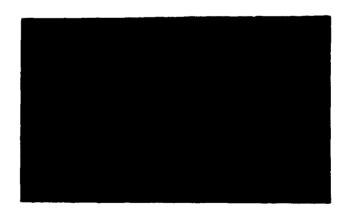
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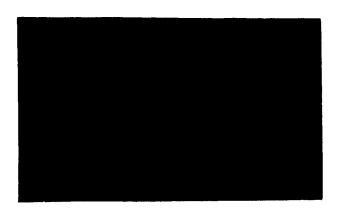




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SEISMIC ANALYSIS OF AXISYMMETRIC\* SHELLS

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\* To be presented at Conference on Structural Analysis and Design of Nuclear Power Plants. \*\* Instituto de Engenharia Nuclear, CNEN.

## SUMMARY

Dynamic analysis of shell structures subjected to seismic loadings is one of the basic requirements in the safety assessment of nuclear power plants. Typical examples of such structures are steel containment vessel that houses the primary cooling system, cooling towers and the concrete reactor building that serves as a missile and biological shield for the containment wessel. In this work we study Axisymmetric and shells subjected to multiple support excitation. The shells state are spatialy discretized by the finite element method and in order to obtain estimatimes for the maximum values of displacements and stresses the response spectrum technique is used. Finally, some numerical results are presented and discussed in the case of a shell of revolution with vertical symmetry axis, subjected to seismic ground motions in the horizontal, vertical and rocking directions.

# **RESUMO**

A análise dinâmica de cascas sujeitas a cargas constitui um dos requisitos básicos de segurança em contrais nucleares. Casos típicos de tais estruturas são por o vaso de contenção em aço, que abriga o sistema primário de esfriamento, as terres de resfriamento e o vaso de compreto do reator que representa para o vaso de contenção um anteparo bio lógico e uma proteção contra misseis.

Necta Labrillo studam-se cascas axisimétricas sujeitas a excitação múltipla de apoios.

As cascas são discretizadas espacialmente pelo método dos elementos finitos e o conceito de espectro de resposta é utili zado na obtenção de estimativas para os valores máximos deslocamentos e tensões.

Finalmente, são apresentados e discutidos alguns resulta dos numéricos para o caso de uma casca de revolução com eixo vertical de simetria, sujeita a excitação sísmica de nas direções horizontal, vertical e rocking.

#### 1. INTRODUCTION

The design of thin shells is one of the most important structural problem in a nuclear power plant. An analyst is often confronted with the task of performing rather complicated analysis in order to satisfy the available safety codes.

An important part of such an analysis deals with the response of shells when subjected to time depedent loads produced by seismic support motions.

Seismic analysis of thin shells of revolution is carried out in this work making use of response spectrum concepts.

The equations derived herein stems from the Principle of Virtual Work applied to axisymmetric thin shells according to Flügge and Love's theory, obtaining a general formulation for the treatment of multiple support excitation problems for this kind of structures.

Application to an actual design case of a steel containment vessel presently under construction in Brazil, is obtained and the results are shown in good agreement with those obtained with computer code ANSYS.

# 2. STATEMENT OF THE PROBLEM

In this work the problem of finding the response of axisymmetric shells to given motions of its foundations is considered. The problem consists of finding the solution of the following boundary value problem: Given a shell of revolution, and a model for its deformation, find its stresses and displacements when the structure is subjected to precribed time dependent boundary conditions.

The equations governing the problem can be stated if we consider an elastic body, in a regular bounded region  $\Omega$  of a 3-D Euclidean space - E', free from body and surface forces and subjected to a prescribed field displacement in part  $(\Gamma_{\mathbf{v}})$  of its boundary.

It is not difficult to show that to find the displacement field to this kind of loading is equivalent to solving

the following variational problem, derived from the Principle of Virtual Work:

Find the field v & Kin, such that:

$$\int_{\Omega} \rho \nabla \cdot \Phi d\Omega + \int_{\Omega} DE(V) \cdot E(\hat{V}) d\Omega = 0 \qquad \qquad \forall \hat{V} \in Var_{V}$$
 (1)

where: D - Elasticity tensor

p - material density

 $\mathbf{E}(\mathbf{v})$  - infinitesimal strain tensor and

$$\text{Var}_{\mathbf{v}} = \{\hat{\mathbf{v}}, \hat{\mathbf{v}} = 0 \text{ at } \Gamma_{\mathbf{v}}\}$$

 $\lim_{x \to \infty} = \{v; v = g(x,t) \text{ and } g(x,0) = 0 \text{ both at } \Gamma_v\}$ 

v(x,0) = v(x,0) = 0 - initial conditions

Employing the usual technique to treat this kind of problem one decomposes the original displacement field in two new fields  $-v=u^{+}+u$  - in such way that  $u^{+}\in \mathrm{Kin}_{V}$ , the stresses  $\mathrm{DE}(u^{+})$  are self equilibrated and  $u\in \mathrm{Var}_{V}$ . The original problem (1) readily becomes two new (P1,P2) problems i.e.:

$$P1 - \int_{\Omega} DE(u^*) \cdot E(\tilde{v}) d\Omega = 0 \quad \forall \ \tilde{v} \in Var_{v}$$
 (2)

$$P2 - \int_{\Omega} \rho \ddot{\mathbf{u}} \cdot \ddot{\mathbf{v}} \, d\Omega + \int_{\Omega} \mathbf{D} \, \mathbf{E}(\mathbf{u}) \cdot \mathbf{E}(\hat{\mathbf{v}}) \, d\Omega = -\int_{\Omega} \rho \ddot{\mathbf{u}}^* \cdot \ddot{\mathbf{v}} d\Omega \quad \forall \, \ddot{\mathbf{v}} \in \mathrm{Var}_{\mathbf{v}} \quad (3)$$

In order to make use of this formulation in the case of thin shells of revolution one adopts a cylindrical coordinate system r,0,z, to describe the geometry of the shell middle surface. Under the simplifications—retained in the Flügge's theory—and Love's theory as well — the displacement field of a shell subjected to the action of arbitrary loading is uniquely defined by the displacement field of its middle surface.

Let  $v_1$ , is 1,2,3, be the components of the displacement vector associated to an arbitrary point of the middle surface, in the local basis  $\{t_1,t_2,n\}$  (fig.1). Let moreover,  $v_4 = v_1/R_1 - (1/s') 2v_3/3\xi$  represent the rotation of the normal n around  $t_2$  and where:

R<sub>1</sub> - One of the principal radiiof curvature

 $s' = (r'^2 + z'^2)^{1/2}$  (.)' indicating derivative respect to the parameter  $\xi$  adopted along the meridian.

Defining the generalized displacement vector  $\mathbf{v}_{=}(\mathbf{v}_{1}\ \mathbf{v}_{2}\ \mathbf{v}_{3}\ \mathbf{v}_{4})^{T}$  and expanding this displacement field  $\mathbf{v}$  of the original problem in Fourier series and noting that doing so  $\hat{\mathbf{v}}$ ,  $\mathbf{u}^{*}$ ,  $\mathbf{u}$ , of the Pl, P2 problems will have similarly expressions, one writes for  $\mathbf{v}$ 

$$v = \sum_{n=0}^{\infty} (q_n^{s_n} v_n^{s_n} + q_n^{a_n} v_n^{a_n})$$
 (4)

where the superscripts s and a indicate, respectively, the symmetric and antisymmetric components of v(and  $\hat{v}$ ,  $u^*$ , u). The diagonal matrices  $q_n^1$  for l=s,a are defined as follows:

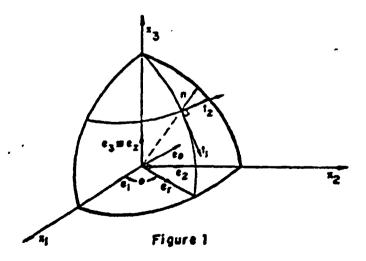
$$q_n^s = 1 \cos n\theta - \sin n\theta \cos n\theta \cos n\theta$$
 (5)

$$q_n^2 = 1 \sin n\theta \cos n\theta \sin n\theta \cos n\theta$$
 (6)

and where

$$v_n = (v_{1n}^1 v_{2n}^1 v_{3n}^1 v_{4n}^1)^T$$
 1 = s,a.

The constitutive relations assumed here allow for Hookean orthotropic materials, provided that the planes of orthotropy contain the principal lines of curvature.



With the above assumptions, and considering the orthogonality relations:

$$\int_{0}^{2\pi} \sin m\theta \sin n\theta = \int_{0}^{2\pi} \cos m\theta \cos n\theta = 0 \quad \text{for } m \neq n$$

$$= \pi \quad \text{for } m = n \neq 0$$

$$= 2\pi \quad \text{for } m = n = 0$$

it is easy to show that the problems Pl and P2 become:

Pl - Pind the vector field  $u^* = 0$   $\mathbb{R}$   $\mathbb{N}$   $\mathbb{N}$   $\mathbb{N}$   $\mathbb{N}$   $\mathbb{N}$   $\mathbb{N}$  for any  $\hat{\mathbf{v}}_n^1 \in \mathbb{N}$   $\mathbb{N}$   $\mathbb{N}$ 

P2 - Find the vector field  $\mathbf{u}_n^1 \in \mathrm{Var}_n^2$  for any  $\hat{\mathbf{v}}_n^1 \in \mathrm{Var}_n^2$  such that:

$$\int_{\xi_{0}}^{\xi_{1}} F_{n} \tilde{u}_{n}^{1} . \tilde{v}_{n}^{1} s'rd\xi + \int_{\xi_{0}}^{\xi_{1}} E(B_{n}u_{n}^{1}) .B_{n}(\tilde{v}_{n}^{1}) s'rd\xi = \\
= -\int_{\xi_{0}}^{\xi_{1}} F_{n} \tilde{u}_{n}^{*1} .\tilde{v}_{n}^{1} s'rd\xi \qquad (8)$$

Both problems with the colateral condition:

$$v_{4n}^{1} = \frac{v_{1n}^{1}}{R_{1}} - \frac{1}{s}, \frac{dv_{3n}^{1}}{d\zeta}$$
 (9)

where:

- Rin<sup>1</sup> is the space of the kinematically admissible displacements related to the n-th harmonic, symmetric or antisymmetric according to superscript 1.
- Var<sup>1</sup> is the space of virtual displacements related to the n\_th harmonic, symmetric and antisymmetric according to the superscript 1.
- 2,7<sub>n</sub> are matrices depending on the shell geometry and material properties. Their explicit forms are given in reference<sup>2</sup>.

- B<sub>n</sub> is a differential operator matrix depending on the shell geometry. Its explicit form can be found in reference<sup>3</sup>.
- $\ddot{u}_n^1, \ddot{u}_n^{*1}$  are the second derivative with respect to time of the vector fields  $u_n^1$  and  $u_n^{*1}$ .

For sake of simplicity it will be dropped, in the sequel, the superscript 1 and subscript n. The development for any harmonic, symmetric or antisymmetric part, is entirely similar and the following equations will cover all cases.

#### 3- APPROXIMATE SOLUTIONS - FINITE ELEMENT DISCRETIZATION

Using the standard Finite Element procedures in the construction of the approximation spaces, the region of interest  $I = \{\xi_0, \xi_1\}$  can be divided in sub-regions (finite elements) conected by nodal points. One can easily obtain the global interpolation functions constructed by assemblage of the suitable local interpolation functions selected to represent the displacement variation along the meridian direction. Then the original problem defined in a space of infinite dimensions can straightfoward be written in a finite dimension space.

The finite element employed here is a curvilinear one developed by Feijóo et al<sup>1</sup>, with three nodal points. The displacement field has four components - three displacements and the rotation of the normal of the shell around  $\mathbf{r}_1$  - and is interpolated with cubic and quintic polinomials. The geometry of the shell is also approximated and the element satisfies all the continuity requirements for  $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$  and  $\mathbf{v}_4$ , preserving moreover the colateral condition (9) for every point of the shell surface.

Hence the problem Pl can readily be written:

$$P1 - x_{\ell\ell} U^* = G = -x_{\ell p} \bar{U}^* \to U^* = -x_{\ell\ell}^{-1} x_{\ell p} \bar{U}^*$$
 (10)

\*\*LL - Partition of the global stiffness matrix associated to the free d.o.f. of the structure.

- K<sub>Lp</sub> Partition of the global stiffness matrix representing the coupling between the prescribed and the free d.o.f. of the structure.
- U\* Nodal free displacement vector of the structure composed of the suitable assemblage of all elements nodal global displacement vectors.
- U+ Prescribed nodal displacement vector of the structure.

All quantities related above are obtained by suitable assemblage of the matrices and vectors at element level in the usual way of the finite element technique and explicit forms can be found in Jospin<sup>2</sup>.

It must be pointed out that from the knowledge of the prescribed function g(x,t) in  $\Gamma_V$  one must identify the Fourier series terms envolved in the analysis of (10) and then construct the vector  $\bar{\mathbb{U}}^*$  of amplitudes of the prescribed motions in each harmonic and solve Pl problem for each one of these harmonics and obtain the total solution using (4) in the superposition of the results thus obtained.

It is worth noting that  $\overline{U}^*$  can be decomposed as written:

$$\vec{U}^* = \sum_{k=1}^{p} v_k \vec{U}_k (t) \quad (11) \quad \text{where } v_k = \begin{bmatrix} 0 \\ 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix} + k\text{-th component} \quad (11)$$

where the scalar function  $\overline{U}_{k}(t)$  is the prescribed time history in the k-th d.o.f.

Hence from the linearity of the problem Pl, U\* can be obtained by the superposition of the solutions  $U_{k}^{*}$  of p "static" problems corresponding to the application of a unit displacement in each one of the p prescribed d.o.f., multiplied by its associated time-histories i.e.:

where

 $\mathbf{U}_{\mathbf{X}}^{*}$  is the solution of Pl for each term in the summation (11). For the problem P2 in similar way and making use of the

solution for Pl one obtains:

$$M_{\ell\ell}\ddot{U} + K_{\ell\ell}U = - |M_{\ell\ell}K_{\ell\ell}^{-1}K_{\ell p} + M_{\ell p}|\ddot{U}^*$$
(13)

where

M<sub>22</sub>- Partition of the global mass matrix associated with the free d.o.f. of the structure.

M<sub>Lp</sub> -Partition of the global mass matrix representing the coupling between the prescribed and the free d.o.f.

With the decomposition (11) the prescribed accelerations  $\overline{U}^*$  can be conveniently written

$$\ddot{\ddot{\mathbf{U}}}^* = \sum_{k=1}^{p} \mathbf{v}_k \ddot{\mathbf{U}}_k(\mathbf{t}) \tag{14}$$

From time-histories  $\tilde{U}_{k}$  one can obtain the forcing term in (13) and determine the solution U. The superposition of the solutions U and U\* in each time t will finally give the solution to the problem initially stated in (1).

In solving the equations (13), one can employ any desired method like direct integration, modal superposition and so on.

#### 4- SPECTRAL ANALYSIS

If a deterministic response to a known ground motion is desired one can use any of the above mentioned methods in order to provide the time dependence of the response of the structure. However seismic design is concerned with a ground motion that is expected to occur in the future, so that its deterministic time record cannot be anticipated. The use of an acceleration record of a previous earthquake is possible, but, as far as the design of an unbuild shell structure is concerned, that is not very meaningful. A logical procedure to follow in seismic design is the response spectrum approach.

In this section the response spectrum concepts are used in order to obtain estimatives for the maximum values of each

response of interest (displacement, stresses, etc).

The natural frequencies  $w_r$  and the free vibration mode shapes of problem P2 can be obtained from the eigenvalue problem

$$(\mathbf{R}_{II} - \mathbf{u}^2 \mathbf{H}_{II}) \mathbf{X} = \mathbf{0} \tag{15}$$

Considering the orthogonality relations

$$x_{i}x_{\ell}x_{j} = \delta_{ij} \tag{16}$$

where  $\delta_{ij}$  is the kronecker delta.

Then it is possible to obtain the uncoupled form of equation (13) through the use of the linear transformation:

$$\mathbf{U} = \chi \, \mathbf{n}(\mathbf{t}) \tag{18}$$

Where  $\chi$  is the modal matrix with the first m vibration mode shapes of the system and  $\eta=\left(\eta_1\ \eta_2\dots\eta_m\right)^T$  contains the modal amplitudes.

The contribution to the total response, of the r-th vibration mode, when one has a prescribed support motion in the k-th d.o.f. is given by:

$$\ddot{\eta}_{x}^{k} + \omega_{x}^{p} \eta_{x}^{k} = I_{x}^{k} \ddot{\overline{v}}_{k}(t)$$
 (19)

where  $\Gamma^k_T$  is the modal participation factor of the mode r due to the motion in the k-th prescribed d.o.f., and can be determined by

$$\Gamma_{\mathbf{x}}^{\mathbf{k}} = \gamma_{\mathbf{x}}^{\mathbf{T}} [\mathbf{M}_{\boldsymbol{\ell}\boldsymbol{\ell}} \mathbf{X}_{\boldsymbol{\ell}\boldsymbol{p}}^{-1} \mathbf{X}_{\boldsymbol{\ell}\boldsymbol{p}} + \mathbf{M}_{\boldsymbol{\ell}\boldsymbol{p}}] \quad \mathbf{V}_{\mathbf{k}}$$
 (20)

The maximum value of  $\eta_{\chi}^{k}$  that occurs in time  $t_{\chi}^{k}$  not known is given by

$$\eta_{x,\text{max}}^{k} = \Gamma_{x}^{k} \frac{\text{SA}^{k}(\omega_{x})}{(\omega_{x})^{2}}$$
 (21)

where

 $SA^k(\omega_r)$  - is the spectral acceleration value of the r-th mode with respect to excitation  $\ddot{U}_k(t)$ .

Taking into account the simultaneous application of all support motions one possible estimate of the maximum response in the r-th mode can be:

$$\eta_{r,\text{max}} = \left[\sum_{k} (\eta_{r,\text{max}}^{k})^{2}\right]^{1/2}$$
 (22)

or conservatively, by absolute summation

$$\eta_{r,\max} = \sum_{k} |\eta_{r,\max}^{k}| \tag{23}$$

The corresponding displacement of the structure are  $U_r = \eta_{r,\max} X_r. \quad \text{With these values the modal stresses, moments, etc.} \\ \text{can promptly be obtained.}$ 

Once obtained the representative values of the maximum for each response of interest in each mode subjected to all prescribed support motions, or these values are usually combined in order to evaluate the representative value of their maximum value taking into account the contribution of all vibration modes. Two common ways of doing so are:

a) Absolute sum – 
$$Q = \sum_{r=1}^{m} |Q_r|$$
 (24)

b) Square root of sum of the squares (RQSQ)

$$Q = \begin{bmatrix} m \\ \sum_{r=1}^{m} Q_r^2 \end{bmatrix}^{1/2}$$
 (25)

where .

Qr - the peak response of interest due to k-th mode

In the case of vertically axisymmetric shells the known response spectra are in the horizontal, vertical, rocking and torsional "directions". Hence in this case the d.o.f. of the nodal point at each basis, if there are more than one, of the shell must be combined in order to represent each one of those above mentioned motions.

In this case one defines analogously with (20) and (21) the

following:

$$(r_x^d)_n = \sum_{k=1}^4 r_x^k (e_n^1)_k \tag{26}$$

and

$$\left(\eta_{x}^{d}\right)_{n}^{1} = \left(\Gamma_{x}^{d}\right)_{n} \frac{\operatorname{SA}^{d}\left(\omega_{x}^{n}\right)}{\left(\omega_{x}^{n}\right)^{2}} \tag{27}$$

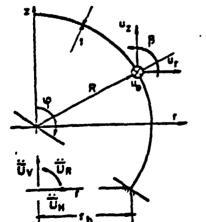
where the index n is relative to the excited harmonic; l=s,a = means symmetric or antisymmetric part of then-th harmonic

 $\mathbf{c}_{r}^{n}$  = r-th frequency of the n-th excited harmonic.  $(\mathbf{c}_{n}^{1})_{k}$  is a constant suitably choosed that relates the k-th d.o.f. of the basis of the shell, to the "d" directions previously defined.

Finally, the harmonics involved may be combined by absolute sum.

## 5- NUMERICAL RESULTS

The results here analysed are from a spherical steel containment vessel (fig.2) under the action of a vertical, horizontal and rocking ground acceleration



Material properties

Figure 2

The estimatives for the maximum modal responses, in each

"direction" d, according to (26), are given by:

- vertical motion 
$$(\eta_r^V)_{0,\text{max}}^S = \Gamma_r^V \frac{SA^V(\omega_r^0)}{(\omega_r^0)^2}$$
 (28)

- horizontal motion 
$$(\eta_{\underline{r}}^{H})^{S} = \Gamma_{\underline{r}}^{H} \frac{S\lambda^{H}(\omega_{\underline{r}}^{1})}{(\omega_{\underline{r}}^{1})^{2}}$$
 (29)

- rocking motion 
$$(\eta_T^R)^s = \Gamma_T^R \frac{S\lambda^R(\omega_T^1)}{(\omega_T^1)^2}$$
 (30)

where explicit expressions for the participation factors in the r-th mode  $\Gamma_{x}^{V}$ ,  $\Gamma_{x}^{H}$ ,  $\Gamma_{x}^{R}$  are:

$$\Gamma_r^V = \Gamma_r^{-3} (c_\theta^S)_3$$
 with  $(c_\theta^S)_3 = 1$  (31)

$$\Gamma_r^H = \Gamma_r^1 (c_3^S)_1 + \Gamma_r^2 (c_3^S)_2 \text{ with } (c_3^S)_3 = (c_3^S)_2 = 1$$
 (32)

$$I_r^R = I_r^3 (c_1^S)_3 + I_r^6 (c_1^S)_4 \text{ with } (c_1^S)_3 = -r_b \text{ and } (c_1^S)_4 = 1$$
 (33)

The constants  $(c_n^1)_k$  in (31), (32) and (33) are defined as indicated in figure 3.

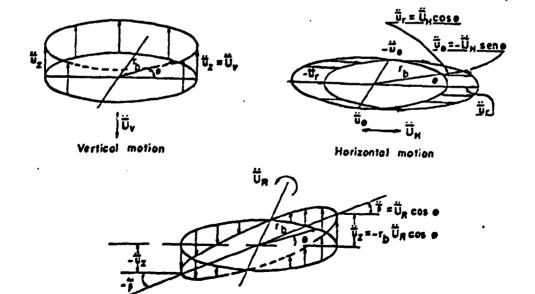


Figure 3

Rocking motion

All results presented as being from the ANSYS code were obtained with a discretization employing 100 elements. Only the first vibration mode shape was considered in the comparisons made from hereafter.

A camparison between the participation factors determined by ANSYS code and those obtained herein is shown in Table 1. Good agreement is achieved even in the case where it was used only four of the finite elements employed in this work.

	w <sub>1</sub> (H <sub>2</sub> )	w1 (H <sub>2</sub> )	L'A	Γ <sub>1</sub> <sup>H</sup>	Γ <sub>2</sub> <sup>R</sup> *10 <sup>-5</sup>
ANSYS	13.75	6.35	15.81	18.73	7.53
50 ELEM.	13.71	6.31	15.74	20.87	7.56
20 ELEM.	13.76	6.33	15.71	20.87	7.56
4 ELEM.	14.62	6.67	15.10	20.77	7.47

Table 1 - Participation factors for vertical, horizontal and rocking motions

Next the radial, axial displacement as well as the rotation of the normal of the shell around  $t_{\rm s}$  (Fig.1), are also compared in Table 2 and 3, for the circumferential wave number n=0 and n=1 respectively. In both Tables the presented results here, were obtained with a 50 elements mesh.

	U_*102		U <sub>2</sub> *10 *		g*10°	
Ģ	ANSYS	P.W.	ANSYS	P.W.	ANSYS	P.W.
123.47	15.60	16.03	14.60	14.91	7.99	8.00
91.02	9.28	9.34	19.34	19.42	3.95	4.38
61.00	3.16	3.01	24.13	24.34	5.32	5.21

Table 2 - Maximum displacements (m m), Rotations (rad) for circumferencial wave n=0.

	U_+1	03	U	*10'	β *10°	
¢	ANSYS	P.W.	ANSYS	P.W.	ANSYS	P.W.
 123.43	192.44	205.06	176.95	187.38	46.45	45.72
91.02	347.27	366.25	276.49	292.31	115.02	121.95
61.00	507.63	538.78	257.69	270.11	111.70	117.71

Table 3 - Maximum displacements (mm) and rotations (rad) for circumferential wave n=1

In the Table 3 the absolute sum (23) was used to obtain maximum response estimations for the case of simultaneous action

of the horizontal and rocking excitations.

Nø						
	ANSYS	50 EL.	20 EL.	Ansys	50 EL.	20 EL.
n = 0	28.83	24.08	37.28	8.65	7.22	11.19
n=l	392.69	328.93	491.76	118.14	98.68	147.53

Table 4 - Maximum sectional forces at clamping point (N/mm)

	Мф			Me		
	ANSYS	50 EL.	20 EL.	ansys	50 EL.	20 EL.
n=0	340.50	395.26	286.14	102.13	118.58	85.84
n=1	4428.10	4942.55	3644.04	1328.63	1482-77	1093.21

Table 5 - Maximum moments at the clamping point (N.mm)

Finally it can be concluded from Table 4 on 5 that although a poor mesh (4 elements) can give good results for the participation factors (Table 1), this is not the case for the sectional values  $(N_{\phi},N_{\theta})$  and moments  $(M_{\phi},M_{\theta})$  at the clamping point.

### CONCLUSIONS

Although the usual problems in seismic analysis of shells seem to be that of rigid basis excitation, the approach used in this work was that of multiple support excitation. The advantages of this kind of approach comes from the clarity given to all aspects involved: formulation of the problem, assumed decomposition criteria conducting to problems Pl and P2 and the numerical algorithms used to obtain approximated solutions. The generality gained enables the study of any problem of prescribed time-histories motions for this kind of structures, using the available techniques in dynamic analysis such as direct integration, modal superposition and so on. And last but not least is worth mentioning that the solution of problem Pl in the case of rigid basis excitation can be seen as a good test for the finite element employed in the discretization.

ACKNOWLEDGEMENTS: The authors would like to thank CNEN, IEN, LCC with through the existing joint projects have supported this work.

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