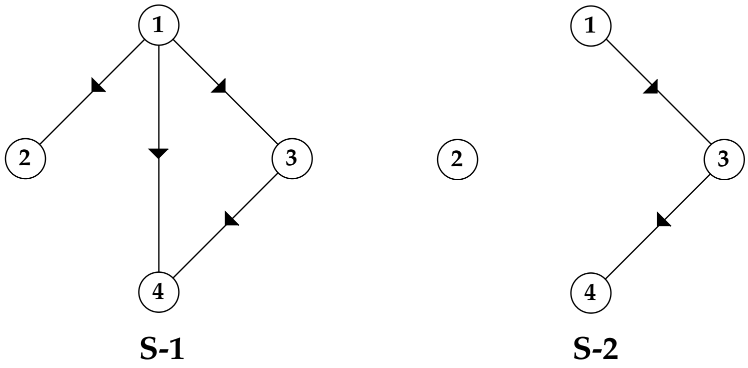

Figure 1: The permission structures with Example 2.6.

Example 2.6 We discuss a situation with four agents, $N=\{1,2,3,4\}$, and consider two games with an authority structure (v,S_1) and (v,S_2). The authority structures S_1 and S_2 are given by $S_1(1)=\{2,3,4\},\ S_1(2)=S_1(4)=\varnothing,\ S_1(3)=\{4\}$, and $S_2(1)=\{3\},\ S_2(2)=S_2(4)=\varnothing,\ S_2(3)=\{4\}$. These authority structures are depicted in Figure 1. We let the cooperative game v be given by $v(E)=3$ if $4\in T$ and $v(E)=0$ otherwise.
We remark that authority structure S_1 is not transparent since $S_1(1)\cap\widehat{S}_1(S_1(1))=\{4\}\neq\varnothing$. Hence, agent 1 dominates agent 4 directly, although 1 also dominates 4 indirectly through 3. On the other hand, authority structure S_2 is transparent. Furthermore,

$$\mathscr{R}(v,S_1)(E)=\mathscr{R}(v,S_2)(E)=\begin{cases}3 & \text{if } \{1,3,4\}\subset E\\ 0 & \text{otherwise.}\end{cases}$$

Hence, the restriction of v on both authority structures is the same. This is due to the fact that there are superfluous relationships in non-transparent hierarchies. Deleting these relationships does not affect the restriction of a game. This is the case for the relationship between agents 1 and 4 in S_1.
Furthermore, Agent 2 is an inessential agent in (v,S_1). Removing relationships with inessential agents does not affect the restriction either. (We also refer to van den Brink and Gilles [8] for more elaborate discussions.) □

Next we address the question whether the restriction $\mathscr{R}$ is an appropriate description of the explicit enforcement of authority. The next theorem gives a normative justification for its use. We identify the restriction $\mathscr{R}$ as the unique mapping $\mathscr{F}:\mathscr{G}^N\times\mathscr{S}^N\to\mathscr{G}^N$ satisfying four regularity assumptions and one descriptive hypothesis. This descriptive property, stated as 2.7(v), states that an agent $i\in N\setminus W_S$ "vetoes" her direct subordinates $j\in S(i)$ in the sense that the contribution of agent j to a team is non-trivial only if agent i herself is a member of that team. This exactly describes that a superior can deny a subordinate access to the firm's productive asset.

Theorem 2.7 *A mapping $\mathscr{F} : \mathscr{G}^N \times \mathscr{S}^N \to \mathscr{G}^N$ is equal to the restriction $\mathscr{R}$ if and only if the mapping $\mathscr{F}$ satisfies the following properties:*

(i) *For every $(v,S) \in \mathscr{G}^N \times \mathscr{S}^N$ it holds that $\mathscr{F}(v,S)(N) = v(N)$;*

(ii) *For every $(v,S),(w,S) \in \mathscr{G}^N \times \mathscr{S}^N$ it holds that $\mathscr{F}(v+w,S) = \mathscr{F}(v,S) + \mathscr{F}(w,S)$;*

(iii) *For every $(v,S) \in \mathscr{G}^N \times \mathscr{S}^N$ and $i \in N$ such that all $j \in \widehat{S}(i) \cup \{i\}$ are null players in v it holds that i is a null player in $\mathscr{F}(v,S)$;*

(iv) *For every $(v,S) \in \mathscr{G}^N \times \mathscr{S}^N$ and $i \in N$ such that $v(E) = 0$ for all $E \subset N \setminus \{i\}$ it holds that $\mathscr{F}(v,S)(E) = 0$ for all $E \subset N \setminus \{i\}$;*

(v) *For every $(v,S) \in \mathscr{G}^N \times \mathscr{S}^N$, $i \in N$, $j \in S(i)$, and $E \subset N \setminus \{i\}$ it holds that $\mathscr{F}(v,S)(E) = \mathscr{F}(v,S)(E \setminus \{j\})$.*

The proof of this theorem can be found with all other proofs in Appendix A. Without proof we mention that the five properties in Theorem 2.7 are independent, and thus this axiomatization is proper.

The remainder of this paper discusses two game theoretic approaches to the exercise of authority in hierarchical organizations using the restriction $\mathscr{R}$. In these approaches the individual agents decide whether to exercise authority over their subordinates based on the preferences of these agents as represented by the utility structure u.

Throughout this paper we consider a given authority situation (u,v,S) in which there are no inessential agents. We make this assumption solely for notational convenience. Without exception, our results can be restated to include inessential agents. We consider which of the formal authority relations in S agent $i \in N \setminus W_S$ chooses to enforce. Thus, each agent $i \in N \setminus W_S$ selects a subset $T(i) \subset S(i)$ of formal authority relations that she decides to enforce. If each potential superior has selected such a set of explicitly enforced authority relations we arrive at an authority structure consisting of exactly *all* explicitly enforced authority relations. The resulting authority structure is an element in the following collection of authority structures:

$$\mathbb{H}(S) := \left\{ T \in \mathscr{S}^N \mid T(i) \subset S(i) \text{ for every agent } i \in N \right\}. \tag{2}$$

An authority structure $T \in \mathbb{H}(S)$ thus describes those authority relations that are *enforced*. In comparison, the relations described by $S - T \in \mathbb{H}(S)$, where for every $i \in N$ we define $(S-T)(i) = S(i) \setminus T(i)$, only have a *latent* or non-enforced quality.

Our next result states that under certain regularity conditions, agents indeed prefer to exercise as much authority as possible if it is costless to do so.

Theorem 2.8 *Assume that v is a monotone game. Let $h \in N \setminus W_S$ and $T \in \mathbb{H}(S)$ be such that $T(h) \neq S(h)$. Finally, let $Z \in \mathbb{H}(S)$ be given by*

$$Z(i) = \begin{cases} S(h) & \text{if } i = h \\ T(i) & \text{otherwise.} \end{cases}$$

Then:

(a) *If utility structure u satisfies dual monotonicity, then $u_h(\mathscr{R}(v,Z)) \geqslant u_h(\mathscr{R}(v,T))$.*

(b) *If utility structure u satisfies strong dual monotonicity and $\mathscr{R}(v,Z) \neq \mathscr{R}(v,T)$, then $u_h(\mathscr{R}(v,Z)) > u_h(\mathscr{R}(v,T))$.*

(c) *Suppose that v is strictly monotone, S is acyclic, and u satisfies strong dual monotonicity. If $T(h) = \varnothing$ or S is transparent, then $u_h(\mathscr{R}(v,Z)) > u_h(\mathscr{R}(v,T))$.*

The proof of Theorem 2.8 is relegated to Appendix A. Theorem 2.8 forms the foundation for further analysis of the enforcement of authority within a hierarchical organization.

3 Exercising explicit authority

In this section we analyze the decision-making processes of myopically rational agents who decide which of their formal authority relations to enforce explicitly. We model this by means of a non-cooperative normal form game.

Every agent $i \in N \setminus W_S$ has strategy set given by $\Gamma_i = \{E \subset N \mid E \subset S(i)\}$. (Clearly, for every worker $j \in W_S$ it holds that $\Gamma_j := \{\varnothing\}$.) A strategy $E_i \in \Gamma_i$ describes those subordinates over which agent i explicitly enforces her authority. Let $\mathscr{E} = (E_1, \ldots, E_n) \in \Gamma := \prod_{i \in N} \Gamma_i$ be a strategy tuple. Then the resulting authority structure is the one given by $T_{\mathscr{E}} \in \mathbb{H}(S)$ with $T_{\mathscr{E}}(i) := E_i$ for all agents $i \in N$.

Since the explicit exercise of authority usually induces a cost to monitor these subordinates, we introduce a fixed monitoring cost parameter $c \geqslant 0$. We impose that monitoring any subordinate $j \in S(i)$ by an agent $i \in N \setminus W_S$ will cost $c \geqslant 0$. This leads to the following formalization:

Definition 3.1 *The **authority game** induced by (u,v,S) and monitoring cost parameter $c \geqslant 0$ is defined by the tuple $\Theta^c = \left(N, \{\Gamma_i, u_i^c\}_{i \in N}\right)$ with for every strategy tuple $\mathscr{E} = (E_1, \ldots, E_n) \in \Gamma$*

$$u_i^c(\mathscr{E}) = u_i(\mathscr{R}(v, T_{\mathscr{E}})) - c\,|E_i|\,. \tag{3}$$

For the authority game Θ^c with monitoring cost $c \geqslant 0$ we consider the standard equilibrium concepts. A strategy tuple $\widehat{\mathscr{E}} = \left(\widehat{E}_1, \ldots, \widehat{E}_n\right) \in \Gamma$ is a *Nash equilibrium* in Θ^c if for every $i \in N$ and every $E_i \in \Gamma_i$ we have that $u_i^c\left(\widehat{\mathscr{E}}\right) \geqslant u_i^c\left(\widehat{\mathscr{E}}_{-i}, E_i\right)$, where $\left(\widehat{\mathscr{E}}_{-i}, E_i\right) \in \Gamma$ is a modification of the strategy tuple $\widehat{\mathscr{E}}$ in which agent i selects E_i and each agent $j \neq i$ selects $\widehat{E}_j$. We denote by $\mathscr{N}(\Theta^c) \subset \Gamma$ the set of all Nash equilibria of the authority game Θ^c.

A Nash equilibrium $\widehat{\mathscr{E}} \in \mathscr{N}(\Theta^c)$ is called *strict* if for every $i \in N$ and every $E_i \in \Gamma_i$ with $E_i \neq \widehat{E}_i$ it holds that $u_i^c\left(\widehat{\mathscr{E}}\right) > u_i^c\left(\widehat{\mathscr{E}}_{-i}, E_i\right)$. The set of strict Nash equilibria of Θ^c is denoted by $\mathscr{N}_s(\Theta^c) \subset \mathscr{N}(\Theta^c)$.

For ease of notation we denote for every authority structure $T \in \mathbb{H}(S)$ the corresponding strategy tuple by $\mathscr{E}^T = \left(E_1^T, \ldots, E_n^T\right)$, where $E_i^T := T(i)$ for every $i \in N$. Now the strategy tuple $\mathscr{E}^S$ refers to the complete exercise of authority within the given structure S.

Definition 3.2 *An authority structure $T \in \mathbb{H}(S)$ is (v,S)-**equivalent** if $\mathscr{R}(v,T) = \mathscr{R}(v,S)$. We denote by $\mathbb{M}(v,S)$ the collection of (v,S)-equivalent authority structures.*
*An authority structure $T \in \mathbb{H}(S)$ is (v,S)-**minimal** if T is (v,S)-equivalent and*

$$|T| = \min\left\{ \left|T'\right| \;\middle|\; T' \in \mathbb{M}(v,S) \right\} \tag{4}$$

where $|T'| = \sum_{i \in N} |T'(i)|$ is the total number of authority relationships in the authority structure $T' \in \mathbb{H}(S)$. We denote by $\widehat{\mathbb{M}}(v,S)$ the set of (v,S)-minimal authority structures.

We remark that $S \in \mathbb{M}(v,S)$ and therefore $\widehat{\mathbb{M}}(v,S) \neq \varnothing$ for any game with an authority structure. Using these auxiliary concepts we are able to give a partial characterization of the (strict) Nash equilibria of the authority game with costless monitoring and dual monotone utility structures. Nash equilibria under costless monitoring exist for dual monotone utility structures, while for *strong* dual monotonicity even complete characterizations can be given. The proofs of the following two theorems are again relegated to Appendix A.

Theorem 3.3 *Let (u,v,S) be an authority situation such that u is a dual monotone utility structure and v is a monotone game. Then:*

(a) $\left\{\mathscr{E}^T \mid T \in \mathbb{M}(v,S)\right\} \subset \mathscr{N}(\Theta^0)$, *and*

(b) $\mathscr{N}_s\left(\Theta^0\right) \subset \left\{\mathscr{E}^S\right\}$.

Next we address the (strict) Nash equilibria under costless monitoring and strong dual monotonicity.

Theorem 3.4 *Let (u,v,S) be such that u is a strongly dual monotone utility structure and v is a monotone game.*

(a) *If* $S \in \widehat{\mathbb{M}}(v,S)$*, then* $\mathscr{N}_s\left(\Theta^0\right) = \mathscr{N}\left(\Theta^0\right) = \left\{\mathscr{E}^S\right\}$.

(b) *If* $S \notin \widehat{\mathbb{M}}(v,S)$*, then* $\mathscr{N}_s\left(\Theta^0\right) = \varnothing$ *and* $\mathscr{N}\left(\Theta^0\right) = \left\{\mathscr{E}^T \mid T \in \mathbb{M}(v,S)\right\}$.

We remark that the assertions of Theorem 3.4 are no longer valid if the utility structure is merely dual monotone instead of strongly dual monotone. This can be illustrated by the egalitarian utility structure $\bar{u}$ given by $\bar{u}_i(v) = \frac{v(N)}{|N|}, i \in N$, based on the equal division of the total output value of the grand coalition N. The egalitarian utility structure $\bar{u}$ is dual monotone, but not strongly dual monotone. For any (v,T), $T \in \mathbb{H}(S)$, $\bar{u}(\mathscr{R}(v,T)) = \bar{u}(v)$, i.e., the utilities received are equal regardless the authority structure implemented. This implies that $\mathscr{N}\left(\Theta^0\right) = \left\{\mathscr{E}^T \mid T \in \mathbb{H}(S)\right\}$.

For sufficiently low monitoring costs we derive the following insight. A proof of Theorem 3.5 is given in Appendix A.

Theorem 3.5 *Let* (u,v,S) *be such that* u *is a strongly dual monotone utility structure and* v *is a monotone game. Then there exists a cost level* $c^* > 0$ *such that for every* $0 < c < c^*$ *it holds that*

$$\mathscr{N}\left(\Theta^c\right) = \left\{\mathscr{E}^T \,\middle|\, T \in \widehat{\mathbb{M}}(v,S)\right\}.$$

It is evident that every minimal authority structure is transparent, i.e., there are no superfluous authority relationships in such structures. This immediately leads to the following corollary of Theorem 3.5.

Corollary 3.6 *If the utility structure is strongly dual monotone, the potential productive output values are monotone, and the monitoring costs are sufficiently low, then the resulting Nash equilibrium authority structures are transparent.*

We illustrate this analysis by referring to Example 2.6. Let the game v and the authority structures S_1 and S_2 be as given. Then for any authority situation (u,v,S_1) with the utility structure u strongly dual monotone, the unique resulting Nash equilibrium authority structure for sufficiently low monitoring costs is S_2. (In fact, S_2 is the unique (v,S_1)-minimal authority structure.) Clearly in S_2 neither the redundant authority relationship 14 nor the ineffective authority relationship 12 are enforced.

4 Exercising latent authority

In the previous section we discussed the explicit exercise of authority. Next we consider a more advanced form of reasoning on part of the agents in the authority situation. Under this type of advanced rationality there might result situations in which superiors abstain from the explicit exercise of authority, but in which their authority remains effective. Here, even though authority is not exercised explicitly, subordinates might nevertheless perceive a potential, or latent, threat that a superior is willing to exercise that authority explicitly and incur monitoring costs if they do not voluntarily restrict their productive activities. Thus, these subordinates might act *as if* authority was exercised explicitly. If such behavior results, we talk about *latent authority* to distinguish it from explicit authority.

It is clear that such latent authority cannot be described properly by the game theoretic structure introduced in the previous section. In those authority games the only way for an agent to profit from her formal authority is to explicitly enforce it. In this section we present an approach in which agents can choose to enforce authority explicitly as well as not to enforce any authority at all. This allows us to define an equilibrium concept that incorporates that the subordinates perceive threats that their superiors will enforce authority relationships with them. Thus, the resulting equilibria describe outcomes that are based on implicit considerations rather than explicit considerations. This approach is based on the theory of social situations developed in Greenberg [15].

For every authority structure $T \in \mathbb{H}(S)$ we define the set of *potential authorizers* in T by

$$\psi(T) = \{i \in N \mid T(i) = \varnothing \text{ and } S(i) \neq \varnothing\}$$

Here, the agents in $\psi(T) \subset N \setminus W_S$ are the ones who are undecided regarding the explicit enforce of their authority. From this it might be clear that the set of *explicit authorizers* in T can be introduced as $\psi'(T) = N \setminus (\psi(T) \cup W_S)$. Note that for $S_0 \in \mathbb{H}(S)$ given by $S_0(i) = \varnothing$ for every $i \in N$, it holds that $\psi(S_0) = N \setminus W_S$ and $\psi'(S_0) = \varnothing$.

To describe the ability of a superior $i \in N \setminus W_S$ to enforce authority, we introduce an auxiliary tool. Namely, as long as agent i does not enforce any authority, she still has the ability to execute her authority over any subset of her direct subordinates. Hence, agent i can induce from any authority structure in which she does not enforce any authority, another authority structure in which she (partially) enforces the formal authority that is assigned to her within S.

The point-to-set mapping $\gamma_i : \mathbb{H}(S) \to 2^{\mathbb{H}(S)}$ is the *veto correspondence* for agent $i \in N$ on $S \in \mathscr{S}^N$ if

$$\gamma_i(T) = \begin{cases} \left\{T_i^F \mid \varnothing \neq F \subset S(i)\right\} & \text{if } i \in \psi(T) \\ \varnothing & \text{if } i \notin \psi(T) \end{cases}$$

where for every $F \subset S(i)$ we define $T_i^F \in \mathbb{H}(S)$ by

$$T_i^F(j) = \begin{cases} T(j) & \text{if } j \neq i \\ F & \text{if } j = i. \end{cases}$$

The multidimensional mapping $\gamma := (\gamma_1, \ldots, \gamma_n) \colon \mathbb{H}(S) \to 2^{\mathbb{H}(S) \times N}$ is called the *veto structure* on S.

Let $\gamma = (\gamma_1, \ldots, \gamma_n)$ be a veto structure on S. It is obvious that γ defines a configuration that describes the exact enforce of authority within the boundaries of a given authority structure. Remark that each agent in $N \setminus W_S$ can announce only once over which subordinates she is exercising explicit authority. Within the veto structure γ we are now able to construct equilibria that describe the stable states of the latent exercise of authority. We define for every $T \in \mathbb{H}(S)$

$$\Lambda(v, T) = \{\mathscr{R}(v, Z) \mid T(i) \subset Z(i) \subset S(i) \text{ for all } i \in N\}$$

as the set of all games that can potentially result from T within (v, S).

Definition 4.1 *Let (u, v, S) be some authority situation on N.*

(i) *A point-to-set mapping $\Sigma \colon \mathbb{H}(S) \to 2^{\mathscr{G}^N}$ is an **authority protocol** for (u, v, S) if for every $T \in \mathbb{H}(S)$ it holds that $\Sigma(T) \subset \Lambda(v, T)$.*

(ii) *Let a monitoring cost $c \geqslant 0$ be given. An authority protocol $\Sigma^c \colon \mathbb{H}(S) \to 2^{\mathscr{G}^N}$ is **stable** for (u, v, S) if for every $T \in \mathbb{H}(S)$ it holds that $w \in \Sigma^c(T)$ if and only if $w \in \Lambda(v, T)$ and there is no agent $i \in \psi(T)$, authority structure $T' \in \gamma_i(T)$ and $w' \in \Sigma^c(T')$ with*

$$u_i(w') - c|T'(i)| > u_i(w) - c|T(i)|. \tag{5}$$

An authority protocol assigns to every authority structure T within S a set of games that can emerge within (u, v, T) given the formal authority structure S. In this respect an authority protocol is a potential solution for the latent exercise of authority within (u, v, S).

From the definition, a *stable authority protocol* is an equilibrium concept that describes the latent exercise of authority within (u, v, S). Namely, it incorporates the individual incentives to explicitly veto subordinates. However, it formalizes the potential, or latent, development of the exercise of authority, not *how* it is actualized. Hence, it exactly formalizes the notion of a *perceived* exercise of authority within (u, v, S). We remark that a stable authority protocol satisfies the von Neumann–Morgenstern notions of internal and external stability. For convenience we indicate a stable authority protocol by SAP.

The next theorem addresses the existence of a stable authority protocol. It is shown that there is a unique SAP for authority situations with a monotone game and an acyclic authority structure. Moreover, if the utility structure u is strongly dual monotone, all direct subordinates of agents that have not yet explicitly exercised their authority act *as if* they were monitored by their superiors. In particular, if no agent has explicitly enforced her authority to monitor and veto, every subordinate acts *as if* all agents fully enforce their authority. Hence, in equilibrium full latent authority is enforced. For a proof we again refer to Appendix A.

Theorem 4.2 *Let (u,v,S) be an authority situation such that $v \in \mathscr{G}^N$ is monotone and $S \in \mathscr{S}^N$ is acyclic. Then:*

(a) *For every monitoring cost level $c \geqslant 0$ there exists a unique stable authority protocol Σ^c_* for (u,v,S).*

(b) *If the utility structure u is strongly dual monotone, then there exists a cost level $c^* > 0$ such that for every $0 \leqslant c < c^*$ and every $T \in \mathbb{H}(S)$ it holds that $\Sigma^c_*(T) = \{\mathscr{R}(v,Z)\}$ where $Z \in \mathscr{S}^N$ is given by*

$$Z(i) = \begin{cases} T(i) & \text{if } i \notin \psi(T) \\ S(i) & \text{if } i \in \psi(T). \end{cases}$$

In particular, $\Sigma^c_*(S_0) = \{\mathscr{R}(v,S)\}$ for $0 \leqslant c < c^*$, where $S_0(i) = \varnothing$ for every $i \in N$.

5 The case of high monitoring costs

In this section we consider the consequences of higher monitoring costs for the explicit and latent exercise of authority. We use a simple example to clarify some of these consequences. A general analytical study is rather involved and therefore subject of future research.

Throughout this section we consider a three agent situation with $N = \{1,2,3\}$. Furthermore, we impose the authority situation (φ^S, v, S), where (1) the utility structure $\varphi^S \colon \mathscr{G}^N \to \mathscr{R}^N$ is equal to the Shapley value, (2) the formal authority structure S is given by $S(1) = S(2) = \{3\}$ and $S(3) = \varnothing$, and (3) the output values v is given by $v(E) = 1$ if $3 \in E$ and $v(E) = 0$ otherwise. The authority structure S is depicted in Figure 2.

We develop the analysis of this authority situation in three steps: explicit exercise of authority, latent exercise of authority, and a comparison between these two models of behavior.

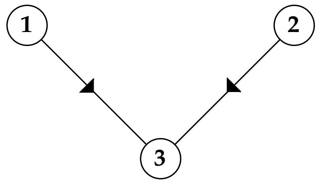

Figure 2: The authority structure S.

5.1 The explicit exercise of authority

The strategies of the two agents 1 and 2 in the authority game Θ^c are given by $\Gamma_1 = \Gamma_2 = \{\{3\}, \varnothing\}$. Since agent 3 has no subordinates we treat the authority game as a two-person game. For convenience we denote these two basic strategies as $V = \{3\}$ (veto) and $N = \varnothing$ (no veto).

Given positive monitoring cost $c > 0$ the payoffs for the four possible strategy profiles are $u(V,V) = \left(\frac{1}{3} - c, \frac{1}{3} - c, \frac{1}{3}\right)$, $u(V,N) = \left(\frac{1}{2} - c, 0, \frac{1}{2}\right)$, $u(N,V) = \left(0, \frac{1}{2} - c, \frac{1}{2}\right)$, and $u(N,N) = (0,0,1)$. The Nash equilibria for different values of c are now represented in the following table:

Cost level	Equilibria	Utilities
$c < \frac{1}{3}$	$\mathcal{N}(\Theta^c) = \{(V,V)\}$	$u = \left(\frac{1}{3} - c, \frac{1}{3} - c, \frac{1}{3}\right)$
$c = \frac{1}{3}$	$\mathcal{N}(\Theta^c) = \{(V,V), (V,N), (N,V)\}$	$u \in \left\{\left(0,0,\frac{1}{3}\right), \left(\frac{1}{6},0,\frac{1}{2}\right), \left(0,\frac{1}{6},\frac{1}{2}\right)\right\}$
$\frac{1}{3} < c < \frac{1}{2}$	$\mathcal{N}(\Theta^c) = \{(V,N), (N,V)\}$	$u \in \left\{\left(\frac{1}{2} - c, 0, \frac{1}{2}\right), \left(0, \frac{1}{2} - c, \frac{1}{2}\right)\right\}$
$c = \frac{1}{2}$	$\mathcal{N}(\Theta^c) = \{(V,N), (N,V), (N,N)\}$	$u \in \left\{\left(0,0,\frac{1}{2}\right), (0,0,1)\right\}$
$c > \frac{1}{2}$	$\mathcal{N}(\Theta^c) = \{(N,N)\}$	$u = (0,0,1)$

So, if $c < \frac{1}{3}$ or $c > \frac{1}{2}$ there is a unique Nash equilibrium (both veto, respectively, not veto), and for intermediate values there are multiple Nash equilibria.

5.2 The latent exercise of authority

Next we consider the latent exercise of authority and the corresponding notion of a stable authority protocol. For convenience we denote by T_1, T_2, and S_0 the authority structures given by $T_1(1) = \{3\}$, $T_1(2) = T_1(3) = \varnothing$ (only agent 1 enforces explicit authority over agent 3), $T_2(1) = T_2(3) = \varnothing$, $T_2(2) = \{3\}$ (only agent 2 enforces explicit authority over agent 3), and $S_0(1) = S_0(2) = S_0(3) = \varnothing$ (neither 1 nor 2 enforce explicit authority over agent 3). For S, T_1, T_2, and S_0 we have

$$\Lambda(v,S) = \{\mathscr{R}(v,S)\}$$
$$\Lambda(v,T_1) = \{\mathscr{R}(v,T_1),\mathscr{R}(v,S)\}$$
$$\Lambda(v,T_2) = \{\mathscr{R}(v,T_2),\mathscr{R}(v,S)\}$$
$$\Lambda(v,S_0) = \{\mathscr{R}(v,S_0),\mathscr{R}(v,T_1),\mathscr{R}(v,T_2),\mathscr{R}(v,S)\}$$

For any cost $c \geqslant 0$ the unique SAP assigns to the full authority structure S its restriction $\mathscr{R}(v,S)$ because nothing else can be induced from that situation. For the other situations we distinguish three possibilities:

- $c < \frac{1}{3}$: Suppose that in situation T_1 the game $\mathscr{R}(v,T_1)$ with payoffs $(\frac{1}{2}-c,0,\frac{1}{2})$ is played. Since agent 2 can induce situation S with payoffs $(\frac{1}{3}-c,\frac{1}{3}-c,\frac{1}{3})$, the SAP Σ_*^c cannot assign $\mathscr{R}(v,T_1)$ to this situation (agent 2's payoff if he induces S is $\frac{1}{3}-c$ which exceeds it payoff 0 in situation T_1). So, $\Sigma_*^c(T_1) = \{\mathscr{R}(v,S)\}$ with payoffs given by $(\frac{1}{3}-c,\frac{1}{3},\frac{1}{3})$ (Note that agent 2 does not actually has to pay its monitoring cost if $\mathscr{R}(v,S)$ is played in situation T_1). By a similar argument $\Sigma_*^c(T_2) = \{\mathscr{R}(v,S)\}$ with payoffs given by $(\frac{1}{3},\frac{1}{3}-c,\frac{1}{3})$.

  Now, suppose that in situation S_0 the game $\mathscr{R}(v,S_0)$ with payoffs $(0,0,1)$ is played. Since agent 1 can induce T_1 and the SAP assigns $\mathscr{R}(v,S)$ to situation T_1 (with payoffs $(\frac{1}{3}-c,\frac{1}{3},\frac{1}{3})$), the SAP cannot assign the game $\mathscr{R}(v,S_0)$ to situation S_0 (agent 1's payoff if he induces T_1 is $\frac{1}{3}-c$ which exceeds it payoff 0 in situation S_0). Suppose that in situation S_0 the game $\mathscr{R}(v,T_1)$ with payoffs $(\frac{1}{2},0,\frac{1}{2})$ is played. Since agent 2 can induce T_2 and the SAP assigns $\mathscr{R}(v,S)$ to T_2 (with payoffs $(\frac{1}{3},\frac{1}{3}-c,\frac{1}{3})$), the SAP cannot assign the game $\mathscr{R}(v,T_1)$ to situation S_0. Similarly, $\mathscr{R}(v,T_2) \notin \Sigma_*^c(S_0)$. So, also in this situation $\Sigma_*^c(S_0) = \{\mathscr{R}(v,S)\}$. Thus according to the SAP, in situation S_0 agents act *as if* both agents 1 and 2 enforce full authority over agent 3 with corresponding payoff vector given by $(\frac{1}{3},\frac{1}{3},\frac{1}{3})$.

- $\frac{1}{3} < c < \frac{1}{2}$: Suppose that in situation T_1 the game $\mathscr{R}(v,T_1)$ with payoffs $(\frac{1}{2}-c,0,\frac{1}{2})$ is played. Since agent 1 cannot induce any other situation and agent 2 can only induce situation S (with payoffs $(\frac{1}{3}-c,\frac{1}{3}-c,\frac{1}{3})$), the SAP assigns $\mathscr{R}(v,T_1)$ to this situation. Also, if in situation T_1 the game $\mathscr{R}(v,S)$ with payoffs $(\frac{1}{3}-c,\frac{1}{3},\frac{1}{3})$ is played, agent 2 cannot induce a situation in which it can do better. So, $\Sigma_*^c(T_1) = \{\mathscr{R}(v,T_1),\mathscr{R}(v,S)\}$. (Note the difference with $c < \frac{1}{3}$ considered above in which only $\mathscr{R}(v,S)$ was stable in this situation.) By a similar argument $\Sigma_*^c(T_2) = \{\mathscr{R}(v,T_2),\mathscr{R}(v,S)\}$.
  Now, suppose that in situation S_0 the game $\mathscr{R}(v,S_0)$ with payoffs $(0,0,1)$ is played.

Since agent 1 can induce T_1 to which the SAP assigns $\mathscr{R}(v,T_1)$ (with payoffs $(\frac{1}{2}-c,0,\frac{1}{2})$), the SAP cannot assign the game $\mathscr{R}(v,S_0)$ to situation S_0. Suppose that in situation S_0 the game $\mathscr{R}(v,T_1)$ with payoffs $(\frac{1}{2}-c,0,\frac{1}{2})$ is played. Since agent 2 can induce T_2 to which the SAP assigns $\mathscr{R}(v,T_2)$ (with payoffs $(0,\frac{1}{2}-c,\frac{1}{2})$), the SAP cannot assign the game $\mathscr{R}(v,T_1)$ to situation S_0. Similarly, $\mathscr{R}(v,T_2) \notin \Sigma^c_*(S_0)$. No agent can induce an advantageous situation if $\mathscr{R}(v,S)$ with payoffs $(\frac{1}{3},\frac{1}{3},\frac{1}{3})$ is played. So, in this situation $\Sigma^c_*(S_0) = \{\mathscr{R}(v,S)\}$. Note that, although in the intermediate situations T_1 and T_2 the latent exercise of authority is different for the cases $c > \frac{1}{3}$ and $\frac{1}{3} < c < \frac{1}{2}$, for both cases in situation S_0 agents act *as if* both agents 1 and 2 enforce full authority over agent 3 with corresponding payoff vector given by $(\frac{1}{3},\frac{1}{3},\frac{1}{3})$.

- $c > \frac{1}{2}$: In a similar way as above, it can be shown that $\Sigma^c_*(S) = \{\mathscr{R}(v,S)\}$, $\Sigma^c_*(T_1) = \{\mathscr{R}(v,T_1),\mathscr{R}(v,S)\}$ and $\Sigma^c_*(T_2) = \{\mathscr{R}(v,T_2),\mathscr{R}(v,S)\}$. Finally, it can be determined that everything is stable in the situation in which all authority is latent, $\Sigma^c_*(S_0) = \{\mathscr{R}(v,S_0),\mathscr{R}(v,T_1),\mathscr{R}(v,T_2),\mathscr{R}(v,S)\}$.
- For $c = \frac{1}{3}$ and $c = \frac{1}{2}$ intermediate cases apply.

5.3 A comparison

Comparing the Nash equilibria of the authority game and the stable authority protocol describing the latent exercise of authority allow us to conclude that there is a difference of the equilibrium utility levels when $c < \frac{1}{2}$. Namely, under explicit exercise of authority the monitoring cost is actually realized, while this is not the case under the latent exercise of authority. For $\frac{1}{3} < c < \frac{1}{2}$ even the attitude towards exercising authority is different, as described by these equilibrium concepts.

Namely, in the Nash equilibrium of the authority game authority is not enforced fully, while under the SAP the agents act as if this authority is enforced fully if no agent has announced whether it is going to enforce its authority. This significant difference indicates that if agents are myopic — as modelled in the authority game —, there would be no full exercise of authority in equilibrium. However, a more advanced form of rationality on part of the subordinates — as modelled by the concept of a stable authority protocol —, would induce them to accept full (latent) authority.

6 Concluding remarks

In this paper we have developed a theory of the nature of authority within a given firm, described as a hierarchical authority structure with team production. We introduced two models of the exercise of authority in a framework including a description of team production, an arbitrarily complex authority structure of decision makers who are principal to certain decision makers and agents to other decision makers, and a utility structure. The first model addresses the explicit enforcement of authority through costly monitoring. The second model describes a latent form of the exercise of authority, namely the rational acceptance of authority even though this authority is not enforced explicitly.

We emphasize that at the foundation of our theory, we consider the question of ownership of the firm's asset to have no bearing on the study of the nature of authority. Indeed, we base our modelling on the hypothesis that ownership and control are fundamentally separated and that "control" is represented by the authority structure. Here decision makers in the authority structure have delegated control over the firm's asset in the sense that a decision maker can deny the access of her subordinates to the asset. This modelling principle corresponds to observed practices; firms are either publicly traded or the owner exercises his or her control through managers with delegated powers. In either case the question who exactly owns the firm's asset is of no consequence for the practices that result with regard to the control of the firm's asset. In our analysis there emerged two practices: directly or explicitly exercised control and latently exercised control.

Finally, we emphasize that our model of the latent exercise of authority represents the elusive concept of *loyalty* of subordinates to the firm and its objectives. Indeed, as modelled, at a higher level of rationality, intelligent subordinates voluntarily submit themselves to the objectives of their superiors to avoid being subjected to enforced monitoring. This standard of behavior can in this respect be interpreted as a game theoretic formulation of "loyalty".

Relation to the literature

Our approach to the notion of authority is in line with the typology of authority relations considered in Aghion and Tirole [1]. They distinguish *formal* from *real* authority within a hierarchical production organization. Formal authority can be seen as the "right to decide" while real authority is the "effective control over decisions." In our theory the concept of formal authority is represented by the given structure of formal authority relations between agents. In our framework the notion of real authority is then further developed into two distinct forms: *explicit* and *latent*.

Related is the distinction made in Baker, Gibbons and Murphy [4] between formal ("the organizational chart") and informal ("the way things really work") aspects of organizational structures. They study the interaction between asset ownership (which they consider to be formal) and relational contracts (which they consider to be informal). The study of differences and interaction between formal and informal aspects of economic organizations seems to be an important and growing topic for future research.

We emphasize that the formal authority structure of the hierarchical production organization in our model is exogenously given. Further research will be directed towards endogenously determining the formal authority structure of the organization. In this paper we restrict ourselves the question what game will be played within the organization given a particular formal authority structure. And, thus, what real authority structures emerge endogenously within the production organization. In this sense our model is complementary to the literature that studies the endogenous formation of hierarchical authority structures such as principal-agent models (see, e.g., Grossman and Hart [17] and Kessler [24]), models on vertical integration (see, e.g., Klein, Crawford and Alchian [25]), and models on incomplete contracts (see, e.g., Grossman and Hart [16], and Hart and Moore [18, 19]). As mentioned in the introduction, these models assume rather simple authority structures while we allow for arbitrarily complex formal authority structures.

To study the formation of hierarchies, our model can be extended in various ways. One extension is introducing risk as has been pursued by Prescott and Townsend [30] who study how risk sharing can be a reason to form collective organizations. They study why these collective organizations form by using principal-agent relations between these organizations and outsiders.

Beggs [6] uses techniques from queueing theory to determine the optimal structure of hierarchies when workers differ in the range of tasks they can perform. He studies how the complexity of tasks influences the organizational structure. He explains why many organizations have a hierarchical structure by the economies of skilled workers. Skilled workers can make decisions without consulting other workers, while unskilled workers need to ask (superior) more skilled workers for advice or approval. In our model, the skills of different workers are not specified. Only their contribution in the production process is characterized by the cooperative team production game, and their position in the authority structure determines their formal authority which can be exercised explicitly or latent. By extending our model with differences in skills we can require that the implicit exercise of latent authority is only possible if the subordinate worker is skillful enough to do the work on its own. Unskilled workers always have to ask for explicit approval.

Garicano [12] develops a similar model in which he uses specialization instead of differences in worker skills. In a “knowledge-based hierarchy” easy problems are solved by lower (production) levels, while more exceptional or harder problems need to be passed on to higher levels. In his model the decision “who must learn what and whom each worker should ask when confronted with an unknown problem” is part of the organization. We quote from Garicano [12]: “The organization is characterized by the *task design*, as defined by the scope of discretionality of production workers and problem solvers and *structure of hierarchy*, given by the span of control of problem solvers and the number of layers in the organization”. Where our model takes the hierarchical organization structure as given and explains which authority relations are actually activated, Garicano explains the formation of hierarchies by a trade off between communication versus knowledge acquisition costs. In our model (like in Beggs [6]) there is no distinction between different knowledge levels necessary to perform different tasks. A future direction in research is to make this distinction in our model, and see what is the effect on the exercise of authority. One would expect that more easy tasks are suitable to be performed under latent authority, while more difficult tasks need more explicit authority.

Like our model, the above two mentioned papers set aside incentive problems since (as Beggs argues) to get more insight in the functioning of hierarchical organizations it is best to focus on one of many aspects. In this sense these models are complementary to the models which focus on incentive problems such as Qian [31] who endogenously determines the number of hierarchical levels, the span of control and the wage scales by using optimal control techniques, and in that way extends the seminal work of Keren and Levhari [22, 23]. However, these papers do not address the question what authority is actually exercised within a hierarchy.

Another aspect that we do not address here is the organizational form of a hierarchy. Maskin, Qian and Xu [26] compare an M-form (multi-divisional form in which the organization goes along institutional lines) with a U-form (unitary form in which the organization goes along regional lines) with respect to their effectiveness in giving incentives to managers. In their terminology an organization is a “hierarchy of managers built on top of technology” where the technology is present in productive plants. It would be interesting to see if the games that are played within organizations are affected by their organizational form. For example, we might consider the question whether latent exercise of authority appears more often in M-form organizations (which each act more independent from each other in their own region), while in U-form organizations authority is exercised more explicitly (because the stronger dependence between the different organizational units).

Another strand of literature that we mentioned earlier is the incomplete contracts literature which tries to answer the question how to distribute ownership over residual rights, i.e., who has the authority over assets that are non-contractible. While the incomplete contracts literature focusses on the ownership over residual rights to explain the formation of firms, Rajan and Zingales [32] focus on the control of access to critical resources. In this respect we follow in our modelling a similar principle. Rajan and Zingales define access as "the ability to use, or work with, a critical resource". We quote: "The agent who is given privileged access to the resource gets no new residual rights of control. All she gets is the opportunity to specialize her human capital to the resource and make herself valuable. When combined with her preexisting residual right to withdraw her human capital, access gives her the ability to create a critical resource that she controls, her specialized human capital, control over this resource is a source of power."

Rajan and Zingales [33] develop this idea further by relating the control of access to resources to specialization of employees (managers) and try to explain the formation of (firm) hierarchies.[9] This is in line with our model in which we explain the exercise of authority over subordinate employees. Assets are comparable with positions in our authority structure, and control over assets is exercised by vetoing the access to the productive asset by agents in subordinate positions. Although their hierarchical structures are much simpler than ours, also in their model different positions in a hierarchy have different *positional power*. Where Rajan and Zingales [33] use positional power to explain the formation of firm hierarchies (by managers splitting off from a firm and by doing so constructing a new firm), we use positional power to explain how authority is exercised (i.e. what game is played) within a given hierarchical production organization.

References

[1] Aghion, P., and J. Tirole (1997), "Formal and Real Authority in Organizations," *Journal of Political Economy* 105, 1–29.

[2] Alchian, A.A., and H. Demsetz (1972), "Production, Information Costs, and Economic Organization", *American Economic Review*, 62, 777–795.

[3] Aumann, R.J., and J.H. Drèze (1974), "Cooperative Games with Coalition Structure," *International Journal of Game Theory* 3, 217–237.

[4] Baker, G., R. Gibbons, and K.J. Murphy (2002), "Relational Contracts and the Theory of the Firm", *Quarterly Journal of Economics*, 117, 39-84.

[5] Banzhaf, J.F. (1965), "Weighted Voting Doesn't Work: A Mathematical Analysis", *Rutgers Law Review* 19, 317–343.

[6] Beggs, A.W. (2001), "Queues and Hierarchies," *Review of Economic Studies* 68, 297–322.

[9] The wages proposed by Rajan and Zingales [33] can be extended to the hierarchical structures considered in this paper in a way so that they satisfy dual monotonicity.

[7] Brink, R. van den (1997), "An Axiomatization of the Disjunctive Permission Value for Games with a Permission Structure", *International Journal of Game Theory* 26, 27–43.

[8] Brink, R. van den, and R.P. Gilles (1996) "Axiomatizations of the Conjunctive Permission Value for Games with Permission Structures", *Games and Economic Behavior* 12, 113–126.

[9] Brink, R. van den, and G. van der Laan (1998), "Axiomatizations of the Normalized Banzhaf Value and the Shapley Value," *Social Choice and Welfare* 15, 567–582.

[10] Coase, R.H. (1937), "The Nature of the Firm," *Economica* 4, 386–405.

[11] Derks, J.J.M., and R.P. Gilles (1995), "Hierarchical Organization Structures and Constraints on Coalition Formation," *International Journal of Game Theory* 24, 147–163.

[12] Garicano, L. (2000), "Hierarchies and the Organization of Knowledge in Production," *Journal of Political Economy* 108, 874–904.

[13] Gilles, R.P., and G. Owen (1999), "Cooperative Games and Disjunctive Permission Structures," *CentER Discussion Paper* 9920, CentER for Economic Research, Tilburg University, Tilburg.

[14] Gilles, R.P., G. Owen and R. van den Brink (1992), "Games with Permission Structures: the Conjunctive Approach", *International Journal of Game Theory* 20, 277–293.

[15] Greenberg, J. (1990), *The Theory of Social Situations*, Cambridge University Press, Cambridge.

[16] Grossman, S.J., and O.D. Hart (1986), "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration," *Journal of Political Economy* 94, 691–719.

[17] Grossman, S.J., and O.D. Hart (1983), "An Analysis of the Principal-Agent Problem," *Econometrica* 51, 7-45.

[18] Hart, O., and J. Moore (1990), "Property Rights and the Nature of the Firm," *Journal of Political Economy* 98, 1119–1158.

[19] Hart, O., and J. Moore (1999), "Foundations of Incomplete Contracts," *Review of Economic Studies* 66, 115–138.

[20] Herstein, I.N., and J. Milnor (1953), "An Axiomatic Approach to Measurable Utility", *Econometrica* 21, 291–297.

[21] Ichiishi, T. (1993), *The Cooperative Nature of the Firm*, Cambridge University Press, Cambridge.

[22] Keren, M., and D. Levhari (1979), "The optimum span of control in a pure hierarchy", *Management Science*, 25, 1162-1172.

[23] Keren, M., and D. Levhari (1983), “The internal organization of the firm and the shape of average costs”, *Bell Journal of Economics*, 14, 474-486.

[24] Kessler, A.S. (2000), “On Monitoring and Collusion in Hierarchies”, *Journal of Economic Theory*, 91, 280-291.

[25] Klein, B., R.G. Crawford, and A. Alchian (1978), “Vertical Integration, Appropriable Rents, and the Competitive Contracting Process,” *Journal of Law and Economics* 21, 297-326.

[26] Maskin, E., Y. Qian, and C. Xu (2000), “Incentives, Information and Organizational Form”, *Review of Economic Studies*, 67, 359-378.

[27] Maskin, E., and J. Tirole (1999), “Unforeseen Contingencies and Incomplete Contracts,” *Review of Economic Studies* 66, 83–114.

[28] Myerson, R.B. (1977). “Graphs and Cooperation in Games,” *Mathematics of Operations Research* 2, 225–229.

[29] Pérez-Castillo, D., and D. Wettstein (2001), “Bidding for the Surplus: A Non-Cooperative Approach to the Shapley Value”, *Journal of Economic Theory*, 100, 274-294.

[30] Prescott, E.S., and R.M. Townsend (2002), “Collective Organizations versus Relative Performance Contracts: Inequality, Risk Sharing and Moral Hazard”, *Journal of Economic Theory*, 103, 282-310.

[31] Qian, Y. (1994), “Incentives and Loss of Control in an Optimal Hierarchy,” *Review of Economic Studies* 61, 527–544.

[32] Rajan, R.G., and L. Zingales (1998), “Power in a Theory of the Firm”, *Quarterly Journal of Economics*, 113, 387–432.

[33] Rajan, R.G., and L. Zingales (2001), “The Firm as a Dedicated Hierarchy: A Theory of the Origins and Growth of Firms”, *Quarterly Journal of Economics*, 116, 805-851.

[34] Roth, A.E. (1977), “The Shapley Value as a von Neumann-Morgenstern Utility,” *Econometrica* 45, 657–664.

[35] Roth, A.E. (1977), “Utility Functions for Simple Games,” *Journal of Economic Theory* 16, 481–489.

[36] Roth, A.E. (1988), “The Expected Utility of Playing a Game”, in A.E. Roth (editor), *The Shapley Value: Essays in Honor of Lloyd S. Shapley*, Cambridge University Press, Cambridge, MA.

[37] Shapley, L.S. (1953) “A Value for *n*-Person Games,” in H.W. Kuhn and A.W. Tucker (editors), *Annals of Mathematics Studies 28: Contributions to the Theory of Games*, Volume 2, Princeton University Press, Princeton, NJ.

[38] Shubik, M. (1962), "Incentives, Decentralized Control, the Assignment of Joint Costs, and Internal Pricing", *Management Science* 8, 325–343.

[39] Simon, H.A. (1951), "A Formal Theory of the Employment Relationship," *Econometrica* 19, 293–305.

[40] Weber, R.J. (1988) "Probabilistic Values for Games", in A.E. Roth (editor), *The Shapley Value: Essays in Honor of Lloyd S. Shapley*, Cambridge University Press, Cambridge, MA.

[41] Williamson, O.E. (1967), "Hierarchical Control and Optimum Firm Size," *Journal of Political Economy* 75, 123–138.

[42] Williamson, O.E. (1979), "Transaction-cost Economies: the Governance of Contractual Relations," *Journal of Law and Economics* 19, 223–261.

[43] Young, H.P. (1985), "Monotonic Solutions of Cooperative games", *International Journal of Game Theory* 14, 65–72.

Appendix A: Proofs of the main results

Proof of Theorem 2.7

First, we show that the restriction $\mathscr{R}$ indeed satisfies the five properties stated in the assertion. Let $S \in S^N$ and $v, w \in \mathscr{G}^N$. Since $\sigma_S(N) = N$ it holds that $\mathscr{R}(v,S)(N) = v(\sigma_S(N)) = v(N)$, and thus $\mathscr{R}$ satisfies property (i). $\mathscr{R}$ satisfies (ii) since $\mathscr{R}(v+w,S)(E) = (v+w)(\sigma_S(E)) = v(\sigma_S(E)) + w(\sigma_S(E)) = \mathscr{R}(v,S)(E) + \mathscr{R}(w,S)(E)$ for all $E \subset N$. If $i \in N$ is such that all $j \in \widehat{S}(i) \cup \{i\}$ are null players in v then $\mathscr{R}(v,S)(E) = v(\sigma_S(E)) = v(\sigma_S(E) \setminus (\{i\} \cup \widehat{S}(i))) = v(\sigma_S(E \setminus \{i\})) = \mathscr{R}(v,S)(E \setminus \{i\})$ for all $E \subset N$, and thus $\mathscr{R}$ satisfies property (iii). If $i \in N$ is such that $v(E) = 0$ for all $E \subset N \setminus \{i\}$ and $E \subset N \setminus \{i\}$ then $i \notin \sigma_S(E)$ and thus $\mathscr{R}(v,S)(E) = v(\sigma_S(E)) = 0$, which implies that $\mathscr{R}$ satisfies property (iv). Finally, property (v) follows from the fact that $j \in S(i)$ and $E \subset N \setminus \{i\}$ implies that $\sigma_S(E) = \sigma_S(E \setminus \{j\})$ and thus $\mathscr{R}(v,S)(E) = v(\sigma_S(E)) = v(\sigma_S(E \setminus \{j\})) = \mathscr{R}(v,S)(E \setminus \{j\})$.

Next suppose that $\mathscr{F} : \mathscr{G}^N \times S^N \rightarrow \mathscr{G}^N$satisfies the five properties, and let $S \in S^N$. Consider the game $w_T = c_T u_T$ with $c_T \geqslant 0$, and u_T the unanimity game of $T \subset N$ given by

$$u_T(E) = \begin{cases} 1 & \text{if } T \subset E \\ 0 & \text{otherwise.} \end{cases}$$

Property (i) now implies that $\mathscr{F}(w_T, S)(N) = c_T$. Define $\alpha_S(T) = T \cup \widehat{S}^{-1}(T)$. We distinguish the following cases with respect to $E \subset N, E \neq N$:

- $E \supset \alpha_S(T)$. Since for all agents $i \in N \setminus \alpha_S(T)$ it holds that all $j \in \widehat{S}(i) \cup \{i\}$ are null players in w_T, property (iii) implies that $\mathscr{F}(w_T,S)(E) = \mathscr{F}(w_T,S)(N) = c_T$.

- $E \not\supset T$. Since for all agents $i \in T$ it holds that $w_T(E) = 0$ for all $E \subset N \setminus \{i\}$, property (iv) implies that $\mathscr{F}(w_T,S)(E) = 0$.

- $E \supset T, E \not\supset \alpha_S(T)$. Then there exist $i \in \alpha_S(T) \setminus E$ and $j \in S(i) \cap T$. Properties (iv) and (v) then imply that $\mathscr{F}(w_T,S)(E) = \mathscr{F}(w_T,S)(E \setminus \{j\}) = 0$.

So, $\mathscr{F}(w_T,S) = \mathscr{R}(w_T,S)$. The theorem then follows with property (ii) and the fact that v can be expressed as a linear combination of the unanimity games u_T in a unique fashion.

This completes the proof of Theorem 2.7.

Proof of Theorem 2.8

We proof each of the three assertions stated in the theorem.

(a) Let $F \subset N$ be such that $h \in F$. Then for every $i \in F$ it holds that $Z^{-1}(i) \subset F$ if and only if $T^{-1}(i) \subset F$. From this it follows that $\sigma_Z(F) = \sigma_T(F)$. Thus, for every $F \subset N$ with $h \in F$ we have that

$$\mathscr{R}(v,Z)(F) = v(\sigma_Z(F)) = v(\sigma_T(F)) = \mathscr{R}(v,T)(F). \tag{6}$$

Suppose that $F \subset N$ is such that $h \notin F$. Then $Z^{-1}(i) \supset T^{-1}(i)$ for all $i \in F$, and thus $\sigma_Z(F) \subset \sigma_T(F)$. From the monotonicity of v it then follows that for every $F \subset N$ with $h \notin F$ it holds that

$$\mathscr{R}(v,Z)(F) = v(\sigma_Z(F)) \leqslant v(\sigma_T(F)) = \mathscr{R}(v,T)(F). \tag{7}$$

These two properties together with dual monotonicity of u establish assertion (a) in Theorem 2.8.

(b) Together with the properties shown under (a), $\mathscr{R}(v,Z) \neq \mathscr{R}(v,T)$ now implies that there exists some $F \subset N$ with $h \notin F$ for which it holds that $\mathscr{R}(v,Z)(F) < \mathscr{R}(v,T)(F)$. Together with (a) and strong dual monotonicity of u this establishes assertion (b) in Theorem 2.8.

(c) Suppose that v is strictly monotone and that S is acyclic. Furthermore, suppose that $T(h) = \varnothing$ or S is transparent. Then we show that there exists a team $F \subset N$ with $h \notin F$ for which it holds that $R(v,Z)(F) < R(v,T)(F)$.
We now show that under these conditions $S(h) \setminus \widehat{T}(h) \neq \varnothing$. First, suppose that $T(h) = \varnothing$. Then $\widehat{T}(h) = \varnothing$ and since $h \in N \setminus W_S$ it then follows that $S(h) \setminus \widehat{T}(h) = S(h) \neq \varnothing$.
Second suppose that S is transparent. Now, we proceed by contradiction and assume that $S(h) \setminus \widehat{T}(h) = \varnothing$. Then $S(h) \subset \widehat{T}(h)$ and, thus, $\varnothing \neq S(h) \setminus T(h) \subset \widehat{T}(T(h)) \subset \widehat{S}(T(h)) \subset \widehat{S}(S(h))$, implying that $S(h) \cap \widehat{S}(S(h)) \neq \varnothing$. This contradicts the transparency of S.
Next consider the team

$$F := \widehat{T}^{-1}\left(S(h) \setminus \widehat{T}(h)\right) \cup \left[S(h) \setminus \widehat{T}(h)\right]. \tag{8}$$

Remark that $S(h) \setminus \widehat{T}(h) \neq \varnothing$ implies that $F \neq \varnothing$. Since S is acyclic, $T \in \mathbb{H}(S)$ is acyclic as well. This implies that $h \notin F$. Furthermore, $\sigma_T(F) = F \in \Phi_T$. Thus, since v is strictly monotone and $F \neq \varnothing$, it follows that $\mathscr{R}(v,T)(F) = v(F) > 0$.
Finally, we note that $\sigma_Z(F) \subset F \setminus \left[S(h) \setminus \widehat{T}(h)\right]$ since $h \notin F$. Hence, since $S(h) \setminus \widehat{T}(h) \neq \varnothing$, $\sigma_Z(F) \neq F$, and thus by strict monotonicity of v it holds that

$$\mathscr{R}(v,Z)(F) = v(\sigma_Z(F)) < v(F) = \mathscr{R}(v,T)(F). \tag{9}$$

Assertion (c) of Theorem 2.8 now follows with assertions (a) and (b) shown above and the strong dual monotonicity of the utility structure u in combination with Lemma 2.2.

This completes the proof of Theorem 2.8.

Proof of Theorem 3.3

Throughout this proof we let $E_i^S := S(i)$, $i \in N$, define $\mathscr{E}^S = (E_1^S, \dots, E_n^S)$.

(a) Let $T \in \mathbb{M}(v,S)$ and consider the corresponding strategy $\mathscr{E}^T$. Let $i \in N$ be arbitrary. Now define

$$Z(j) = \begin{cases} T(j) & \text{for } j \neq i \\ S(i) & \text{for } j = i. \end{cases}$$

By definition of the restriction $\mathscr{R}$ and monotonicity of v it now can be concluded that $\mathscr{R}(v,Z) = \mathscr{R}(v,S) = \mathscr{R}(v,T)$. Hence, $Z \in \mathbb{M}(v,S)$.
Now let $\mathscr{E} := \left(\mathscr{E}^T_{-i}, E_i\right)$ be given, where $E_i \subset S(i)$ is arbitrary. From dual monotonicity of u, Theorem 2.8(a), and the definition of $\mathbb{M}(v,S)$ it now follows for agent $i \in N$ that

$$u_i^0\left(\mathscr{E}^T\right) = u_i(\mathscr{R}(v,T)) = u_i(\mathscr{R}(v,Z)) \geqslant u_i(\mathscr{R}(v,T_{\mathscr{E}})) = u_i^0(\mathscr{E}).$$

Hence, since $i \in N$ and $E_i \subset S(i)$ are arbitrary, $\mathscr{E}^T \in \mathscr{N}(\Theta^0)$.

(b) Let $\mathscr{E} \in \mathscr{N}_s(\Theta^0)$ and suppose that $\mathscr{E} \neq \mathscr{E}^S$. Then there exists some $j \in N$ with $E_j \subsetneq S(j)$. Now consider $\overline{\mathscr{E}} := \left(\mathscr{E}_{-j}, S(j)\right)$, then by dual monotonicity and Theorem 2.8(a) we have that $u_i^0\left(\overline{\mathscr{E}}\right) \geqslant u_i^0(\mathscr{E})$. This contradicts the strict Nash condition for $\mathscr{E}$. This implies that $\mathscr{N}_s(\Theta^0) \subset \left\{\mathscr{E}^S\right\}$.

This completes the proof of Theorem 3.3.

Proof of Theorem 3.4

We develop the proof of Theorem 3.4 through a sequence of intermediate results. These lemmas are put together to form a proof of the assertions stated in the two main theorems. Throughout this and the next subsection we let (v,S) be a game with an authority structure such that v is monotone.

Lemma A.1 *Let u be a strongly dual monotone utility structure. If $T \notin M(v,S)$ then $E^T \notin N\left(\Theta^0\right)$.*

Proof. If $T \notin \mathbb{M}(v,S)$ then there exist $j \in N$, $h \in \widehat{S}^{-1}(j) \setminus \widehat{T}^{-1}(j)$ and $H \subset N$ with $\Delta_v(H) \neq 0$, $H \cap \widehat{T}(h) = \varnothing$ and $H \cap \widehat{T}(j) \neq \varnothing$. (If such a j, h and H would not exist then $\mathscr{R}(v,T) = \mathscr{R}(v,S)$ and thus $T \in \mathbb{M}(v,S)$.) But then there exists a sequence of agents $h_1, \dots, h_p$ such that $h_1 = j$, $h_p = h$, $h_k \in S(h_{k+1})$ for all $k \in \{1, \dots, p-1\}$, and $j \notin \widehat{T}(h_k)$ for at least one $k \in \{2, \dots, p\}$. Let $m \in \{2, \dots, p\}$ be the lowest label for which $j \notin \widehat{T}(h_m)$ and there exists $H \subset N$ with $\Delta_v(H) \neq 0, H \cap \widehat{T}(h_m) = \varnothing$ and $H \cap \widehat{T}(j) \neq \varnothing$. (Note that such a label exists because it holds for label p.) Then, for $Z \in \mathbb{H}(S)$ given by

$$Z(i) = \begin{cases} T(i) & \text{for } i \neq h_m \\ T(h_m) \cup \{h_{m-1}\} & \text{for } i = h_m, \end{cases}$$

it holds that $\mathscr{R}(v,Z) \neq \mathscr{R}(v,T)$. Since $\mathscr{R}(v,Z)(E) \leqslant \mathscr{R}(v,T)(E)$ for all $E \subset N$, and $\mathscr{R}(v,Z)(E) = \mathscr{R}(v,T)(E)$ for all $E \subset N$ with $h_m \in E$, it follows from strong dual monotonicity of u that $\mathscr{E}^T \notin \mathscr{N}\left(\Theta^0\right)$. □

The next lemma discusses situations in which the full authority structure S is (v,S)-minimal.

Lemma A.2 *Let u be a strongly dual monotonic utility structure and let $S \in \widehat{\mathbb{M}}(v,S)$. Then*

(a) $\mathscr{E}^S \in \mathscr{N}_s(\Theta^0)$, *and*

(b) $\mathscr{N}(\Theta^0) = \{\mathscr{E}^S\}$.

Proof. Under the assumptions, by definition, $\widehat{\mathbb{M}}(v,S) = \mathbb{M}(v,S) = \{S\}$.

(a) Let $i \in N$ be arbitrary and let $\mathscr{E} := \left(\mathscr{E}^S_{-i}, E_i\right)$, where $E_i \subsetneq S(i)$ is arbitrary as well. The resulting authority structure is given by $T_{\mathscr{E}} \notin \mathbb{M}(v,S)$. Hence, $\mathscr{R}(v,T_{\mathscr{E}}) \neq \mathscr{R}(v,S)$. From strong dual monotonicity of u and Theorem 2.8(b) it now follows that

$$u_i^0\left(\mathscr{E}^S\right) = u_i\left(\mathscr{R}(v,S)\right) > u_i\left(\mathscr{R}(v,T_{\mathscr{E}})\right) = u_i^0(\mathscr{E}).$$

Hence, $\mathscr{E}^S \in \mathscr{N}_s(\Theta^0)$.

(b) This assertion follows from Lemma 6 and the fact that $S \in \widehat{\mathbb{M}}(v,S)$ implies that $T \notin \mathbb{M}(v,S)$ for all $T \in \mathbb{H}(S)$ with $T \neq S$.

This shows Lemma A.2. □

Now Theorem 3.4(a) follows immediately from Proposition 3.3(b) and Lemma A.2. Next we turn to the proof of assertion 3.4(b).

Proof of Theorem 3.4(b). Now the assertion that $\mathscr{N}\left(\Theta^0\right) = \left\{\mathscr{E}^T \mid T \in \mathbb{M}(v,S)\right\}$ is a simple consequence of the properties given in Proposition 3.3(a) and Lemma A.1.

It remains to be shown that $\mathscr{N}_s(\Theta^0) = \varnothing$. From Proposition 3.3(b) it only remains to be shown that $\mathscr{E}^S$ is not a strict Nash equilibrium. Namely, by assumption there exists some $T \in \mathbb{M}(v,S)$ with $T \neq S$. Then it follows that there is some $j \in N$ with $T(j) \subsetneq S(j)$. Consider the authority structure Z given by

$$Z(i) = \begin{cases} S(i) & \text{if } i \neq j \\ T(j) & \text{if } i = j. \end{cases}$$

From a repeated application of Theorem 2.8(a) it can be concluded that $\mathscr{R}(v,Z) = \mathscr{R}(v,S)$, i.e., $Z \in \mathbb{M}(v,S)$. Now it can immediately be concluded that $\mathscr{E}^S$ cannot be a strict Nash equilibrium of the authority game Θ^0. □

Proof of Theorem 3.5

Lemma A.3 *Let u be a strongly monotone utility structure. For every (v,S)-minimal authority structure $T \in \widehat{\mathbb{M}}(v,S)$ there exists a cost level $c_T > 0$ such that $E^T \in N(\Theta^c)$ for every $0 \leqslant c \leqslant c_T$.*

Proof. Let $T \in \widehat{\mathbb{M}}(v,S)$ be (v,S)-minimal. Then by Theorem 2.8(a) and (b) we have for every $i \in N$ that

$$u_i^0\left(\mathscr{E}^S\right) = u_i^0\left(\mathscr{E}^T\right) = u_i^0\left(\mathscr{E}^T_{-j}, S(j)\right).$$

for any agent $j \in N$. Now define for $j \in N \setminus W_S$

$$\delta_j = \min \left\{ u_j^0 \left(\mathscr{E}_{-j}^T, S(j)\right) - u_j^0 \left(\mathscr{E}_{-j}^T, E_j\right) \;\middle|\; \begin{array}{c} E_j \subset S(j) \text{ such that} \\ \mathscr{R}(v,S) \neq \mathscr{R}\left(v, T_{\left(\mathscr{E}_{-j}^T, E_j\right)}\right) \end{array} \right\}$$

We remark that if $E_j^T \neq \varnothing$, $\delta_j > 0$ due to the fact that $T \in \widehat{\mathbb{M}}(v,S)$. Finally we introduce

$$c_T := \min \left\{ \frac{\delta_j}{|T(j)|+1} \;\middle|\; j \in N \setminus W_S \text{ with } \delta_j > 0 \right\} > 0. \tag{10}$$

Let $i \in N \setminus W_S$ and let $\mathscr{E} = \left(\mathscr{E}_{-i}^T, E_i\right)$ with $E_i \subset S(i)$. Now we consider two cases:

Case A $\mathscr{R}(v,T) = \mathscr{R}(v,T_{\mathscr{E}})$

Then by definition of (v,S)-minimality of T it follows that $|T(i)| \leqslant |E_i|$. Hence for any $c > 0$ we conclude that

$$\begin{aligned} u_i^c\left(\mathscr{E}^T\right) - u_i^c(\mathscr{E}) &= u_i^0\left(\mathscr{E}^T\right) - u_i^0(\mathscr{E}) + c\,(|E_i| - |T(i)|) \\ &= c\,(|E_i| - |T(i)|) \geqslant 0. \end{aligned}$$

Case B $\mathscr{R}(v,T) \neq \mathscr{R}(v,T_{\mathscr{E}})$

Then by strong dual monotonicity of u and Theorem 2.8(b) we conclude that $u_i^0\left(\mathscr{E}^T\right) = u_i^0\left(\mathscr{E}_{-i}^T, S(i)\right) > u_i^0(\mathscr{E})$. Hence, $\delta_i > 0$. Let $0 < c < c_T$. Now we derive by definition of c_T that

$$\begin{aligned} u_i^c\left(\mathscr{E}^T\right) - u_i^c(\mathscr{E}) &= u_i^0\left(\mathscr{E}^T\right) - u_i^0(\mathscr{E}) + c\,(|E_i| - |T(i)|) \\ &\geqslant \delta_i - c_T\,|T(i)| > 0. \end{aligned}$$

Cases A and B now complete the proof of the assertion stated in Lemma A.3. □

Proof of Theorem 3.5. Consider any $\mathscr{E} \in \Gamma$ such that $T_{\mathscr{E}} \notin \widehat{\mathbb{M}}(v,S)$. We now distinguish two possible cases:

Case A: $T_{\mathscr{E}} \notin \mathbb{M}(v,S)$

Then by Theorem 3.4 $\mathscr{E} \notin \mathscr{N}\left(\Theta^0\right)$. Hence, there exists some $j_{\mathscr{E}} \in N \setminus W_S$ with $u_{j_{\mathscr{E}}}^0(\mathscr{E}) < u_{j_{\mathscr{E}}}^0(\mathscr{E}')$, where $\mathscr{E}' = \left(\mathscr{E}_{-j_{\mathscr{E}}}, S(j_{\mathscr{E}})\right)$. Define

$$c_{\mathscr{E}} = \frac{1}{|S(j_{\mathscr{E}})|}\left(u_{j_{\mathscr{E}}}^0\left(\mathscr{E}'\right) - u_{j_{\mathscr{E}}}^0(\mathscr{E})\right) > 0.$$

Then for any $0 \leqslant c < c_{\mathscr{E}}$ we have that

$$\begin{aligned} u_{j_{\mathscr{E}}}^c\left(\mathscr{E}'\right) - u_{j_{\mathscr{E}}}^c(\mathscr{E}) &= u_{j_{\mathscr{E}}}^0\left(\mathscr{E}'\right) - u_{j_{\mathscr{E}}}^0(\mathscr{E}) + c\left(\left|E_{j_{\mathscr{E}}}\right| - |S(j_{\mathscr{E}})|\right) \\ &\geqslant (c_{\mathscr{E}} - c)\,|S(j_{\mathscr{E}})| > 0. \end{aligned}$$

Thus, $\mathscr{E} \notin \mathscr{N}(\Theta^c)$.

Case B: $T_{\mathscr{E}} \in \mathbb{M}(v,S)$

By Theorem 3.4(b) we know that $\mathscr{E} \in \mathscr{N}\left(\Theta^0\right)$. Since $T_{\mathscr{E}} \notin \widehat{\mathbb{M}}(v,S)$, we conclude from the definition of the restriction $\mathscr{R}$ that there exists some $\widehat{T} \in \widehat{\mathbb{M}}(v,S)$ with $\left|\widehat{T}(i)\right| \leqslant |T_{\mathscr{E}}(i)|$ for all $i \in N$ and $\left|\widehat{T}(j)\right| < |T_{\mathscr{E}}(j)|$ for some $j \in N \setminus W_S$. Also from Theorem 3.4(b) we conclude that $\mathscr{E}^{\widehat{T}} \in \mathscr{N}\left(\Theta^0\right)$. Thus, for any $c > 0$ we conclude that

$$\begin{aligned} u_j^c\left(\mathscr{E}^{\widehat{T}}\right) - u_j^c(\mathscr{E}) &= u_j^0\left(\mathscr{E}^{\widehat{T}}\right) - u_j^0(\mathscr{E}) + c\left(|T_{\mathscr{E}}(j)| - \left|\widehat{T}(j)\right|\right) \\ &= c\left(|T_{\mathscr{E}}(j)| - \left|\widehat{T}(j)\right|\right) > 0. \end{aligned}$$

Hence, $\mathscr{E} \notin \mathscr{N}(\Theta^c)$.

Now define using the constructions in Lemma A.3 and Case A

$$c^* = \min\left\{c_T \,\middle|\, T \in \widehat{\mathbb{M}}(v,S)\right\} \cup \{c_{\mathscr{E}} \mid \mathscr{E} \in \Gamma \text{ with } T_{\mathscr{E}} \notin \mathbb{M}(v,S)\} > 0.$$

Now for any $0 < c < c^*$ it follows that

(i) from Lemma A.3: $\left\{\mathscr{E}^T \,\middle|\, T \in \widehat{\mathbb{M}}(v,S)\right\} \subset \mathscr{N}(\Theta^c)$, and

(ii) from Case A and Case B: $\mathscr{N}(\Theta^c) \subset \left\{\mathscr{E}^T \,\middle|\, T \in \widehat{\mathbb{M}}(v,S)\right\}$.

This concludes the proof of Theorem 3.5. □

Proof of Theorem 4.2

The proof of Theorem 4.2 is based on results from the theory of social situations, developed in Greenberg [15]. Greenberg develops the notion of a *stable standard of behavior* as the main equilibrium concept within this theory. In this proof we transform our notion of a stable authority protocol into a stable standard of behavior of an appropriately constructed social situation. The proof of the existence of the SAP then becomes a straightforward application of the main existence theorem developed by Greenberg.

Let (u,v,S) and $c \geqslant 0$ be as in Theorem 4.2. Hence, $v \in \mathscr{G}^N$ is a monotone game and $S \in \mathscr{S}^N$ is an acyclic authority structure. Furthermore, (u,v,S) does not have any inessential agents. We now construct a social situation from (u,v,S). (For an exhaustive discussion and definition of a social situation we refer to Chapter 2 in Greenberg [15], in particular Definitions 2.1.1 and 2.1.3.)

First, for every $T \in \mathbb{H}(S)$ we define

$$X^T = \left\{\mathscr{R}(v,Z) \in \mathscr{G}^N \,\middle|\, Z \in \mathbb{H}(S) \text{ and } Z(i) = T(i) \text{ for all } i \in \psi'(T)\right\},$$

for every $i \in N$ the restricted utility function $f_i^T : X^T \to \mathbb{R}$ is for every $w \in X^T$ given by $f_i^T(w) = u_i(w) - c\,|T(i)|$, and for every $E \subset N$ and $w \in X^T$ we define

$$\gamma^T(E,w) = \begin{cases} \gamma_i(T) & \text{if } E = \{i\} \\ \varnothing & \text{otherwise} \end{cases}$$

where γ_i is the veto correspondence for agent $i \in N$.

We remark that these definitions imply that every agent can announce to enforce her authority at most once. Now the tuple $\Upsilon^c = \left(\mathbb{H}(S), \left(X^T, f^T, \gamma^T\right)_{T \in \mathbb{H}(S)}\right)$ defines a social situation introduced by Greenberg [15]. We now develop the proof of Theorem 4.2 through a series of intermediate results.

From the definition of an Optimistically Stable Standard of Behavior[10] (OSSB) and a stable authority protocol the next lemma follows trivially. A proof is therefore omitted.

Lemma A.4 *Any OSSB of the social situation Υ^c corresponds to an SAP for (u, v, S). Furthermore, any SAP for (u, v, S) corresponds to an OSSB of social situation Υ^c.*

The set of positions in Υ^c corresponds to the set of authority structures $\mathbb{H}(S)$ in the authority situation. For the next lemma we remark that the notions of hierarchical and strictly hierarchical social situations are given in Definitions 5.1.1 and 5.3.2 in Greenberg [15].

Lemma A.5 *The social situation Υ^c is strictly hierarchical.*

Proof. Let $n_0 := |N \setminus W_S|$ and let for every $k \in \{0, 1, \ldots, n_0\}$

$$\mathbf{P}_k := \{T \in \mathbb{H}(S) \mid |\psi(T)| = n_0 - k\}.$$

Clearly, $\mathbf{P}_0 = \{S_0\}$ and $\mathbf{P}_{n_0} = \{T \in \mathbb{H}(S) \mid T(i) \neq \varnothing \text{ for } i \in N \setminus W_S\}$. Now, the collection $\{\mathbf{P}_0, \ldots, \mathbf{P}_{n_0}\}$ forms a partition of $\mathbb{H}(S)$. Also, from above $\gamma^T(E, w) = \varnothing$ for all $E \subset N$ and $w \in X^T$ if $T \in \mathbf{P}_{n_0}$.

Let $k \in \{0, 1, \ldots, n_0 - 1\}$ and take $T \in \mathbf{P}_k$. Then for every $i \in \psi(T)$ and $w \in X^T$ obviously $\gamma^T(\{i\}, w) \subset \mathbf{P}_{k+1}$, since $|\psi(T')| = |\psi(T)| - 1$ for $T' \in \gamma^T(\{i\}, w)$. Furthermore, $\gamma^T(E, w) = \varnothing$ for all $E \subset N$ such that there is no $i \in \psi(T)$ with $E = \{i\}$. So, we conclude that

$$\left(\left(\bigcup_{t=k+1}^{n_0} \mathbf{P}_t\right) \cup \{T\}, \left(X^H, u^H, \gamma^H\right)_{H \in \left(\bigcup_{t=k+1}^{n_0} \mathbf{P}_t\right) \cup \{T\}}\right)$$

is indeed a social situation. Hence, we conclude that Υ satisfies requirement H.1 of Definition 5.1.1 in Greenberg [15], pages 43–44. Furthermore, requirement H.2 of that definition is satisfied as well by Υ^c. So, Υ^c is indeed hierarchical.

Finally we observe that there is no $E \subset N$ and $w \in X^T$ for which $T \in \gamma^T(E, w)$. Hence, Υ^c satisfies Definition 5.3.2 in Greenberg [15], page 52. □

The next lemma follows immediately from Lemma A.5 and Corollary 5.3.3 in Greenberg [15], page 52. A proof is therefore omitted.

Lemma A.6 *The social situation Υ^c admits a unique OSSB $\sigma_*^c : \mathbb{H}(S) \to 2^{X^T}$.*

Assertion (a) of Theorem 4.2 now follows immediately from Lemmas A.4 and A.6. To show assertion (b) as well, we define for $T \in \mathbb{H}(S)$ and $h \in \psi(T)$ the authority structure $T_h \in \mathbb{H}(S)$ by

[10] For the definition of an Optimistically Stable Standard of Behavior, or OSSB, we again refer to Greenberg, Section 2.3 and Definitions 2.4.1, 2.4.2, and 2.4.3.

$$T_h(i) = \begin{cases} T(i) & \text{if } i \in N \setminus \{h\} \\ S(i) & \text{if } i = h, \end{cases}$$

and $\pi(T) = \{h \in \psi(T) \mid u_h(\mathscr{R}(v, T_h)) - u_h(\mathscr{R}(v, T)) > 0\}$.

Lemma A.7 *Let the utility structure u be strongly dual monotone, let there be at least one agent $i \in N$ with $S(i) \neq \varnothing$, and let $\overline{c} := \min_{T \in \mathbb{H}(S), h \in \pi(T)} \{u_h(\mathscr{R}(v, T_h)) - u_h(\mathscr{R}(v, T))\}$. For $c^* := \frac{\overline{c}}{\max_{i \in N} |S(i)|}$ it then follows that*

1. $c^* \geqslant 0$*, and*
2. $c^* = 0$ *if and only if* $\mathscr{R}(v, T) = \mathscr{R}(v, S)$ *for.all* $T \in \mathbb{H}(S)$.

Proof. From the definition of c^* the fact that u satisfies strong dual monotonicity, and Theorem 2.8(a) it immediately follows that $c^* \geqslant 0$.
It is also easy to see that $c^* = \overline{c} = 0$ if $\mathscr{R}(v, T) = \mathscr{R}(v, S)$ for all $T \in \mathbb{H}(S)$.
Now suppose that $\mathscr{R}(v, T) \neq \mathscr{R}(v, S)$ for some $T \in \mathbb{H}(S)$. Then there exists a $T \in \mathbb{H}(S)$ and $h \in \psi(T)$ such that $\mathscr{R}(v, T_h) \neq \mathscr{R}(v, T)$. Since u satisfies strong dual monotonicity it follows from Theorem 2.8(b) that $u_h(\mathscr{R}(v, T_h)) - u_h(\mathscr{R}(v, T)) > 0$. But then $\overline{c} > 0$, and thus $c^* > 0$. □

Our final step in the proof of assertion (b) in Theorem 4.2 is the following:

Lemma A.8 *Let the utility structure u be strongly dual monotone and let the monitoring cost satisfy $c < c^*$, where c^* is as defined in Lemma A.7. Then for every $T \in \mathbb{H}(S)$ the unique OSSB σ^c_* of the social situation Υ^c is given by $\sigma^c_*(T) \equiv \{\mathscr{R}(v, Z)\}$ where $Z \in S^N$ is given by*

$$Z(i) = \begin{cases} T(i) & \text{if } i \notin \psi(T) \\ S(i) & \text{if } i \in \psi(T). \end{cases}$$

Proof. The proof consists of two steps, constituting a proof by induction on the partition discussed in the proof of Lemma A.5.
First, let $T \in \mathbf{P}_{n_0}$. Using the notion of the Optimistic Dominion given in Greenberg [15], page 19, and Greenberg [15] Definition 2.4.7 plus the fact that $\gamma^T(E, w) = \varnothing$ for all $E \subset N$ and $w \in X^T$, we compute the unique OSSB for Υ^c to be given by

$$\sigma^c_*(T) = X^T = \{\mathscr{R}(v, Z) \mid Z \in \mathbb{H}(S) \text{ and } Z(i) = T(i),\, i \in N\}.$$

We note that $\psi(T) = \varnothing$. Thus,

$$\sigma^c_*(T) = \left\{ \mathscr{R}(v, Z) \;\middle|\; \begin{array}{ll} Z(i) = T(i) & \text{for } i \notin \psi(T) \\ Z(i) = S(i) & \text{for } i \in \psi(T) \end{array} \right\}$$

Second, suppose that for all $T \in \mathbf{P}_t$ with $t \in \{k, \ldots, n_0\}$, where $k \geqslant 1$, it holds that

$$\sigma^c_*(T) = \left\{ \mathscr{R}(v, Z) \;\middle|\; \begin{array}{ll} Z(i) = T(i) & \text{for } i \notin \psi(T) \\ Z(i) = S(i) & \text{for } i \in \psi(T) \end{array} \right\}$$

Let $T \in \mathbf{P}_{k-1}$. Choose $h \in \psi(T)$ and let $Z \in \mathbb{H}(S)$ be given by

$$Z(i) = \begin{cases} S(i) & \text{if } i = h \\ T(i) & \text{otherwise.} \end{cases}$$

Note that $T(h) = \varnothing$. Since, u is strongly dual monotone, it follows by definition of c^* that $u_h(\mathscr{R}(v,Z)) - u_h(\mathscr{R}(v,T)) \geqslant \frac{\bar{c}}{|S(h)|} \geqslant c^*$ if $\mathscr{R}(v,Z) \neq \mathscr{R}(v,T)$. Since $\mathscr{R}(v,Z) \in X^T \cap \gamma^T(\{h\}, \mathscr{R}(v,T))$ and $c < c^*$ it can be concluded that $\mathscr{R}(v,T) \notin \sigma_*^c(T)$ if $\mathscr{R}(v,Z) \neq \mathscr{R}(v,T)$. Thus,

$$\sigma_*^c(T) \subset \left\{ \mathscr{R}(v,Z) \;\middle|\; \begin{array}{ll} Z(i) = T(i) & \text{for } i \notin \psi(T) \\ Z(i) = S(i) & \text{for } i \in \psi(T) \end{array} \right\}$$

From Theorem 2.8(b) it also follows that this inclusion can be reversed as well. This shows the assertion. □

To complete the proof of Theorem 4.2 we remark that from Lemma A.8 it can immediately be concluded that for $0 \leqslant c < c^*$ it holds that

$$\Sigma_*^c(S_0) = \sigma_*^c(S_0) = \{\mathscr{R}(v,Z) \mid Z(i) = S(i) \text{ for } i \in N\} = \{\mathscr{R}(v,S)\}.$$

Hence, we have established assertion (b) of Theorem 4.2. Since we already established assertion (a), we have completed the proof of Theorem 4.2.

Appendix B: Regarding dual monotonicity

From the analysis in this paper we conclude that the dual monotonicity condition on the utility structure is of great significance. For both explicit as well as latent exercise of authority, the dual monotonicity and strong dual monotonicity properties introduced in Definition 2.1 are identified as the main hypotheses under which complete control of access to the productive asset is established. In this appendix we investigate the (strong) dual monotonicity properties and compare them to some more familiar monotonicity concepts from the literature. Our main insight is that the dual monotonicity concept is not stronger than the monotonicity concepts used in the literature.

First, we compare our dual monotonicity concepts with the notion of strong monotonicity discussed in Young [43]. A utility structure $u\colon \mathscr{G}^N \to \mathbb{R}^N$ satisfies *Young's strong monotonicity* property if for every $v, w \in \mathscr{G}^N$ and $i \in N$ it holds that $u_i(v) \geqslant u_i(w)$ whenever $v(E \cup \{i\}) - v(E) \geqslant w(E \cup \{i\}) - w(E)$ for all $E \subset N \setminus \{i\}$.

Proposition B.1 *If $u\colon G^N \to R^N$ satisfies Young's strong monotonicity property, then it satisfies dual monotonicity.*

Proof. Suppose that $u\colon \mathscr{G}^N \to \mathbb{R}^N$ satisfies Young's strong monotonicity and let $v, w \in \mathscr{G}^N$ satisfy the condition stated in Definition 2.1.3, i.e., for some $F \subset N$ it holds that $v(F) \leqslant w(F)$ and for all other teams $E \in 2^N \setminus \{F\}$ it holds that $v(E) = w(E)$. For every $i \in N \setminus F$ it then holds that $v(F \cup \{i\}) - v(F) \geqslant w(F \cup \{i\}) - w(F)$ and $v(E \cup \{i\}) - v(E) = w(E \cup \{i\}) - w(E)$ for all $E \in 2^N \setminus \{F\}$.
From Young's strong monotonicity of u it then follows that $u_i(v) \geqslant u_i(w)$. Thus, u satisfies dual monotonicity. $\square$

Dual monotonicity does not imply Young's strong monotonicity property as the following example shows.

Example B.2 Let $\bar{u}\colon \mathscr{G}^N \to \mathbb{R}^N$ be the egalitarian utility structure given by $\bar{u}_i(v) = \frac{v(N)}{n}$ for all $i \in N$. Obviously the utility structure $\bar{u}$ is dual monotone.
Consider the games $v, w \in \mathscr{G}^N$ with $N = \{1, 2, 3\}$ given by $v(E) = |E|$ for all $E \subset N$, and $w(E) = 1$ if $1 \in E$, and $w(E) = 0$ otherwise. Then $v(E \cup \{1\}) - v(E) = w(E \cup \{1\}) - w(E)$ for all $E \subset N \setminus \{1\}$. But $\bar{u}_1(v) = 1 > \frac{1}{3} = \bar{u}_1(w)$. This shows that u indeed does not satisfy Young's strong monotonicity property. $\square$

A utility structure $u\colon \mathscr{G}^N \to \mathbb{R}^N$ satisfies *coalitional monotonicity* if for every $v, w \in \mathscr{G}^N$ such that there is an $F \subset N$ for which $v(F) \geqslant w(F)$, and $v(E) = w(E)$ for all $E \in 2^N \setminus \{F\}$, it holds that $u_i(v) \geqslant u_i(w)$ for all $i \in F$. Coalitional monotonicity has been considered by Shubik [38] and in some sense can be perceived as a dual formulation of dual monotonicity. The following example shows that in general these two properties do not imply one another.

Example B.3 Let $g_i(v) = \max\{\max_{E \ni i} v(E), 0\}$ for all $i \in N$ and $v \in \mathscr{G}^N$, and let $G(v) = \sum_{i \in N} g_i(v) \geqslant 0$.
Let the utility structure $u\colon \mathscr{G}^N \to \mathbb{R}$ distribute the worth $v(N)$ proportional to the values

$g_i(v)$ over the agents if $G(v) > 0$, and according to the egalitarian rule that is considered in Example B.2 if $G(v) = 0$, i.e.,

$$u_i(v) = \begin{cases} \frac{g_i(v)}{G(v)} v(N) & \textit{if } G(v) > 0 \\ \frac{v(N)}{n} & \textit{if } G(v) = 0. \end{cases}$$

This utility structure satisfies dual monotonicity but does not satisfy coalitional monotonicity. Consider the games $v, w \in \mathscr{G}^N$ with $N = \{1,2,3\}$ given by

$$v(E) = \begin{cases} 1 & \text{if } E \in \{\{1\},\{1,2\}\} \\ 0 & \text{otherwise,} \end{cases} \quad \text{and } w(E) = \begin{cases} 1 & \text{if } E = \{1\} \\ 0 & \text{otherwise.} \end{cases}$$

Then $v(\{1,2\}) > w(\{1,2\})$ and $v(E) = w(E)$ for all $E \in 2^N \setminus \{\{1,2\}\}$. But $u_1(v) = \frac{1}{2} < 1 = u_1(w)$.
Similarly, by taking $g_i(v) = \max\{\min_{E,\ i\in E}\ v(E), 0\}$ it can be shown that coalitional monotonicity does not imply dual monotonicity. □

It turns out that dual and coalitional monotonicity are equivalent under the assumption that the utility structure satisfies additivity and the null player property.

Proposition B.4 *Let the utility structure* $u\colon G^N \to R^N$ *satisfy additivity and the null player property. Then* u *satisfies dual monotonicity if and only if it satisfies coalitional monotonicity.*

Proof. Let $u\colon \mathscr{G}^N \to \mathbb{R}^N$ satisfy additivity and the null player property. According to Theorem 3 in Weber [40] it then holds that for every $i \in N$ there exists a collection of constants p_E^i, $E \subset N \setminus \{i\}$, such that (i) $\sum_{E \subset N\setminus\{i\}}\ p_E^i = 1$, and (ii) $u_i(v) = \sum_{E\subset N\setminus\{i\}}\ p_E^i(v(E\cup\{i\}) - v(E))$ for every $v \in \mathscr{G}^N$.
We now show that u satisfies dual monotonicity if and only if $p_E^i \geqslant 0$ for all $i \in N$ and $E \subset N \setminus \{i\}$.

Only if
Suppose that u satisfies dual monotonicity. Let $i \in N$, $F \subset N \setminus \{i\}$, and let $v \in \mathscr{G}^N$ be such that $v(F) \leqslant v_0(F)$ and $v(E) = v_0(E)$ for all $E \in 2^N \setminus \{F\}$, where v_0 denotes the null game, i.e., $v_0(E) = 0$ for all $E \subset N$.
From Weber's result it follows that $u_i(v) = p_F^i(v(F\cup\{i\}) - v(F))$. According to dual monotonicity and the null player property it holds that $u_i(v) \geqslant u_i(v_0) = 0$. Since $v(F \cup \{i\}) - v(F) \geqslant 0$ it must hold that $p_F^i \geqslant 0$.

If
Suppose that $p_E^i \geqslant 0$ for all $i \in N$ and $E \subset N \setminus \{i\}$. Let $v, w \in \mathscr{G}^N$ satisfy the condition stated in Definition 2.1.3 for team $F \subset N$, and let $i \in N \setminus F$. Further, let $w' \in \mathscr{G}^N$ be given by $w'(E) = w(E) - v(E)$ for all $E \subset N$.
Since $p_F^i \geq 0$, $w'(F\cup\{i\}) = 0$, and $w'(F) \geqslant 0$ it holds that $u_i(w') = p_F^i(w'(F\cup\{i\}) - w'(F)) \leqslant 0$.
Since u satisfies additivity and $w = v + w'$ it holds that $u_i(w) = u_i(v) + u_i(w') \leqslant u_i(v)$. Thus, u satisfies dual monotonicity.

In a similar fashion it can be shown that u satisfies coalitional monotonicity if and only if $p^i_E \geqslant 0$ for all $i \in N$ and $E \subset N \setminus \{i\}$. Combining these two equivalence properties yields that u satisfies dual monotonicity if and only if it satisfies coalitional monotonicity. $\square$

In the previous sections we also used strong dual monotonicity in our analysis. A utility structure $u\colon \mathscr{G}^N \to \mathbb{R}^N$ satisfies this property if it satisfies the dual monotonicity condition stated with the inequalities replaced by strict inequalities. (This is stated in Definition 2.1 (iv).) Similarly we can replace the inequalities in the definitions of strong and coalitional monotonicity by strict inequalities. Propositions B.1 and B.4 also hold if we replace the monotonicity concepts by these strict monotonicity concepts.

It is easy to see that, for example, all utility structures $u\colon \mathscr{G}^N \to \mathbb{R}^N$ for which there are constants $p_k > 0$, $0 \leqslant k \leqslant n$, such that for every $v \in \mathscr{G}^N$ it holds that $u_i(v) = \sum_{E \subset N} p_{|E|}(v(E \cup \{i\}) - v(E))$, satisfy strong dual monotonicity as well as strong coalitional monotonicity. Familiar examples of such solution concepts are the Shapley value, for which $p_k = \frac{(k-1)!(n-k)!}{n!}$, and the Banzhaf value, for which $p_k = \frac{1}{2^{n-1}}$, for all $1 \leqslant k \leqslant n$. (For an elaborate discussion of this class of utility structures we also refer to Weber [40].)