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RESILIENCE: AN EVOLUTIONARY APPROACH TO SPATIAL ECONOMIC SYSTEMS

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ABSTRACT

The concept of resilience has received a great deal of attention in the past decades. Starting from the first fundamental definitions offered by Holling, Pimms and Perrings in an economic-ecological modeling context, the present paper explores the 'evolution' of the resilience concept – as well as related different measures – in both a continuous and discrete time setting.

From this perspective, the paper explores the relevance of the resilience concept in socio-economic systems, by focussing the attention on the relationships among resilience, transition dynamics and lock-in effects, in particular in the light of the dynamics of technological innovation diffusion and adaptive behaviour of firms. In this framework we will describe an empirical application, in which the resilience and dynamics of the West-German labour market will be investigated. This empirical illustration is offered by making use of an algorithm constructed for detecting Lyapunov exponents, so as to classify the resilience among employment sectors in our case study.

1 Setting the Scene

The analysis of dynamic systems has become a fashionable research topic in the past decade. Several disciplines, such as biology, ecology, sociology, and increasingly also economics, have in particular drawn attention to behavioural aspects of non-linear dynamic models, with reference to positive and negative feedback relations, synergetic responses and adaptive behaviour. The methodology of the social sciences has recently increasingly directed its attention to the analysis of complex systems, often characterised by non-linear dynamics and unpredictable time-paths and outcomes.

Furthermore, many dynamic systems are increasingly examined from a multi-layer and multi-actor perspective, and address issues like evolutionary behaviour, endogenous growth, resilience and the like. Particularly the concept of resilience is at present the subject of interesting and controversial debates, as is witnessed by the following statement: "The modern study of stability in ecology can be said to have begun with the appearance of 'Fluctuations of Animal Populations and a Measure of Community Stability', by R.H. MacArthur in 1955. Since the publication of this influential paper, ecologists have investigated the properties of a number of different stability and stability-related concepts; the concepts of persistence, resilience, resistance, and variability readily come to mind. Of these various concepts, the concept of resilience itself appears to have been rather resilient. Indeed, as Neubert and Caswell (1997) and others have noted, today there is a vast literature on resilience" (Batabyal, 1998, p.235).

In addition, recently several authors have argued that the concept of resilience is not only applicable to ecosystems, but also to socio-economic systems. Consequently, it seems worthwhile to create a perspective on the state of the art in this context (e.g., definitions and measurements of resilience) as well as on recent advances concerning the potential applications in the space-economy.

This paper is organised as follows. We will first offer in the next Section an introduction to the different definitions of resilience in the scientific literature (Section 2). Then, in Section 3 – after offering some methodological reflections on resilience, particularly, by analysing the two interpretations of engineering and ecological resilience – we will give a concise description of the possibility to employ the resilience concept in spatial-economic systems. Next, in Section 4 we will show an empirical illustration concerning the application of the (engineering) resilience concept in the context of regional employment in West-Germany. Finally, in Section 5 we will offer some retrospective and prospective remarks.

2 The Concept of Resilience: Definitions and Measurements

The concept of resilience is heavily based on the hypothesis that different states of a system involve different equilibria. In other words, it is assumed that the evolution of (ecological, economic, etc.) systems is formed by the 'switch' of these systems from one equilibrium-state (or stability domain) to another one. For example, ecologists believe that "change in most territorial systems is not continuous and gradual, but is punctuated by the sudden reorganisation of the stock resources. This often occurs after long period of apparent stability, and often after some ‘exogenous’ perturbation of the systems” (Perrings, 1994, p. 36).

In this context we may distinguish two different ways of defining resilience (see Perrings, 1998, p. 505): “One refers to the properties of the system near some stable equilibrium (i.e. in the neighbourhood of a stable focus or node). This definition, due to Pimm (1984), takes the resilience of a system to be a measure of the speed to its return to equilibrium. The second definition refers to the perturbation that can be absorbed before the system is displaced from one state to another. This definition, due to Holling (1973, 1986, 1992), does not depend on whether a system is at or near some equilibrium. It assumes that ecological systems are characterised by multiple locally stable equilibria, and the measure of a system’s resilience in any local stability domain is the extent of the shocks it can absorb before being displaced into some other local stability domain. Perturbation may induce the system to change from one attractor (stability domain) to another, or not. If not, the system may be resilient with respect to that perturbation.”

From these conceptual description it is clear that both definitions are based on the following assumptions:

a) the existence of local stability for these equilibria;
b) the existence of sudden ‘exogenous’ perturbations.

Given these hypotheses, *resilience* in both cases refers to the “capacity of a systems to retain its organisational structure following perturbation of some state variable from a given value” (Perrings, 1994, p. 30).

Clearly, these definitions have some intrinsic limitations, as nothing is said on the question whether:

a) the system is conceived of in continuous or discrete terms;
b) the system is deterministic or stochastic;
c) these definitions can be assumed also for systems with more than two state variables.

The *first definition* by Pimm, more ‘traditional’, focuses on the property of the systems near some stable equilibrium point. Perrings (1994) identifies this equilibrium point with a stable focus or nodus, which clearly belongs to a continuous system of two dimensions, as demonstrated by the Poincare’-Bendixon theorem (see Nijkamp and Reggiani, 1992).

Consequently, while on the one hand the measurement of Pimm resilience is certainly easier – from an empirical viewpoint – than Holling resilience, on the other hand it appears to be more ‘restrictive’, since it regards only the equilibrium points, rather than the stability domains or basins of attraction.

The *second definition* by Holling focusses on the property of the systems further away from the stable state (i.e., the size of the stability domain). The measure of resilience by this definition is the perturbation that can be absorbed before the system converges on another equilibrium state, by crossing an unstable manifold (again Perrings, 1994).

A clear representation of the measure of resilience according to Holling definition is again given by Perrings (1994), as displayed in Figure 1 below.

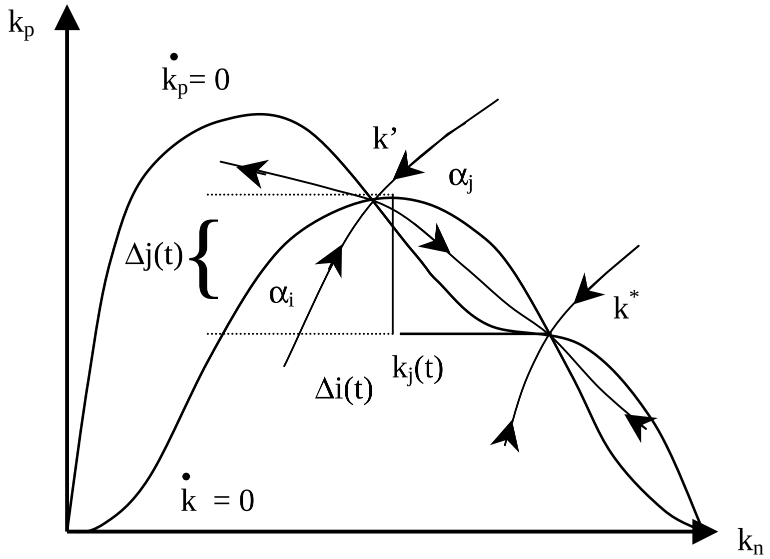

Figure 1: The measure of system resilience according to Holling. Source: Perrings (1994, p. 35)

In concise terms, in Figure 1 $k_p(t)$, $k_n(t)$, are the state variables, while **k*** represents the equilibrium point surrounded by its attraction basin with coordinates α_i, α_j, etc.. From this Figure we can immediately derive an interesting element, which differentiates the resilience definition from the usual stability definition, i.e. *that the resilience measure can be different in the same system, by varying according to the direction of the perturbation.*

Let us consider the formal definition given by Perrings here (1994, p. 37):

"The resilience of a system at some point in the basin of a locally stable equilibrium, **k***, with respect to change in any of the state variables of that system, is the maximum perturbation that can be sustained in those variables without causing the system to leave the α_j -neighbourhood of **k***."

The measure of the system's resilience in direction i or j is simply $\alpha_i - \mathbf{k_i(t)}$ or $\alpha_j - \mathbf{k_j(t)}$. It then follows that:

(a) a point in the system close to the boundary of the attraction basin is less resilient than one near the equilibrium point;

(b) the resilience is different for different directions of the perturbation;

(c) if the system loses resilience with respect to some perturbation (e.g., $\alpha_i - \mathbf{k_i(t)}$ is negative), then the system switches from one basin to another one (**k'** in Figure 1), via the manifold α_i.

From this analysis, it is then evident that the possibility of the system to remain in the unstable manifold is not considered by Perrings, at least not in the short run. This point has been also addressed by Batabyal (2000a) who has derived a measure of *short run or transient resilience*. This is undoubtedly an intriguing issue worth to be further explored.

In conclusion, from a *methodological viewpoint* both definitions are interesting and complementary. Particularly, we observe that on the one side (Pimm) resilience depends on the *strength of the perturbation*; on the other side (Holling) on the *size of the attractor/stability domain*. Clearly, a formulation of resilience integrating these two aspects is extremely important. A first interesting contribution in this respect has been given by Dalmazzone (1998), who integrates both these two definitions by considering the system no longer as deterministic, but as stochastic, subject to continuous disturbances of a Brownian motion type. Consequently, this author identifies as a measure of resilience the potential (the Hamiltonian equation) associated with the dynamic system involved.

In this context it should be noted that also various stochastic measures of resilience have been proposed in the literature as underlined by Batabyal (1998). This author also provides a theoretical and operational probability characterisation of ecological resilience (Batabyal, 1999a).

From an *empirical viewpoint* it is clear that difficulties emerge when we want to measure empirically the Holling resilience. How to measure the size of the basin of attraction? Perrings mentions the Lyapunov function in this respect; this is however, quite a critical issue. In this context Pimm definition is certainly more practical. However, the problem of measuring resilience – especially in socio-economic terms – remains a rather difficult one and certainly deserves further attention.

The previous analysis has brought to light the formal definitions and measurements of resilience, stemming from an ecologically-based framework. Recently several authors (see among others, Batabyal, 1998, 1999a, 1999b; Levin et al., 1998) have underlined that the concept of resilience can be used to effectively describe and study not only ecological systems, but also socio-economic systems, since all these systems can be viewed as only one system. In this context it may turn to be interesting to explore what resilience means for socio-spatial-economic systems, especially in the light of the stability paradigm. This will be concisely illustrated in the next section.

3 Resilience: A New Concept beyond Stability?

3.1 A New Perspective

In the previous sections resilience has been interpreted as a ‘measure’ of the system stability in the presence of external shocks, mainly related to the size of the basin of attraction surrounding the equilibrium point.

In the present section we will explore a different interpretation of resilience, viz., how resilience could transcend the ‘myth’ of stability, by addressing a new concept, i.e., the ‘myth’ of multiple stability. As a consequence, following Holling (1986), we will pinpoint – in this first step of our research – a ‘dynamic’ view of a system by considering:

1. a multiplicity of resting points instead of one (multi-equilibrium structures);
2. the evolution conceived of as the movement of the variables from one domain to another (and this behaviour may be discontinuous);
3. the irrelevance of the kind of equilibrium;
4. variability patterns in space and time which may change the systems parameters.

Clearly, a joint dynamic-stochastic perspective is more suitable for analysing the resilience notion. However, in this paper we will focus our attention on a dynamic-deterministic view, given also the deterministic character of our empirical experiments.

In the light of the four mentioned aspects, *resilience* points out *the 'possibility to change'*, while *stability* emphasises the '*impossibility to change'*. An idea, which was also put forward by Timmerman (1986, p. 444): "Equilibrium myths are way of picturing nature as *natura naturata* – i.e., nature as object, fixed or fixable. The myth of resilience, on the other hand, sees nature as *natura naturans* – nature naturing – i.e., nature actively altering and responding in various ways to predictable or unpredictable stresses". We may also refer here to Holling (1986, p. 297) for a similar view: "Stability, as here defined, emphasises equilibrium, low variability, and resistance to and absorption of change. In sharp contrast, resilience emphasises the boundary of a stability domain and events far from equilibrium, high variability, and adaptation to change".

By using the above two definitions as a starting point, it is then clear that the structures with multiple equilibria, that are able to maintain their structural features in the presence of parameters changes, become more relevant than the ones with only one steady-point.

In this context an interesting interpretation of resilience is underlined by Peterson et al. (1998).

a) *Engineering Resilience*: Rate at which a system returns to a single steady or cyclic state following perturbation (Holling, 1996). This interpretation is clearly according to Pimm's definition.
b) *Ecological Resilience*: The amount of disturbance that can be sustained before the system changes its structure by changing the variables and processes that control behavior (see also Gunderson and Holling, 2001).

If we then consider resilience in the light of these two above perspectives, we obtain a *clearer* view of the resilience concept and the related measurement in a deterministic system.

An interesting example in this respect may be offered by the *two-dimensional prey-predator system* – in a continuous setting – in its oscillating form (displaying in the phase-port the so-called neutral stability). This system shows a multiplicity of limit cycles, without changing its 'organisational structure', by varying the parameter values of the model. In this case ecological resilience is possible, but also engineering resilience can be identified here, since the system will return – in the short or long run – to the limit cycle attractor. In other words, both engineering and ecological resilience legitimate structures ending up with cycles, which have been always considered as a 'transition-phase' between stable and chaotic domains.

More ambiguous is the situation in which a system displays chaos. In this case the engineering resilience definition is not fulfilled (the system is far from the equilibrium steady-state), while the ecological resilience can occur here (instabilities can flip the system into another regime behaviour). Also recently Gunderson and Holling (2001, p. 21) emphasised this distinction between engineering and ecological resilience by underlining the two related contrasting aspects of stability: "One focuses on maintaining <u>efficiency</u> of function (engineering resilience). In contrast, the other focuses on maintaining <u>existence</u> of function (ecosystem resilience)."

Let us consider, for example, the well-known (Verhulst) logistic equation in discrete terms:

$$y(t+1) = r\, y(t)\,(1 - y(t))$$

by performing the related phase-space analysis (y vs. r). It is well-known that for the values $3 < r < 3.824...$ the logistic equation gives rise to cycles, before starting the chaotic period for r =

3.824… The engineering resilience, defined above, can be then identified with the stability period as well as with the period of cycles for the logistic equation under analysis. In addition, the ecological resilience can reflect, also in the chaotic period, the property of a system to persist, i.e. its capability of absorbing extreme waves of fluctuations.

We can now summarise the interpretation of the resilience concept, by using the discrete logistic equation as a concrete example (see Figure 2):

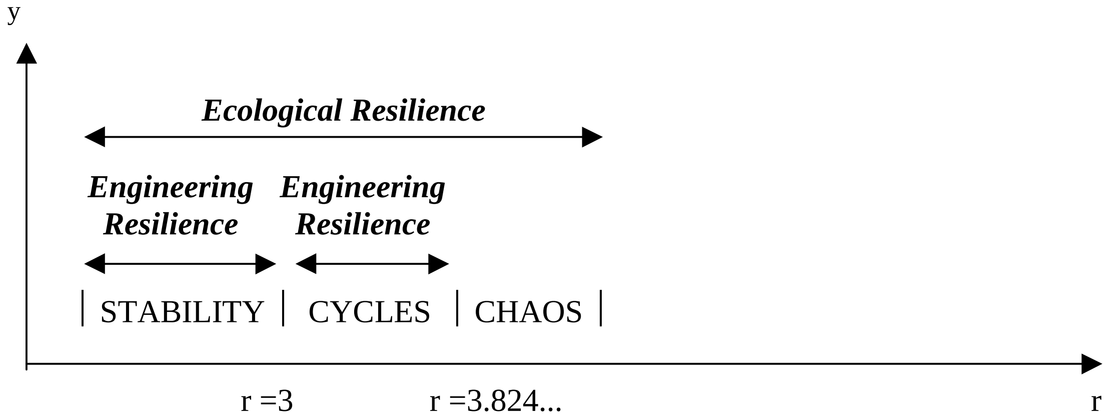

Figure 2. The engineering and ecological resilience for the logistic equation in discrete terms.

But how to measure the capacity of a system of displaying and/or facing this multi-equilibria region (resilience domain) before changing its structure? Clearly, difficulties in analysing resilience may emerge for high-dimensional dynamic systems or networks with a consequential number of parameters (the so-called ‘complex’ systems), and mostly on empirical studies.

3.2 Resilience in Spatial Economic Systems

It becomes clear from the previous considerations that both the engineering and ecological resilience concepts are extremely appealing for spatial economic systems, and useful for exploring the characteristics of their evolutionary stages.

In an interesting article, Levin et al. (1998) indicate that socio-economic and spatial systems have to be adequate on addressing new challenges and sudden qualitative shifts, while it is usually difficult to detect strong signals of change early enough to motivate and induce effective solutions. The authors state: “To deal with such problems, one needs a response system that is flexible and adaptive” (Levin et al., 1998, p. 224), and also: “In ecological and socioeconomic systems alike, human activities can lead to qualitative shifts in structure and function, evidence that the system concerned has lost resilience: that is no longer capable of absorbing the stresses and shocks imposed by human activity without undergoing a fundamental change involving loss of function and, often, loss of productivity. Resilience, the ability to experience change and disturbances without catastrophic qualitative change in the basic functional organisation, is a measure of the system’s integrity (Holling, 1973)” (Levin et. al, 1998, p. 224).

Perrings (1998) explores the relevance of resilience in joint economic-environmental systems, by emphasising the necessity of carrying out research in the area of *stochastic evolutionary theory*, by adopting renewal theory and /or Markov theory. This is undoubtedly a significant step forward, since, in this way, one focuses the attention also on the *transition dynamics* of stochastic multiple equilibrium systems, by overcoming the implicit drawbacks emerging from the Pimm and Holling definitions (see the previous section). In this context Perrings also makes a distinction between *absorbing states and transient states*: "… the greater the probability that the system in one state will change to some other state, the less resilient is the system in the first state. … Absorbing states are more resilient than transient states. It also follows that the system will evolve from less resilient transient groups of states to more resilient absorbing groups of states, and that their transition will be irreversible. Another way of describing the same things is that the system will get locked-in to a particular group of absorbing states. In its future evolution will then involve only transition between states in the group" (Perrings, 1998, p. 510).

It should be noted that a theoretical characterisation of resilience – based on renewal theory – has also been proposed by Batabyal (1999c), for the study of the optimal management of ecological-economic systems.

From the above considerations the following question clearly emerges: How much is resilience desirable for a socio-economic and spatial system? On the one hand, 'too much stability' or 'too strong absorbing states' could lock-in people or actors in pattern behaviours precluding any positive evolution, by reducing also their resilience (i.e., their capacity of absorbing shocks). Firms, companies, group styles, who are subject to small continuous changes and have to fight for survival, probably develop a 'better' resilience than very stable groups. On the other hand, 'very low resilience' may lead the system towards unstable states, likely irreversible.

In other words, it might be worth to analyse not only resilience, but also the *'optimum' resilience* for a certain group, company, or state of the system. In this respect, the Holling measure could certainly be useful in understanding 'how much' the system is able to absorb shocks. In addition, the conceptualisation of resilience as stationary probability – identified by Batabyal (2000b) – may offer optimal measures of resilience. Another way could be, by following Perrings (1998), to reduce the probability of transition into undesirable states or to increase the probability of transition into desirable states. Certainly, the measure of the 'optimum' resilience should deserve further attention. An interesting direction in this perspective, worth to be explored, is the concept of *entropy*, as indicated by Dalmazzone (1998). Here the author refers entropy to dynamic stochastic continuous systems. The following step in this respect could be then the 'search' for optimum entropy, by means of stochastic optimal control. Consequently, the use of 'entropy' for measuring resilience should be thoroughly investigated in future scientific work.

A still ongoing debate also concerns the relationships between resilience, density dependence, field effects and diversity, in order to evaluate the adaptive behaviour of a system. Mainly, *density dependence* and *field effects* are based, respectively, on the growth dynamics of distinct populations (depending on their size) and on the interdependence of people's preferences (concentration of activities). Both these two factors – at a high dimensional level – seem to lock-in the system into a particular technology or market choice, by reducing then the related resilience. Concerning *diversity*, one view sees complex systems less resilient than simple systems; the second view gives more attention to the number of 'alternative' populations which can take over a particular function when a system is perturbated (the so-called 'passenger species' which can take the role of 'drivers or keystone species'). The presence of these species insures then the resilience of systems, that is, their ability to adapt to new conditions (see van den Bergh and Gowdy, 2000; Perrings, 1998). It then follows that: "Evolving economic systems may also be described as being more or less resilient, in terms of being adaptable to economic changes as well as environmental changes, due to some redundancy of capacity and information present in them" (van den Bergh and Gowdy, 2000, p. 20).

Finally, it may be worthwhile to explore the link of the resilience concept with other concepts currently appearing in spatial economics, like the concept of 'consilience' (Wilson, 1998), 'persistence' (Batabyal, 2000c), 'adaptive learning', 'learning behaviour', 'path dependence', 'emergence' (see, e.g., Arthur, 1994a, 1994b; Batten, 2000; van Geenhuizen and Nijkamp, 1998; Holland, 1998; Kaufmann, 1995; Wilson, 2000), 'survival of the fittest' (Holland, 1975; Nijkamp and Reggiani, 1998; Reggiani et al., 2000), 'small-world phenomena' (Schintler and Kulkarni, 2000; Watts and Strogatz, 1998). In conclusion, more research is certainly needed, *theoretically* and *methodologically*, on the resilience concept, as well as on its integration with all the above mentioned approaches adopted in spatial economics. *Empirically*, applications of resilience measures to real case studies in economic science are still missing to our knowledge. In this context, given the feasibility of measuring the engineering resilience, empirical applications in this perspective are a first step towards the measurement of ecological resilience. An attempt in this respect is offered in the next section, showing an empirical illustration related to the dynamics of regional labour markets in West-Germany. Particularly, we use the method of Lyapunov exponents in order to identify non-resilient (unstable) trends in the context of engineering resilience. The remaining trends are then further considered by us in order to extract some 'typology' of 'resilience', by looking – empirically – at their return time to stability in the face of external shocks (Pimm's approach).

Before discussing our empirical illustration, we will then briefly depict the method of Lyapunov exponents used for identifying (non)resilient systems (in the engineering sense).

4 An Empirical Illustration: The Labour Market in West-Germany

4.1 Introduction

In this section we will adopt the technique of finding Lyapunov exponents in order to identify – among eleven sectors of employment in West-Germany – the ones more able to absorb shocks, i.e. the more resilient ones. In this contest a first screening will be carried out by the method of Lyapunov exponents, since this method may be used to classify and help determining the type of stability of an attractor.[1] In the previous section we argue that the occurrence of chaotic dynamics

[1] "In order for an attractor to exists the sum of the exponents must be negative, and for attractors other than an equilibrium point, at least one exponent must be zero. For non-chaotic attractors, all exponents must be negative for an asymptotically stable equilibrium point, while for an asymptotically stable *k*-torus, *k* exponents must be zero and the remainder negative." (Donaghy, 2000, p. 251)

precludes a system from being resilient in the engineering sense. Measuring Lyapunov exponents in a system was one of the first methods to detect chaotic dynamics, first in physics (see e.g. Wolf et al., 1985, and Holzfuss and Lauterborn, 1988) and later on in the social sciences (see e.g. Brock, 1986). The largest Lyapunov exponent being positive indicates the presence of a strange attractor, and, thus, chaos. Therefore, we use Lyapunov exponents to identify whether a system is resilient in the engineering sense.

4.2 Measuring 'Engineering Resilience' by Lyapunov Exponents

4.2.1 Measuring Lyapunov Exponents from a Time Series

Though it is relatively easy to compute Lyapunov exponents out of a known map, computing Lyapunov exponents out of empirical (time series) data may be somewhat more difficult and trickier. This subsection will briefly discuss an evolutionary algorithm (see Wolf et al., 1985), which is used to obtain the largest Lyapunov exponent from a time series. The next subsection will apply this algorithm in order to extract the largest Lyapunov exponents from German regional employment data.

First, for each embedding dimension m the time-series w_i will be used to form the residual-series $\{w_i^m\}$ or the so-called m-histories, for $i = 1,\ldots, (T - m + 1)$, where:

$$w_i^m = \{w_i, w_{i+1}, \ldots, w_{i+m-1}\}. \tag{1}$$

We will now concisely describe the computational algorithm. Start the algorithm to locate the nearest neighbour $w_j^m \neq w_1^m$, for j = 2,…, $(T - m + 1)$, to the initial m-history w_1^m. Now let $d_1^{(1)}$ be the smallest positive distance, set t_1 equal to j and let, for a positive integer q, $d_2^{(1)}$ be:

$$d_2^{(1)} = \left\| w_{t_1}^m - w_1^m \right\|$$

and (2)

$$d_2^{(1)} = \left\| w_{t_1+q}^m - w_{1+q}^m \right\|$$

Now store $g_1(q) = d_2^{(1)} / d_1^{(1)}$ and define q as the evolution time. This ends the first iteration. For finding the new m-history $w_{t_2}^m$ near $w_{t_1+q}^m$, we have to construct a penalty function, which should minimise the angle between the new m-history and the last founded m-history, compared with the m-history of the status of the evolutionary process. So, in formal equations, we store for iteration k,

$$d_1^{(k)} = \left\| w_{t_k}^m - w_{1+(k-1)q}^m \right\|, \qquad d_1^{(k)} = \left\| w_{t_k}^m - w_{1+kq}^m \right\|$$

and

$$g_k(q) = \frac{d_2^k}{d_1^k}. \tag{3}$$

The point t_k in (3) may then be found by minimising the following penalty function:

$$p_{k,m,q,\hat{w}} \equiv \left\| w_{t_k}^m - w_{1+(k-1)q}^m \right\| + \hat{w} \left| \theta\left(w_{t_k}^m - w_{1+(k-1)q}^m , w_{t_{k-1}+q}^m - w_{1+(k-1)q}^m \right) \right|,$$

where $\hat{w}$ is a chosen penalty weight on the deviation of the angle θ from zero, and where t_k is subject to $w_{t_k}^m \neq w_{1+(k-1)q}^m$. Just continue this procedure until $k = K$, where K solves $\max\{k \mid 1 + kq \leq T - m + 1\}$. Finally, to calculate the empirical Lyapunov exponent $\hat{\lambda}_q$, we set:

$$\hat{\lambda}_q = \frac{1}{K} \sum_{k=1}^{K} \frac{\ln(g_k(q))}{q}. \tag{4}$$

Wolf et al. have shown in 1985 that the empirical $\hat{\lambda}_q$ will converge to the real largest Lyapunov exponent, when $T \to \infty$ and $m \to \infty$. Numerical experiments have confirmed that this algorithm indeed identifies the largest Lyapunov exponent. However, we note that these experiments usually consisted of more than 10,000 replications, whereas empirical socio-economic applications usually deal with less than 1,000 observations.

We can make the algorithms described above clear by using Figure 3:

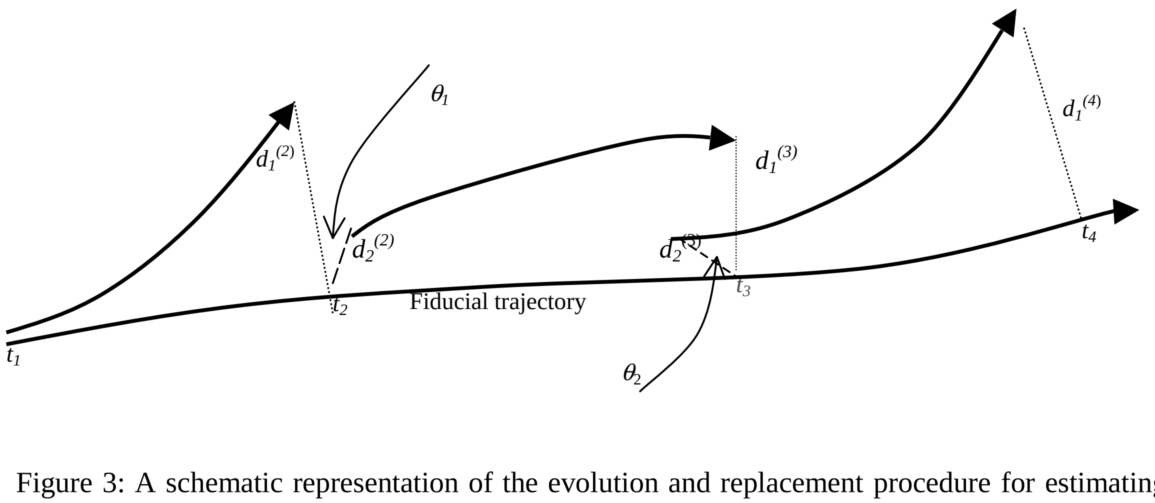

Figure 3: A schematic representation of the evolution and replacement procedure for estimating the largest Lyapunov exponent. The largest Lyapunov exponent is computed here from the growth of length elements. When the length of the vector between two points becomes large, a new point is chosen near the reference trajectory, minimising both the replacement length $d^{(k)}$ and the orientation angle θ (see Grassberger and Procaccia, 1984).

In Figure 3 one can see the working of the algorithm schematically. Note that Figure 3 resembles both discrete and continuous cases. The length of a vector between two points in time is determined by the evolution time, and the angle θ should keep the successive vectors as close to the fiducial trajectory as possible.

4.2.2 Empirical Illustration: The Labour Market in West -Germany

In order to illustrate the algorithm above we will apply the concept of engineering resilience to the case of the German labour market. We have a rich data set, which consists of the absolute number of employment in the 327 regions (the so-called Kreisen) in the former BRD. We have information over these regions for 11 years (1987-1997) and these employment figures are subdivided in 11 sectors (see for a list of sectors Table 1).

Figure 4 gives the time paths of the absolute figures of employment in Germany for the 11 sectors.

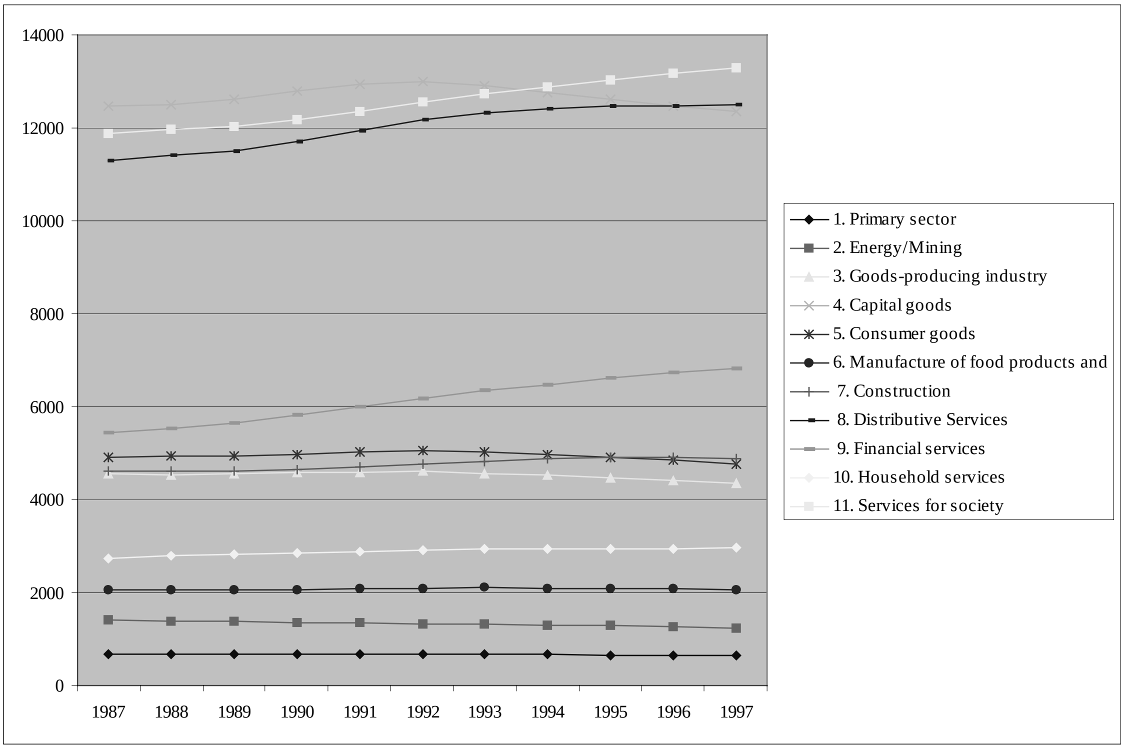

Figure 4: The time paths of German employment in the various industrial sectors between 1987 and 1997.

Figure 4 shows us that the main characteristic of the German labour market between 1987 and 1997 is that it is rather stable across the industrial sectors. This could be expected because of the nature of the German labour market. According to most scales the German labour market is rather corporatistic, has a centralised union wage bargaining and has an inflexible and protected labour force (see, for a characterisation of the German labour market from a corporatistic perspective, Teulings and Hartog, 1998).

On the other hand, one may expect to find some dynamics on the German labour market, not merely because of the wide-scale introduction of new technologies in the 90s, but mainly because of the German unification in 1990 and especially the subsequent mass migration from former East-Germany to West-Germany. Nevertheless small changes are found in Figure 4 and the only sectors that grew in the observed 11 years are distributive services, financial services and services for society.

Even though it seems that there is little dynamics on the German labour market, employment changes on the regional scale may be more fluctuating. Therefore, we have calculated the Lyapunov exponents for all 11 sectors, where each w_i represents a vector of regional employment for each sector. The distance between w_i and w_j is then easily calculated as:

$$\left\| w_i - w_j \right\| = \sqrt{\left(w_{i1} - w_{j1}\right)^2 + \cdots + \left(w_{ir} - w_{jr}\right)^2 + \cdots + \left(w_{iR} - w_{jR}\right)^2},$$

for each $i,j \in \{1,\ldots,T\text{-}m+1\}$ and $r \in \{1,\ldots,R\}$. The lack of temporal observations is made up by the large amount of regional observations. If we consider our data as an extended panel data set[2], then we may even say that the amount of data is rather sizeable. Unfortunately, we cannot say much about the absolute value of the Lyapunov exponent, because we are not able to compute standard errors and because of the small size of the data set compared to the controlled experiments in physics.[3] Nonetheless, comparing between sectors could be valuable and may give additional insights into the dynamics of the German labour market. In Table 1 we present the empirical Lyapunov exponents for different m-histories for the various sectors.

Table 1: Lyapunov exponents of employment trajectories of the different industrial sectors in Germany (1987-1997).

	m-histories		
Industrial Sectors	$m = 2$	$m = 3$	$m = 4$
Primary Sector	0.03	0.02	0.04
Energy/Mining	-0.09	-0.02	0.00
Goods-Producing Industry	0.12	0.15	0.16
Capital Goods	0.02	-0.03	0.03
Consumer Goods	0.10	0.08	0.08
Manufacture of Food Products	0.07	0.02	-0.04
Construction	0.09	0.07	0.06
Distributive Services	0.06	0.05	0.03
Financial Services	-0.02	-0.05	-0.12
Household Services	0.03	0.00	-0.04
Services for Society	0.01	-0.04	-0.09

[2] Panel data usually is two-dimensional (temporal and another dimension), while our data set is three-dimensional (temporal, sectoral and regional).

[3] An even more troublesome feature of this method is the fact that we do not know its power against *other* kinds of dependencies. From Monte-Carlo simulations performed on other tests designed to detect non-linearities and chaotics we know that these are sensitive to almost all kinds of dependencies. However, if we assume that all sectors are affected equally by other forms of inter-temporal and inter-sectoral dependencies, then relative comparisons are still allowed.

From table 1 we observe that the Lyapunov exponents seem to depend heavily on the chosen m-history, a feature which is known from the literature. Ideally, *m* should converge to ∞, however, at a much slower rate than *T*. Therefore we will set *m* at 2.

As mentioned before, we cannot say much about the absolute value of the Lyapunov exponents, especially with the low numbers of *m* and *T*, but we are able to compare the relative values. From Table 1 it seems that the goods-producing industry, the consumer goods, and to a lower extent construction are the less stable sectors, whereas financial services, household services, services for society and energy/mining seem to be the more stable or resilient (in the engineering sense) sectors across time. Not surprisingly, the former group of sectors is more dependent on the business cycle, whereas the latter group is not. Note that stable behaviour could also contain continually up- or downward movements, as is the case for financial services and energy/mining respectively.

To gain a better understanding in what Table 1 actually displays, we have drawn the growth rates of the West-German employment in the various sectors in Figures 5 and 6. In Figure 6 we see that the more unstable sectors have all experienced growth rates that are both positive and negative. In Figure 5 this is only true for capital goods and manufacture of food products. The other sectors have all more or less steady growth paths, or are at least only positive or negative. Furthermore, note that there is a break in all growth paths from the fourth or fifth observation; this is around the year of the German unification. This is probably the shock that the resilient systems have absorbed, since they returned to stability after a certain period; on the contrary, the same shock seems to have led the non-resilient systems to instability.

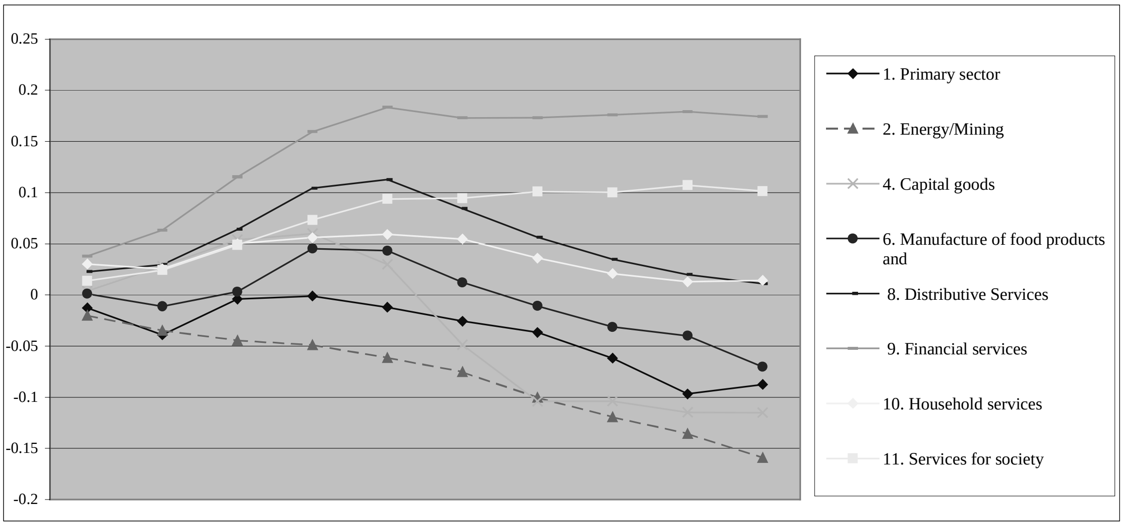

Figure 5: The growth rates of employment in selected sector in Germany (1987-1997).

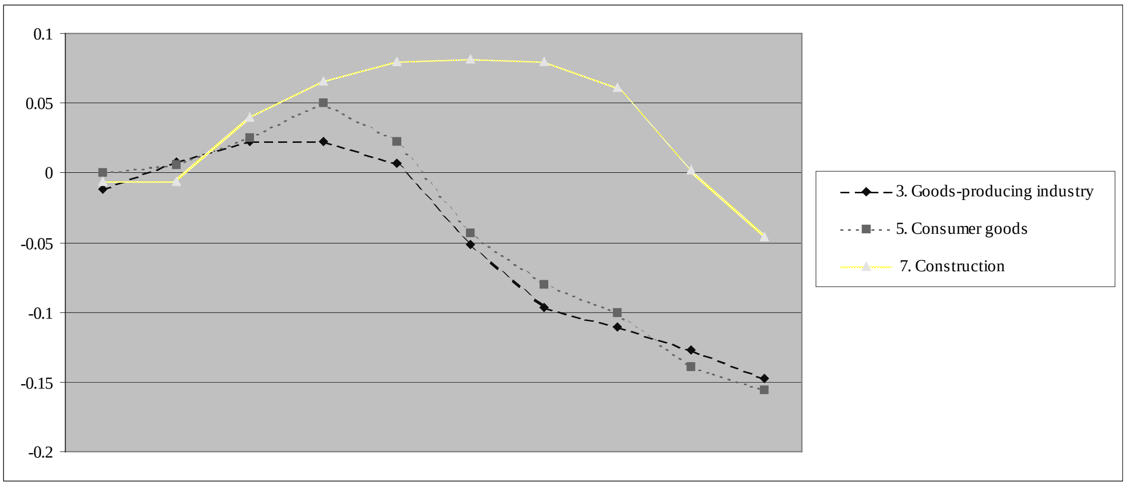

Figure 6: The growth rates of employment in selected sector in Germany (1987-1997).

Also notable is the large similarity in growth paths between consumer goods and the goods producing industry, reflecting the complementary nature of these two industries. The absolute value of employment decline for especially consumer goods and the goods producing industry is rather high and equals even that of the energy/mining sector, an industry which is rapidly fading away in the last three decades in Western Europe.

Based on this scarce evidence, we may say that the consumer goods and the goods producing industry (and to a lesser account construction) seem to be less resilient (in the engineering sense, as underlined in Section 3.1) than the other sectors, but we must emphasise that the result are still preliminary and, until we have standard errors, statistically not precise.

Another way to compare the Lyapunov exponents, is the comparison between groups of regions. The 327 regions in West Germany may be subdivided into groups, depending on their urbanisation rate and the type of cities they contain. Table 2 displays a regional decomposition of the Lyapunov exponents for the 11 different sectors. Again we see relatively high Lyapunov exponents for the goods producing industry, consumer goods and construction, but in the same time we see regional instability emerging. Normally, we will not observe these patterns, because the large majority of German employment is working in regions with urban agglomerations. However, we can now observe relatively large dynamics of energy/mining in regions with rural features. Furthermore, it seems that employment in the primary sector is also rather unstable in regions with an urban agglomeration. That both sectors are in a decline is widely known, but the decline is likely not linear. Remarkable is also the relative stability of the distributive services, financial services, household services and services for society, also because these sectors are the largest employers in West-Germany (Figure 4). This may raise the question whether stability or resilience is not only dependent of the intrinsic nature of a certain sector, but also upon the size of that sector (see also Section 3). After all, a large sector is by its very nature better equipped to absorb external shocks than smaller sectors.

Table 2: Lyapunov exponents (*m* =2) for different types of regions, for the 11 industrial sectors.
The numbers in the heading represent the industrial sectors below.

		1	2	3	4	5	6	7	8	9	10	11
A. Regions with urban agglomeration	**1. Central cities**	0.09	-0.17	0.17	0.05	0.08	0.11	0.12	0.10	-0.05	0.02	0.04
	2. Highly urbanized districts	0.03	0.08	0.15	0.01	0.20	0.19	0.07	0.02	0.09	0.04	0.01
	3. Urbanized districts	0.03	-0.05	0.13	0.01	0.12	0.11	0.14	0.00	0.05	0.01	-0.01
	4. Rural districts	0.12	0.10	0.07	-0.05	0.13	0.14	0.04	0.00	0.07	-0.01	-0.02
B. Regions with tendencies towards agglomeration	**5. Central cities**	0.04	0.22	0.40	0.06	0.11	0.12	0.13	0.06	0.03	-0.08	0.02
	6. Highly urbanized districts	0.07	0.03	0.09	-0.01	0.07	0.03	0.03	0.00	-0.04	0.07	0.01
	7. Rural districts	-0.02	0.07	0.21	-0.02	0.06	0.07	0.10	0.00	0.08	0.05	0.03
C. Regions with rural Features	**8. Urbanized districts**	0.06	0.25	0.08	-0.05	0.08	0.12	0.10	0.01	-0.01	0.04	0.02
	9. Rural districts	0.06	0.20	0.12	0.04	0.07	0.04	0.07	0.03	0.02	-0.02	0.03

1. Primary sector
2. Energy/Mining
3. Goods-producing industry
4. Capital goods
5. Consumer goods
6. Manufacture of food products and
7. Construction
8. Distributive Services
9. Financial services
10. Household services
11. Services for society

5 Conclusions

In this paper we have explored the concept of resilience, stemming from ecological sciences, in the context of spatial economic systems.

At first glance, it appears that resilience, conceived of as the capacity/ability of the system in absorbing shocks without catastrophic changes in its basic functional organisation, is a potentially effective tool in understanding the evolutionary paths of complex spatial systems. As such, this concept has a great potential also in other, non-ecological systems, in particular socio-economic systems. Moreover resilience seems to offer a new approach to evolutionary networks, where systems able to shift – in the presence of external shocks – to new equilibria become relevant. But there also limitations in terms of operationalisation and measurement. And therefore, an integration of this concept with other recently developed, novel concepts in economics, like emergence, consilience, adaptive behaviour, path analysis, selection behaviour, small world phenomenon, might certainly be fruitful in this respect.

Empirically, the measure of resilience is still a rather critical issue. Several measures have been proposed in the literature (e.g., time of recovering from shocks, size of the stability domain absorbing the shocks, potential, entropy, transition probabilities, etc.,); however, they still remain at a formal-theoretical level. In this framework we conclude that the two different interpretations (engineering and ecological resilience) advocated in the literature are certainly useful for grasping and deepening the resilience concept.

In the context of measuring resilience we have investigated the possibility of using the method of Lyapunov exponents in order to identify whether (non-)resilience exists in employment concerning the regional labour market in West-Germany.

Our preliminary findings, although satisfying in detecting the more resilient sectors, underline the difficulties in implementing, in practice, a resilience measure. More research would have to be done to explore this issue. Particularly the relationships – and related measures – between resilience and sustainability, in both a deterministic and a stochastic perspective, are worth to be further investigated, also in the light of institutional and policy landscapes.

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