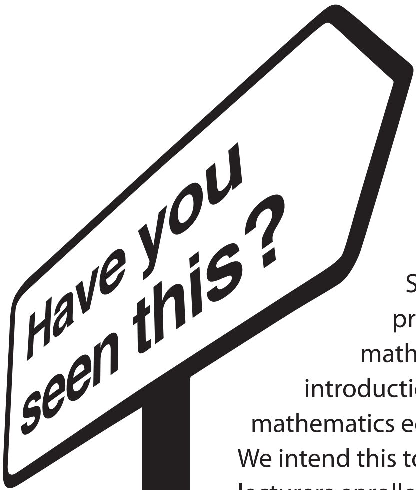

Ideas from Mathematics Education: An Introduction for Mathematicians

Lara Alcock and Adrian Simpson

Using funding made available by the Maths, Stats & OR Network we have produced a guide to provide mathematicians with an accessible introduction to some ideas from the mathematics education research literature. We intend this to be particularly useful for new lecturers enrolled on PGCHE programmes, but we also think it would be of interest for experienced lecturers looking for further insight into their students' thinking.

The guide focuses upon three ideas that we use to structure our own thinking, as university mathematics lecturers, about what students need to learn in order to succeed in undergraduate mathematics. Specifically, we concentrate on what they need to learn in order to make sense of their early encounters with abstract definitions and proofs, which we see as key to making a successful transition to university-level mathematical thinking.

In brief, the main points of the three chapters within the guide are:

1. Definitions: Students are often unaware of the status of formal definitions within mathematical theory and attempt to work with concepts informally instead;
2. Mathematical objects: Many mathematical constructs can be understood as processes, but need to be understood as objects in order for higher level reasoning to make sense;
3. Two reasoning strategies: We can distinguish two sensible ways of approaching a proof-related task; each demands a range of skills and some individuals may have preferences for one or the other.

Each of these ideas provides us with a lens through which to view our students' thinking and, rather than having to consider 200 students with, potentially, 200 different ways of thinking, we can consider two or three broad categories of ways that students might respond to the mathematics they encounter.

Our discussion of each of these issues is based on research involving close observations of students' learning of undergraduate mathematics. We include a number of quotations from students to illustrate their thinking; the majority of these are from relatively successful students attending high ranking institutions. Like other researchers, we have found it illuminating to ask students to think aloud, without interruption, as they work on mathematical tasks. This often uncovers uncertainties and misconceptions among even the strongest students.

The guide is very much about theories of learning *mathematics*, as opposed to general theories of learning or generic advice on "good practice" in teaching. We believe that this is important because mathematical content is central to a lecturer's work in constructing lectures, notes, problem sheets etc., and because understanding students' likely interpretations of this content is therefore directly relevant to this day-to-day design work. We find the ideas described in the guide helpful in planning our own mathematics teaching and in responding to students' individual questions, because they allow us to think systematically about underlying difficulties that manifest themselves in a variety of misunderstandings and errors. We do not, of course, suggest that there is a unique "best way to teach", but many teaching strategies can be thought of in terms of how they address the issues raised, and throughout the three chapters we provide specific illustrations of things that we and others have tried in our lectures and seminar classes.

In presenting these ideas we hope to show that students face genuine challenges in learning to think like mathematicians and that some of the difficulties they face are inherent in making the transition to undergraduate mathematics, and are not attributable solely to lack of effort or preparation. We also hope to convey our belief that these difficulties are not insurmountable, and that having more insight into students' thinking is interesting for its own sake, and can make the work of teaching easier and more enjoyable.

The guide is available for electronic download by visiting:
http://mathstore.gla.ac.uk/index.php?pid=257