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HEDGING WITH STOCK INDEX FUTURES: DOWNSIDE RISK VERSUS THE VARIANCE*

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Abstract. In this paper we investigate hedging a stock portfolio with stock index fùtures. Instead of defining the hedge ratio as the minimum variance hedge ratio, we consider several measures of downside risk: the semivariance according to Markowitz [1959] and the various lower partial moments according to Fishbum's [1977] at model (α>0). Analytically we show that for normal returns and biased futures markets there is an extra cost associated with hedging lower partial moments if the minimum variance hedge ratio instead of the optimal hedge ratio is used. We prove that the extra cost is different from zero if ail. Furthermore, in case fùtures markets are positively biased minimum lower partial moment hedge ratios are smaller than the minimum variance hedge ratio (strictly smaller in case $\alpha \geq 1$).

We used the Dutch FTI contract to hedge three Dutch stock market indexes. The in-sample analysis shows that (i) minimum semivariance and minimum variance hedge ratios are almost the same in size, (ii) minimum lower partial moment hedge ratios are smaller than minimum variance hedge ratios (only slightly smaller for $\alpha \geq 1$) and (iii) except for the lower partial moment with α=0.5, hedging downside risk using the minimum variance hedge ratio instead of the optimal hedge ratio is appropriate. For both strategies risk can

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be reduced in the same proportion whereas the extra cost of using the minimum variance hedge strategy is negligible. In contrast to (iii), out-of-sample results show that the extra cost of hedging lower partial moments with the minimum variance hedge strategy can be significant (statistically as well as in size).

Keywords. Downside risk, Hedging, Futures

1 INTRODUCTION

Since the foundation of an organized market in trading foreign currency futures in 1972 and after that the successful introduction of interest rate futures and stock index futures, there has been published an enormous amount of literature on financial fùtures. One specific concern of this literature is the economic rationale for tùtures markets, viz. that these markets facilitate hedging. Hedging involves the transfer of price change risk of an asset from the owner of the asset to others who are willing to bear this risk. Reasons for bearing this risk could be speculative (bearing more risk) or neutralizing an opposite risk exposure. There have been developed several hedging theories in the Iiterature. The classical constant equal and opposite hedge strategy emphasizes the risk avoidance potential of futures markets. It implicitly assumes that price movements in the futures contract match perfectly the price movements in the spot. As this is generally not the case, the risk of price changes cannot be eliminated entirely. Furthermore, Johnson [1960] argued that the maximum reduction in risk (i.e., the variance) is often accomplished with a hedge unequal to one instead of the traditional 1: 1 hedge. He showed that the minimum risk hedge equals the covariance of the spot and the futures divided by the variance of the futures. Fmally, Rolfo [1980], among others, applied the portfolio theory of Markowitz [1959] to tùtures hedging and formulated the hedging problem in a mean-variance (MV) framework. The decision maker chooses the optimal number of futures contract by maximizing an expected utility function with linear indifference curves in the MV space. Hence, the Rolfo model considers both expected return and risk.

To date almost all academic literature on tùtures hedging assumes that the MV framework is valid, i.e., it is consistent with the utility theory of Von Neumann-

Morgenstem (see, e.g., Ingersoll [1987]), and consequently the variance is the correct risk measure. However, MV analysis should not be taken too seriously unless a quadratic utility fùnction is assumed or the probability distributions used in the analysis are normal.[1] A quadratic utility fùnction has two well-known undesirable properties (see, e.g., Ingersoll [1987, p. 96]): first, all concave quadratic utility fùnctions are decreasing after a certain point and second, they display increasing absolute risk aversion. However, even when distributions are (approximately) normal, there is still a contention, set forth by Markowitz [1959], among others, that investors frequently associate risk with failure to attain a target return. Mao [1970a] was the first who reported that risk defined as failing to meet a target leve1 of returns is consistent with the practitioner's view of risk. Also, Fishbum [1977] showed that in several published empirical studies of risk-taking behavior, below target returns is a rather well description of risk The findings of Fishbum were confirmed by Laughhum, Payne and Crum [1980]. Hence, decision makers should at least consider altemative (downside) risk measures instead of the variance only.

Motivated by the evidence on practitioner's view of risk and the fact that the questionable minimum variance hedge ratio is still being used (theoretically and in practice), in this study we investigate several downside risk measures in relation to hedging with futures. The first measure of downside risk we adopt is the semivariance originally suggested by Markowitz [1959] and more fully developed by Mao [1970b]. The semivariance can come in two different ways. One approach is, like the variance, to measure it as the expected value of squared deviations below the mean. The second version measures the semivariance as the expected value of squared deviations below a fixed critical (or target) value. In the remainder of the paper the target semivariance refers to the second whereas the semivariance refers to the first measure. The theoretical basis for acceptance of the target semivariance is, first, that it is consistent with maximization of expected utility where the utility function is quadratic below the target and linear above the target (see Markowitz [1959]), and, second, that for arbitrary return distributions mean-target semivariance (MS_t) efficient portfolios belong to the second

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1 The class of distributions that justify MV analysis is broader than only normal distributions. For example, Ingersoll [1987, pp. 104-107] shows that for elliptical distributions (with finite variance) MV analysis is appropriate. However, in the remainder of the paper we concentrate on normal distributions.

order stochastic dominance (SSD) efficient set (see Porter [1974]).[2,3] Fishbum [1977] and Bawa [1978] extended the MS, model to a generalized lower partial moment fiamework. The lower partial moment, the second downside risk measure we adopt, measures the risk as the expected deviation below a target (t) with the deviations raised to the ath power with $\alpha>0$ a prespecified constant.[4] Fishbum showed that mean-lower partial moment, MLPM(α,t), models of choice are also consistent with utility theory and that for all t, all $\alpha>0$ and arbitrary return distributions the MLPM(a,t) efficient set is a subset of the first order stochastic dominance (FSD) efficient set (Theorem 3, p. 123). For $\alpha\geq1$ this is true for the set of SSD efficient portfolios. Notice that the target semivariance (but not the semivariance versus the mean) is a particular lower partial moment, viz. for $\alpha=2$.

To our knowledge there is only one published paper that considers futures hedging in a semivariance or lower partial moment framework.[5,6] Ahmadi, Sharp and Walther [1987] evaluated the hedging effectiveness of foreign currency options and futures, where the hedging effectiveness is defined as the relative reduction of various lower partial moments according to Fishbum. In comparing the performance of both markets they used the traditional 1: 1 fùtures hedge ratio (also for options). However, like in the MV framework, such a hedge ratio does in general not yield the maximum reduction of risk. Hence, a comparison of two markets based upon non-optimal hedging strategies is very limited.

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2 It is important to note that both these properties do not hold for the semivariance with respect to the mean.

3 If returns have a normal distribution, then the set of MV efficient portfolios together with the minimum variance portfolio constitutes the SSD efficient set. However, if distributions are not normal, as empirical research suggests, then there are MV efficient portfolios that are not SSD efficient and/or, conversely, there are SSD efficient portfolios (other then the minimum variance portfolio) that are not MV efficient. Hence, this shortcoming of the MV model is also a reason to question the variance as appropriate risk measure.

4 Sarin and Weber [1993] described that from the large body of empirical research on risk-taking behavior, a few stylized facts emerge consistently. One of these facts is that risk decreases if a constant positive amount is added to all outcomes. This holds for the lower partial moment, but clearly not for the variance. This is another reason to doubt the variance as the relevant risk measure.

5 Telser [1955] was the first who combined hedging with futures and downside risk. In his model, he assumed that an entrepreneur maximizes expected income subject to the constraint that he does not want the probability of his income falling short of a fixed disaster level of income to exceed a prespecified level.

6 Another risk measure, as an altemative to hedging with futures in the MV fiamework, is described in the papers by Hodgson and Okunev [1992], Kolb and Okunev [1993] and Lien and Luo [1993]. They use the mean-(extended-)Gini (MG) setting where the (extended-)Gini coefficient is the measure of risk.

The purpose of this paper is twofold. First, we compare from a theoretical point of view minimum risk hedge ratios for different measures of downside risk and the variance. Furthermore, we investigate the consequences of using the minimum variance hedge ratio in a situation where downside risk is the appropriate risk measure. More specifically, we assume an investor who wants to hedge a long spot position with fùtures contracts. The investor is supposed to be highly concemed with returns falling below a target, hence downside risk is the appropriate description of risk for the investor. However, the position taken in the fùtures market is based upon the variance as risk measure, i.e., he uses the minimum variance hedge ratio. We analyze the consequences of applying the minimum variance hedge strategy in a situation where downside risk is the correct risk to be hedged. We do this by comparing the expected return and downside risk reduction of both the optimal and the minimum variance hedging strategy. The analysis has been carried out under the assumption of normal and non-normal returns. Although the variance is the correct risk measure in case returns have a normal distribution, we argue that it is still meaningful to adopt a downside risk measure in such a situation.

The second purpose of the paper is to examine the theoretical results empirically. The Dutch FTI contract, this is a futures contract written on the Amsterdam EOE index (AEX), has been used to estimate minimum risk hedge ratios for three Dutch stock market indexes. The analysis has been carried in-sample as well as out-of-sample and for three different hedge durations: one, two and four weeks.

The rest of the paper is organized as follows. In the next section hedging the variance is compared to hedging downside risk. Section 3 describes the data and the methodology we used. In Section 4 we report the empirical results, whereas the final section contains the conclusions.

2 HEDGING DOWNSIDE RISK VERSUS THE VARIANCE

The purpose of this section is, first, to present a model that describes hedging a stock portfolio with stock index futures in a MV framework and in a mean-downside risk framework. For the downside risk we take the semivariance as well as the lower partial moments according to the α-t model of Fishbum [1977]. Then the various minimum risk hedge ratios are compared to each other. In addition, we evaluate the consequences of using the (non-optimal) downside risk hedging strategy that uses the minimum variance hedge ratio. We do this for the situation where the MV framework is correct as well as for the situation where it is not. Throughout the analysis we ignore any market imperfection.

We begin by defining the time 1 rate of return r_H on a hedged portfolio, in which at time zero N stock index futures contracts have been sold short against the long portfolio of stocks (see, e.g., Figlewski [1985]):

$$\begin{aligned} r_H &= \frac{(P_1 - P_0 + D_P) - N(F_1 - F_0)}{P_0} \\ &= r_P - \left(\frac{NF_0}{P_0}\right)\left(\frac{F_1 - F_0}{F_0}\right) \end{aligned} \qquad (1)$$

where P_0 and P_1 denote the beginning and ending market values of the stock portfolio, r_P the rate of return on the stock portfolio at time 1, and F_0 and F_1 denote the futures price at times 0 and 1. D_P represents the cumulative values as of time 1 of the dividends paid out on the portfolio during the period. The index futures are assumed to expire at a date beyond time 1. Furthermore, the 'marking to market' principle of the futures market is ignored in Equation (1).

Defining h as the constant hedge ratio, i.e., the current "value" of the index futures contract as a fraction of the current value of the stock portfolio being hedged, then Equation (1) can be written as:

$$r_H = r_P - hr_F \qquad (2)$$

where r_F is for expository convenience defined as the rate of return on the futures contract. Expected rate of return is one of the two components of the more general mean-risk decision models we consider in this paper. From Equation (2) it follows that the expected rate of return of the hedged stock portfolio u(h) is given by:

$$\mu(h) = \mu_P - h\mu_F \qquad (3)$$

where μ_P and μ_F are the expected rate of return on the stock portfolio and on the lùtures contract, respectively. The other component, risk, can be defined in several ways. In this paper we concentrate on two definitions of risk: variance and several downside risk measures.

The variance of a hedged stock portfolio $\sigma^2(h)$ can be derived from Equation (2):

$$\sigma^2(h) = \sigma_P^2 + h^2\sigma_F^2 - 2h\sigma_{P,F} \tag{4}$$

where σ^2 with a single subscript denotes a variance and σ with two subscripts a covariance. Johnson [1960] showed that the minimum variance hedge ratio h_{var} equals:

$$h_{var} = \frac{\sigma_{P,F}}{\sigma_F^2} \tag{5}$$

In the hedging literature, the minimum variance hedge ratio is usually taken as the optimal hedge ratio. On the one hand, Figlewski [1985] noticed that there is a certain ambiguity in defining optimal hedge ratios in this way. Since investors are in general not infinitely risk averse, they do not always choose the minimum variance portfolio. By altering h, an investor can achieve any combination of expected rate of return and variance of return according to Equations (3) and (4). Hence, the optimal portfolio depends on the specific preferences of the investor. On the other hand, defining the optimal hedge ratio as the minimum variance (or any other risk measure) hedge ratio gives information about the risk reduction potential of a futures contract. When the variance of the optimal hedged portfolio is compared to the variance of the unhedged portfolio, it will show by how much variance risk can be reduced. Therefore, in this paper we also define optimal hedge ratios as the hedge ratio that minimizes the risk of a portfolio.

The other definition of risk we examine in this paper is downside risk. Besides the semivariance versus the mean as a measure of downside risk, we employ the lower partial moment according to Fishbum's [1977] a-t model. The lower partial moment of a hedged portfolio, lpm(h;a,t), is defined as:

$$lpm(h;\alpha,t) = E[-min(t,r_P - hr_F)] = \iint_{r_P - hr_F \leq t} [t-(r_P - hr_F)]^{\alpha} f(r_P,r_F) dr_P dr_F \quad (6)$$

where f(.,.) is the two-dimensional probability distribution of the rate of returns on the stock portfolio and the futures contract and t is the target. The value of a is a measure of risk aversion for returns below the target: a between 0 and 1 implies a risk seeking attitude, a=l implies a risk neutral attitude and α>1 implies risk-averse behavior.[7] The minimum lower partial moment hedge ratio $h_{\alpha,t}$ can be determined by setting the derivative in Equation (6) with respect to h equal to zero. In contrast to the variance, however, an explicit expression for $h_{\alpha,t}$ like Equation (5) cannot be obtained.

Normal returns

In comparing hedging strategies under different risk measures, first assume that the returns of the stock portfolio and the tùtures contract have a bivariate normal distribution. Then the variance is the correct risk measure. Regarding the first measure of downside risk, the semivariance, it is easy to show that for a hedged portfolio it is a factor two smaller than the variance because normal returns are symmetric. Therefore, both minimizing risk hedge ratios must be the same and applying the minimum variance hedge ratio in a situation where the semivariance versus the mean is the appropriate risk measure has no consequences.

With respect to the lower partial moments, first assume that $\alpha \geq 1$. We prove in the Appendix the following two relations:

$$\mu_F > 0 \Rightarrow h_{\alpha,t} < h_{var} \quad (7)$$

$$\mu_F < 0 \Rightarrow h_{\alpha,t} > h_{var} \quad (8)$$

That is, if futures markets are positively biased then minimum lower partial moment hedge ratios are smaller than the minimum variance hedge ratio. In the case of negatively

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7 Loosely speaking, for small values of a an investor is highly concerned for not meeting the target return but has little concern about the size of the deviation, whereas for large values of a there is little concern about small deviations below t but high concern about large deviations below t.

biased markets the opposite holds. Figure 1 depicts the MV opportunity frontier of a stock portfolio P hedged with futures. The opportunity frontier has been drawn for four combinations of μ_F and $\sigma_{P,F}$ (Figures 1 a-ld). In the figures, $h_{\alpha,t}$ and h_{var} denote the minimum lower partial moment hedge ratio and the minimum variance hedge ratio, respectively. The sign of h_{var} is completely determined by the sign of the covariance between the returns on the portfolio and the futures contract (see Equation (5)). The location of $h_{\alpha,t}$ on the opportunity frontier relative to h_{var} is based upon the sign of μ_F in combination with the Equations (7) and (8). Figure la shows that if $\mu_F>0$ and $\rho_{P,F}>0$, as is usual the case, then minimum lower partial moment hedge ratios with $\alpha\geq1$ are less positive than the minimum variance hedge ratio.

Figure 1 reveals that a portfolio hedged with a minimum lower partial moment ratio is always situated on the upper part of the MV opportunity locus, above the minimum variance portfolio. Therefore, the expected return on a minimum lower partial moment hedged portfolio is strictly higher than the expected return on the minimum variance portfolio.[8] The difference between the two expectations determines the extra cost of hedging the lower partial moment with the minimum variance hedge ratio.'

The extra cost can be calculated directly as (writing out Equation (3) for both strategies):

$$\mu(h_{\alpha,t})-\mu(h_{var})=(h_{var}-h_{\alpha,t})\mu_F \tag{9}$$

where $\mu(h_{\alpha,t})$ is the expected return on the minimum lower partial moment hedged portfolio and $\mu(h_{var})$ is the expected return on the minimum variance portfolio. Equation (9) in combination with Equations (7) and (8) shows in a more formal way that if $\mu_F\neq0$ then there is an extra cost, different from zero, associated with hedging the lower partial moment using the minimum variance hedge ratio. Hence, investors who really are

[8] It is easy to show that the expected return on a minimum lower partial moment hedged portfolio is not lower than the mean return on the minimum variance portfolio. Because of the assumed normal returns, the upper part of the opportunity frontier together with the minimum variance portfolio constitutes the SSD efficient set. Fishbum [1977, p. 123] showed in the second part of Theorem 3 that for any return distribution, any target and for all $\alpha\geq1$, the set of MLPM(α,t) efficient portfolios is a subset of the SSD efficient set. Therefore, portfolios hedged with the minimum lower partial moment hedge ratio are situated at or above the minimum variance point.

[9] This definition of extra cost is based upon the general definition of the cost of hedging, i.e., the expected return given up.

concemed with returns falling below a target pay an extra cost if they hedge a lower partial moment of their portfolio with the minimum variance hedge ratio. The extra cost has been pictured in Figure 2a. The lower partial moment of a minimum variance hedging strategy is, by implication, at least as large as the lower partial moment of the minimum lower partial moment strategy. However, it is strictly larger because of the higher expected return on a minimum lower partial moment hedging strategy. Therefore, the investor not only pays an extra cost but is also worse off in terms of the reduction in the lower partial moment. In Figure 2b the difference in downside risk is rendered in the MLPM(α,t) space.

Figure 1

Opportunity frontiers of the stock portfolio P hedged with futures for four different combinations of $\mu_F \neq 0$ and $\sigma_{P,F} \neq 0$

If $\mu_F > 0$ then selling short futures ($h < 0$) comspcnd to points located above the point P on the opportunity frontier and buying futures ($h > 0$) correspond to points under P on the opportunity frontier, see Equation (3). If $\mu_F < 0$ then the opposite is truc. Hence, thc locus of h_{var} relative to P is determined by the sign of h_{var}, which in turn is completely determined by the sign of $\sigma_{P,F}$, see Equation (5). The locus of $h_{\alpha,t}$ is always above h_{var} on the opportunity frontier, see Equations (7) and (8).

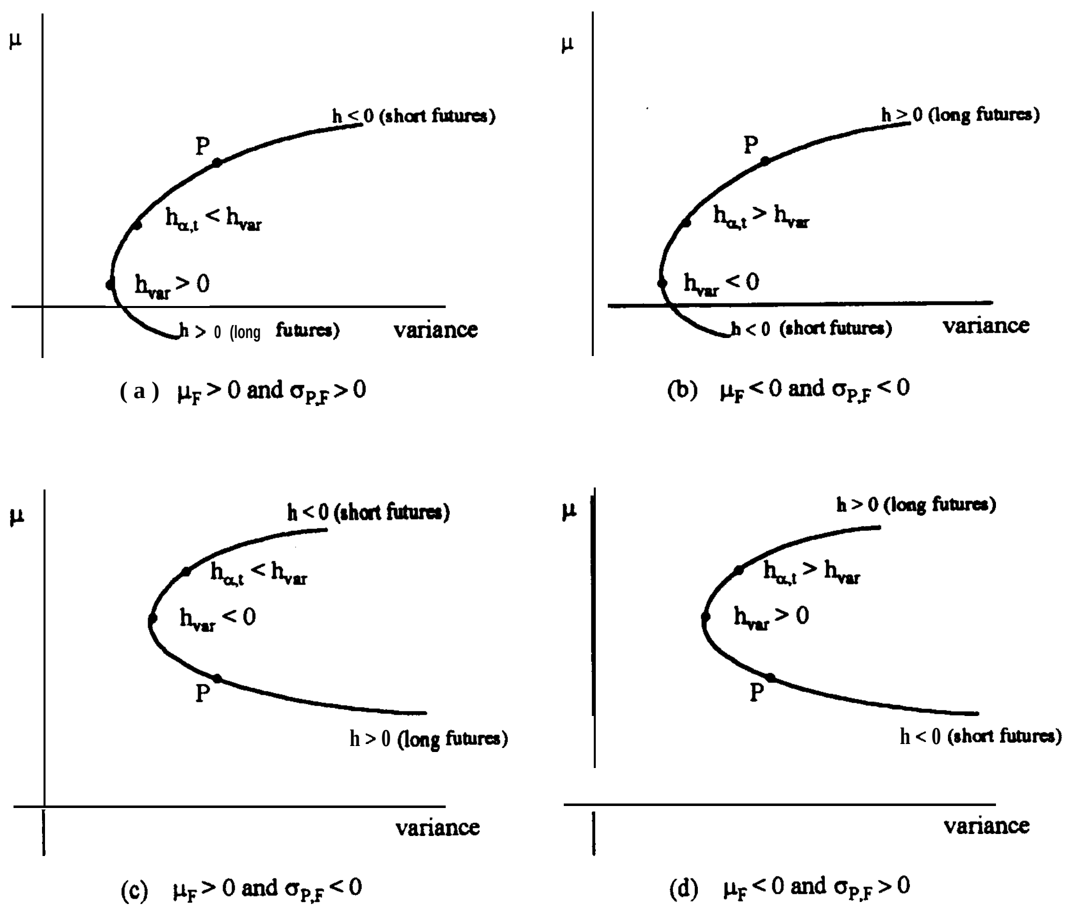
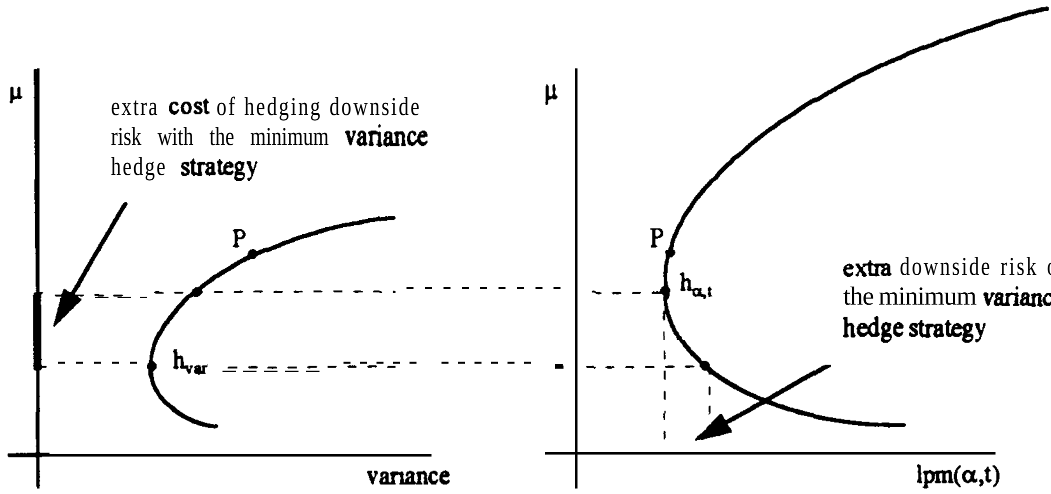


Figure 2

Opportunity frontier of the stock portfolio P hedged with futures ($\mu_F \neq 0$)

Figure (a) shows the extra cost of hedging downside risk using the minimum variance hedge ratio in a MV space. Figure (b) depicts the extra downside risk in a MLPM(α,t) space.



(a) MV space (b) MLPM(α,t) space

Now suppose that $0<\alpha<1$ (and still assuming $\mu_F \neq 0$). The first part of Theorem 3 in Fishbum [1970, p. 123] states that MLPM(α,t) efficient portfolios belong to the first order stochastic dominance (FSD) efficient set. For our investment problem and in the case of normal returns, every portfolio on the lower part of the opportunity frontier is dominated by the portfolio on the upper part of the opportunity frontier with the same variance.[10] Hence, the set of FSD efficient portfolios consists of the upper part of the opporhmity frontier and the minimum variance portfolio. Using Theorem 3 of Fishbum, this implies that minimum lower partial moment hedged portfolios are situated above or at the minimum variance hedged portfolio. Therefore, Equations (7) and (8) with in both equations the second inequality replaced by ≤ and ≥, respectively, are also true for lower partial moments with $0<\alpha<1$. Furthermore, there is an extra cost of hedging a lower partial moment with $0<\alpha<1$ using the minimum variance hedge ratio although the extra cost, in contrast to the situation with $\alpha \geq 1$, can be zero.

If $\mu_F=0$, then there is no cost at all involved in futures hedging. Hence, there is also no extra cost associated with hedging downside risk using the minimum variance hedge

[10] This can be verified by applying the definition of first order stochastic dominance: the cumulative retum distribution function of a portfolio on the upper part of the opportunity locus is strictly below the cumulative distribution of the portfolio with the same variance but located *on* the lower part of the opportunity frontier.

strategy. Regarding the hedge ratios and and the amount of downside risk of a position hedged with the minimum variance hedge ratio, the following can be said. For any risk measure the mean-risk opportunity frontier is a horizontal line (this follows directly from Equation (3)). Then under the condition of normal returns, the minimum variance portfolio is the sole MV efficient portfolio. Furthermore, the minimum variance portfolio is the only portfolio that is not second order dominated by another portfolio, hence it is the only portfolio that is SSD efficient." According to the second part of Theorem 3 in Fishbum [1977, p. 123] (see footnote 8), this implies that all minimum lower partial moment hedge ratios with $\alpha \geq 1$ and the minimum variance hedge ratio must be equal. Moreover, the minimum semivariance hedge ratio is also equal to the minimum variance hedge ratio. This is true because if $\mu_F=0$ then the semivariance is just a special case of Fishbum's α-t model with $\alpha=2$ and a target equal to the fixed expected rate of return on the stock portfolio. Since all hedge ratios are the same, there are no consequences of hedging a lower partial moment with $\alpha \geq 1$ or the semivariance using the minimum variance hedge ratio.

Regarding $0<\alpha<1$ in case $\mu_F=0$, no portfolio is dominated by another one. Hence, all portfolios are FSD efficient. Then from the first part of Theorem 3 of Fishbum [1977] it follows that minimum lower partial moment hedged portfolios can be located anywhere on the horizontal opportunity frontier. Hence, nothing can be said about minimum lower partial moment hedge ratios with $0<\alpha<1$ in relation to the minimum variance hedge ratio.

We must emphasize that although a MV analysis is correct if returns have a normal distribution, it is still meaningful to investigate minimum lower partial moment hedge ratios. In other words, it does make sense to look at lower partial moments in a situation where the variance is the appropriate risk measure. The argument is that pottfolios hedged with minimum lower partial moment hedge ratios are situated above (or possibly at for $0<\alpha<1$) the minimum variance portfolio (see Figure 1). Hence, these portfolios are MV efficient and therefore a meaningful altemative. Moreover they take into account (for $\alpha \geq 1$) a trade off between the mean and the variance of return. In this way it accommodates, to a certain extent, the ambiguity of defining optimal hedge ratios as minimum variance hedge ratios (as described by Figlewski [1986]).

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[11] **Benninga, Eldor and Zilcha [1984] and Bond, Thompson and Lee [1987] showed that the assertion also holds under weaker assumptions about the distribution than normality.**

Non-normal returns

Suppose now that returns do not have a normal distribution. Regarding the semivariance, not much can be said about the minimum variance and semivariance hedge ratios in case of non-normal returns. Only if the non-normal return distribution of an arbitrary hedged portfolio is symmetric, both hedge ratios must be equal.

In the case of non-normal returns, the set of SSD efficient portfolios is in general not the same as the set of MV efficient portfolios. MV efficient portfolios may not belong to the SSD efficient set, whereas MV non-efficient portfolios may belong to the set of SSD efficient portfolios. Consequently, the minimum variance hedged portfolio is not necessarily S SD efficient .

Regarding the MLPM(a,t) framework, first suppose that for al1 a (>0) and t portfolios hedged with the minimum lower partial moment hedge ratio are located above the minimum variance hedged portfolio (on the opportunity frontier in a MV space). Then Fishbum's [1977] first part of Theorem 3 implies that the minimum variance hedged portfolio cannot be FSD efficient. Consequently, such a portfolio is not a very attractive one for a greedy investor. Moreover, if $\mu_F \neq 0$ and the minimum variance hedge ratio is used, the hedged portfolio also comes with an extra cost. On the other hand, however, suppose there is a pair of *a* (>0) and t for which the hedged portfolio is situated below the minimum variance point. Then the minimum variance hedged portfolio must be FSD efficient. Hence, for that pair of a and t there is an extra cost associated w-ith hedging the lower partial moment. However, the extra cost goes with a larger reduction of the lower partial moment.'*

3 DATA AND METHODOLOGY

We use the Dutch FTI fùtures contract to examine empirically hedging under various risk measures. The FTI contract is a stock index futures contract written on the Amsterdam EOE index (AEX) and was introduced on the Dutch financial futures market in

[12] Notice that the reasoning can be applied to a-values larger or equal to one as well. In that case the concern is the SSD efficiency of the minimum variance hedged portfolio in relation to not only greedy investors but also to risk averse investors.

Amsterdam, the FTA, in October 1988. It soon appeared to be very successful with high trading volume and open interest. The AEX is an index composed of twenty-fíve stocks with a fixed number of shares for each stock. The twenty-five stocks are represented by the stocks with the highest trading volume.[13] The stock portfolios that have been hedged were represented by different Dutch market indexes: the AEX and the index that consists of nearly all stocks traded on the Amsterdam stock exchange (General index). Because Royal Dutch has a very large weight in the General index whereas it is restricted in the AEX, we also considered the general index without Royal Dutch (Genera1 ex RD). The General and General ex RD indexes are both market weighted indexes with dividends reinvested. Dividends paid out by stocks in the AEX are not reinvested.

The study uses data from January 4, 1989, through January 26, 1994. Non-overlapping 4-week, 2-week and l-week hedge durations are employed. Hedge durations are defined using Wednesday closing prices for the indexes and Wednesday closing bid (beginning of the hedging period) and closing ask (end of the hedging period) prices for the FTI contract. The fùtures prices are represented by the nearest futures contract not expiring in the hedging period. At any point in time there are six fùtures contracts traded: in the first three consecutive months and in the January-April-July-October cycle. Hence, only the two nearest expiring contracts have been used. An examination of the open interest and volume information on the FTI contract revealed that almost all trading activity occurs in these contracts. Cash and fùtures data on the AEX and on the Genera1 and General ex RD indexes were bought from the European Options Exchange (EOE) located in Amsterdam and the Centraal Bureau voor Statistiek (CBS), respectively.

Measures for downside risk are the sample semivariance and the sample lower partial moments according to the a-t model. Ahmedi, Sharp and Walther [1986] show that the latter is an unbiased estimator of the population lower partial moment. The sample semivariance, however, is unbiased only asymptotically (see Josephy and Aczel [1993]).[14] The values of a we take correspond to risk-seeking behavior for returns below the target, 0.5, risk-neutral behavior, 1, moderate risk-averse behavior, 2 and 3,

[13] At the moment, the number of shares of each stock in the index is updated every February. The new numbers are based upon the trading volume of the stock realized in the preceding three years.

[14] In order to get (asymptotically) unbiased estimators, the multiplier for the sample variance is $1/(n-1)$ (n = number of observations), for the semivariance $n/(n-1)^2$, and for the lower partial moment it is 1\n.

high risk aversion , 5, and extreme high risk aversion for below target returns, 10. These values are based upon earlier findings by Fishbum [1977], who estimated for several empirical studies on risk taking behavior implied *a* values. The a values he found ranged from less than 1 to greater than 4. For the target we used annual rates of 0%, 2.5%, 5% and 7.5 %, which are assumued to be realistic values in practice (although the value of 7.5% is extreme). For the three hedge durations, these numbers were multiplied with the length of the hedge duration in years.

For the three indexes the minimum variance hedge ratios were estimated directly by using Equation (5) with the σ's replaced by the sample estimates. Sample downside risk measures were minimized numerically with the nonlinear optimization routine of the software package Excel (version 4.0). Hedging the three indexes with the FTI contract is evaluated in two ways. First, for the several risk measures the relative size of minimum risk hedge ratios as well as the hedging effectiveness are compared. For all risk measures, effectiveness is defined as minus the relative reduction in normalized risk of the unhedged portfolio:

$$\frac{\textit{normalized risk unhedged portfolio} - \textit{normalized risk hedged portfolio}}{\textit{normalized risk unhedged portfolio}} \qquad (12)$$

where normalized risk is defined as the standard deviation, semi-standard deviation and $\sqrt[\alpha]{lpm(h;\alpha,t)}$ in case variance, semivariance and lower partial moment are the considered risk measures, respectively. Second, the hedge performance is evaluated for the situation that downside risk is the appropriate risk measure, but the portfolio is hedged with the minimum variance hedge ratio. The evaluation has been carried out in terms of differences in return (the extra cost of hedging with the wrong hedge ratio) and hedging effectiveness.

We used an in-sample (i.e., optimization and evaluation are conducted over the same period) as well as an out-of-sample procedure. In-sample hedge ratios were estimated using all data. These constant hedge ratios were applied to the whole sample and the resulting hedge returns were used to determine the evaluation measures. Because of estimation error, all conclusions will only prove useful if they are also valid on a different sample. The out-of-sample results are based upon hedge ratios determined with a moving window procedure (see, e.g., Grammatikos and Saunders [1983]). Initially, hedge ratios were estimated for the three year period January 4, 1989, through January 2, 1992. Then they were re-estimated every four weeks (for the four weeks as well as the two and one week hedge durations) by adding new spot and futures data and deleting the initial four week data (i.e., keeping a three year estimation period). Using this procedure a moving hedge ratio can be derived for every 4-week, 2-week and l-week hedge duration in the two year subsample January 2, 1992, through January 26, 1994. (Notice that for a 2-week and 1-week hedge duration the hedge ratios are constant during the four weeks following an estimation window.) Every hedge ratio was applied to the four weeks following its estimation period. The hedge returns generated in this way (for the last two year of the sample) were used to evaluate the hedge performance.[15]

4 EMPIRICAL RESULTS

In this section we show the empirical results. First, in subsection 4.1 we describe several statistics of the return distribution of the three indexes and the FTI contract. Then the in-sample and the out-of-sample results are summarized in subsections 4.2 and 4.3, respectively.

4.1 Summary statistics

Table 1 shows the relevant summary statistics for the three indexes and the FTI contract. As expected, the AEX has the largest correlation with the FTI contract, 0.99, because it is the index where the contract is written on. Based upon two simple tests, zero skewness and zero excess kurtosis, normality of return distributions is rejected more pronounced if the returns are measured over a shorter interval. This is conform the findings in finance literature. Normality of 4-week returns is, as Table 1 shows, clearly not rejected. According to the results of the previous section, we expect that there is an

[15] We also applied to the last two years of the sample the constant hedge ratios estimated with the first three years of the sample. Based upon the hedging effectiveness, this naive estimation technique performed worse compared to the moving window procedure.

extra cost associated with hedging a lower partial moment (for nomisk-seeking behavior, i.e., $\alpha \geq 1$) using the minimum variance hedge ratio. Furthermore, the fact that the mean return on the FTI contract is positive, although not significant, the minimum lower partial moment hedge ratios should be smaller than the minimum variance hedge ratio. Regarding the I-week returns, all the indexes have a (negative) skewness parameter more than two standard deviations away from zero. This suggests that the return distributions are not symmetric. Based upon Section 3, this implies that minimum semivariance and minimum variance hedge ratio should be different in size.

Table 1
Summary statistics

Statistics are based upon the whole sample, i.e., January 4, 1989, through January 26, 1994. The kurtosis coefficient is in excess of 3. In the case of normal returns, both the skewness and excess kurtosis coeffrcient have a value of 0. Standard error of the mean ($\sqrt{\text{sample variance ofthe mean / number of observations}}$), skewness ($\sqrt{6 / \text{numberof observations}}$) and excess kurtosis ($\sqrt{24 / \text{number of observations}}$) are given between brackets, with an asterisk indicating that the statistic is more than two standard errors away from zero. Means and standard error of the means are annualized, i.e., multiplied by the number of holding periods per year. K-S denotes the Kolmogorov-Smirnov test for normality with a Lilliefors significance level.

Holding period	Index	Mean in %	Skewness coefficient	Excess kurtosis coefficient	pvalue K-S	Corr. coeff. with nearest FTI contract
4-week	AEX	10.8(7.0)	-0.48(0.30)	0.65(0.58)	>0.20	0.99
(66 obs.)	General	15.6(5.3)*	-0.40(0.30)	0.41(0.58)	>0.20	0.91
	Gen. ex RD	14.6(6.0)*	-0.66(0.30)*	1.00(0.58)	>0.20	0.98
	FTI	9.8(7.0)	-0.51(0.30)	0.82(0.58)	>0.20	1
2-week	AEX	10.7(6.8)	-0.41(0.21)	0.74(0.42)	>0.20	0.99
(132 obs.)	General	15.6(5.2)*	-0.51(0.21)*	0.70(0.42)	>0.20	0.91
	Gen. ex RD	14.6(5.7)*	-0.59(0.21)*	1.34(0.42)*	x.20	0.97
	FTI	10.7(6.8)	-0.36(0.21)	0.73(0.42)	>0.20	1
1-week	AEX	10.4(6.8)	-0.53(0.15)*	1.38(0.30)*	>0.20	0.99
(264 obs.)	General	15.6(5.2)*	-0.32(0.15)*	0.71(0.15)*	>0.20	0.90
	Gen. ex RD	14.6(5.7)*	-0.61(0.15)*	1.77(0.15)*	>0.20	0.97
	FTI	13.0(6.8)	-0.50(0.15)*	1.29(0.15)*	>0.20	1

In order to get some insight in the magnitude of the various risk measures, Table 2 presents the annualized standard deviation, semi-standard deviation and normaliied lower partial moments for the three unhedged indexes. For 4-week returns the annualized lower partial moments range from about 7% to 20% for $\alpha \geq 1$. In contrast with this, for $\alpha = 0.5$ the annualized lower partial moments are substantial higher. Notice that for fixed values of a, the lower partial moment increases with a higher target. This must be the case because more returns fall below a higher target. The annualized standard deviations of the three indexes are in agreement with those of a well-diversified portfolio reported in the literature. In terms of the standard deviation the General index is the least risky one. This is not surprising because it consists of all stocks traded on the Dutch stock market implying the highest gain from diversification. The semi-standard deviations of the three indexes are more than $\sqrt{2}$ smaller than the standard deviation. But, according to the normality tests shown in Table 1, the difference should not be significant (from a statistical point of view) for 4-week (and 2-week) returns.

Regarding the relative riskiness, the three indexes are ranked by almost all downside risk measures similar to the standard deviation. For 4-week returns the single exception is the lower partial moment with t=7.5% and risk-seeking behavior. In that case the General ex RD index is the least risky index instead of the General index although the difference in risk is small: 54.9% versus 55.4%.

Table 2

Risk values in %

Risk values are based upon the whole sample, i.e., January 4, 1989. through January 26, 1994. The risk measures are the variance, the semivariance versus the mean (Semivar.) and various lower partial moments: all combinations of 0.0%, 2.5%, 5.0% and 7.5% (annualized target) and 0.5, 1, 2, 3, 5 and 10 (a values). All values are annualized and nonnalized. For example, for the variance as risk measure and a 4-week hedge duration, the square root (normalized value) is taken from 13 times the variance (annualized value). The other risk measures are adjusted accordingly.

Index	Variance	semivar.		Lower partial moment					
4-week holding period (66 obs.)									
				0.5	1	2	3	5	10
AEX	15.8	11.9	0.0%	67.4	16.8	10.3	9.6	9.6	10.1
			2.5%	75.6	17.8	10.6	9.8	9.8	10.3
			5.0%	83.2	18.8	10.9	10.0	10.0	10.5
			7.5%	92.3	19.9	11.3	10.3	10.2	10.7
				0.5	1	2	3	5	10
General	12.1	9.1	0.0%	34.4	10.4	6.9	6.5	6.5	6.6
			2.5%	40.7	11.3	7.2	6.7	6.7	6.8
			5.0%	48.8	12.2	7.5	6.9	6.8	7.0
			7.5%	55.4	13.2	7.8	7.2	7.0	7.2
				0.5	1	2	3	5	10
General ex RD	13.4	10.3	0.0%	37.5	12.1	8.4	8.0	8.1	8.6
			2.5%	43.0	12.9	8.6	8.2	8.3	8.8
			5.0%	49.1	13.8	8.9	8.4	8.5	9.0
			7.5%	54.9	14.7	9.2	8.6	8.7	9.1

Table 2–*Continued*

Index	Variance	Semivar.		Lower partial moment					
2-week holding period (132 obs.)									
				0.5	1	2	3	5	10
AEX	15.0	11.1	0.0%	223	24.4	9.9	8.3	7.9	8.4
			2.5%	242	25.5	10.2	8.4	8.0	8.5
			5.0%	264	26.7	10.4	8.5	8.1	8.6
			7.5%	288	27.9	10.7	8.7	8.2	8.6
				0.5	1	2	3	5	10
General	11.8	8.9	0.0%	121	16.3	7.3	6.2	6.0	6.1
			2.5%	136	17.2	7.6	6.4	6.0	6.2
			5.0%	153	18.2	7.8	6.5	6.2	6.3
			7.5%	173	19.3	8.0	6.7	6.3	6.4
				0.5	1	2	3	5	10
General ex RD	12.9	9.8	0.0%	148	18.8	8.3	7.2	7.1	7.6
			2.5%	163	19.8	8.6	7.3	7.2	7.7
			5.0%	180	20.8	8.8	7.4	7.3	7.8
			7.5%	199	21.9	9.0	7.6	7.4	7.9
1-week holding period (264 obs.)									
				0.5	1	2	3	5	10
AEX	14.8	11.1	0.0%	660	36.0	10.3	7.7	7.0	7.6
			2.5%	699	37.1	10.5	7.8	7.1	7.6
			5.0%	741	38.2	10.6	7.9	7.1	7.7
			7.5%	788	39.4	10.8	8.0	7.2	7.7
				0.5	1	2	3	5	10
General	11.5	8.5	0.0%	434	25.2	7.4	5.4	4.7	4.7
			2.5%	460	26.2	7.5	5.5	4.8	4.7
			5.0%	487	27.2	7.7	5.6	4.8	4.8
			7.5%	517	28.2	7.9	5.7	4.9	4.8
				0.5	1	2	3	5	10
General ex RD	13.0	9.8	0.0%	485	28.8	8.8	6.8	6.2	6.6
			2.5%	520	29.8	8.9	6.8	6.3	6.7
			5.0%	554	30.9	9.1	6.9	6.3	6.7
			7.5%	590	32.0	9.3	7.0	6.4	6.7

4.2 In-sample results

Hedge ratios and hedge effectiveness (in-sample analysis)

In Table 3 the constant risk minimizing hedge ratios are displayed. As can be seen from the table, for all indexes and all hedge durations the minimum semivariance hedge ratios do not differ much from the minimum variance hedge ratios. This is what we expect for 4-week returns (and to a lesser extent for 2-week returns) because for our data 4-week returns exhibit no skewness. Although Table 1 suggests that 1-week returns are skewed from a statistical point of view, the skewness is not large enough to yield diierent hedge ratios. For all indexes the minimum semivariance and the minimum variance hedge ratios are almost the same. The other downside risk measure which weighs the deviations of below target returns with the same exponent as the variance is the target semivariance (i.e., the lower partial moment with $\alpha=2$). Table 3 shows that for the AEX and the General ex RD index the minimum target semivariance hedge ratios are only slightly smaller than the minimum semivariance hedge ratio. Regarding the General index, however, the differences are more pronounced: 0.07, 0.05 and 0.04 for a 4-week, 2-week and 1-week hedge duration, respectively.

Looking at the minimum lower paxtial moment hedge ratios given the risk attitude *a*, the following can be said with respect to all three hedge duration. For $\alpha\geq2$ (risk-averse behavior for returns below the target) the minimum lower partial moment hedge ratios are almost independent of the target. This holds for all three indexes. In contrast with this, a target-independent hedge ratio is not displayed for risk-seeking behavior ($\alpha=0.5$) and risk-neutral behavior ($\alpha=1$). Instead, for both these values of a the hedge ratio has a tendency to decrease with an increase in the target. This dependency is more pronounced for risk-seeking behavior.

Because normality of returns is not rejected for our data (at least for 4-week returns), we expect lower partial moment hedge ratios to be smaller than (or equal to for $\alpha=0.5$) the minimum variance hedge ratio. Table 3 confirms this. Hence, hedging a lower partial moment involves a smaller number of FTI contracts compared to the minimum variance hedge strategy.

Table 3

Hedge ratios (in-sample analysis)

Hedge ratios are estimated for the nearest FTI contract using the whole sample, i.e., January 4, 1989, through January 26, 1994. The risk measures are the variance, the semivariance versus the mean (Semivar.) and various lower partial moments: all combinations of 0.0%. 2.5%, 5.0% and 7.5% (annualized target) and 0.5, 1, 2, 3, 5 and 10 (a values).

Index	Variance	semivar.		Lower partial moment					
4-week holding period (66 obs.)									
				0.5	1	2	3	5	10
AEX	.99	1.00	0.0%	.97	.98	.98	.98	.95	.91
			2.5%	.95	.97	.98	.98	.%	.92
			5.0%	.89	.97	.98	.98	.96	.92
			7.5%	.81	.94	.97	.98	.97	.93
				0.5	1	2	3	5	10
General	.70	.67	0.0%	.67	.67	.63	.63	.64	.63
			2.5%	.60	.64	.64	.63	.64	.63
			5.0%	.61	.62	.63	.63	.64	.63
			7.5%	.63	.63	.63	.63	.64	.63
				0.5	1	2	3	5	10
General ex RD	.83	.82	0.0%						
			2.5%	.80	.82	.80	.77	.75	.74
			5.0%	.74	.81	.80	.78	.76	.74
			7.5%	.70	.81	.80	.78	.76	.74

Table 3—*Continued*

Index	Variance	Semivar.		Lower partial moment					
2-week holding period (132 obs.)									
				0.5	1	2	3	5	10
AEX	.98	.98	0.0%	.96	.97	.97	.97	.95	.94
			2.5%	.93	.96	.97	.97	.96	.94
			5.0%	.90	.96	.97	.97	.96	.95
			7.5%	.88	.94	.97	.97	.96	.95
				0.5	1	2	3	5	10
General	.71	.69	0.0%	.67	.69	.66	.65	.66	.68
			2.5%	.63	.68	.66	.65	.66	.68
			5.0%	.62	.68	.66	.65	.66	.68
			7.5%	.62	.66	.66	.65	.66	.68
				0.5	1	2	3	5	10
General ex RD	.83	.83	0.0%	.82	.85	.82	.79	.75	.72
			2.5%	.80	.84	.82	.80	.75	.72
			5.0%	.81	.82	.82	.80	.75	.72
			7.5%	.76	.81	.82	.80	.76	.72
l-week holding period (264 obs.)									
				0.5	1	2	3	5	10
AEX	.97	.97	0.0%	.92	.95	.96	.96	.95	.93
			2.5%	.90	.94	.96	.96	.95	.93
			5.0%	.84	.94	.96	.96	.95	.94
			7.5%	.80	.93	.96	.96	.95	.94
				0.5	1	2	3	5	10
General	.68	.67	0.0%	.60	.64	.64	.64	.62	.60
			2.5%	.57	.63	.64	.64	.62	.60
			5.0%	.56	.62	.64	.64	.62	.60
			7.5%	.53	.62	.64	.64	.62	.60
				0.5	1	2	3	5	10
General ex RD	.83	.84	0.0%	.79	.81	.82	.81	.78	.74
			2.5%	.79	.80	.82	.81	.78	.74
			5.0%	.74	.80	.82	.81	.78	.74
			7.5%	.72	.80	.82	.81	.78	.75

Table 4 shows that the FTI contract reduces the variance most for the AEX, whereas the variance of General ex RD index can be reduced more than the variance of the General index. This result is in agreement with Table 1: the AEX has the highest correlation coefficient with the FTI contract, followed by the General ex RD and the General index. The semivariance can be reduced (first numbers shown in the 'Semivar' column) in the same proportion as the variance (numbers shown in the 'Variance' column). With respect to the various lower partial moments the maximum reduction in risk that can be achieved is about the same, except for two cases, as the maximum reduction of the variance. The two exceptions are, first, that hedging the lower partial moment with α=0.5 is hardly effective for high targets. This holds for all indexes and all hedge durations. For example, the 4-week lower partial moment of the AEX can be reduced with 90% and 80% for a 0.0% and 2.5% target, respectively, but only 66% and only 48% for a 5% and 7.5% target. Notice that for this example a higher target causes the hedge effectiveness of the **FTI** contract to decline. Table 4 shows that **such** a regularity does not only exists for this special case, but for **all** values of a, all **indexes** and **all** hedge durations. **Second,** for the Genera1 index the lower partial moment **can** be reduced more than the **variance** for high values of ***a*.** For the Genera1 ex RD, this only **holds** for a 4-week hedge duration but in that case it is **true** for **all** values of $\alpha>1$.

Table 4

Hedging effectiveness in % (in-sample analysis)

Hedging effectiveness, (i.e., **the** risk reduction as **a fraction** of **the** risk of the unhedged position) of **the nearest FTI** contract for **several** risk measures using the **whole** sample, i.e., **January 4, 1989, through January 26, 1994.** The risk measures are the **variance, the** semivariance versus the **mean** (Semivar.) and **various** lower partial **moments: all** combinations ofO.O%, **2.5%,** 5.0% and 7.5% **(annualized** target) and 0.5, **1, 2, 3, 5** and 10 (a values). **With** respect to the downside risk measures, the **first** numbers denote the hedging effectiveness using the optimal hedge ratio **whereas** the **second** numbers represent the hedging effectiveness using the minimum **variance** hedge ratio.

Index	Variance	Semivar.	Lower partial moment						
4-week hedge duration (66 obs.)									
				0.5	1	2	3	5	10
AEX	88	86/86	0.0%	90/89	87/87	86/86	85/85	85/85	85/85
			2.5%	80/78	82/82	83/83	83/83	84/83	84/83
			5.0%	66/58	76/75	80/80	81/81	82/82	82/82
			7.5%	48/31	68/66	77/77	79/79	80/80	81/81
				0.5	1	2	3	5	10
General	59	62/62	0.0%	65/64	67/66	71/69	72/71	74/72	74/73
			2.5%	55/55	62/61	67/66	70/68	71/70	72/71
			5.0%	49/48	57/56	64/62	67/65	69/68	70/69
			7.5%	39/39	51/51	60/59	64/63	67/65	68/67
				0.5	1	2	3	5	10
General ex RD	78	80/80	*0.0%*	90/90	87/87	86/85	86/84	86/82	86/80
			2.5%	83/83	83/83	83/83	83/82	84/80	85/79
			5.0%	74/72	*77m*	80/80	81/80	82/79	83/77
			7.5%	59/52	70/70	77/76	78/78	80/77	81/76
2-week hedge duration (132 obs.)									
				0.5	1	2	3	5	10
AEX	88	87/87	0.0%	88/87	86/86	86/86	86/86	87/87	88/87
			2.5%	79/76	82/82	84/84	84/84	86/85	87/86
			5.0%	68/65	77/77	81/81	83/83	84/84	86/85
			7.5%	59/50	72/71	79/79	81/81	83/83	85/84
				0.5	1	2	3	5	10
General	58	61/61	0.0%	63/61	62/62	64/63	66/65	68/67	70/69
			2.5%	56/54	59/59	62/61	65/63	67/66	69/68
			5.0%	49/48	55/55	60/59	63/62	66/65	68/67
			7.5%	43/41	52/51	58/57	61/60	65/63	67/66
				0.5	1	2	3	5	10
General ex RD	76	77/77	0.0%	88/88	84/84	79/79	76/76	75/73	75/70
			2.5%	83/83	*80180*	77/77	75/75	74/72	*74no*
			5.0%	76/75	77/76	76/76	74/74	73/71	73/69
			7.5%	66/62	72/71	74/74	73/73	72/71	72/68

Table 4–*Continued*

Index	variance	Semivar.		Lower partial moment					
1-week hedge duration (264 obs.)									
				0.5	1	2	3	5	10
AEX	88	88/88	0.0%	81/79	84/84	86/86	87/87	87/87	89/88
			2.5%	72/70	81/80	84/84	86/86	87/87	88/87
			5.0%	66/60	77/76	83/82	84/84	86/86	87/87
			7.5%	59/50	73/72	81/81	83/83	85/85	87/86
				0.5	1	2	3	5	10
General	56	59/59	0.0%	55/54	57/56	60/59	62/61	64/63	67/64
			2.5%	50/49	54/54	58/58	61/60	63/62	66/63
			5.0%	45/44	52/51	57/56	59/59	62/61	65/62
			7.5%	39/37	49/49	55/55	58/58	61/60	65/62
				0.5	1	2	3	5	10
General ex RD	76	77/77	0.0%	75/74	77/77	78/78	78/78	77/76	78/75
			2.5%	70/68	74/73	76/76	77/77	77/76	77/74
			5.0%	63/62	71/70	75/75	76/76	76/75	77/74
			7.5%	57/54	67/67	73/73	75/75	76/75	76/73

Implications of hedging downside risk using the minimum variance hedge ratio (in-sample analysis)

Table 1 shows that the expected rate of return on the FTI contract is positive. Consequently, a less positive lower partial moment hedge ratio implies that the expected return on a minimum downside risk hedge strategy is higher than on the minimum variance hedge strategy (see Equation (7)). The extra cost of hedging the lower partial moment using the minimum variance hedge ratio, i.e., difference in expected return of both strategies, can be calculated directly as the difference between the hedge ratios multiplied by the expected return on the FTI contract (see Equation (9)). Hence, a difference of 0.1 in the hedge ratios results in an extra cost of about 1%, 1.1% and 1.3% for a 4-week, 2-week and 1-week hedge duration. Based on the hedge ratios in Table 3, for a 4-week hedge duration the highest extra cost is 1.8%, 1 .O% and 1.2% for the AEX, the General index and the General ex RD index, respectively. For a 2-week and 1-week hedge duration the highest extra costs for the three indexes are 1. 1%, 1 .O% and 1.2% and 2.2%, 2.0% and 1.4%. These extra costs occur for all indexes and all three hedge durations (except the General index for a 4-week hedge duration) for $\alpha=0.5$ and a target of 7.5%. Furthermore, it seems that for $\alpha=0.5$, conform the differences in hedge ratios, the extra cost increases with an increase in the target.

It should be noted, however, that the extra cost is significant from a statistical point of view only if the expected return of the FTI contract is significantly larger than zero (i.e., if the futures market is posively biased). Table 1 suggests that for all three holding periods this is not the case: the mean returns are not significantly larger than zero. Therefore, the extra costs might be in some cases significant in size, it is not automatically true that these extra costs are real. Finally, notice that for most values of a and t the extra cost of hedging the semivariance is not only statistically insignificant but also insignificant in size. The latter follows directly from Table 3 : minimum semivariance and minimum variance hedge ratios are similar for all hedge durations and all indexes.

Hedging downside risk using the minimum variance hedge ratio does not affect the effectiveness very much either. For all three indexes the relative reduction in the semivariance using the minimum variance hedge ratio (second numbers shown in the 'Semivar' column of Table 4) is exactly the same as using the minimum semivariance hedge ratio (first numbers shown in the 'Semivar' column). With respect to the lower partial moments, the downside risk hedging effectiveness of the minimum variance strategy is a little lower compared to the optimal strategy (notice that for an in-sample analysis it cannot be higher). However, for the AEX and for risk-seeking behavior (α=0.5) and a high target there is a tendency of the minimum variance hedging strategy to perform worse. For the General ex RD index the minimum variance hedge strategy also displays a bad performance for the lower partial moment with a=lO, especially for a 4-week hedge duration.

In sum, the in-sample results indicate that hedging the semivariance with the minimum variance hedge ratio is not a serious problem to investors. The extra cost is negligibly small and the semivariance can be reduced in the same proportion as the optimal hedging strategy. With respect to the lower partial moments, the minimum variance strategy seems to be an appropriate one except for risk-seeking behavior and high targets. Then it comes with a significant (in size, not statistically) extra cost and for the AEX it is not be able to reduce the lower partial moment in a proportion similar to the optimal hedge strategy. It should be noted, however, that in these cases the optimal strategy is also not very effective in reducing risk.

4.3 Out-of-sample results

Hedge ratios and hedge effectiveness (out-of-sample analysis)

The averages of the hedge ratios estimated with the moving window procedure are displayed in Table 5. The numbers between the brackets denote standard deviations. This number gives an indication about the volatility of the hedge ratios during the last two years of the sample. Compared to the volatility of the minimum variance hedge ratios, on the one hand, minimum semivariance hedge ratios are about equally volatile. On the other hand, it seems that minimum lower partial moment hedge ratios are (slightly) more volatile. Especially for $\alpha=0.5$ and, to a lesser extent, for $\alpha=10$ the minimum lower partial moment hedge ratios are more volatile. The largest standard deviation of the hedge ratio is exhibited by the General index: 0.18 for $\alpha=0.5$, a 2-week horizon and a 2.5% target, indicating a 90% frequency interval of 0.52- 1.24.

Table 5

Average hedge ratios with standard deviation (out-of-sample analysis)

Hedge ratios, using the nearest FTI contract, were initially estimated for the three year period January 4, 1989, through January 2, 1992. Then they were re-estimated every four weeks (for the 4-week as well as the 2- and 1-week hedge durations) by adding new spot and futures data and deleting the initial four week data (i.e.. keeping a three year estimation period). Using this procedure, for all three hedge durations 27 different hedge ratios were derived in the two year subsample January 2.1992, through January 26, 1994. The risk measures are the variance, the semivariance versus the mean (Semivar.) and various lower partial moments: all combinations of O.O%, 2.5%. 5.0% and 7.5% (annualized target) and 0.5, 1, 2, 3, 5 and 10 (a values).

Index	Variance	semivar.		Lower partial moment					
4-week holding period									
				0.5	1	2	3	5	10
AEX	1.00(.01)	1.00(.02)	0.0%	1.00(.02)	.99(.02)	.99(.02)	.97(.03)	.94(.03)	.92(.04)
			2.5%	.99(.02)	.99(.02)	.99(.02)	.98(.02)	.95(.03)	.92(.04)
			5.0%	.94(.03)	.98(.02)	.99(.01)	.98(.02)	.96(.03)	.93(.04)
			7.5%	.90(.08)	.96(.03)	.98(.01)	.99(.02)	.96(.03)	.93(.04)
				0.5	1	2	3	5	10
Gen.	.71(.03)	.68(.04)	0.0%	.67(.11)	.65(.05)	.64(.02)	.62(.03)	.61(.03)	.59(.04)
			2.5%	.67(.09)	.67(.05)	.64(.02)	.63(.02)	.61(.03)	.59(.04)
			5.0%	.70(.09)	.68(.04)	.64(.03)	.63(.02)	.61(.03)	.59(.04)
			7.5%	.69(.09)	.68(.06)	.65(.03)	.63(.02)	.62(.03)	.59(.04)
				0.5	1	2	3	5	10
Gen. ex RD	.84(.01)	.83(.01)	0.0%	.83(.04)	.84(.03)	.81(.02)	.81(.02)	.80(.03)	.80(.03)
			2.5%	.91(.11)	.83(.03)	.82(.02)	.81(.02)	.80(.03)	.80(.03)
			5.0%	.82(.05)	.83(.04)	.82(.03)	.81(.02)	.80(.03)	.80(.03)
			7.5%	.80(.08)	.82(.05)	.82(.03)	.81(.02)	.81(.03)	.80(.03)

Table 5--*Continued*

Index	Variance	Semivar.		Lower partial moment					
2-week holding period									
				0.5	1	2	3	5	10
AEX	.99(.01)	.98(.01)	0.0%	.98(.02)	.98(.01)	.98(.01)	.98(.01)	.98(.02)	1.00(.03)
			2.5%	.97(.03)	.97(.01)	.98(.01)	.98(.01)	.98(.02)	.99(.02)
			5.0%	.98(.04)	.97(.02)	.98(.01)	.98(.01)	.98(.01)	.99(.02)
			7.5%	.96(.08)	.97(.02)	.98(.01)	.98(.01)	.98(.01)	.99(.02)
				0.5	1	2	3	5	10
Gen.	.70(.02)	.67(.02)	0.0%	.85(.16)	.70(.02)	.65(.02)	.63(.03)	.63(.03)	.64(.04)
			2.5%	.88(.18)	.70(.02)	.65(.02)	.63(.02)	.63(.03)	.64(.04)
			5.0%	.72(.08)	.69(.03)	.65(.02)	.64(.02)	.63(.03)	.64(.04)
			7.5%	.72(.08)	.69(.04)	.66(.02)	.64(.02)	.63(.03)	.64(.04)
				0.5	1	2	3	5	10
Gen. ex RD	.85(.02)	.86(.02)	0.0%	.85(.02)	.85(.02)	.85(.04)	.84(.05)	.84(.06)	.82(.08)
			2.5%	.82(.03)	.85(.02)	.85(.03)	.85(.04)	.84(.06)	.82(.08)
			5.0%	.81(.02)	.84(.02)	.85(.03)	.85(.04)	.84(.06)	.83(.08)
			7.5%	.84(.08)	.84(.02)	.85(.03)	.85(.04)	.84(.06)	.83(.08)
1-week holding period									
				0.5	1	2	3	5	10
AEX	.97(.01)	.97(.00)	0.0%	.94(.03)	.96(.01)	.97(.01)	.97(.01)	.98(.02)	.99(.04)
			2.5%	.93(.05)	.95(.02)	.96(.01)	.97(.01)	.98(.02)	.99(.03)
			5.0%	.97(.06)	.95(.01)	.96(.01)	.97(.01)	.98(.01)	.98(.03)
			7.5%	.89(.09)	.95(.01)	.96(.01)	.97(.01)	.97(.01)	.98(.03)
				0.5	1	2	3	5	10
Gen.	.68(.01)	.65(.02)	0.0%	.61(.02)	.64(.02)	.63(.01)	.62(.02)	.60(.03)	.59(.02)
			2.5%	.61(.05)	.63(.02)	.63(.01)	.62(.02)	.60(.03)	.59(.02)
			5.0%	.62(.06)	.63(.02)	.63(.01)	.62(.02)	.60(.03)	.59(.02)
			7.5%	.62(.07)	.63(.03)	.63(.01)	.62(.02)	.60(.02)	.59(.02)
				0.5	1	2	3	5	10
Gen. ex RD	.84(.01)	.85(.01)	0.0%	.82(.04)	.83(.02)	.84(.02)	.84(.03)	.83(.04)	.82(.06)
			2.5%	.82(.04)	.83(.02)	.84(.02)	.84(.03)	.83(.04)	.83(.06)
			5.0%	.80(.06)	.83(.02)	.84(.02)	.84(.03)	.83(.04)	.83(.06)
			7.5%	.77(.06)	.83(.02)	.84(.02)	.84(.03)	.84(.04)	.83(.06)

In comparing Tables 5 and 3, it is obvious that the average hedge ratios do not differ very much from the constant hedge ratios based upon all data. But similar average hedge ratios do not guarantee that the hedge effectiveness based on the out-of-sample analysis and the hedge effectiveness based on the in-sample analysis are similar. An indication that the results may differ can be inferred from the high volatility of some of the hedge ratios. The optimal hedging effectiveness based upon the out-of-sample analysis are depicted in Table 6 (regarding the downside risk measures, the first numbers shown). Comparing these values with Table 4, it is obvious that the out-of-sample and in-sample based hedging effectiveness are similar. For example, like the in-sample analysis, hedging the lower partial moment with $\alpha=0.5$ is again hardly effective for high targets.

Table 6
Hedging effectiveness in % (out-of-sample analysis)

Hedging effectiveness (i.e., the risk reduction as a Gaction of the risk of the unhedged position) using the nearest FTI contract of the subsample January 2, 1992, through January 26, 1994. The risk measures are the variance, the semivariance versus the mean (Semivar.) and various lower partial moments: all combinations of 0.0%, 2.5%, 5.0% and 7.5% (annualized target) and 0.5, 1, 2, 3, 5 and 10 (a values). With respect to the downside risk measures, the fust numbers denote the hedging effectiveness using the out-of-sample optimal hedge ratios whereas the second numbers represent the hedging effectiveness using the out-of-sample minimum variance hedge ratios.

Index	Variance	semivar.	Lower partial moment						
4-week holding period (27 obs.)									
				0.5	1	2	3	5	10
AEX	84	82/83	0.0%	88/88	85/84	80/81	79/80	78/80	76/80
			2.5%	84/81	81/79	78/78	77/78	76177	74/77
			5.0%	76/59	74/71	75/74	74/75	74/75	72/75
			7.5%	64/21	66/60	71/70	72/72	72/73	71/73
				0.5	1	2	3	5	10
General	66	65/64	0.0%	86/77	81/77	79M	78/78	76/79	74/79
			2.5%	72/64	75/69	75/72	74/74	73/75	71/76
			5.0%	55/51	68/61	71/67	71/70	70/72	69/73
			7.5%	40/33	58/53	66/62	67/66	68/68	66/70
				0.5	1	2	3	5	10
General ex RD	78	79/79	0.0%	96/94	92/91	91/90	90/90	90/90	90/90
			2.5%	78/84	88/85	87/85	86/85	86/86	86/86
			5.0%	82173	81/77	82/80	82/81	82/82	83/83
			7.5%	63/57	72169	77/75	78/76	79/78	80/80
2-week holding period (54 obs.)									
				0.5	1	2	3	5	10
AEX	87	85/86	0.0%	93/92	89/89	85/85	82/82	79/80	78/78
			2.5%	82/82	85/84	83/82	80/81	77/78	76/76
			5.0%	67/68	79/78	80/80	79/79	76/76	74/74
			7.5%	57/50	73/71	77M	77M	74/75	72/73
				0.5	1	2	3	5	10
General	65	67/66	0.0%	58/75	78/77	73/75	68/72	65/68	63/64
			2.5%	35/61	73/71	70/72	67/70	63/67	61/62
			5.0%	43144	67/65	68/69	65/68	62/65	59/61
			7.5%	25/28	59/58	65/66	63/66	60/64	58/59
				0.5	1	2	3	5	10
General ex RD	75	75/76	0.0%	90/91	85/85	77/76	71/71	65/65	61/61
			2.5%	85/83	81180	75/74	69/69	63/63	59/59
			5.0%	76/75	77/76	73/72	68/68	62/62	57/58
			7.5%	68/59	71/70	70/69	66/66	61/61	56/56
1-week holding period (108 obs.)									
				0.5	1	2	3	5	10
AEX	88	88/88	0.0%	90/89	89/89	89/89	88/88	88/88	87/87
			2.5%	84/81	86/85	87/87	87/87	87/87	86/86
			5.0%	70/71	82/81	85/85	86/86	86/86	85/85
			7.5%	68/61	78/76	83/83	84/84	85/85	85/85
				0.5	1	2	3	5	10
General	64	64164	0.0%	69/69	69/68	67/66	65/64	61/59	59/55
			2.5%	66/64	66/65	65/65	63/63	60/59	58/54
			5.0%	57/55	63/62	63163	62/62	60/58	57/53
			7.5%	51/47	59/58	62/61	61/60	59/57	56/53
				0.5	1	2	3	5	10
General ex RD	75	75/75	0.0%	78/77	78/77	78/77	78/78	77/78	77M
			2.5%	72/72	75/74	76/75	76/76	76/76	76/76
			5.0%	70/68	72/71	74/74	75/75	75/75	75/75
			7.5%	63/61	69/68	72/72	73/73	74/74	74/74

Implications of hedging downside risk using minimum variance hedge ratios (out-of-sample analysis)

In contrast to the in-sample results, the out-of-sample minimum variance hedge strategy may perform better in reducing downside risk than the out-of-sample optimal hedge strategy. However, Table 6 shows that only in a few cases the minimum variance hedge strategy reduces risk more than 5% points compared to the optimal. The most noticeable ones are the lower partial moment with α=0.5 and the two lowest target for the General index and a 2-week hedge duration. In these two cases the minimum variance strategy performs far more better than the out-of-sample optimal strategy.

In the cases where the optimal hedge strategy performs better than the minimum variance strategy, the latter is still an appropriate strategy in reducing downside risk. In particular, this is true for the semivariance and the lower partial moments with nonrisk-seeking behavior (regarding the latter it does not hold for the General index with a=l and a 4-week hedge duration). However, like the in-sample results, it is not valid for lower partial moments with risk-seeking behavior (α=0.5), especially for the AEX. The most dramatic example regarding this index is a 4-week hedge duration: hedging the lower partial moment with α=0.5 and a target of 7.5% using the minimum variance hedge ratio reduces the lower partial moment only 21% whereas the optimal strategy reduces it 64%. Finally, notice that, also like the in-sample results, there is a tendency of the minimum variance hedging strategy to perform worse the higher the target.

We now turn to the question whether or not there is an extra cost associated with hedging downside risk using the minimum variance hedge ratio. Table 7 displays the average annualized extra costs. As for the lower partial moments, there is an extra cost present in almost all cases (either or not statistically significant). In the few cases for which the average return on the minimum variance hedging strategy is higher than on the minimum lower partial moment strategy (i.e., a negative extra cost), the difference is smal1 in size (the largest value is 0.2%) and not statistically significant different from zero. Regarding the semivariance, only hedging the AEX for four weeks involves a negative extra cost. Although the extra cost is significant from a statistical point of view, it is not significant in size (0.4%, annualized).

In those cases where the average extra cost is positive (i.e., the average return on the out-of-sample optimal hedge strategy is higher than on the out-of-sample minimum variance hedge strategy), the average extra cost is negligible for the semivariance. Although it is statistically significant in two cases, it is small in size (0.6%). In contrast to the semivariance, the average (positive) extra costs associated with the lower partial moments can be significant from a statistical point of view as well as in size. This is true for all three indexes.

Hedging the AEX involves a significant extra cost for $\alpha=0.5$. Furthermore, the extra cost seems to be higher if the target is higher. In the case of a 7.5% target the extra costs are 3.6%, 1 .0% and 3.3% for a 4-week, 2-week and l-week hedge duration, respectively. Combining this observation with the worse effectiveness of the minimum variance strategy in reducing the lower partial moment with $\alpha=0.5$ (see Table 6), it must be concluded that the minimum variance strategy is not appropriate for hedging lower partial moments with risk-seeking behavior for below target returns. And this conclusion is more compelling the higher the target. With respect to $\alpha=1$, there is also an extra cost associated with hedging the lower partial moment using the minimum variance hedge strategy. Although it is statistically significant for a 2-week and 1-week hedge duration, it is not significant in size (less than 1.0%).

Different from the AEX, hedging the lower partial moment of the General index entails an extra cost for risk-averse behavior. For example, consider an investor who has a long position in the General index. The investor wants to hedge his portfolio for the next four weeks. If he is highly concemed with returns below any target ($\alpha=5$) but uses minimum variance hedge ratios, the return on his hedged portfolio will be reduced by more than 25% (annualized) on average. Notice also that the extra costs are present for all three hedge durations.

Finally, hedging the lower partial moment of the General ex RD index with the minimum variance hedge ratio exhibits an extra cost which seems to be highest for extreme high aversion for returns below a target (a=lO). The extra costs are about 0.6%, 1.5% and 1 .O% for a 4-week, 2-week and 1-week hedge duration, though only for the 2-week hedge duration it is statistically significant. Also, there is a tendency for an extra cost if the lower partial moment with $\alpha=0.5$ and a high target of the General ex RD index is hedged with the minimum variance hedge strategy. However, this tendency is weak because only for a 2-week hedge duration the extra cost is statistically significant.

Table 7

Average optimal hedge return and the extra cost of hedging downside risk with the minimum variance hedge strategy in % (out-of-sample analysis)

The fust numbers denote the annualized average hedge return using the out-of-sample minimum downside risk hedge ratios of the nearest FTI contract (subsample January 2, 1992, through January 26, 1994). The downside risk measures are the semivariance versus the mean (Semivar.) and various lower partial moments: all combinations of 0.0%, 2.5%, 5.0% and 7.5% (annualized target) and 0.5, 1, 2, 3, 5 and 10 (α values). The second numbers represent the difference of the annualized average hedge return between the out-of-sample optimal hedge strategy and the out-of-sample minimum variance hedge strategy (i.e., the extra cost). An asterisk indicates that the extra cost is different from zero at a 5% significance leve1 (based on a normal distribution for a large number of observations). A + sign indicates a negative extra cost, i.e., a higher average return when hedged with the minimum variance hedge ratio. Underlined numbers emphasize a statistical significant positive extra cost equal to or higher than 1% point (arbitrarily chosen). The numbers in the fust column are the ammalized average return of the indexes (unhedged).

Index	Semivar.		Lower partial moment					
4-week holding period (27 obr)								
			0.5	1	2	3	5	10
AEX	2.7+0.4*	0.0%	3.3-0.2	3.5-0.3	3.1+0.0	3.1+0.0	3.6-0.4	3.948
21.1		2.5%	3.6-0.4*	3.6-0.5	3.2-0.1	3.1+0.0	3.5-0.3	3.8-0.7
		5.0%	5.1-2.0*	3.8-0.7	3.4-0.2	3.2-0.0	3.4-0.2	3.8-0.7
		7.5%	6.7-3.6*	4.3-1.2	3.6-0.4*	3.3-0.2	3.3-0.2	3.8-0.6
			0.5	1	2	3	5	10
Genera1	11.1-0.6*	0.0%	13.3-2.8*	12.5-1.9	12.5-1.9*	12.9-2.3*	13.4-2.8*	13.7-3.1*
23.4		2.5%	12.4-1.8	12.3-1.7*	12.5-1.9*	12.8-2.2*	13.3-2.7*	13.7-3.1*
		5.0%	12.2-1.6	11.9-1.3	12.5-1.9*	12.7-2.1*	13.2-2.6*	13.6-3.0*
		7.5%	12.4-1.8	12.1-1.5	12.4-1.8*	12.7-2.1*	13.1-2.5*	13.6-3.0'
			0.5	1	2	3	5	10
General	9.04 1	0.0%	9.4-0.5	8.9+0.0	9.344	9.445	9.546	9.5-0.6
ex RD		2.5%	8.7+0.2	9.244	9.344	9.4-0.5	9.546	9.546
23.5		5.0%	9.648	9.344	9.344	9.4-0.5*	9.5-0.6	9.546
		7.5%	10.3-1.4	9.749	9.4-0.5	9.4-0.5*	9.5-0.6*	9.546
2-week holding period (54 obs.)								
			0.5	1	2	3	5	10
AEX	3.3-0.0	0.0%	3.5-0.3	3.6-0.4*	3.5-0.2*	3.4-0.2*	3.2-0.0	3.1+0.1
21.0		2.5%	3.5-0.3	3.6-0.4*	3.5-0.3*	3.4-0.2*	3.3-0.1	3.1+0.1
		5.0%	3.3-0.1	3.7-0.4*	3.5-0.3*	3.4-0.2*	3.3-0.1	3.1+0.1
		7.5%	4.2-1.0	3.9-0.7*	3.6-0.3*	3.5-0.2*	3.3-0.1	3.1+0.1
General	11.4-0.7		0.5	1	2	3	5	10
23.4		0.0%	11.0-0.3	11.0-0.2	12.1-1.3*	12.5-1.8*	12.6-1.9*	12.5-1.8*
		2.5%	10.7+0.1	11.2-0.4	12.0-1.3*	12.5-1.7*	12.6-1.9*	12.5-1.8*
		5.0%	11.5-0.7	11.4-0.7*	12.0-1.2*	12.4-1.7*	12.6-1.9*	12.6-1.8*
		7.5%	10.7-0.0	11.4-0.7	11.9-1.2*	12.4-1.6*	12.6-1.8*	12.6-1.8*
			0.5	1	2	3	5	10
General	8.2-0.0	0.0%	8.3-0.1	8.2-0.0	8.5-0.3	8.8-0.6*	9.2-1.0*	9.8-1.6*
ex RD		2.5%	9.0-0.8*	8.4-0.2	8.5-0.3	8.7-0.6	9.1-1.0*	9.8-1.6*
23.4		5.0%	8.9-0.7	8.5-0.4*	8.5-0.3	8.7-0.5	9.1-0.9*	9.7-1.5*
		7.5%	9.6-1.4*	8.7-0.5*	8.5-0.3	8.7-0.5*	9.0-0.8*	9.6-1.5*

Table 7–*Continued*

Index	semivar.		Lower partial moment					
1-week holding period (108 obs.)								
			0.5	1	2	3	5	10
AEX	2.6+0.0	0.0%	3.6–1.0*	3.0-0.4'	2.8-0.1	2.7-0.0	2.7-0.1	3.0-0.4
21.2		2.5%	4.2–1.5*	3.2–0.6*	2.8-0.2	2.7-0.1	2.7-0.1	3.0-0.3
		5.0%	3.3-0.7	3.2–0.5*	2.9-0.2	2.7-0.1	2.7-0.1	3.0-0.3
		7.5%	5.9–3.3*	3.2–0.6*	2.9-0.2	2.8-0.1	2.7-0.1	2.9-0.3
			0.5	1	2	3	5	10
General	10.9–0.6*	0.0%	11.9–1.5	11.4–1.1*	11.6–1.2*	12.0–1.6*	12.4–2.0*	12.5-2.1
23.4		2.5%	12.6–2.2*	11.4–1.1*	11.6–1.2*	12.0–1.6*	12.4–2.0*	12.5-2.1
		5.0%	12.1-1.8	11.8–1.5*	11.6–1.2*	11.9–1.6*	12.4–2.0*	12.5-2.1
		7.5%	12.1-1.7	11.8–1.4*	11.6–1.2*	11.9–1.5*	12.3-2.0'	12.5-2.1
			0.5	1	2	3	5	10
General	7.5+0.0	0.0%	7.8-0.2	7.8–0.3	7.9-0.3	7.9-0.4	8.1-0.6	8.5-1.0
ex RD		2.5%	7.6-0.1	7.9-0.3	7.9-0.3	7.w.4	8.1–0.5	8.5-0.9
23.6		5.0%	8.3-0.7	7.9-0.4	7.9–0.3	7.9-0.4	8.1-0.5	8.5-0.9
		7.5%	8.8-1.3	7.9-0.4	7.9–0.3	7.9-0.4	8.1-0.5	8.4–0.9

Based upon the above observations, essentially all in-sample conclusions applies to the out-of-sample analysis. Except for the extra cost. The out-of-sample results suggests that hedging the lower partial moment with $\alpha \geq 1$ using the minimum variance hedge ratio may come with an extra cost.

5 SUMMARY AND CONCLUSIONS

In this paper we compared theoretically minimum risk hedging strategies with risk defined as the variance and various measures of downside risk. We showed that if return distributions are symmetric, then minimum semivariance (versus the mean) hedge ratios are equal to minimum variance hedge ratios. We proved that if return distributions are normal and futures markets are positively biased, then minimum lower partial moment hedge ratios are smaller (or possibly equal to in case $0<\alpha<1$) than minimum variance hedge ratios. Based on this finding, we showed that hedging such a lower partial moment using the minimum variance always comes with an extra cost (or possibly zero cost in case $0<\alpha<1$). Moreover, the minimum variance hedge strategies reduces the lower partial moment less than (or perhaps equal to in case $0<\alpha<1$) the optimal strategy.

The Dutch FTI contract has been used to hedge three Dutch stock market indexes: the AEX (the index underlying the futures contract), the General index and the General ex RD index. Three hedge durations were used: 4-week, 2-week and 1-week. Minimum risk hedge strategies were analyzed in-sample as well as out-of-sample. The in-sample results show that minimum semivariance hedge ratios are in magnitude almost the same compared to the minimum variance hedge ratio. Furthermore, as theory suggests for normal return distributions, minimum lower partial moment hedge ratios with $\alpha>0$ are less positive (only slightly in case $\alpha\geq1$) than the minimum variance hedge ratio. We also found that minimum lower partial moment hedge ratios with $\alpha\geq1$ are nearly independent of the target. On the other hand, for a=l and $\alpha=0.5$ there is a tendency for minimum lower partial moment hedge ratios to become less positive the higher the target. This tendency is more pronounced for $\alpha=0.5$. The FTI contract is able to reduce downside risk in the same proportion as the variance. Only for lower partial moments with $\alpha=0.5$ and high targets, reducing risk with the FTI contract seems to be not appropriate.

Regarding the reduction in downside risk, hedging it with the minimum variance hedge ratio instead of the optimal hedge ratio is not a serious problem. According to theory, the extra cost of hedging downside risk using the minimum variance hedge strategy is completely determined by the difference between the hedge ratios and expected return on the fùtures contract. Because the difference in hedge ratios is small for the semivariance and the lower partial moments with $\alpha\geq1$, the extra cost is not significant in size. For the lower partial moments with $\alpha=0.5$ and a target of 7.5% the extra cost ranged from 1% to 2.2% (annualized). In this case, however, reducing the lower partial moment with the optimal hedge ratio is also not appropriate. Whether or not these extra costs are significant from a statistical point of view, that depends on the significance of the returns on the futures contract, A simple statistical test shows that for all three hedge durations the expected return on the fùtures contract is not statistically different from zero (at 5% significance level).

The out-of-sample analysis has been based upon hedge ratios estimated using a four-week moving window. In agreement with the in-sample results, the effectiveness of the FTI contract in reducing the semivariance with the ex-ante optimal hedge ratio is almost the same compared to the variance. Furthermore, hedging the semivariance can be carried out very well with a minimum variance strategy. The reduction in the semivariance is nearly the same as the optimal strategy and the extra cost can be neglected.

Different from the semivariance are the results regarding lower partial moments with risk-seeking behavior ($\alpha=0.5$) and high targets. Hedging this type of risk with the optimal hedge ratio is not very effective in terms of risk reduction. Furthermore, the minimum variance hedge strategy is not appropriate in reducing lower partial moments with risk-seeking behavior ($\alpha=0.5$) and high targets (especially for the index underlying the fùtures contract, the AEX). Moreover, there is also a tendency for an extra cost. A lower partial moment with nonrisk-seeking behavior ($\alpha\geq1$) can be reduced similar to the variance using the corresponding optimal hedge ratio. In addition, the same amount of risk reduction can be achieved with the minimum variance hedge strategy. However, for the General index (and to a lesser extent for the AEX) it comes with a significant extra cost. The annualized extra cost ranges from 1.7% to 3.1%, 1.2% to 1.9% and from 1.1% to 2.0% for a 4-week, 2-week and 1-week hedge duration, respectively.

The main conclusion of the paper is that investors who really care about returns below a fixed target, should not use the minimum variance hedge ratio. Especially for investors who have little concern about the size of the deviation ($0<\alpha<1$). Regarding lower partial moments with $\alpha\geq1$ the minimum variance hedge strategy seems to be appropriate in reducing downside risk, but it can come with an extra cost. On the other hand, if the mean portfolio return is used as a target, the semivariance can be hedged very well with the minimum variance hedge ratio.

APPENDIX

In this appendix we show that if returns have a normal distribution and if $\mu_F>0$ (<0), then minimum lower partial moment hedge ratios with $\alpha\geq1$ must be strictly smaller (larger) than the minimum variance hedge ratio. The outline of the prove is as follows. First, we show that lpm(h;a,t) is a strict concave function of h. In other words, the first derivative of $lpm(h;\alpha,t)$ is strictly increasing in h. Second, we derive an expression for the first derivative of the lower partial moment lpm(h;a,t) evaluated at the minimum variance hedge ratio (h_{var}). Based upon the sign of this expression it is easy to show that the above assertion is true.

Strict concavity follows directly from the **fact** that the second order derivative of **lpm(h;α,t),** which **can** be obtained by differentiating Equation **(6),** is strictly positive. For **α>1** the second order derivative is given by:

$$\frac{d^2 lpm(h;\, a,\, t)}{dh^2} = \int_{-\infty}^{\infty} \int_{-\infty}^{t+hr_F} \alpha(\alpha-1) r_F^2 \left[t-(r_P - hr_F)\right]^{\alpha-2} f\,(r_P, r_F)\, dr_P\, dr_F \qquad \text{(A1)}$$

with **f(.,.)** the two-dimensional distribution **function** of the **rate** of return on the stock portfolio and the futures contract. For a=l it is:

$$\frac{d^2 lpm(h;\alpha,\, t)}{dh^2} = \int_{-\infty}^{\infty} r_F^2 f\,(t + hr_F, r_F)\, dr_F \qquad \text{(A2)}$$

Because the expressions to be integrated are strictly positve for **all values** on the integration interval, the integral value itself must be strictly positive. [16]

If the return on the stock portfolio and the fùtures contract have **a** bivariate **normal** distribution, then **any** portfolio consisting of the stock portfolio and the fùtures is distributed **normally**. The **mean μ(h)** and standard **deviation σ(h)** of **a** hedged portfolio are given by Equations (3) and **(4),** respectively. Hence, the lower partial moment of **a** hedged stock portfolio is given by:

$$lpm(h;\alpha,t) = \int_{-\infty}^{t} (t - r_H)^{\alpha} \frac{1}{\sigma(h)\sqrt{2\pi}} exp\left[-\tfrac{1}{2}\left(\frac{r_H - \mu(h)}{\sigma(h)}\right)^2\right] dr_H \qquad \text{(A3)}$$

It **can** be shown that the first derivative of Equation (A3) with respect to h **evaluated** at the minimum **variance** hedge ratio h_{var} equals:

$$\left.\frac{dlpm(h;\alpha,t)}{dh}\right|_{h_{var}} = \frac{-\mu_F}{\left[\sigma(h_{var})\right]^{1-\alpha}} \int_{-\infty}^{t(h_{var})} \left[t(h_{var}) - x\right]^{\alpha} x \frac{1}{\sqrt{2\pi}} exp(-\tfrac{1}{2}x^2)\, dx \qquad \text{(A4)}$$

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[16] **Notice that strict** concavity **holds for arbitrary return** disnibutions **and not only for normal distributions.**

where

$$t(h_{var}) = \frac{t - \mu(h_{var})}{\sigma(h_{var})} \qquad \text{(A5)}$$

Now consider the sign of the integral in Equation (A4). If $t(h_{var}) \leq 0$, then for all values of x in the interval $-\infty$ to $t(h_{var})$ the expression to be integrated is negative. Hence the value of the integral is negative. For $t(h_{var}) > 0$, the integral in Equation (A4) can be divided into three parts: $-\infty$ to $-t(h_{var})$, $-t(h_{var})$ to 0 and 0 to $t(h_{var})$. The integral evaluated over the first two intervals is negative, whereas over the last interval it is positive. However, it can be shown that the integral evaluated over the second interval plus the integral over the last interval is negative. Consequently, the value of the whole integral is negative and for all values of $t(h_{var})$ Equation (A4) can be written as:

$$\left.\frac{dlpm(h; a, t)}{dh}\right|_{h_{var}} = \frac{\mu_F}{[\sigma(h)]^{1-\alpha}} \times \text{positive number} \qquad \text{(A6)}$$

From this expression it follows that if $\mu_F > 0$, then the first order derivative of the lower partial moment evaluated at the minimum variance hedge ratio is positive. Because the first order derivative of lpm(h;a,t) is a strict increasing fùnction of h, minimum lower partial moment hedge ratios must be smaller than the minimum variance hedge ratio. If $\mu_F < 0$ then, along the same line of reasoning, minimum lower partial moment hedge ratios are larger than the minimum variance hedge ratio. Hence, for $\alpha \geq 1$ the following two relations hold:

$$\mu_F > 0 \Rightarrow h_{\alpha,t} < h_{var} \qquad \text{(A7)}$$

$$\mu_F < 0 \Rightarrow h_{\alpha,t} > h_{var} \qquad \text{(A8)}$$

Q.E.D.

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