UNIVERSITI PUTRA MALAYSIA

OPTIMAL STRATEGIES OF PLAYERS IN LINEAR DIFFERENTIAL GAMES

MEHDI SALIMI

FS 2011 81

OPTIMAL STRATEGIES OF PLAYERS IN LINEAR DIFFERENTIAL GAMES

By

MEHDI SALIMI

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

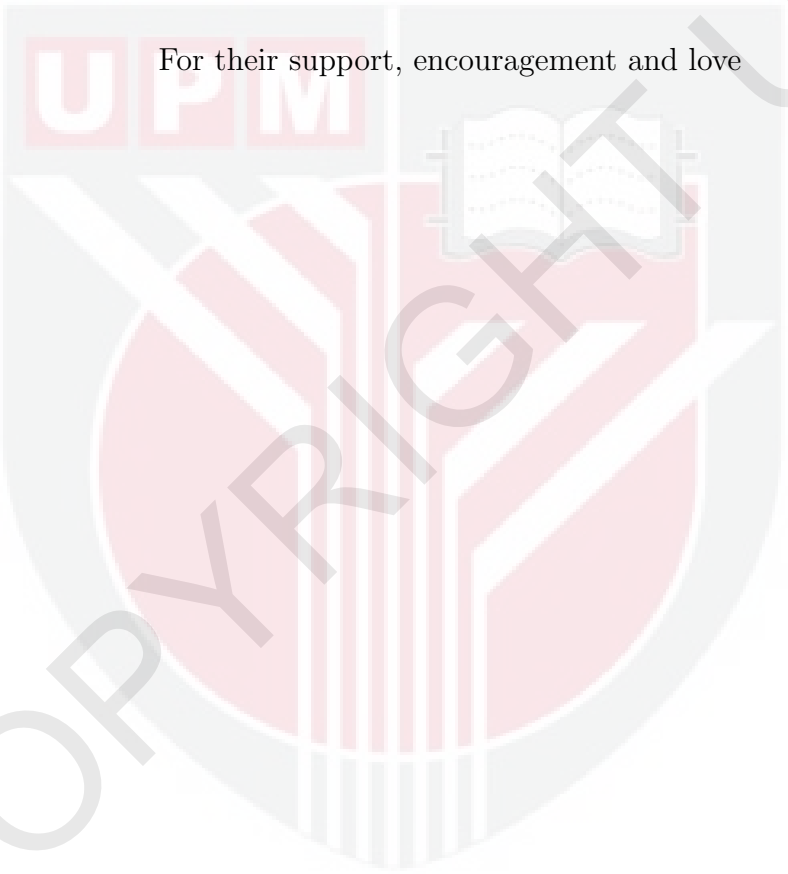
June 2011

DEDICATION

To

My Father and My Mother

For their support, encouragement and love



Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Doctor of Philosophy

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June 2011

Chair: Associate Professor Gafurjan Ibragimov, PhD

Faculty: Science

A game involves a number of players, says N, a set of strategies for each player, and a payoff that quantitatively describes the outcome of each play of the game in terms of the amount that each player wins or loses. A common type of game is often called the pursuit-evasion game. Pursuit-evasion game is about how to guide one or a group of pursuers to catch one or a group of moving evaders. In the general definition of a pursuit-evasion game, there will typically be N players with opposing goals, each of them conflicts the other. Each player tries to fulfill his or her goals, and it is assumed that all players always do their best to fulfill their goals. These goals are formally expressed in terms of minimizing or maximizing a payoff functional.

In this thesis, we study a pursuit-evasion differential game of countably many players in Hilbert space. Motions of the players are described by the ordinary differential equations of first and second order. The control functions of players are subject to geometric and integral constraints. Resource for the control

of each pursuer is greater than that of the evader. Duration of the game is fixed. The payoff functional is the greatest lower bound of the distances between the pursuers and evader when the game is terminated. The pursuers try to minimize the payoff functional, and the evader tries to maximize it. We give a formula to calculate the value of the game and construct optimal strategies of the players. To solve the first part of the problem, the pursuit game, we use the method of fictitious pursuers.

In addition, we consider an evasion differential game of several pursuers and one evader with simple motions and integral constraints on control functions of players. We find the sufficient condition for the evader to escape from all pursuers. We present explicit strategy for the evader and show that the proposed escape is possible, no matter what control is adapted by the pursuers. We prove the admissibility of our strategy as well.

Finally, an application of pursuit-evasion game in a missile guidance system is introduced by constructing optimal strategy of pursuer missile which guarantees capturing of the evader missile.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

STRATEGI OPTIMUM BAGI PEMAIN DALAM PERMAINAN PEMBEZAAN LINEAR

Oleh

MEHDI SALIMI

Jun 2011

Pengerusi: Profesor Madya Gafurjan Ibragimov, PhD

Fakulti: Sains

Permainan yang melibatkan beberapa orang pemain N dengan set strategi untuk setiap pemain, dan satu keputusan yang kuantitatif yang menerangkan hasil dari setiap permainan melalui jumlah pemain samada menang atau kalah. Jenis permainan umum dikenali sebagai permainan mangsa-pemangsa. Permainan mangsa-pemangsa adalah berkenaan panduan satu atau sekumpulan pemangsa untuk menangkap satu atau sekumpulan mangsa yang bergerak. Dalam takrifan umum permainan mangsa-pemangsa lazimnya N pemain mempunyai matlamat yang bertentangan, dan setiap mereka juga berkonflik antara satu sama lain. Setiap pemain cuba untuk mencapai matlamat masing-masing dengan andaian semua pemain sentiasa melakukan yang terbaik untuk memenuhi matlamat mereka. Matlamat ini secara formalnya adalah mengenai miminimumkan atau memaksimumkan satu fungsi ganjaran.

Dalam tesis ini, kami mengkaji permainan pembezaan mangsa-pemangsa dengan bilangan pemain yang boleh dikira dalam ruang Hilbert. Pergerakan pemain ini dijelaskan melalui persamaan pembezaan biasa peringkat pertama

dan kedua. Fungsi kawalan bagi pemain adalah tertakluk kepada kekangan geometrik dan kekangan kamiran. Sumber kawalan untuk setiap pemangsa adalah lebih besar berbanding mangsa. Tempoh permainan adalah ditetapkan. Fungsi ganjaran adalah batas bawah terbesar dari jarak pemangsa dan mangsa apabila permainan ditamatkan. Pemangsa cuba meminimumkan fungsi ganjaran dan mangsa cuba memaksimumkannya. Kami mencadangkan satu formula untuk mengira nilai permainan dan membina strategi yang optimum bagi pemain-pemain. Bagi menyelesaikan masalah bahagian pertama, permainan pengejaran, kami menggunakan kaedah pemangsa khayalan.

Sebagai tambahan, kami mempertimbangkan permainan pengelakan pembezaan bagi sebahagian pemangsa untuk satu mangsa dengan gerakan mudah dan kekangan kamiran ke atas fungsi kawalan pemain. Kami memperolehi satu syarat cukup untuk mangsa melepaskan diri dari semua pemangsanya. Kami mencadangkan strategi yang jelas untuk mangsa berkemungkinan melepaskan diri dengan tidak mengira apa juga strategi yang digunakan oleh pemangsa. Kami juga membuktikan kesahihan strategi ini.

Akhir sekali, aplikasi permainan mangsa-pemangsa dalam sistem kawalan peluru berpandu diperkenalkan dengan membina strategi optimum bagi peluru berpandu pemangsa yang pasti dapat memintas peluru berpandu mangsa.

ACKNOWLEDGEMENTS

First and foremost, thanks to God for His grace, wisdom and comfort throughout my thesis.

I hereby would like to express my deepest gratitude to Assoc. Prof. Dr. Gafurjan Ibragimov, chairman of the supervisory committee, for his excellent supervision, valuable guidance, helpful discussion, and continuous encouragement. He has taught me how to find an interesting problem, given me a great degree of freedom to explore problems independently and provided me with valuable feedbacks. I am grateful for having the opportunity to work under his supervision. His valuable assistance and comments in the preparation and completion of this thesis are also highly appreciated. I would like to thank the members of supervisory committee, Prof. Mohamed Suleiman, Assoc. Prof. Dr. Fudziah Ismail, and Assoc. Prof. Dr. Massoud Amini for their valuable discussion, comments and helps.

My deepest gratitude and love to my parents, my dear father and mother, my sisters and all of my relatives, for their supports and encouragements.

My thanks also go to the School of Graduate Studies (SGS) of Universiti Putra Malaysia for the Graduate Research Fellowship (GRF) that supported me throughout my studies. I also wish to express my thanks to all of my friends, especially Dr. Farzin Modarres Khiyabani, Hadi Eslami, and colleagues during my studies at Universiti Putra Malaysia. I would like to thank all staffs of the Department of Mathematics.

This thesis is the result of almost three years of work where I have been accompanied by some people, I now have the pleasant opportunity to express my gratitude to all of them.

I am always indebted to Assoc. Prof. Dr. Javad Laali for encouraging me to pursue my studies.

I certify that a Thesis Examination Committee has met on 14 June 2011 to conduct the final examination of Mehdi Salimi on his thesis entitled "Optimal Strategies of Players in Linear Differential Games" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy. The members of the Supervisory Committee were as follows:

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

MEHDI SALIMI

Date: 14 June 2011

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