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Growth and Risk

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Growth and Risk:

Methodology and Micro Evidence

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Abstract

There has been a revival of interest in the effect of risk on economic growth. We quantify both *ex ante* and *ex post* effects of risk using a stochastic version of the Ramsey model. We develop a simulation-based econometric methodology which allows us to estimate the model in the structural form suggested by theory. The methodology is applied to micro data from a remarkable long-running panel data set for rural households in Zimbabwe. We find that risk substantially reduces growth: in the ergodic distribution the mean (across households) capital stock is 46% lower than in the absence of risk. This is, we believe, the first micro-based estimate of the effect of shocks on growth. About two-thirds of the impact of risk is due to the *ex ante* effect (i.e. the behavioral response to risk) which is usually not taken into account in policy design. Our results suggest that the effectiveness of policy interventions which reduce exposure to shocks or help households in risk management may be seriously underestimated.

1 Introduction

Growth and risk are central issues in development. While the two phenomena are usually studied separately it has often been suggested that they are closely linked. For example, Collier and Gunning (1999) argue on the basis of micro-economic evidence that the responses of agents to risk are an important part of the explanation for Africa's poor growth performance. While the significance of growth-reducing responses to risk is recognised, neither the theoretical nor the empirical literature provides much guidance for quantifying the effect of risk on growth.

A large part of growth theory is, of course, deterministic so that the issue cannot be addressed. In stochastic growth models an increase in risk usually[1] affects an agent's policy function, i.e. agents adjust their investment behaviour in response to risk. In addition to this *ex ante* effect there also is an *ex post* effect: actual shocks affect accumulation for a *given* policy function. Recently there has been a revival of interest in growth under uncertainty (e.g. Binder and Pesaran, 1999, de Hek, 1999; after early contributions such as Levhari and Srinivasan, 1969) but this has not led to empirical work, presumably because closed form solutions are rarely possible. The theoretical contributions typically treat rather special cases. For example, there is no *ex post* effect in the Levhari-Srinivasan model and no *ex ante* effect in the Binder-Pesaran paper. To study the effect of risk on growth we clearly need less restrictive models.

Turning to the empirical growth literature, the equation estimated in growth regressions is typically derived from a deterministic growth model with a stochastic component added only as an afterthought. However, some authors go beyond this and attempt to measure the effect of

[1] There are exceptions. For example, the canonical, loglinear growth regression can be derived from a growth model in which shocks affect growth *ex post* but not *ex ante* (see section 2).

Lucas (1987, 2003) in his famous back-of-envelope calculation of the welfare effect of eliminating business cycles implicitly assumes away any *ex ante* effect of risk. He assumes that the elimination stabilizes aggregate consumption at its expected value. Krusell and Smith follow this neutrality assumption (1999, p. 251).

risk on growth by including measures of shocks as regressors (e.g. Easterly, *et al.* 1993 include terms of trade shocks). Such regressions go some way towards estimating the effect of risk on growth. However, at best they can identify the *ex post* effect: the *ex ante* effect will be missed. Whether this is a serious limitation is one of the issues we address in this paper.[2]

Modern growth theory is based on an intertemporal optimisation model for an individual agent (e.g. Barro and Sala-i-Martin, 1995). In spite of this microeconomic basis empirical applications of growth theory have almost invariably used macro datasets. The alternative of testing micro theory on micro datasets has intuitive appeal but has so far been used rarely. There are two reasons for this. First, growth regressions require time series and panel datasets for individual households are relatively rare. Secondly, most panel data sets are for advanced economies with well-developed financial markets so that volatility across times or states can be smoothed relatively easily. Micro data sets can then be used to test the Euler equation, under the assumption that the agent faces a given interest rate (e.g. Gourinchas and Parker, 2001). Treating the interest rate as exogenous makes the agent's saving decisions independent of the production technology. This removes the concavity of the production function, a central component of growth theory, from the analysis.

By contrast developing countries are often characterised by very imperfect capital markets and also by high risk. This combination would appear to make micro data sets for developing countries ideal for estimating a stochastic growth model. While there exists an extensive literature on the micro effects of risk in developing countries this literature often misses effects on growth. For example, in the model of Deaton (1991) there is a capital market imperfection (the agent faces a borrowing constraint) but the return on assets is fixed. The model leaves the

[2] There are a few exceptions. For example, Dehn (2000) attempts to capture both the *ex ante* and the *ex post* effect of risk (terms of trade volatility) on growth: in a growth regression he includes both a measure of the risk a country is exposed to and a measure of shocks experienced in the current period.

production technology off stage: the agent's non-asset income is exogenous and asset income does not reflect diminishing returns to investment. Also, the agent is impatient (the time preference rate exceeds the interest rate) so that in the absence of risk there will be no growth: the agent will not want to hold a postive level of assets. These assumptions are suitable for analysing the short-run effects of shocks but they are clearly inappropriate for testing growth theory on micro data.

In this paper we estimate a microeconomic model of growth under uncertainty, a stochastic version of the Ramsey model.[3] We use a unique long-running panel dataset for rural households in Zimbabwe. We assume initially that there are *no* financial assets and that accumulation takes the form of investment in livestock, the capital stock used in the agent's own production process. While obviously extreme this assumption (which we later relax) is realistic for the households in our sample: they make little use of financial assets and of informal insurance and their investment largely takes the form of building up the own livestock herd. The return to this asset is stochastic[4] (households are exposed to shocks which affect their income and assets) and endogenous: the marginal productivity of capital in the agricultural production process decreases with capital accumulation. The panel data span the period 1980-2000, a period in which these households' assets and incomes grew very rapidly, in spite of exposure to massive shocks, including a severe drought.

The model has in general no closed form solution. We derive the optimal accumulation of

[3] A similar model is used in a macroeconomic context by Krusell and Smith (1998, 1999). There are three key differences. First, they do not estimate their model but use calibrated parameter values. Secondly, the risk they consider (reflecting US data) is tiny compared to that in our empirical application. Thirdly, in their model agents can lend or borrow at a given interest rate (equal to the marginal productivity of aggregate capital). This perfect capital market assumption implies that idiosyncratic shocks (while uninsurable) have very little aggregate effect. In their simulation experiments the removal of risk affects growth but the effect is trivial.

[4] Dercon (2002) stresses that models of consumption smoothing (e.g. Deaton, 1991) often assume that agents have access to a safe asset. This unrealistic assumption overstates the effectiveness of consumption smoothing as a risk coping strategy. The Dercon critique does not apply to our model: the asset is modelled as risky.

the households' capital stock by simulation and also estimate the model by simulation, using a pseudo maximum likelihood method. Since the policy function can be written in recursive form this methododology is computationally not unduly demanding.

The estimated micro growth model is used to address two questions. The first concerns convergence. In our model agents are heterogeneous: households differ in total factor productivity and in initial wealth. In addition, they are exposed to idiosyncratic (as well as to covariant) shocks. In the absence of shocks growth is an error correction response to the difference of the capital stock from its (household-specific) steady state value. This generates conditional convergence, as in deterministic macro growth models. Our first question is whether in our micro data set convergence is empirically important. We find that it is. In the absence of shocks the aggregate capital stock of the households in our sample would have grown at an average annual rate of 11.3% per capita (in efficiency units) over the first 10 years.

Our second question concerns the effect of risk on growth. We estimate this by comparing three agent-specific growth rates, calculated by solving the model over a 50-year period (a) in the absence of risk, (b) when agents correctly perceive the distribution of shocks but do not actually experience any shocks, and (c) when agents experience shocks, drawn from the correctly perceived distributions. In case (c) the growth rates are calculated over the mean over 100,000 household-specific simulated paths. The differences between the growth rates identify the effect of risk on growth and allow a decomposition into the *ex ante* and *ex post* effects. We find a very strong effect of risk on growth (in the sense of transitional dynamics): the expected value of the capital stock at the end of a 50-year period is 46% lower than in the deterministic case. Notably about two-thirds of the reduction is accounted for by the *ex ante* effect. This suggests that methodologies which assess the effect of risk on growth on the basis of realised shocks only may seriously underestimate the effect.

The structure of the paper is as follows. The next section sets out the model. Section

3 describes the survey data. Estimation and simulation results are presented in Section 4. Section 5 concludes.

2 The Model

Our starting point is a Ramsey model: there is a single good, used for consumption, as a store of value and as a productive asset and agents optimize over an infinite horizon. Household h solves:

$$V(w(k_{h0}, s^y_{h0}, s^k_{h0})) = \max_{\{c_{ht}, k_{ht+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_{ht}) \quad (1)$$

subject to

$$\begin{aligned} c_{ht} &= w_{ht} - k_{h,t+1} \\ w_{ht} &= s^y_{ht} a_{ht} f_h(k_{ht}) + s^k_{ht}(1-\delta)k_{ht} \\ \text{for } t &= 0, 1, 2, .. \text{ and } k_{h0}, s^y_{h0}, s^k_{h0} \text{ given} \end{aligned}$$

where c denotes consumption, k the capital stock, w wealth on hand, u the instantaneous utility function, β a discount factor, and δ the depreciation rate. Households are indexed by h and t denotes time. We assume that $0 < \beta < 1$. Note that wealth on hand, w, is a function of the capital shock and the current shocks:

$$w_{ht} = w(k_t, s^y_{ht}, s^k_{ht}).$$

Unlike in the original Ramsey model, the household is exposed to risk: income $af(k)$ and assets $(1-\delta)k$ are both affected by shocks: s^y, s^k where $Es^y = Es^k = 1$. These income and asset shocks have both idiosyncratic and covariant components:

$$\begin{aligned} s^y_{ht} &= (\varepsilon^r_t)^{\pi_1} \varepsilon^y_{ht} \\ s^k_{ht} &= (\varepsilon^r_t)^{\pi_2} \varepsilon^k_{ht}. \end{aligned}$$

In our application we identify the covariant shocks ε^r with rainfall (denoted by the superscript r) and measure their importance by the elasticities π_1, π_2. We assume that the distributions of $\varepsilon_{ht} = (\varepsilon^y_{h,t}, \varepsilon^k_{h,t})$ and ε^r_t are lognormal, independent of each other and across time and that $\ln \varepsilon_{ht}$ has correlation matrix

$$\begin{pmatrix} a_1^2 & a_1 b_1 \\ a_1 b_1 & b_1^2 + b_2^2 \end{pmatrix}.$$

When the household decides on c_t and k_{t+1} both k_t and the realizations (s^y_{ht}, s^k_{ht}) are known. Future shocks are, of course, unknown but the household does know the distributions of these shocks.

If a solution exists the model can be written in recursive form as the Bellman equation:

$$V(w(k, s^y, s^k)) = \max_{\tilde{k}} u(w(k, s^y, s^k) - \tilde{k}) + \beta EV(w(\tilde{k}, \tilde{s}^y, \tilde{s}^k)) \tag{2}$$

with associated policy function

$$\varphi(w(k, s^y, s^k)) = \arg\max_{\tilde{k}} u(w(k, s^y, s^k) - \tilde{k}) + \beta EV(w(\tilde{k}, \tilde{s}^y, \tilde{s}^k))$$

where k and $\tilde{k}$ denote the capital stock at the beginning and the end of the current period and similarly s and $\tilde{s}$ denote current and future shocks. Equation (2) applies to every period so that time subscripts can be suppressed. Note that the policy function φ maps the current

(k, s^y, s^k) into $\tilde{k}$, next period's k. Hence φ can be seen as an investment function, giving k_{t+1} as a function of wealth on hand w_t (itself a function of the capital stock k_t) and the current shocks s_t^y and s_t^k.

A finite value function V which satisfies the Bellman equation (2) for all (k, s^y, s^k) is a solution to the original maximization problem (1) . V and φ satisfy the first order condition:

$$u'(w(k, s^y, s^k) - \varphi(w(k, s^y, s^k))) = \beta E V'(w(\tilde{k}, \tilde{s}^y, \tilde{s}^k)) w_k(\tilde{k}, \tilde{s}^y, \tilde{s}^k)$$

and the envelope condition

$$V'(w) = u'(w - \varphi(w))$$

where w_k denotes a partial derivative. The first condition equates the current marginal utility of consumption to the expected discounted value of a future extra unit of wealth on hand. The second condition states that the marginal value of wealth on hand (w) is equal to the marginal utility of the corresponding consumption $(w - \varphi(w))$.

It is typically *not* possible to solve the two conditions analytically. We approximate the solution to the Bellman equation by restricting and rounding outcomes to a finite grid of (w, k, s^y, s^k) values. The key to the solution of the discrete system is the observation that program value $V(\cdot)$ and the policy function $\varphi(\cdot)$ are functions of a single variable w only:

$$V(w) = \max_{\tilde{k}} u(w - \tilde{k}) + \beta E[V(\tilde{w}) | \tilde{k}],$$

where $\tilde{w} = w(\tilde{k}, \tilde{s}^y, \tilde{s}^k)$. With only finite sets of values for k, s^y and s^k, and $\tilde{w}$ rounded to the nearest grid value for wealth on hand, it is easy to calculate the probabilities p_{ij} =Prob$[w(k_i, s^y, s^k) =$

w_j] so that the equation to solve becomes

$$V(w_\ell) = \max_i u(w_\ell - k_i) + \beta \sum_j p_{ij} V(w_j), \text{ for all } \ell.$$

This equation can be solved by iteration, with arbitrary initial values for $V(w_\ell)$, $\ell = 1, 2, \ldots$. With $\beta < 1$ the iteration converges.[5] Given the solution $V(w_\ell)$ it is straightforward to derive the corresponding policy function $\varphi(w_\ell)$.

In this stochastic form of the Ramsey model a change in risk affects household behaviour in two ways. First, if the household perceives a change in the distribution of shocks (*e.g.* an increase in rainfall risk in the form of a mean preserving spread of the covariant shock ε_t^r) then it will, in general, adjust its policy function φ. It will then (for the same values of the capital stock k_t and the shocks s_t^y, s_t^k) choose different values of k_{t+1} (and hence c_t). This effect of a change in risk on the household's policy function we term the *ex ante* effect. There also is an *ex post* effect: the change in risk will affect the size of the realised shocks so that the optimal values of k_{t+1} and c_t (controlling for k_t) are affected, for a given policy function.

The effect of risk is illustrated in Figure 1. The curve labelled φ_{det} depicts the deterministic case (corresponding to the textbook Ramsey model), when the shocks are drawn from degenerate distributions concentrated at $s^y = s^k = 1$. Starting at the initial capital stock k_0 the agent's capital stock will converge to the steady state value k^*. Such 'transitional' dynamics may imply substantial growth. For example, in our empirical application (where each agent starts with a capital stock far below the steady state) it amounts to almost 5.75% growth per annum of k over a 20-year period.

The curve labelled $\varphi(w(k_t, 1, 1))$ depicts the case of optimal growth under uncertainty where shocks are drawn from distributions with positive variance. To facilitate comparison with the

[5] See Stokey and Lucas (1989, pp. 82–83).

deterministic case we have drawn the curve for the case when the household receives shocks in period t which happen to be equal to the mean: $s_t^y = s_t^k = 1$. In Figure 1 the effect of an increase in risk is to shift the policy function downward: the *ex ante* effect of risk on saving is shown as negative (hence $k^{**} < k^*$).[6] As is well-known the sign of this *ex ante* effect is ambiguous: while the danger of having to deal with a negative shock at $t+1$ provides an incentive to increase saving (the precautionary motive) this is offset by the danger that current savings have a low return if the agent experiences a positive shock in $t+1$. The net effect is determined by the characteristics of the distribution of shocks, the curvature of the utility function and of the production function. In our application it is negative, as in the Figure.

Now consider the *ex post* effect of risk, *i.e.* the effect on capital accumulation of the actual shocks s_t^y, s_t^k given the policy function $k_{t+1} = \varphi(w(k_t, s_t^y, s_t^k))$. The expected value of k_{t+1} (given k_t) is

$$E\varphi(w(k, s^y, s^k)) = \int_{s^k}\int_{s^y} \varphi(w(k, s^y, s^k))F(ds^y, ds^k)$$

where F is the joint distribution of the shocks. While the shocks have unitary mean the policy function is in general not linear in the shocks s^y, s^k. Hence the $E\varphi(w(k, s^y, s^k))$ curve does not coincide with the $\varphi(w(k, 1, 1))$ curve; in the Figure we have drawn it lower. As before this need not be the case but corresponds to our empirical application.[7]

It is tempting to consider the fixed point k^{***} as the expected long run value of the capital

[6] This is the case in our empirical application but the result is, of course, not general. Levhari and Srinivasan (1969) show for the special case of a CRRA utility function and a linear production function $f(k) = k$ that the effect of risk on savings is positive (negative) if and only if the degree of relative risk aversion R exceeds (is less than) unity. Hence in that model risk has no effect on the policy function if the utility function is logarithmic ($R = 1$). The same is true for the special case of equation (3) below, defined by $u(c) = \ln c$, $\delta = 1$ and $f(k) = k^\alpha$.
In models with given returns to assets the sign of the effect is determined by the curvature of the utility function: savings increase iff marginal utility is convex (concave) in consumption (e.g. Deaton, 1992, p. 29). When the return on investment is endogenous (as in our model) this condition is neither necessary nor sufficient.

[7] Our assumptions do not imply concavity or convexity of φ in the shocks s hence we cannot apply Jensen's inequality to establish the relative position of the two curves.

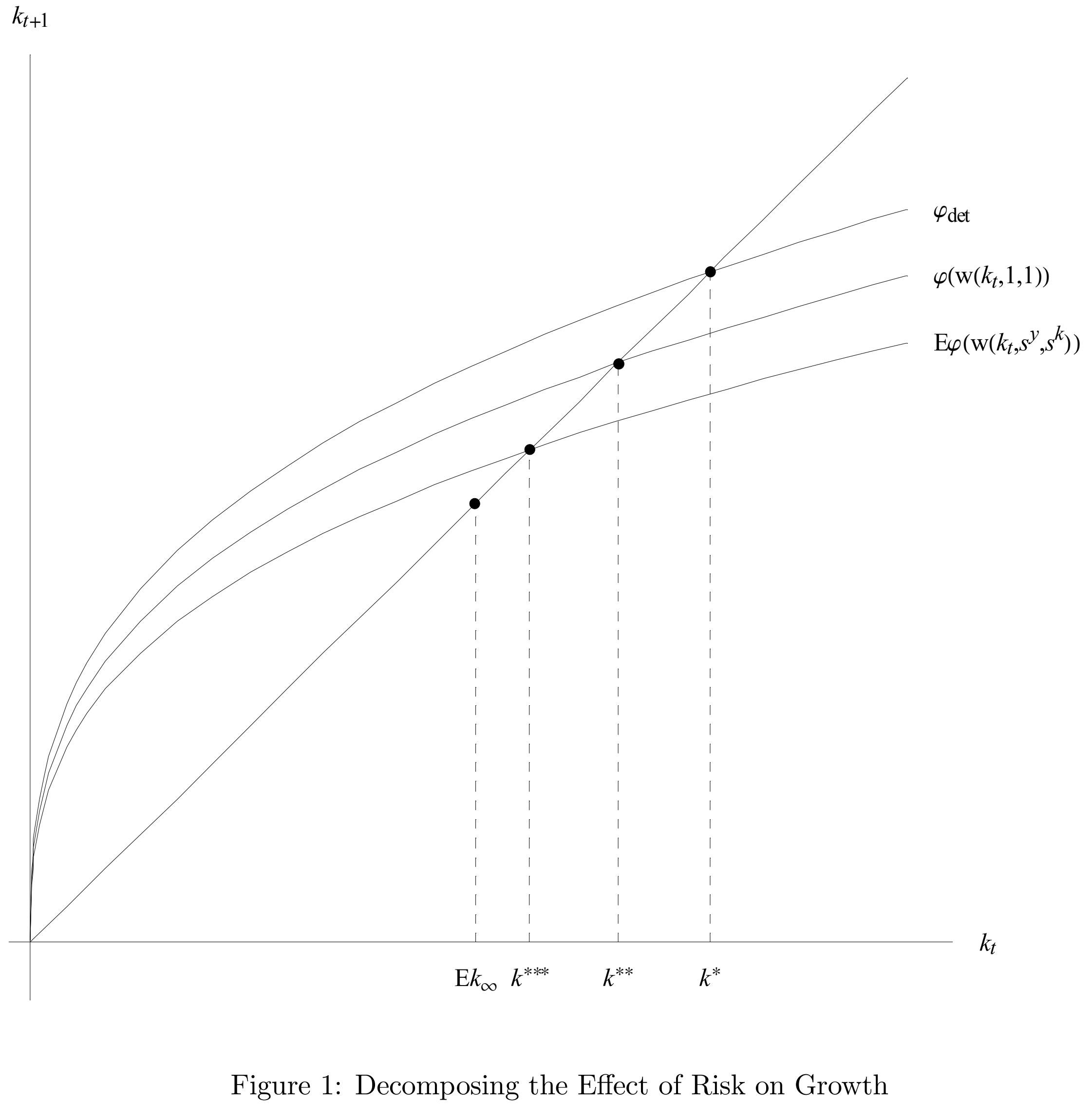

Figure 1: Decomposing the Effect of Risk on Growth

stock. However, this would be wrong: φ is a one-period ahead mapping so $E\varphi(w(k^{***}, s^y, s^k))$ does not represent the mean of the ergodic distribution. The individual accumulation paths follow a Markov process. Denote the ergodic distribution of k by $G(k)$:

$$G(k) = \int_{s^k} \int_{s^y} \int_{\{\varphi(x,s^y,s^k) \leq k\}} G(dx) F(ds^y, ds^k)$$

with mean Ek_∞. Note that

$$\begin{aligned} Ek_\infty &= \int_k kG(dk) = \int_k \int_{s^k} \int_{s^y} \varphi(w(k, s^y, s^k)) F(ds^y, ds^k) G(dk) \\ &= \int_{s^k} \int_{s^y} \int_k \varphi(w(k, s^y, s^k)) G(dk) F(ds^y, ds^k) \end{aligned}$$

$$< \int_{s^k} \int_{s^y} \varphi(w(Ek_\infty, s^y, s^k)) F(ds^y, ds^k) = E\varphi(w(Ek_\infty, s^y, s^k))$$

where the strict inequality follows from the fact that in our application $\varphi(w(k, s^y, s^k))$ is strictly concave in k. It follows that at $k = Ek_\infty$ the curve $E\varphi(w(k, s^y, s^k))$ lies above the 45° degree line. Hence $Ek_\infty < k^{***}$.

The Figure illustrates the two effects of risk on growth. The *ex ante* effect reduces the long-run value of the capital stock from k^* to k^{**}. The *ex post* effect further reduces (the mean of) the long-run value of k from k^{**} to Ek_∞.

In our application we allow for three types of heterogeneity: agents differ in their initial capital stock (k), in productivity (a) and in shocks (s^y, s^k). In section 4 we use the estimated model in simulation experiments to derive the distributions of k_h^*, k_h^{**}, and $Ek_{h\infty}$, where h denotes the agent. We then calculate the mean (across households) of the capital stock ($\overline{k}$) and interpret the growth from $\overline{k_0}$ to $\overline{k^*}$ as *potential* growth (that is the growth which would occur in the absence of shocks). Similarly, the growth from $\overline{k_0}$ to $\overline{k^{**}}$ is potential growth corrected for the *ex ante* effect of shocks and the growth from $\overline{k_0}$ to $\overline{Ek_\infty}$ incorporates both the *ex ante* and the *ex post* effects of shocks.

Example: the loglinear growth model

An interesting special case[8] arises if (a) capital depreciates fully ($\delta = 1$) so that we need to consider only income shocks, (b) the production function is Cobb-Douglas, $f(k) = k^\alpha$, and (c) the utility function has unitary relative risk aversion, $u(c) = \ln c$. Under these assumptions the policy function is

$$\tilde{k} = \varphi(w(k, s^y)) = \alpha\beta s^y k^a. \tag{3}$$

A striking implication of equation (3) is that the policy function is independent of the distribution of s^y. Hence an increase in risk (e.g. a mean preserving spread in the distribution of s^y) has (rather implausibly) no *ex ante* effect.

Except for the full depreciation assumption this model is identical to the Solow growth model with production function $s^y k^a$ and the savings rate equal to $\alpha\beta$. Substituting $y_t = s_t^y k_t^\alpha$, k_{t+1} for $\tilde{k}$ and k_t for k, and taking logs gives

$$\ln y_{t+1} = [\alpha \ln \alpha\beta + E \ln s^y] + \alpha \ln y_t + [\ln s_{t+1}^y - E \ln s^y].$$

Defining $\varepsilon_t = [\ln s_t^y - E \ln s^y]$ we obtain the canonical growth regression:

$$\ln y_{t+1} - \ln y_t = [\alpha \ln \alpha\beta + E \ln s^y] + (\alpha - 1) \ln y_t + \varepsilon_{t+1}.$$

It is instructive to consider the magnitude of the *ex post* effect in this special case where an analytical solution is feasible. Substituting A for $\alpha \ln \alpha\beta + E \ln s^y$ and z for $\ln y - A/(1-\alpha)$

[8] See e.g. Stokey and Lucas, 1989, section 2.2 or Obstfeld and Rogoff, 1996, section 7.4.

gives

$$z_t = \alpha z_{t-1} + \varepsilon_t = \sum_{i=0}^{\infty} \alpha^i \varepsilon_{t-i}.$$

If s^y is distributed lognormally with $\ln s^y \sim N(-\sigma^2/2, \sigma^2)$ so that $Es^y = 1$ and $E \ln s^y = -\sigma^2/2$ then the ergodic distribution of z is normal: $z_\infty \sim N(0, \sigma^2/(1-\alpha^2))$. Hence $\ln y_\infty \sim N(A/(1-\alpha), \sigma^2/(1-\alpha^2))$ and

$$\begin{aligned} Ey_\infty &= \exp\left(\frac{A}{(1-\alpha)} + \frac{\sigma^2}{2(1-\alpha^2)}\right) \\ &= \exp\left(\frac{\alpha \ln \alpha\beta}{(1-\alpha)} - \frac{\alpha\sigma^2}{2(1-\alpha^2)}\right) \\ &= y^* \exp\left(\frac{-\alpha}{(1-\alpha^2)} \frac{\sigma^2}{2}\right) \end{aligned}$$

where $y^* = (\alpha\beta)^{\alpha/(1-\alpha)}$ is the riskless steady state income level. Similarly, with $k^* = (\alpha\beta)^{1/(1-\alpha)}$ we find

$$Ek_\infty = k^* \exp\left(\frac{-\alpha}{(1-\alpha^2)} \frac{\sigma^2}{2}\right).$$

This may imply a substantial *ex post* effect of shocks on accumulation. For example, if $\alpha = .7$ and $\sigma = .5$ then $Ek_\infty = .84k^*$: the mean of the ergodic distribution of the capital stock falls 16% short of what it would be without shocks. If the initial position was, say, half of the deterministic steady state value ($k_0 = .5k^*$) then the effect of shocks would be to eliminate about one third of the growth (in terms of k) which would otherwise have occurred.[9] Hence even though in this special case risk has no *ex ante* effect, growth is quite sensitive to changes in risk.

[9] $(k^* - Ek_\infty)/(k^* - k_0) = (1 - .84)/(1 - .5) = .32.$

3 Data[10]

In the early 1980s the government of Zimbabwe undertook a land reform programme which involved resettlement of peasant farmers and landless labourers on land formerly owned by commercial white farmers. To be eligible for resettlement household heads had to married (or widowed), not in formal employment, and not younger than 18 years or older than 55. They were randomly assigned to resettlement schemes and had to renounce any claims to land elsewhere. Initial landholdings were identical: each settler was assigned 5 ha. of arable land. Resettled households could engage only in farming.[11]

In 1983/84 one of us, Bill Kinsey, surveyed a sample of about 400 of the resettled households. The sampling frame consisted of all resettlement schemes established in the first two years of the programme. The sample was restricted to the three most important natural regions (NRs) or agro-climatic zones. In Zimbabwe these are designated as NR II ("moderately high agricultural potential"), III ("moderate potential") and IV ("restricted potential"). One scheme was selected randomly for each zone: Mupfurudzi in Mashonaland Central (north of the capital Harare) for NR II, Sengezi in Mashonaland East (south east of Harare) for NR III and Mutanda in Manicaland (also south east of Harare) in NR IV. Stratified sampling was then used to select 20 villages within these schemes, and for each selected village in two of the areas a complete census was attempted, while in the third area 10 households were randomly selected from each village.

The households were first interviewed in 1983/84, shortly after their resettlement and re-interviewed first in 1987 and then annually since 1992. They have now been followed for two

[10] This section is based on Gunning *et al.* (2000) and Hoogeveen (2001).

[11] For the purpose of this paper this is very fortunate: while rural households in Africa typically engage in a range of non-agricultural activities the resettled households engage only in farming.

decades, making this the only long-running panel dataset in Africa.[12] The questionnaire now includes questions on crop production, sales, labour hiring, credit, food storage and antropometrics but initially the scope of the survey was more limited. The questionnaire includes detailed information on livestock ownership.[13] The questions were partly retrospective; for example, the first survey round in 1983/84 asked about initial livestock holdings in 1980. We have observations on k_{ht} for five points in time: 1980, 1992, 1993, 1996 and 2000. We have information on crop income for two points in time: 1993 and 1996.[14]

The empirical study of economic growth is riddled by measurement error problems (Bliss, 1999 and Carroll, 2001). We expect measurement errors to be less serious in our application. First, by using a micro data set we use a single method of measurement unlike growth regressions which have to rely on data collected by different institutions. Secondly, we can base our estimations on asset (livestock) rather than income data. While income and expenditure data are notoriously noisy the importance of livestock in most African societies suggests that it is measured fairly accurately.

[12] In this paper we use the NR I data only. There is remarkably little sample attrition. Approximately 90% of households interviewed in 1983/84 were re-interviewed in 1997. There is no systematic pattern to the few households that dropped out. Some were inadvertently dropped during the re-surveys, a few disintegrated (such as those where all adults died) and a small number were evicted by government officials. It should be noted that what is tracked is the land assigned to the original settlers, not the household itself: the household is retained even if its composition changes. The most important such change is the death of the household head, but even this is rare (Hoogeveen, 2001, pp. 45-46).

[13] The survey collected data data on various types of livestock (oxen, heifers, goats, etc.). These were aggregated using constant market prices.

[14] We are grateful to Trudy Owens who provided us with the aggregate crop income data which she constructed.

4 Estimation and Simulation Results[15]

In applying the stochastic Ramsey model to household data we make the following assumptions. First, household preferences are defined over per capita consumption. Secondly, the utility functions and discount rates are identical across households. Thirdly, the capital variable, k, is identified with livestock. Fourthly, households are heterogeneous in terms of productivity and the production function is linearly homogeneous in livestock and labour.[16]

We assume that the utility function is of the CRRA-type, $u(c) = c^\gamma$, with parameter $\gamma < 1$ (risk aversion) and that the production function is CES with parameters ρ and ψ:

$$f(k) = (1 + \psi(k^{-\rho} - 1))^{-1/\rho} \tag{4}$$

and that total factor productivity is a function of the household's size (n_h) and the highest educational attainment of its adult members (e_h):

$$a_h = (\alpha_0 + \alpha_1 n_h + \alpha_2 e_h). \tag{5}$$

Demographic change (birth, death and disability) in the context of Zimbabwean farmers is largely unplanned; we therefore incorporate it in the idiosyncratic part of the shocks.

We first estimate the function $a_h f(k_h)$.[17] The dependent variable, crop income, is available for two years, 1993 and 1996. Denote crop output $a_h f(k_{ht}) s_t^y$ by y_{ht}. Esssentially we estimate

[15] A detailed description of our estimation procedures is given in Elbers and Gunning (2003). This includes tests of robustness and regression diagnostics.

[16] This allows us to write the production function in the intensive form $f(k)$ as in section 2.

[17] The usual objection to direct estimation of the production function: that outputs and inputs are determined simultaneously does not carry much force in the present situation. Since households are exposed to shocks the optimal use of inputs is continually disturbed. Elbers and Gunning (2002) show that the model is essentially identified from income dynamics alone. This would suggest that it is unnecessary to estimate the production function separately. However, our two-step procedure is more practical since it reduces the dimensionality of the estimation problem.

the parameters ψ and ρ in (4) by non-linear regression of y_{h96}/y_{h93} on $f(k_{h96})/f(k_{h93})$.[18] The parameters of (5) are estimated by regressing $\ln(y_{ht}/f(k_{ht};\hat{\psi},\hat{\rho})$ on household size and education, allowing for household random effects. The results are shown in Table 1.[19]

Table 1: Production Function Estimates

Parameter	Point estimate	Standard error[20]	
α_0	1429	347	
α_1	-9.842	19.9	household size
α_2	54.038	34.3	education
ψ	0.5315	.153	capital share
ρ	-0.5394	1.01	

Table 2: Other Model Parameters

Parameter	Point estimate	Standard error	
γ	0.0082	0.0008	close to log utility
β	0.7490	0.0367	discount rate 34%
λ	0.1969	0.0064	conversion parameter
δ	0.1330	0.0070	depreciation rate
π	0.0330	0.0039	rain elasticity
a_1	0.2691	0.0568	σ of $\ln \varepsilon^y$
b_1	0.2394	0.0465	
b_2	0.1389	0.0145	
τ	0.0089	0.0220	rate of tech. progress

[18] Details are provided in Elbers and Gunning (2003).

[19] The results are virtually the same if no random effect is included: in that case the estimates for θ and ρ are 0.5340 and -.4713 respectively. Similarly, specifying a_h as a loglinear function of education and household size does not lead to different results.

[20] The standard errors of $\alpha_0, \alpha_1, \alpha_2$ are based on simulation to take into account the sampling variance of $\hat{\psi}$ and $\hat{\rho}$.

Note that the effect of household size is negative and that productivity is increasing in education. The estimated value of ρ implies a substitution elasticity of about 2. It does not differ significantly from 0, i.e. from the Cobb-Douglas case. However, in all our estimations we find a negative value and we have decided to retain the point estimate rather than imposing the Cobb-Douglas value of 0.

Before estimating the remaining parameters of the model we make a number of changes to the general model in equation (1). First, household total factor productivity, as estimated in table 1 has been renormalized so that average tfp in the sample is 5. Second, note that the production function has dimension 'income', whereas the capital accumulation equation has dimension 'cattle'. An extra parameter λ is used for the conversion. Instead of fixing to a particular base year value, we have decided to estimate it along with the other parameters. Finally, to capture trends in labour productivity and/or labour availability at the farm we have added a parameter τ which we refer to as 'rate of technical progress'. With technical progress, c, k and w refer to consumption, capital and wealth per efficient labour unit. The revised model becomes:

$$V(w(k_{h0}, s^y_{h0}, s^k_{h0})) = \max_{\{c_{ht}, k_{ht+1}\}} E_0 \sum_{t=0}^{\infty} (\beta(1+\tau)^\gamma)^t c^\gamma_{ht} \tag{6}$$

subject to

$$\begin{aligned} (1+\tau)k_{h,t+1} &= w_{ht} - c_{ht} \\ w_{ht} &= \lambda s^y_{ht} a_{ht} f_h(k_{ht}) + s^k_{ht}(1-\delta)k_{ht} \\ \text{for } t &= 0, 1, 2, .. \text{ and } k_{h0}, s^y_{h0}, s^k_{h0} \text{ given} \end{aligned}$$

The Bellman equation is changed accordingly.

There now are 10 parameters to estimate: γ, the parameter of the utility function; β, the discount factor; λ, the conversion parameter; δ, the rate of depreciation; π_1, π_2 the rainfall elasticity; (a_1, b_1, b_2) the parameters of the distribution of the idiosyncratic shocks ε^y and ε^k; and τ, the rate of technical progress. We estimate these parameters by Simulated Pseudo Maximum-Likelihood.[21] For a given choice of parameter values, a vector θ, we solve, for each household h, the Bellman equation, deriving the policy function φ_h. Using the policy function we generate paths of accumulation over the time intervals between dates for which we have observations: 1980, 1992, 1993, 1996, and 2000. Given rainfall, the shock component common to households, the changes in capital stocks between observation dates are statistically independent across households and time intervals. Household-and-interval specific means and variances are calculated by repeating the simulations sufficiently often, each time using independent household idiosyncratic shocks ε^y_{ht} and ε^k_{ht}. Assuming that the distribution of stock changes is lognormal, this is sufficient to calculate the likelihood $\mathcal{L}(\theta)$ of the observations for the given parameter vector θ. We use hill-climbing to maximize the likelihood with respect to the parameters. The results of this procedure are reported in Table 2. As before, the standard errors are based on simulation to take into account the sampling variance of the production function parameters.

The estimated value of γ is very close to zero, implying log utility and a unitary degree of relative risk aversion. The estimate of β suggests a high degree of impatience: a discount rate of 34%. The depreciation rate δ should be interpreted as a net rate, reflecting not just the aging and death of animals but also livestock births.[22] Our attempts to estimate the elasticities π_1 and π_2 separately suggested a high degree of correlation. We therefore imposed $\pi_1 = \pi_2 = \pi$. The estimated coefficient is very significant but remarkably low. The parameters of the distribution

[21] See e.g. Gouriéroux and Montfort (1996), section 3.2.

[22] Note that our estimates do not support the assumptions underpinning the canonical growth regression: notable our estimate of δ is far below unity.

of idiosyncratic shocks (a_1, b_1, b_2) are highly significant. The esimates imply that the standard deviation of $\ln s^y$ is equal to 0.27 and that of $\ln s^k$ 0.28. The correlation between the two types of shocks is .86. It should be noted that these estimates imply a very high level of idiosyncratic risk. For example, the probability that in any year a household experiences a shock of less than 10% of its income is only 28%. This is a much riskier environment than the US macrodata studied by e.g. Lucas (2003). The rate of technical progress is only imprecisely estimated. Earlier estimates (Gunning *et al.,* 2000) are higher but within the 95%-confidence interval around our point estimate of slightly below 1% p.a.

From the discussion of the model it should be clear that, before shocks in the initial period are known, a household's welfare (as measured by the expected value of the dynamic program after realization of the period-0 shocks) is completely determined by its total factor productivity and initial capital holdings. Figure 2 illustrates this by drawing contour lines of household welfare in the *(capital,tfp)* plane. The dots indicate observations for the initial year, 1980. A striking element in the figure is the steepness of the contour lines stressing the importance of cattle holdings for current household welfare. Similar plots for expected program value at more distant points in time show increasing importance of tfp: the contour lines rotate counter clockwise as one considers household welfare in future periods. Convergence and shocks eventually make initial cattle holdings irrelevant; long-run household welfare is determined by tfp only.

We now use the estimated parameters to simulate accumulation paths k_{ht} for a household with average tfp and initial capital stock equal to the sample average of 0.56.

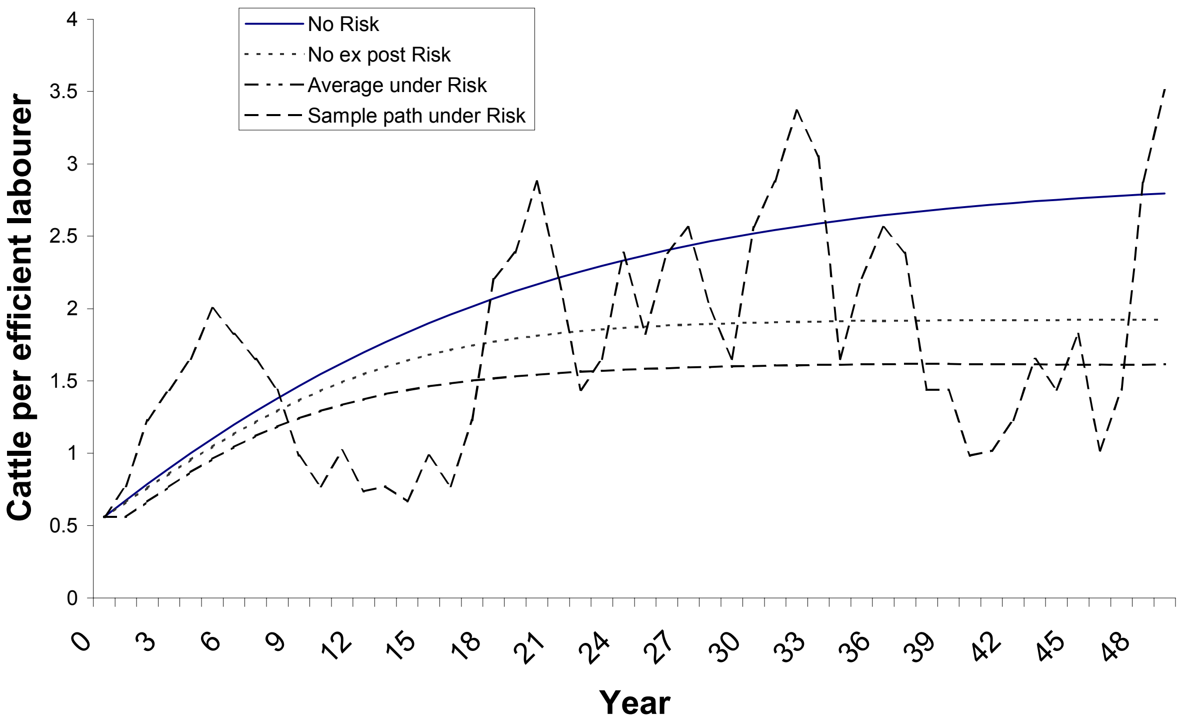

Figure 2: Growth and Risk (selected household)

Table 3: Decomposing the effect of risk on growth

Simulation type	$t = 0$	$t = 5$	$t = 10$	$t = 50$	**annual growth over first 10 years**	**annual growth over first 50 years**
Without risk	0.56	1.13	1.62	3.46	11.3%	3.7%
With risk (*ex ante* only)	0.56	1.02	1.44	2.33	9.9%	2.9%
With risk (including *ex post* risk)	0.56	1.00	1.37	1.88	9.5%	2.5%

Source: authors' calculations. Numbers in the table are cattle per efficient labourer, averaged across households.

Figure 2 shows four 50-year paths of livestock ownership (scaled by labour in efficiency units). The sample path represents a particular (randomly drawn) series of shocks.[23] Note that the shocks are very large: for much of the period asset ownerships changes by 50% in one or two years. The path denoted "average under risk" represents the mean over 100,000 such paths. This shows that in this average sense the household grows very rapidly, starting at 0.56 and reaching a level very close to the steady state value of (1.6) after about 20 years. The remaining two paths show the effect of risk. This is massive: risk reduces the mean of the ergodic distribution from 2.8 (the steady state value in the deterministic case) to 1.6.

Figure 3 shows a similar decomposition for the mean across households rather than for a selected individual household. The "average under risk" path is calculated as the mean across households, where for each household the average over 100 simulated paths is used. The results are summarized in Table 3. In the absence of risk the sample shows very rapid growth: in the first 10 years the per capita capital stock grows at over 11.3% per year. Under risk growth is substantially reduced. While in the deterministic case the per capita capital stock approaches a

[23] The 50-year period does not refer to a particular time period so rainfall data have also been simulated.

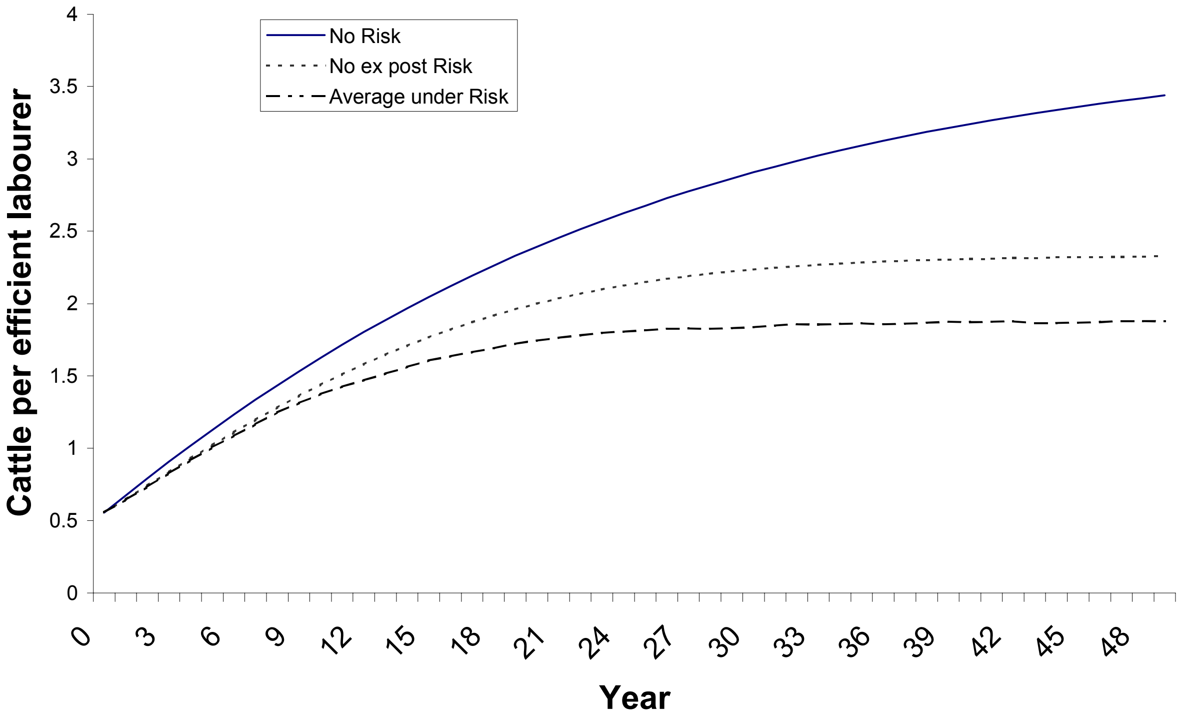

Figure 3: Growth and Risk (averages over all households).

steady state value of 3.46, the mean of the ergodic distribution is only 1.88. Hence risk reduces the long run value of the capital stock by almost half (46%). More than two-thirds of this is the *ex ante* effect. Studies which treat the mean over time of a household's consumption as the riskless counterfactual (e.g. Ravallion, 1988) would in this case miss most of the story: they would erroneously treat the "*ex ante* only" long run value (2.33) as the deterministic value (of 3.46). This is important for the study of chronic poverty which is often diagnosed as the result of poor endowment, as opposed to transient poverty which is seen as the result of risk. Our calculations show that risk has very substantial effect on mean consumption as well and hence is a structural determinant of chronic poverty. The implication is that policies which are designed reduce the exposure of household to risk or help households to cope with risk may well have the added advantage of substantially reducing chronic poverty.

Figure 4 makes the same point graphically. The points in the scatter diagram represent the households in the sample. The vertical axis measures total productivity as defined by equation (5) and scaled to an average of 5. In the long run TFP differences are the only reason for differences between households in the deterministic case (k^*). The horizontal axis measures initial capital holdings. The contour lines represented combinations which yield an equal welfare in the stochastic Ramsey model. It should be noted that these lines are very steep, indicating that TFP differences have very limited explanatory power in the short run when the purpose is to identify vulnerable households.[24]

The model has been explained and estimated under the admittedly extreme assumption that households do not share inputs and outputs in any way. Every household is essentially a single-agent economy. In particular, under the model's assumptions households do not pool idiosyncratic risk. A weak test of risk pooling can be developed as follows.

[24] This point is pursued further in Elbers and Gunning (2003a), where the model is used to improve methods of estimating vulnerability.

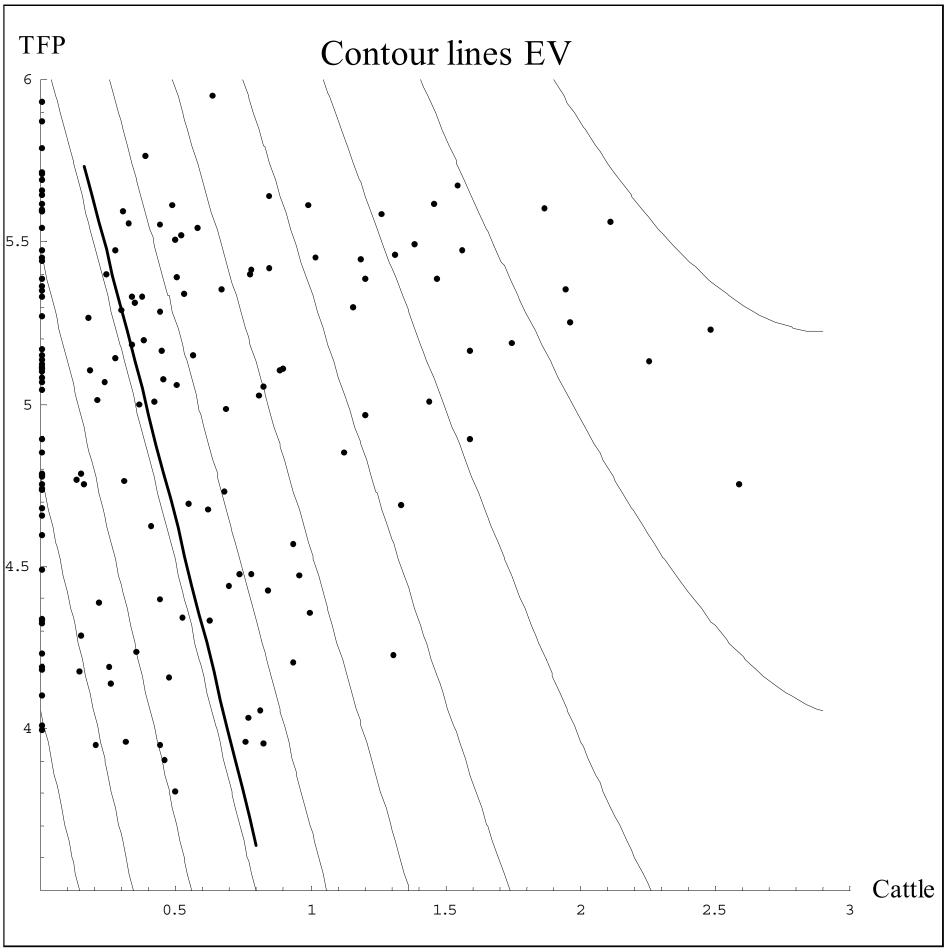

Figure 4: Expected one-year ahead program value (equation 1). Bold line separates 50% of sample. Dots indicate sample points.

From the individual household's perspective, risk pooling involves a change in the relationship between the expected value and the variance of idiosyncratic shocks. Consider partial insurance whereby shock s is replaced by insurance-modified shock $A + Bs$, where $-A$ can be interpreted as the 'insurance premium' and B as the degree of risk mitigation. Note that with log-normally distributed shocks s and $A, B \neq 0$, $A + Bs$ is not log-normally distributed so that the model would be misspecified for this type of risk pooling. If risk sharing is important, we would therefore expect a significantly improved fit by allowing A and B to vary. The signs of A and $B - 1$ would depend on whether the sample households' average position is 'long' or 'short'. The unrestricted estimation result give a value of A which is not significantly different from zero and a value of B which is not significantly different from 1. Also, the likelihood shows very little improvement as a result of dropping the restrictions $A = 0$, $B = 1$.[25] We conclude that in this sample there is very little insurance. This may reflect the particular nature of the sample consisting of farmers originating from various parts of the country.[26]

5 Conclusion

The paper makes, we believe, four contributions. *First*, we have proposed a framework for analysing the effect of risk on growth, distinguishing between the *ex ante* and *ex post* effects of shocks. Much of the theoretical literature makes restrictive assumptions, ruling out either the *ex ante* or the *ex post* effect. In empirical work the effects of shocks on growth are usually either assumed away (by modelling income as a stochastic but exogenous process) or only the *ex post* effect is identified. Our results suggest that this may seriously underestimate the welfare cost of risk and hence the potential benefits of policy interventions to reduce exposure to risk or

[25] Significance is determined on the basis of bootstrapping and simulation. For details see Elbers and Gunning (2003).

[26] In other circumstances our model must be adjusted to allow for risk pooling or factor market transactions.

promote insurance or credit.

Secondly, we have shown that it is feasible to estimate a stochastic growth model using simulation-based econometric methods. The simulation approach has the great advantage that it allows estimating a growth model in the form suggested by theory. For our purpose this was essential since a rigorous quantificaton of the effect of risk on growth is otherwise not possible. But more generally, the use of simulation methods in estimating growth models allows one to abandon the simplifications (e.g. linearization around the steady state) which are usually adopted in applied work to make the estimation problem tractable. Such simplifications have widened the gap between theoretical and applied work on growth. The paper suggests that the gap can be bridged through the use of simulation-based estimation methods. This would seem a promising approach both for macro research on growth (using country data) and for micro work (using household data, as in our Zimbabwe example).

Thirdly, turning from the methodology to the micro evidence of our subtitle, our application showed that for a sample of rural households in Zimbabwe (observed for almost a generation) risk has a very substantial effects on capital accumulation, incomes and poverty. We estimate that the mean of the ergodic distribution of the households' capital stock is 46% lower than it would be in the absence of risk. This result confirms the suggestion in the literature that self-insurance and other microeconomic responses to risk may substantially reduce growth. We have argued that part of the observed poverty which is classified as chronic reflects this impact of risk. This runs counter to the usual, descriptive methodology for decomposing poverty into structural and transient components and associating only the transient component with risk. Our results indicate that this approach may miss a large part of the effect of risk on poverty or vulnerability. We believe this is the first micro-based estimate of the empirical importance of shocks in the process of growth. Its magnitude in Zimbabwe suggests that policy makers may need to reconsider the balance between interventions which address "structural" determinants of poverty (e.g. raising productivity through education or improvements in farming practices) and interventions which reduce exposure to shocks or help households in risk management.

Finally, our empirical application recognised household heterogeneity in terms of initial assets, total factor productivity and exposure to idiosyncratic shocks. We found that with such heterogeneity cross-section data on household consumption are a very poor guide to household welfare. Households classified (*ex ante*) as vulnerable or (*ex post*) as poor may be better off in terms of expected utility than other households. This suggests a redirection of the research on poverty dynamics, taking into account the use of assets in households' optimal responses to risk.

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