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Extremal spillovers in financial markets

Stefan Straetmans

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Stefan Straetmans
Free University Amsterdam

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Abstract

We analyze the interdependency between different financial markets by using multivariate extreme value theory. This permits one to focus on the occurrence of simultaneous financial market crises, whereas standard covariance analysis is less suitable for studying extreme interdependencies. The analysis builds on the so-called stable tail dependence function which measures the amount of interdependency between the tail probabilities of multiple random variables. The empirical implementation of this semiparametric approach relies on order statistics. With these estimates one can calculate conditional spillover probabilities or other VaR-related multivariate risk measures for vectors of asset returns and for chosen crash levels. An empirical illustration shows relatively low stock market spillovers which is not in line with the presumption that stock markets are fairly good integrated and that integration has risen over time.

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1 Introduction

The scope for stock market and currency crashes to spill over across assets or markets - so-called systemic risk - has been hotly debated in the wake of recent speculative attacks and stock market crashes. For example in the aftermath of the 1992-1993 crisis in the European Monetary System (EMS) financial analysts argued that the French Franc and the Irish Punt came under attack as a result of the earlier crises experienced by the British Pound and the Italian Lira. A similar story seems to hold for stock markets. All major world markets declined substantially in 1987 which is an exceptional occurrence given the usually modest correlations of returns across countries: out of 23 markets, 19 declined more than 20 per cent, see e.g. Roll (1988). More recently, the Asian crisis and the worldwide correction in 'new economy' shares - although to a lesser extent in Europe - render additional casual evidence that 'domino' effects are present in financial markets.

Knowledge of this systemic risk may provide economic theorists, central bankers, regulatory authorities and portfolio risk managers with valuable information. Macroeconomists are interested in measuring extremal spillovers within currency markets or stock markets as a first step to pinning down common fundamentals of speculative attacks and stock market crashes. Assessing the degree of extremal market dependence also has practical implications for macroeconomic policy makers. Central bankers might use it as a kind of performance measure because extremal spillovers reveal the degree of credibility of national monetary policies and international monetary policy cooperation. If e.g. the domestic currency comes under speculative pressure following a sharp depreciation in foreign currency markets, this may signal some common macroeconomic imbalance/ mismanagement. As such the potential for speculative currency attacks to spread may discipline monetary authorities. Last but not least the modelling of simultaneous extremal market risks is highly relevant for risk managers and regulators for the sake of testing stress scenarios like financial market crises abroad. Indeed, such systemic events may drive domestic portfolios into bankruptcy and might eventually destabilize the whole financial sector.

The number of authors that have tried to estimate spillover effects is still limited. Most studies in the area employ correlation analysis in one form or another, see e.g. King, Sentana and Whadwani (1994), Longin and Solnik (1995), Connolly and Wang (1998) or Bodart and Reding (1999). Most of these studies use some variant of the multivariate GARCH framework in order to model the time variation in volatilities and correlations. This framework has also been used in order to test for volatility spillovers from one market to another. In some of the papers the correlation structure is conditioned on macrofactors, periods of high or low volatility or the sign of the returns.

A variance-covariance approach, like multivariate GARCH analysis, towards identifying extremal spillovers requires choosing a parametric form for the multivariate distribution of the asset returns. This, in turn, is used to estimate the model parameters using Maximum Likelihood optimization. It follows that estimated model parameters, like the conditional correlation measure will depend on the chosen distribution for the asset returns. Moreover, the candidates for the multivariate distribution to be chosen (multivariate Normal, Student-t etc.) are all nonnested in the parameter space, i.e., we cannot choose the 'right' distribution by statistical testing of the alternatives against each other. Thus, if one erroneously estimates the GARCH model for the 'wrong' multivariate distribution, the resulting conditional correlation measure gives almost surely a distorted view of the real extent of extremal spillover. We will therefore renege from choosing a parametric distribution to fit our spillover measure but instead opt for a semi-parametric approach based upon extreme value theory (see below). For a more thorough overview of the correlation pitfalls in finance, see Embrechts, McNeil and Straumann (1999).

Currency attacks or stock market crashes may spread internationally via numerous transmission channels. Extreme linkages may be present due to common underlying macrofactors like interest rate changes or operating procedures of governments and regulatory authorities. Financial domino effects may also be triggered by information revealing events. For example, some suggest that the Asian crisis started by the abolishment of implicit government guarantees to Thai banks and the resulting bankruptcy of some of them. This reminded foreign investors that the same could happen in other - comparably immature - financial systems in the Far East.

This article does not contain a structural model that explicitly specifies transmission channels; we believe that it is nearly impossible to disentangle empirically which channels are most important. Moreover, trying to explicitly link systemic events with potential underlying causes may give rise to misspecification error and biased estimates of the extreme linkages. We rather limit ourselves to a reduced-form approach, i.e., we do not care about possible causes of financial crises. To proceed with the empirical analysis we need an operational definition of market linkages during times of stress. A natural measure for extremal spillovers would be the likelihood or probability that several assets or markets crash at the same point in time. To be more precise, let (X, Y) represent a pair of asset returns. Asset price fluctuations become crashes if X (or Y) drop beneath critical levels x and y. These may reflect VaR numbers calculated separately for X and Y, i.e., without taking into account that extremal returns may co-move. Denote by κ the number of assets/portfolios/market indexes that crash simultaneously $(\kappa = 0, 1, 2)$.

Then the probability that two assets crash simultaneously given a crash of at least one of them boils down to:

$$P\{\kappa = 2 \,|\kappa \geq 1\} := \frac{p_1 + p_2}{p_{12}} - 1, \tag{1}$$

with

$$\begin{aligned} p_1 &:= P\{X > x\}, \\ p_2 &:= P\{Y > y\}, \\ p_{12} &:= \mathrm{P}\{\mathrm{X} > \mathrm{x} \text{ o r } \mathrm{Y} > \mathrm{y}\}. \end{aligned}$$

For sake of convenience the returns and spillover measure are mapped into the first quadrant. The statistical theory and estimation procedures to be discussed furtheron will also be defined on this area. Studying e.g. negative extremal spillovers between stocks and bonds can proceed using the formulas in the paper by putting the right minus signs, see e.g. Hartmann et al. (2000). A related bivariate measure of extreme links is $E\{\kappa \,|\kappa \geq 1\}$, i.e., the expected number of crashes that may simultaneously occur given that at least one crash happens. This alternative measure is directly related to the conditional crash probability in (1). Indeed, using elementary probability theory we can write down the following chain of equalities:

$$\begin{aligned} E\{\kappa|\kappa \geq 1\} &:= \frac{P\{\kappa = 1\} + 2P\{\kappa = 2\}}{P\{\kappa \geq 1\}} \\ &:= \frac{E\{\kappa\}}{P\{\kappa \geq 1\}} \\ &:= \frac{P\{\kappa \geq 1\} + P\{\kappa = 2\}}{P\{\kappa \geq 1\}} \\ &:= P\{\kappa = 2|\kappa \geq 1\} + 1. \end{aligned} \tag{2}$$

As a final extreme linkage measure, one could also calculate the likelihood that one asset crashes given another asset crash. Note that univariate excess probabilities p_1 and p_2 will generally differ from each other such that the resulting conditional probabilities will be unequal too:

$$P\{Y > y|X > x\} := \frac{p_1 + p_2 - p_{12}}{p_1}, \tag{3}$$

and

$$P\{X > x|Y > y\} := \frac{p_1 + p_2 - p_{12}}{p_2}. \tag{4}$$

One should be aware that the eventual difference between these conditional probabilities does not point towards causality in the spillover dynamics because the

wedge between them is solely induced by unequal marginal distributionspl and p_2.

These measures of extremal spillover both have advantages and pitfalls. Compared to the earlier defined measures in (1) and (2), spillover indicators that condition on a specific crash event cannot be defined for more than two assets. Moreover one needs to report two spillover numbers instead of one if the marginals in (3) and (4) are unequal. On the other hand, investors might be more interested in calculating the spillover probability or systemic risk of their portfolio w.r.t. a specific systemic event such as the Asian crisis or a drop in the NASDAQ index by some percentage amount. We will therefore pay attention to both types of spillover measures in our empirical application.

The reduced-form character of the above probability measures perhaps constitutes their main appeal because they make no reference to underlying causes and theories. On the other hand, this also constitutes their Achilles heel because the absence of a structural model of market behavior implies that the measures may fall victim to the Lucas critique, especially during institutional changes. For example, 'fire walls' might be in place in order to protect a market crash form spreading. Think e.g. of restrictions such as limited market entry for foreign investors, limited currency convertibility etc. These legal-institutional arrangements may bias downward our conditional probability measures of extreme linkage. Moreover, policymakers and risk managers might erroneously conclude that market regulation intended to prevent spillovers is no longer necessary. There is evidence that we should not worry too much about the severity of the Lucas critique for our spillover measures. Indeed it is often argued that financial market prices exhibit great similarity regarding their extremal behavior across different policy regimes and time periods. For example, VaR measures constructed using univariate extreme value theory (see further below) typically show remarkable stability over different time periods and asset classes.

How to estimate the proposed extremal spillover measures? We opted for extreme value theory in order to estimate p_1, p_2 and p_{12}. This approach enables one to identify the marginal tails of X and Y and their bivariate dependence structure without having to know or specify a parametric model for the returns. Thus, extreme value theory is a robust technique insofar as we do not have to choose a particular parametric distributional model for (X, Y) that may be wrong; see also the earlier discussion on multivariate GARCH models and the pitfalls of correlation analysis. Consequently, the estimators of the univariate tail probabilities p_1, p_2 and the bivariate tail probability p_{12} also have a sound statistical interpretation that does not depend on the specification of any parametric probabilistic model for the tails. As will be more thoroughly explained in the theory section of this article, estimation of eqs. (1)-(4) by means of extreme value theory proceeds in two steps. First, in order to estimate p_1 and p_2 we take into account the now well recognized fact that asset prices do at times exhibit sharp fluctuations (heavy tails), see e.g. Jansen and de Vries (1991), Campbell, Lo and

MacKinlay (1997) and Embrechts, Kluppelberg and Mikosch (1997). Univariate extreme value theory learns that such heavy tails can best be approximated by a Pareto-type law, e.g. for X :

$$p_1 \approx ax^{-\alpha}, \tag{5}$$

with a and α unknown parameters and x a large but bounded quantile or VaR level. The parameter α is called the tail index and determines the maximal number of bounded distributional moments of X that exist. Note that p_1 is decreasing in α, i.e., the lower the tail index the more probability mass in the tails of X. For distributions with an infinitely large a, all statistical moments exist and power tail behavior such as in (5) does not apply; instead the tail of univariate risks can be approximated by an exponential ('thin') tail like e.g. the Normal distribution. A procedure based upon Hill's estimator will be applied in order to estimate the tail behavior of X and Y as parametrized in (5). In order to calculate the joint excess likelihood p_{12} we use a result form multivariate extreme value theory. More specifically, the bivariate excess probability can be expressed as a function of the excess likelihoods obtained in the univariate step, i.e.,

$$p_{12} \approx l\,(p_1, p_2),$$

where the $\approx$ arises because we do not want to go infinitely far into the distributional tail, i.e., we are interested in assessing the probability of extremal but bounded spillovers. This so-called stable tail dependence function $l\,(.,.)$ exists under fairly general conditions and its curvature determines the dependence structure of the extremes. If the extreme dependence structure expressed by $l\,(.,.)$ is found to be linear, the univariate risks X and Y are said to be tail independent. On the other hand, the stronger the curvature of the tail dependence function the higher the degree of tail dependence between both assets. A nonparametric estimator for l based on order statistics will be introduced and discussed.

In the empirical application we estimate extremal spillover probabilities for our different conditional spillover measures using a set of daily stock indices for the 5 largest industrialized nations (G-5) and over the last **12** years. We argue that VaR analysis of stock market portfolios while conditioning on systemic events like stock market crashes abroad might provide additional insights for portfolio selection. Moreover we investigate whether extremal linkages between stock markets - and thus the degree of systemic stock market risk - have changed over time. We also calculate the conditional spillover measure (1) under a bivariate Normal parametrization as a warning against the use of parametric models for evaluating extreme market spillovers.

The remainder of this article goes as follows. Section 2 provides a short introduction into univariate and multivariate extreme value theory. The tail dependence function is introduced as a device to identify the extremal dependence structure between multiple risks. Estimation procedures are discussed in section

3. Estimated conditional crash probabilities for pairs of stock index returns are presented in the fourth section. In section 5 we investigate whether it is reasonable to assume that the dependence structure of the extremes remained constant over the considered sample period. Section 6 contains a comparison of extreme-value based conditional probability estimates of systemic risk and estimates based upon a bivariate normal parametrization. We end with a summary and conclusions.

2 Probability theory

We start this section by recapitulating the basics of univariate extreme value theory; this provides us with a parametric form for the tails of single risks (2.1). In order to capture the dependency between the tails of single risks in a portfolio we introduce the so-called stable tail dependence function which constitutes the key concept in bivariate extreme value theory (2.2).

2.1 The distribution of univariate extreme returns

Consider a stationary sequence Xi, $X_2, \ldots . X_n$ of i.i.d. random variables with a common distribution function $F_X(x)$. Think of the random variable X as a single time series of daily changes in asset prices as will be the case in our empirical application. Let $X_{(i)}$ represent the i-th ascending order statistic of X with $X_{(1)} \leq \ldots \leq X_{(n)}$. Suppose one is interested in the probability that the maximum return $X_{(n)}$ falls below a certain level x. This probability is given by

$$P\left\{X_{(n)} \leq x\right\} = F_X^n(x). \tag{6}$$

Loosely speaking, univariate extreme value theory investigates the properties of this distribution when the sample size grows large Suppose there exists constants $a, > 0$, b_n such that

$$\lim_{n \to +\infty} P\left\{\frac{X_{(n)} - b_n}{a_n} \leq x\right\} = G_X(x), \tag{7}$$

where G_X (x) is a probability distribution function; then G_X is called an *extreme value distribution* and F is said to be in the *Maximum domain of attraction* of G_X, i.e., $F_X \in \mathrm{MDA}(G_X)$. Both Leadbetter et al. (1983) and Embrechts et al. (1997) provide comprehensive introductions to univariate extreme value theory.

The key result of univariate extreme value theory concerns the shape of $G_X(x)$ and is presented in the following theorem:

Theorem 1 (Extremal Types) *If (properly scaled) maximum returns converge to an extreme value distribution, the latter takes one of the following parametric families:*

$$\text{Type I:} \quad G_X(x) = \exp\left(-e^{-x}\right) \qquad -\infty < x < +\infty;$$

$$\text{Type II:} \quad G_X(x) = \begin{cases} 0 & x \leq 0, \\ \exp\left(-x^{-\alpha}\right) & x > 0,\ \alpha > 0; \end{cases}$$

$$\text{Type III:} \quad G_X(x) = \begin{cases} \exp\left(-(-x)^{\alpha}\right) & x < 0,\ \alpha > 0, \\ 1 & x \geq 0; \end{cases}$$

The great appeal of these limit laws lies in the fact that, without specifying a specific distribution for the original returns, we know that the properly scaled maxima converge to some parametric distributional form under fairly general conditions. Thus, the limit laws are not postulated but exact parametrizations for the tails of distributions exist! For the Type II limit law it reflects the number of distributional moments that are finite (and thus exist). As for the Type III limit law all moments exist irrespective of the value α because the distributional support is finite. For proofs of the theorem and other claims we refer to the cited references.

A complication for our analysis of extreme market risks is the fact that there are three limit laws. Fortunately, the qualitative characteristics of financial return data point to the relevant limit law. Because financial returns are measured as log differences of original prices, both negative and positive returns are in principle unbounded. Thus, Type III cannot correctly describe market risks because of its finite support. Note, however, that it can play a role in credit risk management because maximal credit risk losses always stay bounded. Moreover it can be shown that the Type III limit law correctly describes the tail of portfolio credit losses under fairly general conditions, see Lucas et al. (2000). As for the remaining two candidate tail models for market risk, the Type II tails decline by a power which means that they contain relatively more probability mass than the Type I tails which decline exponentially. Distributions that lie in the MDA of the Type II or Type I limit law are therefore called fat-tailed or thin-tailed, respectively. In this article we assume that maximum returns follow a type II limit law because the fat tail feature best captures the stylized fact that asset prices do relatively frequently exhibit sharp fluctuations (see earlier).

2.2 Characterizing the asymptotic dependence structure

A joint distribution $F(x, y) = P\{X < x, Y < y\}$ of asset price changes X and Y both nests information on the marginal distributions of separate asset returns, $F_X(x)$ and F_Y (y) , and the dependence structure. In this paper we do not identify the dependence structure by a variance-covariance approach, see our earlier argumentation that correlations may be misleading indicators of extremal spillover. The starting point of our dependence analysis constitutes the so-called dependence function or copula:

$$D(u,v) = F\left(F_X^{-1}(u), F_Y^{-1}(v)\right), \ 0 \le u \le 1, \ 0 \le v \le 1, \tag{8}$$

where

$$F_X^{-1}(\mathrm{u}) = \inf\{z \,|F_X(z) \ge x\},$$

is the general inverse function of F_X ; the general inverse of F_Y is defined in the same way. By definition

$$F_X^{-1}(F_X(x)) = x, \ x \in \mathbf{R},$$

and for (x, y) $\in \mathbf{R}^2$,

$$F(x,y) := D(F_X(x), F_Y(y)).$$

Through the transformation (8) *we* have *uniformized* the marginal distributions of F. This follows from the following chain of equalities:

$$\begin{aligned} F(x,y) &= P\{X \le F,\text{-}'(u) \ , Y \le F_Y^{-1}(v)\} \\ &= P\{F_X(X) \le u, F_Y(Y) \le v\} \\ &= P\{U \le u, V \le v\} \\ &= D(u,v), \end{aligned}$$

where (U, V) are uniform $(0,1)$ variables. Copulas represent a way to extract the dependence structure from the joint distribution and to distinguish between dependence and marginal behavior. A copula may be thought of in two equivalent ways: as a function (with some technical restrictions) that maps values in the unit hypercube to values in the unit interval or as a multivariate distribution with standard uniform marginal distributions. In either case, it makes sense to interpret D as the dependence structure of F.

In order to make inferences about the dependence of *extremal* events, however, we define the following tail version of the dependence function D :

Definition 1 *(Huang (1992)) Suppose F is the d.f. of (X,Y) with $Q_X := (1 - F_X)^{-1}$ and $Q_Y := (1 - F_Y)^{-1}$ representing the marginal quantile functions for X and Y, respectively. Assume there exists a function $l : \mathbf{R}_+^2 \to \mathbf{R}_+$, such that for all $u, v \geq 0$,*

$$l(u,v) := \lim_{t \to +0} t^{-1}[1 - D(1 - tu, 1 - tv)] \tag{9}$$

$$= \lim_{t \to +0} t^{-1} P\{X > Q_X(tu) \text{ or } Y > Q_Y(tv)\}, \tag{10}$$

then l is called the stable tail dependence function (STDF) of F.

Multivariate extreme value theory deals with existence conditions, properties and estimators for the tail dependence function. For elementary introductions into multivariate extreme value theory, see e.g. Johnson and Kotz (1972), de Haan and Resnick (1977), Tawn (1988, 1990), Huang (1992) and Sinha (1997). In contrast to correlation analysis, the curvature of $D(u, v)$ and $l(u, v)$ completely determines the dependence structure of joint risks over the whole distributional support and in their tails, respectively. The tail dependence expression above greatly resembles the expression for the bivariate excess probability p_{12} in our spillover definitions (l), (3) and (4). However, the tail dependence function is defined for infinitely large quantile values Q_X and Q_Y whereas investors may be more interested in the values taken by our spillover indicators for extremal but bounded spillovers. Nonetheless, we argue in the empirical section that estimates of the asymptotic probability in (10) provide a fairly good approximation of p_{12}.

We end this section by mentioning the homogeneity property of $l(u, v)$. More specifically it can be shown that the tail dependence function is homogeneous of degree one, i.e.,

$$l(\lambda u, Xv) = \lambda l(u,v), \quad \lambda > 0. \tag{11}$$

This property will proof to be very useful for estimation purposes (see next section). Moreover, using this property one can easily show that the bivariate excess probability p_{12} and the marginal probabilities p_1 and p_2, as defined in (1), are related via the tail dependence function. In order to see this, suppose that the tail dependence function is still fairly good approximated by the right hand side probability if one does not go infinitely far into the bivariate tail, i.e.,

$$l(u, v) \approx t^{-1} P\{X > Q_X(tu) \text{ or } Y > Q_Y(tv)\},$$

for $t > 0$ but small. Without loss of generality we can now set $tu = p_1$ and $tv = p_2$ in the above expression in order to get:

$$l\left(t^{-1}p_1, t^{-1}p_2\right) \approx t^{-1} P\{X > Q_X(p_1) \text{ or } Y > Q_Y(p_2)\}. \tag{12}$$

Because of the homogeneity property, the factor t^{-1} can be divided away in the above expression. Rename $x = Q_X(tu)$ and y = Q_Y **(tv)** in order to obtain:

$$P\{X > x \text{ or } Y > y\} \approx l(P\{X > x\}, P\{Y > y\}).$$

Stated otherwise, the joint probability p_{12} only depends on the marginal probabilities p_1 and p_2 once l is known.

3 ESTIMATION PROCEDURES

The conditional spillover measures in (1), (2), (3) and (4) reveal that they should be estimated in a two-step estimation procedure. Estimators of p_1 and p_2 based upon extreme value theory are considered in the first subsection (3.1). An estimator of the tail dependence function based upon bivariate extreme value theory is discussed in the next subsection (3.2). We close this section by a note on estimating systemic risk for different time horizons (3.3).

3.1 Univariate step

In order to identify likelihoods of extremal events in a univariate context, we exploit the earlier mentioned stylized fact of heavy-tailed time series in financial markets, i.e., returns lie in the maximum domain of attraction of the heavy-tailed Type II limit law. Equivalently, this implies that the excess probability is approximately evolving as a power function of the quantile provided the quantile is 'large enough' , i.e., at the boundary or outside the historical sample. For e.g. the univariate risk X this would imply:

$$p_1 \approx ax^{-\alpha}, \tag{13}$$

with ***a*** and α unknown parameters and large x. From this property it directly follows that such distributions have only bounded moments up to α, where α is known as the tail index. In contrast, exponential decaying tails, or distributions with finite endpoints, have all moments bounded. Univariate excess probabilities like p_1 can be estimated by either the peaks-over-threshold (POT) method, the method of block maxima or the method we propose here. A nice description of the former two methods can be found in Embrechts et al. (1997). We employ the estimator of De Haan et al. (1994) which hinges upon the fat tail feature:

$$\hat{p}_1 = \frac{m}{n}\left(\frac{X_{(n-m)}}{x}\right)^{\alpha}, \tag{14}$$

where $X_{(n-m)}$ is the $(m + 1)$ -th descending order statistic from a sample of size ***n,*** and $(\hat{p}_1, x)$ is the probability-quantile combination that we are interested in. The idea behind this estimator is that it extends the empirical distribution

function outside the domain of the sample by means of the Pareto tail in (13) which it must approach eventually. A more thorough discussion of this estimator is provided in Danielsson and de Vries (1995).

The estimator (14) is conditional upon knowledge of the tail index a and the choice of the higher order statistic $X_{(n-m)}$. We estimate the tail index by means of the popular Hill (1975) estimator:

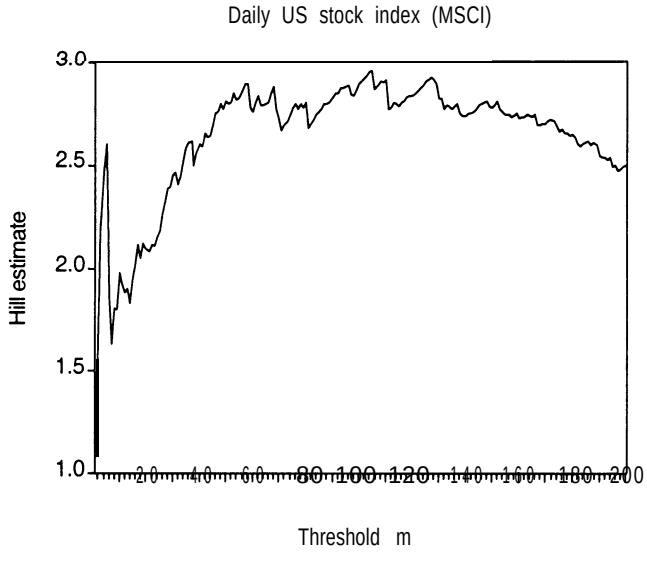
$$\frac{1}{\hat{\alpha}} = \frac{1}{m} \sum_{j=0}^{m-1} \ln \left(\frac{X_{(n-j)}}{X_{(n-m)}} \right), \tag{15}$$

where m equals the number of highest order statistics used in estimation.

In order to select the number of highest order statistics m, one can exploit the asymptotic normality of the Hill estimator in combination with the fact that the estimator is biased at the m-values that minimize the asymptotic mean-squared error, see Goldie and Smith (1987). The idea is to select m in such a way that the bias-squared and variance vanish at the same rate when the sample size n increases. Consequently, minimizing the sample Mean Squared Error (MSE) is an appropriate selection criterion for m. A heuristic procedure for this is to make a so-called Hill plot by computing $\hat{\alpha}$ at different m levels and to select the threshold in the region over which $\hat{\alpha}$ is more or less constant. There exists such a region, because when one uses too few order statistics, then $\hat{\alpha}$ will vary heavily due to inefficiency. In the opposite case when one goes too deep into the center of the distribution, the first order Pareto approximation to the tail is no longer appropriate and the bias from the second order parameters kicks in. Figure 1 plots Hill estimates against m for a market index of US stocks. Because the magnitude of the Hill estimates varies only slightly across the international stock market we only present the US results as a representative example.

Figure 1 shows that a horizontal range for the estimated tail index exists; it suggests an optimal threshold value that lies approximately in between 50 and 130. We decided to be quite conservative and to condition our tail index estimates on m = 55 given the sample size of 3319 daily returns. See also Jansen and de Vries (1991), Hols and de Vries (1991) and Koedijk et al. (1990) for applications of the same methodology.

Figure 1: Hill estimates for varying threshold m



3.2 Bivariate step

Roughly two approaches are available in order to estimate the bivariate excess probability p_{12} in the spillover indicators (1), (3) and (4). Some authors specify a bivariate parametric model of the tail dependence function of (X, Y) , see e.g. Tawn (1988) or Longin and Solnik (1998). However, and in contrast to the univariate limit laws for extremes, bivariate extreme value theory does not provide an explicit parametric form for the tail dependence function. Because the chosen parametrization might be too restrictive or simply wrong we do not opt for this approach but instead propose a nonparametric method based on order statistics. We now give a short, intuitive derivation of the relevant estimator.

As a starting point recall the definition (10) of the tail dependence function, and replace (u, v) by $(\hat{p}_1, \hat{p}_2)$, estimated in the univariate step:

$$l\,(\hat{p}_1, \hat{p}_2) = \lim_{t \to +0} \frac{1}{t} P\{X \geq \mathbf{Qx}\,(t\hat{p}_1) \text{ or } Y \geq \mathrm{Q_Y}\,(t\hat{p}_2)\}\,. \qquad (16)$$

Clearly, an estimator based upon this limit expression should necessarily be based upon the 'larger' observations, as only these can tell us something about the asymptotic dependence structure. So only a small number, say k, of the original n observations will be used for estimation. The number k must tend to infinity with n in order to enable us to apply the law of large numbers to get consistency

of the estimator. But it should also be small relative to n, since we are only interested in the tail. Hence we require

$$k = k(n) \to \infty \text{ and } \frac{k(n)}{n} \to 0 \text{ as } n \to \infty.$$

Replacing t by k/n in **(16)** renders

$$l(\hat{p}_1, \hat{p}_2) = \lim_{n\to\infty} \frac{n}{k} P\left\{ X \geq Q_X\left(\frac{k\hat{p}_1}{n}\right) \text{ or } Y \geq Q_Y\left(\frac{k\hat{p}_2}{n}\right)\right\}. \tag{17}$$

In order to turn **(17)** into an estimator we replace the bivariate excess probability in **(17)** by its empirical counterpart. This is simply the number of points in the area

$$\left\{ (X_i, Y_i)_{i=1,\ldots,n} \middle| X_i > Q_X\left(\frac{k\hat{p}_1}{n}\right) \text{ or } Y_i > Q_Y\left(\frac{k\hat{p}_2}{n}\right)\right\}, \tag{18}$$

divided by the size of the sample n.
According to our assumptions, however, Q_X and Q_Y are so large that there may be few observations left in this area. Note, however, that the tail dependence function's homogeneity property allows one to scale up its arguments in order to increase the number of observations in the area **(18).** The estimator should then be premultiplied by the inverse of this scaling factor in order to leave $\hat{l}$ invariant. A handy candidate for this operation is the polar transformation:

$$\hat{p}_1 = \hat{\rho}\cos\hat{\theta} \text{ and } \hat{p}_2 = \hat{\rho}\sin\hat{\theta}. \tag{19}$$

Exploiting the approximate homogeneity of $\hat{l}$, one obtains:

$$\hat{l}(\hat{p}_1, \hat{p}_2) \approx \hat{\rho}\hat{l}\left(\cos\hat{\theta}, \sin\hat{\theta}\right). \tag{20}$$

Conditional upon knowledge of $\hat{p}_1$ and $\hat{p}_2$, the angle θ and corresponding radius ρ can be consistently estimated by:

$$\hat{\theta} = \arctan(\hat{p}_2/\hat{p}_1) \text{ and } \hat{\rho} = \sqrt{\hat{p}_1^2 + \hat{p}_2^2}. \tag{21}$$

The pair $(\cos\hat{\theta}, \sin\hat{\theta})$ now lies on the unit circle which leaves more room for excesses in order to estimate l. The function $l\left(\cos\hat{\theta}, \sin\hat{\theta}\right)$ can be estimated by the number of points in the area

$$\left\{ (X_i, Y_i)_{i=1,\ldots,n} \middle| X_i > Q_X\left(\frac{k}{n}\cos\hat{\theta}\right) \text{ and, or } Y_i > Q_Y\left(\frac{k}{n}\sin\hat{\theta}\right)\right\}, \tag{22}$$

divided by n.

Note that Q_X and Q_Y in the above expression are unknown. Estimators of the unknown quantile functions can be found by observing that the expected number of points above $Q_X\left(\frac{k}{n}\cos\hat{\theta}\right)$ and $Q_Y\left(\frac{k}{n}\sin\hat{\theta}\right)$ equals:

$$nP\left\{X > Q_X\left(\frac{k}{n}\cos\hat{\theta}\right)\right\} = k\cos\hat{\theta}, \tag{23}$$

and

$$nP\left\{Y > Q_Y\left(\frac{k}{n}\sin\hat{\theta}\right)\right\} = k\sin\hat{\theta}. \tag{24}$$

Consequently, the order statistic $X_{[n-k\cos\hat{\theta}]}$ is a consistent estimator for Q_X and $Y_{[n-k\sin\hat{\theta}]}$ is a consistent estimator for Q_Y. The vertical brackets imply that the ranks are rounded upwards. Let I(.) stand for the indicator function defined on the area in (22), it then follows from (20), (21), (23) and (24) that

$$\hat{l}(\hat{p}_1,\hat{p}_2) = \frac{1}{k}\hat{\rho}\sum_{i=1}^{n}\mathbf{I}\left(X_i > X_{[n-k\cos\hat{\theta}]} \text{ and, or } Y_i > Y_{[n-k\sin\hat{\theta}]}\right), \tag{25}$$

which is Huang's (1992) estimator.

The estimator in (25) hinges on $\cos\hat{\theta}$ and $\sin\hat{\theta}$, obtained from the univariate step via (21), and the threshold number k. As for the Hill estimator we rely on a heuristic plot method for selecting k. This method requires no parametric prior information and exploits the asymptotic normality of the estimator in (25), see Huang (1992). The method implies calculating the deviation of $\hat{l}(k)$ from homogeneity by

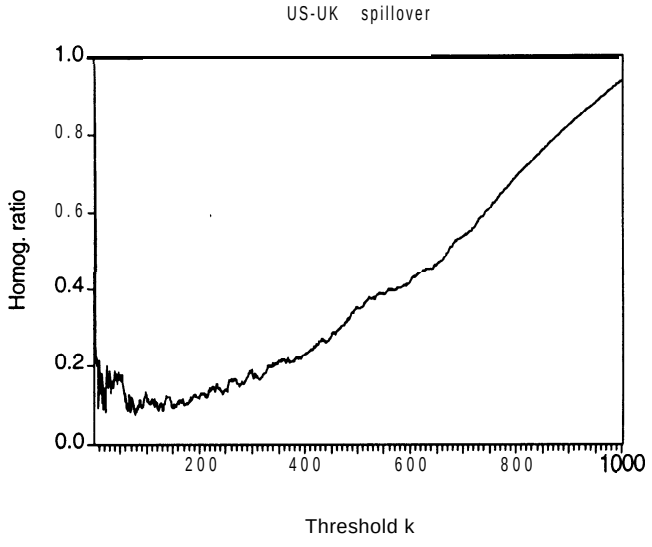
$$\tau(k) = \left|\frac{\hat{l}\left(2\cos\hat{\theta}, 2\sin\hat{\theta}\right)}{\hat{l}\left(\cos\hat{\theta},\sin\hat{\theta}\right)} - 2\right|, \tag{26}$$

and selecting k in the region for which τ is small and more or less constant. Note that the angle $\hat{\theta}$ is predetermined by the univariate estimation results. When one uses a too low number of higher order statistics, then Twill vary heavily due to inefficiency. In the opposite case, $\hat{l}$ becomes more and more downward biased, since it will start to deviate more and more from homogeneity. For additional intuition and examples on the bias-variance trade off when selecting k, *see e.g.* Straetmans (1998a,b) and Hartmann et al. (2000).

Let us now return to implementing the plot method on our dataset. Figure 2 contains the homogeneity ratio for the pair of US-UK stock market indices. We limit ourselves to presenting only one homogeneity ratio because we found that the ratio does not vary much across different pairs of stock indices. This suggests that one can choose more or less the same threshold for all stock index pairs. On the basis of Figure 2 we decided to condition all our estimates of extreme stock market dependence on $k = 100$.

Early applications of this nonparametric approach to extreme dependence include Huang (1992) and de Haan and de Ronde (1998). Very few papers have

Figure 2: Homogeneity ratio for varying threshold k



implemented this multivariate approach into finance, see e.g. Starica (1998) and Straetmans (1998a,b).

3.3 Time horizon and systemic risk

Before turning to the empirical results we want to make a note concerning the desired time horizon for the spillover measure. Recall that risk managers typically calculate VaR numbers for internal purposes on a daily basis; because of e.g. probable market illiquidity during stress periods regulatory authorities also demand the reporting of weekly or biweekly VaR numbers. These values can differ quite dramatically for different risk horizons. Does it make sense to undertake the same exercise with our extremal spillover indicators, i.e., to report estimates of systemic risk measures over different time horizons ? The answer is no !

Using *daily* yields, estimates of e.g. the conditional probability (3) reflect the likelihood that an asset loses y % of its value today given another asset crash by x % on that same trading day. Suppose, however, that we are interested in the likelihood of a daily conditional spillover within the time span of, say, K days. In other words, data frequency and time span do not have to coincide.

How should the estimation of **(1),** (3) or (4) then proceed? Let $\hat{p}_1$ and $\hat{p}_2$ again represent univariate exceedance probabilities estimated with daily data. The *multiperiod* univariate exceedance probabilities $\hat{q}_1$ and $\hat{q}_2$ follow easily from the i.i.d. assumption for X and Y :

$$(1 - \hat{q}_i) = (1 - \hat{p}_i)^K \text{ , } i = 1, 2. \tag{27}$$

Taking logarithms of (27), we obtain (for small $\hat{p}_1$ and $\hat{p}_2$) :

$$\hat{q}_i \approx K\hat{p}_i \text{ , } i = 1, 2. \tag{28}$$

Moreover, approximate homogeneity of $\hat{l}$ implies that

$$\hat{l}(\hat{q}_1, \hat{q}_2) \approx K\hat{l}(\hat{p}_1, \hat{p}_2) . \tag{29}$$

The combined eqs. (28) and (29) leave all our spillover measures invariant for changes in the time span. Thus, in contrast to univariate exceedance probabilities, the time horizon for conditional crash probabilities does not matter.

4 EMPIRICAL RESULTS

4.1 Data description

The data set comprises stock indexes for the G-5 countries (United States, United Kingdom, Germany, France and Japan, abbreviated as US, UK, GE, FR and JP, respectively). The series are daily and run from February 1987 until November 1999, which implies 3319 observations per series. All data were downloaded from Datastream inc. The indices are calculated by Morgan Stanley and exclude dividends. Moreover they are all expressed in local currency. The daily returns are calculated in the usual way, i.e., by taking log first differences of the equity price index.

4.2 Univariate results

Table 1 contains univariate exceedance probabilities for our dataset of daily changes in G-5 stock market indexes. The first column of each Table reports tail index estimates using he Hill estimator (15). This estimator is conditioned upon the portion m/n of the sample used for calculating $\hat{\alpha}$. As justified before, we condition the Hill estimator for all returns on $m = 55$, which is less than 2% of the data. While knowledge of the tail index is interesting in itself, the question of economic interest is how likely extreme returns are; or conversely, how high the loss (Value-at-Risk) will be for a given low probability level. The highest extremal in-sample return for each asset is reported in column 2 of the Tables. These values provide us with a benchmark for choosing values for the out-of-sample quantiles for the bivariate step. The remaining columns in the Tables report probabilities of daily exceedances over a yearly time horizon. We achieve this by calculating daily tail probabilities with the estimator (14) and by subsequently multiplying these values by the number of trading days K in a year,

which we take $K = 260$. As noted earlier this is a correct procedure provided the return series are serially independent. Thus the entry 0.00649 for Japan in Table 1 in the column for the probabilities at the 20% loss level implies that on average once per $1/0.00649 \approx 154$ years there is a year with a day on which the Japanese stock market drops by more than 20%.

Table 1: Left tail probabilities for stock index returns (local currency)

countries	$\hat{\alpha}$	min (%)	-10%	-20%	-30%
US	2.82	-22.32	0.05656	0.00798	0.00254
UK	3.25	-11.95	0.03017	0.00318	0.00085
FR	2.78	-9.410	0.09573	0.01395	0.00452
JP	3.40	-16.67	0.06844	0.00649	0.00164
GE	2.33	-14.07	0.20958	0.04162	0.01616

From a macropoint of view, the Table gives some perspective to the frequency of asset market crashes in isolation. Note that an event like the 1987 crash is very rare. But observe also that stock market drops in the order of 20% over a number of days quickly become more likely as the number of days is increased. Moreover, it can be shown that exceedance probabilities for the same tail points are much smaller under a normal parametrization of financial asset returns. Given the persisting popularity of the normality assumption, it should not surprise that agents tend to underestimate the probability of a financial crisis, see e.g. Friedman and Laibson (1989).

From a microeconomic point of view, the Table can be used as a device for portfolio selection. Suppose that investors are interested in selecting the stock which minimizes the probability of extreme losses (i.e. the minimax strategy). Fixing the extreme loss at 30% or lower, they select the UK stock index. Of course, this selection criterion is very myopic because the investor both ignores the co-movements of asset price changes in portfolio and the limiting dependency of the included assets with large asset fluctuations in the outside world (systemic events). In the next sections we measure these relationships by means of the earlier discussed conditional crash probability measures of systemic market risk. We argue that these may be more relevant and informative to portfolio investors than the univariate excess probabilities form Table 1.

4.3 Reduced form estimates of systemic stock market risk

Table 2 contains systemic risk estimates for pairs of G-5 stock index returns. The first column contains estimates of the systemic risk measure (1). The other two columns report estimates for alternative spillover measures (3) and (4). In contrast to the first column measure the latter measures are conditioned on a

specified crash abroad. All extremal spillover estimates in the Table are conditioned on a common extreme quantile (or VaR level) of +20 %. The Table reveals that the degree of systemic stock market risk is quite sensitive to the chosen spillover measure. Thus one should be careful if one draws conclusions on the basis of one single spillover definition. Regardless of the spillover definition considered, however, note that all probabilities in the Table are much higher than their univariate counterparts in Table 1. This is because one takes into account information on the extreme dependence structure between the markets; this makes crashes in domestic markets more likely if crashes happen abroad. Let us now investigate the results somewhat more carefully. The lrst column estimates are all smaller than the probabilities in the remaining columns. However, this has no economic interpretation because the conditional crash probabilities in the first column are always smaller by construction. Divide e.g. (1) by (3) in order to obtain

$$P\{Y > y|X > x\} = P\{\kappa = 2|\kappa \geq 1\}\frac{p_{12}}{p_1}. \qquad (30)$$

Because $p_{12} < p_1$, the left hand side probability in (30) should be higher than $P\{\kappa = 2|\kappa \geq 1\}$.

Table 2: Estimated conditional crash probabilities across stock markets

MARKETS $(Y - X)$	$P\{\kappa 2|\kappa \geq 1\}$[1]	$P\{Y > y|X > x\}$'	$P\{X > x|Y > y\}$'
US-UK	0.192	0.565	0.225
US-FR	0.208	0.270	0.472
US- JP	0.125	0.248	0.202
US-GE	0.125	0.133	*0.693*
UK-FR	0.162	0.171	0.752
UK-JP	0.134	0.176	*0.359*
UK-GE	0.072	0.073	*0.962*
FR-JP	0.136	0.376	0.175
FR-GE	0.266	0.280	0.837
JP-GE	0.077	0.083	0.533

[1]Quantiles equal $x = y = 20\%$

As for the 2nd and 3rd column differences, these can be explained by differing probability masses in the tails ($p_1 \neq p_2$). More specifically, the definitions (3)-(4) show that the conditional spillover probabilities will be high (low) if the probability of the conditioning event is low (high). For example the univariate

results in Table 1 show that a crash of comparable magnitude is much more likely to happen in Germany than in the UK. Consequently conditioning on a German stock market crash will render a spillover to the UK stock market less likely than the reverse scenario.

Table 2 provides interesting information for policymakers and regulatory authorities in order to assess the vulnerability of the financial system as a whole. Systemic risk estimates can also be informative for portfolio investors for the sake of testing stress scenarios. More specifically, they would like to assess to what extent a certain drop in portfolio value becomes more likely if one conditions on a systemic event abroad. Stated otherwise, they may be interested in conditioning their portfolio VaR on systemic events and compare it with the unconditional portfolio VaR for a common significance level. Note that all returns used for the systemic risk estimates in the Table were expressed in local currency. However, portfolio investors assessing their portfolio systemic risk would like to condition on systemic events abroad expressed in the home currency. This exercise is undertaken in Table 3. Each row corresponds with a representative domestic G-5 investor. We assume that their initial equity holdings only comprise the domestic equity index. We then calculate systemic risk for each of these G-5 domestic portfolios w.r.t. systemic events in the other G-5 stock markets (left panel of Table 3). The estimates are again conditioned on a common extreme quantile (or VaR level) of $+20$ %.

Table 3: Bilateral and diversified systemic portfolio risk

INVESTOR	BILATERAL $P\{Y > y \mid X > x\}$					INTERNATIONAL $P\left\{\sum_{i=1}^{4} \frac{1}{4} Y_i > y \mid X > x\right\}$
	us	FR	GE	JP	UK	
us		0.287	0.240	0.321*	0.309	0.249
FR	0.388	-	0.313	0.200	0.753*	0.612
GE	0.610	0.716	-	0.259	0.949*	0.553
JP	0.097	0.158*	0.122	-	0.151	0.377
UK	0.122	0.324*	0.084	0.232	-	0.465

(*) Nonselected indices for diversified portfolio

Just as in the previous Table the systemic risk estimates are all much higher than their univariate counterparts in Table 1. Thus, univariate VaR numbers will highly understate VaR values conditioned upon systemic events and calculated for the same significance level as their unconditional counterpart. Suppose now that domestic investors want to diversify their systemic portfolio risks by investing abroad. More specifically assume that they form an equally weighed portfolio containing the domestic index and 3 out of 4 foreign stock indices; the index that induced the highest systemic risk w.r.t. the domestic portfolio is not selected.

The right column in Table 3 contains the systemic risk of this 'international' portfolio w.r. t. the index not included. Investors would ideally like to reduce systemic risk beneath the minimum value from the left panel but neither of the investors in the Table achieves this goal. However, three investors (US, France and Germany) manage to reduce systemic portfolio risk beneath the maximum level in the left panel. As for the other two investors (Japan and the UK) the simple diversification strategy does not pay. Summarizing, this stylized example shows that it is quite difficult to diversify portfolio risk if one conditions risk measures upon systemic events. The intuition is that it is not easy to find assets abroad that are weakly dependent in the extremes w.r.t. both the current portfolio and the system ic event.

A general problem of interpretation of Tables 2 and 3 is that stock markets that trade at different places evidently exhibit different trading hours. Thus spillovers may be dampened because a crash in one market occurs when other markets are nearly closing. This enables the latter market to absorb the spillover effect in more than one trading day. Consequently daily stock returns can be less extreme in the latter market and the conditional crash probability will probably be downward biased. Hence, more attention should be paid in the future to the precise timing of the stock index return series and the extent of trading overlap when estimating conditional crash probabilities.

5 Testing for increased stock market integration

Casual observers suggest that markets have become more related over time. In this section we test whether this presumption is true if one measures market linkages in terms of extremal dependencies. If the extreme dependence structure is nonconstant over the considered sample period it could be that the systemic risk estimates in Table 2 are distorted towards some rough average measure of market linkage over time.

In order to test whether the linkages during stress period have remained constant, we exploit the property that the tail dependence estimator $\hat{l}(.,.)$ is asymptotically normal, see Huang (1992):

$$\sqrt{k}\left(\hat{l}(1,1) - l(1,1)\right) \to N\left(0, \sigma^2\right). \tag{31}$$

For sake of convenience we limit ourselves to evaluating and testing the tail dependence function and its estimator in (1,1) because this renders a fairly simple expression for the asymptotic variance. Estimation procedures for the asymptotic variance are discussed in Peng (1999). Note also that the limiting distribution of $\hat{l}$ in (31) becomes degenerate if asset returns are asymptotically independent ($\sigma^2 = 0$). Thus the asymptotic normality property cannot be exploited in order to test for asymptotic independence.

If one selects the degree of tail dependence l **(1,1)** as a measure of stock market integration the relevant null hypothesis boils down to $H_0 : l_1 = l_2$, where the subscripts refer to nonoverlapping subsamples. From the theory sections we know that higher l- values imply a lower asymptotic dependence and vice versa. Hence, the relevant alternative for the international stock market boils down to $H_1 : l_1 > l_2$. Testing for structural breaks can most easily be done by splitting the sample into two parts of equal length $n/2$ = 1659 and by setting the bivariate threshold at half its full sample value of *k*, i.e., *k/2* = 50. Estimating l (1, 1) and σ^2 across the subsamples and exploiting (31) we substitute these numbers into the bivariate test statistic

$$Z = \frac{\sqrt{k/2}\left(\hat{l}_1 - \hat{l}_2\right)}{\sqrt{\sigma_1^2 + \sigma_2^2}}, \tag{32}$$

which is asymptotically standard normal distributed under H_0. Table 4 contains values of the test statistic for the stock market pairs of our dataset. In order to reject H_0 at the 5 % significance level against the one-sided alternative hypothesis H_1 , the values in (32) should be larger than 1.65 which is clearly never the case. Some values are even negative which is rather indicative of a decrease in stock market integration over the considered sample period!

Table 4: Stability testing results on extreme stock market dependence

	us	UK	FR	JP	GE
US	1				
UK	-0.932	1			
FR	-0.505	0.482	1		
JP	0.210	0.203	1.56	1	
GE	-0.625	1.218	0.467	0	1

Our testing results confirm the robustness of extreme value theory over a fairly long sample period. This observation is in line with earlier findings on the stability of tail index cum VaR estimates when applying univariate extreme value theory over fairly long samples see e.g. Jansen and de Vries (1989) or Danielsson and de Vries (1997). Some earlier studies come to similar conclusions concerning the evolvement of stock market integration over time by conditioning on different sample periods and different dependence concepts. The Brady commission (1988) already pointed out that there had been no trend increase in the (rolling) correlations between markets. Estimating conditional correlations by means of a more sophisticated factor ARCH model, King, Sentana and Wadhwani (1994) were also unable to find strong evidence in favor of a trend increase in correlations.

6 Extremal spillovers and the normal distribution

Wide consensus exists among financial economists that fluctuations in asset prices exhibit heavy tails. Moreover, the previous empirical sections revealed a significant degree of dependence between extreme events across stock markets. We also argued that extreme value theory is better-suited for capturing these data properties than a parametric analysis as the latter may be prone to misspecification. In this section we illustrate this point by calculating the conditional crash probabilities under the assumption that market risks follow a multivariate Normal distribution.

Because most statistical packages generate exceedance probabilities for the univariate and bivariate standard normal distribution, it is useful to rewrite (1) as:

$$P\{\kappa = 2 \,|\, \kappa \geq 1\} = \frac{P\left\{Z_1 > \frac{x-\mu_X}{\sigma_X}, Z_2 > \frac{y-\mu_Y}{\sigma_Y}\right\}}{1 - P\left\{Z_1 < \frac{x-\mu_X}{\sigma_X}, Z_2 < \frac{y-\mu_Y}{\sigma_Y}\right\}}, \tag{33}$$

where Z_1 and Z_2 stand for the standardized returns. Note that the correlation coefficient between X and Y is invariant under standardization. The means μ_X and μ_Y and the variancecovariance matrix Σ are estimated by means of the Maximum Likelihood method.

Table 5 presents 'normal' conditional probability estimates for our set of stock index return data. We were unable to condition (1) on one of the crash levels from Table 1 because of numerical instabilities. Once we condition the systemic risk measure on quantiles larger than 7 % the numerator and denominator of (33) become so small that the systemic risk measure can no longer be calculated with satisfactory accuracy. The conditioning quantile is therefore set at +6 %.

Table 5: Conditional crash probabilities under the bivariate normal distribution

	US	UK	FR	JP	GE
US	1				
UK	$1.883*10^{-7}$	1			
FR	$1.141*10^{-5}$	$6.964*10^{-9}$	1		
JP	$4.529*10^{-8}$	$1.797*10^{-10}$	$1.186*10^{-5}$	1	
GE	$1.157*10^{-6}$	$7.399*10^{-10}$	$102*10^{-6}$	$3144*10^{-6}$	1

Conditioning quantiles: $x = y = 6$ %

The estimated probabilities in Table 5 are all extremely small compared to their semi-parametric counterparts in Table 2 although the quantiles x and y on which we condition are chosen less far in the tail!

These results illustrate that the tails of a Normal distribution do not correcly capture the fat tails and extremal dependence observed in the stock market. The

exponentially declining thin tails of a normal distribution severely underestimate the tails of univariate risks X and Y. Moreover, it can be shown that the univariate tails of a multivariate normal distribution exhibit asymptotic independence even when the risks are highly correlated over the full range of the distribution (see e.g. Sibuya (1960) or De Haan and Resnick (1977)). The tail independence of normal risks boils down to a linear dependence function $l(u, v) = u + v$ which renders a zero conditional crash probability upon substitution in our systemic risk measures. Concluding, the Normal distribution both highly underestimates the likelihood of single stock market events as well as the probability that these events occur simultaneously.

7 Conclusions

In this article we proposed a bivariate extreme value framework in order to measure the probability of extremal spillovers (systemic risk) between stock markets. We argued that the approach does not depend on the choice of a particular probabilistic model for the market risks. Moreover, investors might be more interested in knowing this likelihood than in dependence measures that are defined over the whole sample period. Assessing the probability of simultaneous extreme returns may be crucial for the success or failure of portfolio investors or policy makers such as central bankers.

In order to measure extreme linkages, we introduced the concept of the stable tail dependence function. The conditional probabilities that we proposed as measures of systemic risk summarize the amount of dependence between extremes as well as information on the amount of probability mass in the univariate risk tails. We calculated spillover probabilities between stock markets and found that these are lower than expected. Investors could use our conditional probability framework as a device for testing stress scenarios. More specifically, they can investigate the sensitivity of their portfolio VaR to foreign extreme events such as stock market crashes. They can also employ the systemic risk measure as a criterion for international risk diversification and asset allocation.

We also investigated whether the linkages between stock markets during stress periods have changed over our particular sample of 12 years. Contrary to prior beliefs, we found that stock market integration has not risen significantly over the considered sample period.

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