The Design and Production of new Retirement Savings Products:
A Note

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Introduction and summary

With the population in the U.S. and other countries ageing rapidly, the burden of future pension liabilities is ever increasing. In recent years, governments and companies have become much more aware of the inherent risks that are involved. As a consequence, there is a worldwide tendency to shift from defined benefit pension plans to defined contribution plans. The implications for employees are far-reaching: under a defined contribution plan, the employee bears the investment risk: the level of his pension depends on the return on his investments. Under a defined benefit system, the level of pensions is fixed and the sponsor (in many cases the employer) bears the investment risk: the premiums required to fund the pension depend on the return on investments. In this journal, Bodie and Crane (1999) (BC) recognize that the transfer of investment risk from employer to employee calls for easy-to-implement investment strategies that correctly reflect the trade-off between the risk of a poor pension and the joy of a sumptuous pension. They compare investments in traditional equity and bonds with investments in TIPS (inflation linked bonds) and equity with a protective floor. Their results suggest that a series of investments in a product with a protective floor have a much higher chance of reaching a specified retirement income level than investments in a mixture of equity and fixed income securities.

We replicated their analyses but obtain different results: based on their simulation framework, the protective floor strategies do not compare favorably to traditional investment policies. Whether one should prefer a protective floor strategy or a traditional equity and bonds strategy largely depends on the choice of risk-reward framework and prevailing market data.

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Bodie and Crane: setup and risk-return framework

BC analyze the performance of altemative investment strategies in a simple and appealing defined contribution framework. They **focus** on a hypothetical employee, aged 25, who still has 40 years of employment to come. At the end of each year, he invests a fixed percentage of wages in a retirement income fund. In the final year of employment, the hypothetical worker eams USD 50,000; his target retirement income equals 60% of his final wage, or USD 30,000. During the years of employment, wages are supposed to rise at the rate of inflation. The question is how the employee should invest his periodic pension contributions.

Investment Opportunities	return assumptions		
TIPS	total return (μ)	:	10%
S&P 500 index	dividend yield (q)	:	3%
One-Year Protective Floor	volatility (σ)	:	20%
Five-Year Protective Floor	inflation	:	4%
	nominal interest rate :		7%

Table 1: investment opportunities and the assumptions on their returns.

BC evaluate various investment strategies using two criteria: expected pension and the probability of achieving a target pension. Table 1 lists their set of investment opportunities and the assumptions they make regarding their future returns. Table 2 shows the main results reported in Bodie and Crane, in terms of risk and return at the retirement date.

Investment Strategy	Average Value ($000)	% of Results Below Target
100% TIPS	$446	0
100% S&P 500	856	34.3
60% S&P 500/ 40% TIPS	654	28.6
One-Year Floor	581	10.8
Five-Year Floor	950	11.6

Table 2: Results reported by Bodie and Crane

Based on the results in table 2, BC suggest that "avoiding the downside appears to have substantial benefit". Relative to an investment in 100% equity, the five-year protective floor strategy results in a substantially lower probability of not meeting the income target, while yielding a higher expected retirement income. From table 2, it is not difficult to see that for virtually every traditional asset mix, there exists an asset allocation involving a protective floor such that both the expected pension and the probability of achieving the target pension are higher.

What theory tells us

Before we present our simulation results, we provide more intuition for the Bodie and Crane results by a short analysis of the protective floor strategies. We make two simplifying assumptions:

- Inflation is constant at 4%. It can easily be checked that, given the low volatility of inflation, this does not affect the outcome in a material way.
- Instead of having periodic investments, we analyze a five year buy and hold investment. Table 2 shows that the total expected return on all periodic investments in the five year protective floor strategy exceeds the total expected return on the 100% equity product. This implies that the expected return on a single payment of the protective floor strategy over a five year horizon also exceeds the expected return on a single payment of an equity investment over a five year horizon.

Assuming that all dividends on equity are immediately reinvested, a USD 100 investment in equity grows in expectation to USD 161.05 in five years. The expected return on the five year floor product can also be derived: In order to guarantee the real value of the initial investment, we need to invest

$$\frac{100}{(1 + 3\%)^5} = 86.26.$$

in TIPS. The remainder, $100 - 86.26 = 13.74$, can be invested in call options on the S&P 500 index. Given the simulation framework of BC, the call options should have a strike equal to

$$100 \times (1 + 4\%)^5 = 121.67.$$

The Black-Scholes price of this option is 15.37, which leads to a participation rate of

$$\frac{13.74}{15.37} = 0.894$$

BC also report this participation rate. The expected final value of this option can be computed analytically[6]. Given an expected total return on the S&P 500 of 10% and a divided yield equal to 3%, the expected final value of the option equals 32.46. It follows that the expected final value of the portfolio of TIPS and call options is:

$$121.67 \quad +0.894 \times 32.46 \quad = \quad 150.68.$$

This corresponds to an annualized expected return of 8.5% • considerably less than the 10% on the S&P 500 index. To obtain an expected return equal to that of the S&P500, one has to buy 1.21 call options instead of 0.894. This is possible only if call options can be bought at less than 75% of their Black Scholes price. This result is clearly in conflict with BC who report an expected value for the five year protective floor strategy *higher* than the expected return on the S&P500 (table 2).

In table 3, we report simulation results consistent with the theoretical analysis above. There is a marked difference with the results in Bodie and Crane, reported in table 2. The most important outcome is that relative to 100% equity, the expected pension of the five year protective floor strategy is *lower*. In terms of risk, the two strategies look more similar now.

Investment Strategy	Average Value ($000)	% of Results Below Target

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[6] Using the Black-Scholes assumptions, it is easy to derive the following expression for the expected value C of a call option with strike K, time to expiry T, volatility σ and continuous dividend yield q:

$$C = e^{(\mu - q)T}\Phi(d_1) \quad K\Phi(d_2),$$
$$d_1 = \frac{\ln\left(S/K\right) + \left(\mu - q + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}},$$
$$d_2 = d_1 - \sigma\sqrt{T}.$$

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100% TIPS	$446	0
100% S&P 500	853	34.8
60% S&P 500/ 40% TIPS	654	29.0
One-Year Floor	519	29.0
Five-Year Floor	659	34.3

Table 3: Our simulation results

Concluding remarks

We have shown analytically that, contrary to what the BC resuhs suggest, protective floor strategies do not yield higher expected returns than pure equity investments. This theoretical result is corroborated by our simulation results, which have been obtained by replicating the BC simulations. As protective floor products do not result in a higher chance of reaching a target retirement income level either, the BC simulation results do not suggest that there is a lot of potential for these products in detïned-contribution plans.

It is important to notice though that this result only applies to the specific protective floor strategies developed in BC in combination with the specific risk – return framework chosen by BC. Indeed, it has been shown that well chosen option strategies are superior to traditional investments in a variety of risk-return frameworks (see e.g. Carr and Madan (2001)).

Moreover, in a setting of DC pension investments "peace of mind" can be a valuable asset in its own right and protective floor strategies arguably score well on this criterion.

References

Bodie, Zvi and Dwight B. Crane (1999), "The Design and Production of New Retirement Savings Products", The Journal of Portfolio Management, Vol. 25(2), pp 77-82.

Carr, Peter and Philip Madan (2001), "Optimal Positioning in Derivative Securities", Quantitative Finance, Vol. 1, pp 19-37.