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DUOPOLISTIC PRICE COMPETITION WITH CAPTIVES

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ABSTRACT

We extend the Bertrand duopolistic competition to include captives. These are consumers that have no choice between the suppliers. Usual population of shoppers are modeled performing a sequential search in order to decide where to buy a homogenous good. These two simple departures from the original setup have sharp consequences. First, we find that duopolistic price competition is not robust to inclusion of captives. The equilibrium results starkly differ and the only possible equilibrium now includes duopolists charging monopolistic prices. Second, addition of sequential search introduces multiplicity of pure strategy Nash equilibria. In this setup, we observe perverse optimal response to competitor's price changes. Notably, we find that the firm might want to reduce the price in response to the competitor's price increase, which is at odds with the usual undercutting principle. Third, we investigate the behavior of equilibrium prices depending on the heterogeneity in consumer risk attitudes. We find that the higher consumer heterogeneity with respect to acceptance of risky gambles leads to higher prices in equilibrium.

1. INTRODUCTION

In the standard Bertrand competition consumers choose the cheapest product available on the market. All consumers can perfectly observe prices in all shops and are allowed to make the purchase from any of the suppliers. In reality, it is impossible to find such a situation. It is hard for the consumers to know prices in all shops in order to make perfectly informed decision. Moreover, not all consumers will have an opportunity to make the purchase from any of the shops. Up to date, it is not well understood how the predictions of the standard duopolistic price competition model change after these fairly minor alterations.

In this paper, we present the model that introduces two extensions to the Bertrand setup. First, we assume the existence of ‘captives’. Akin to the previous work by Wilde and Schwartz (1979) and Varian (1980), we assume some of the consumers do not have the choice between the two sellers. They have to stick to one of them even if the price it charges is higher than the competitor’s price. However, unlike Wilde and Schwartz (1979) and Varian (1980), who assume that population of captives is randomly distributed across the firms each period, we assume that this distribution is stable. Meaning, that there is a constant market structure dictating that consumer s is a captive of firm i. And this structure is known to the sellers. This is a fairly plausible assumption for many products that are routinely acquired in neighborhood shops. The remainder of the consumers behaves like standard Bertrand consumers. They have access to both shops. We refer to these consumers as ‘shoppers’. We can think of them as people living on the border between neighborhoods and having a choice between the two shops.

The second altercation of the setup is that we oblige shoppers to enter the shops sequentially. Going to the shop is the only way to obtain the information about the price charged by a given producer. In light of this requirement we discuss two versions of the model. One, where shoppers are allowed to visit both shops and then decide where to buy. We refer to this as the ‘return option’. The other, where shoppers have no ‘return option’. In this case, even though they can theoretically enter both shops, they cannot go back to the shop to make a purchase, once they have exited it. In this case shoppers have to take the decision under incomplete information. This is also a plausible assumption for large number of cases where visiting the next shop is costless, while going back to the shop you already visited is not. Consider walking down the shopping street in one direction. Or shopping for gasoline on your way to work.

These two alterations call for the change in the approach to the problem. First, introducing captives with the stable temporal structure, makes it possible to describe the market using buyer–seller networks (Kranton and Minehart, 2001; Corominas-Bosch, 2004). We have two sellers—firm 1 and firm 2—and three types of buyers: shoppers, that are connected to both of the firms, and captives of each of the two firms, who are connected to only one of the firms. Contributions in networked markets literature are mostly concerned with bargaining and use bipartite (Lever, 2011), or even tripartite (Blume *et al.*, 2009) networks where additional traders are introduced in the model. Second, the introduction of the ‘return option’ (or rather the instance when shoppers have no return option), turns the model into a well-developed price search setup (Stigler, 1961; Manning and Morgan, 1982).

Few of works related to ours, like Lever (2011) and Pasini *et al.* (2008), discuss the mixed strategy Nash equilibria. For obtaining meaningful results these models strive for unique equilibrium. This usually involves additional constraints on demand functions and results into an equilibrium in randomized (mixed) strategies. This approach is justified in their case because they are concerned with the analysis of welfare, which is dubious in case of multiplicity of equilibria. Due to the existence of captives, Bertrand setup discussed in this paper becomes a discontinuous game. This complicates significantly the analysis of mixed strategy Nash equilibria (Dasgupta and Maskin, 1986; Reny, 1999). Therefore, we follow the classical works in the discipline (Salop and Stiglitz, 1977, 1982; Stahl, 1996) and concentrate on pure strategy Nash equilibria of the proposed pricing game.

We find that the introduction of captives in the price competition framework changes the outcomes drastically. First, in the setup when consumers have the return option, which is the equivalent to the classical Bertrand setup except the existence of captives, there exists no pure strategy Nash equilibrium (PSNE) for an extensive parameter set. Even more surprisingly, for the parameter set where PSNE does exist, the equilibrium it is characterized by the monopolistic prices instead of the pricing at marginal cost. Second, once no return option is imposed, the possibility of the multiplicity of the PSNE arises. We find the possibility of price dispersion in equilibrium of the duopolistic pricing game. We find incentives for un-orthodox price responses by the firms. Notably, we find that the optimal response from the firm to competitor's price increase might as well be lowering of the price. We also investigate the consequence of the consumer heterogeneity with respect to risk attitudes in their price-search process. We find that the higher heterogeneity leads to higher prices in equilibrium. This sheds a new light at the duopolistic price competition setup.

The paper is structured as follows. Section 2 presents the model. Sections 3 and 4 present the results of the model. Section 5 concludes.

2. THE MODEL

Consider the market where two firms ($i = 1, 2$) are engaging in price competition. Profit-maximizing firms use the same constant returns to scale technology and produce a homogenous good. The unit/marginal cost of production is c, and it can be paid after sales. The homogenous product is indivisible. There are $\bar{S}$ consumers in the economy. Each of them is endowed with funds in amount of m. Consumers can only buy one unit of the product. They can spend all the money for a unit of product, but prefer spending as little as possible.[1]

Distinctive from the original Bertrand setup, we assume that each consumer can either go to one of the shops/producers, or go to both of them. Denote the number of captives of the shop 1 by U_1 and the number of captives of the shop 2 by U_2. Remaining $S=\bar{S}-U_1-U_2$ consumers are shoppers and thus have access to both shops. Only the shoppers can choose between the offers in two shops.

Shoppers make trips to the shops sequentially. The sequence is chosen randomly for every shopper in every period. We further assume that this market structure (U_1, U_2 and S) is known to duopolists. Naturally the market structure is the major consideration when taking the pricing decision. Without loss of generality, for exposition of the results in the paper we will assume that if $U_1 \neq U_2$, then $U_1 > U_2$. We also assume that there always are some shoppers ($S>0$).

As all results are scale free, it is convenient to normalize the size of consumer populations. For this, we use the number of shoppers (S) as the denominator and define $u_i=U_i/S$. In what follows we analyze two variants of the model. One where shoppers have option to return to the producer they have already visited. The other when they have no return option. In the former case, they purchase the product from the shop with the lowest price. In the latter case, this is not necessarily true. In this situation shoppers have to make the decision on where to purchase the product after they have observed only one (rather than both) of the prices.

3. SEQUENTIAL TRIPS WITH RETURN OPTION

In order to set the benchmark for the more interesting setup with no 'return option' for shoppers, we first analyze the version of the model when shoppers can choose the cheapest product offered by the two suppliers.

Recall that in our model there are three groups of consumers. Technically, there are three distinct submarkets and both of the firms have certain freedom to exercise their monopolistic power in one of the three submarkets. They compete only on common/shared submarket (for shoppers). In the situation when shoppers can go back to the shop that they have already visited, the

[1] For demonstration purposes, we can assume that utility function of consumer s has the following form $V_s=v+\delta_s(m-p_s)$, where $\delta_s \in \{0,1\}$ denotes the consumption of the agent, $p_s \in \{p_1,p_2\}$ is the price she has paid for the product, m is his endowment in monetary terms, v is a parameter. It is easy to see that this functional form satisfies all the requirements we have outlined above.

competition on shared submarket collapses to classical Bertrand setup. The problem that producers are facing is that they cannot price discriminate, and thus they have to charge one price for both submarkets they operate on.

Proposition 1: If $0 < u_1 < \frac{m}{c} - 1$, and $u_1 > u_2$, then there is no Pure Strategy Nash Equilibrium in the economy.

Proof: Conditions state that there should be at least one captive in the economy and that the share of captives for each of the competitors should not be too high in order not to discourage duopolists to engage in competition for shoppers. In this situation, prices behave similar to how they would behave in Bertrand model before converging to the equilibrium. Starting from the monopolistic price we will observe a downward spiral. However, due to existence of captives, as price becomes too low firms will have incentive to give up the competition for shoppers and extract monopolistic profit from captives. As a result, prices never reach c (as it is always strictly dominated by m) which is the only candidate for PSNE as it is the only price that cannot be profitably undercut. ■

Remark 1: If $0 < u_1 < \frac{m}{c} - 1$, and $u_1 > u_2$, then prices are confined to the interval $\left[c + \frac{mu_1}{1+u_1}; m\right]$.

In order to see this, consider that there are two reasonable responses to competitor's price. One is to undercut his price by infinitesimally small value ϵ (similar to Bertrand's original model). The other is to abandon competition for shoppers and extract monopolistic profit from captives, thus charging m. This is a direct consequence of existence of captives.

In the first scenario, firm's profit is $\pi_i^c = (S + U_i)(p_i - c)$. In the second scenario, the profit is $\pi_i^m = U_i m$. If prices spiral down below $\underline{p_i} = c + \frac{mu_i}{1+u_i}$, $\pi_i^m > \pi_i^c$, it is optimal for the firm to jump to charging m concentrating on captives. At this point the competitor's best response is to follow and change $m - \epsilon$.

In order to determine which of the two competitors will abandon the shared market first we calculate $\frac{\partial \underline{p_i}}{\partial u_i} = \left(1 - \frac{u_i}{(1+u_i)^2}\right) m > 0$. We see that cut-off price is increasing in the number of captives. Therefore, the firm with the higher share of captives will jump to the monopolistic price first. Thus, the prices will never go down below the cutoff price of the firm with more captives, which as per our assumption is firm 1. The condition $u_1 < \frac{m}{c} - 1$ ensures that the interval $\left[c + \frac{mu_1}{1+u_1}; m\right]$ is of a positive length in order to have room for undercutting.

Despite being different from Bertrand's conclusions, these results are in line with previous literature. There are two aspects to draw parallels here. The first is that the prices are constrained strictly above marginal cost, which echoes the result by Stahl (1996). The second is that in no PSNE situations the likely price dynamics is a perpetual cycle. Starting from the monopolistic price m, they gradually go down as firms undercut each other until they go so low that one of the firms increases the price to monopolize unique consumers. The competitor follows and sets the price high enough to engage the first firm into price cutting again. Similar result has been obtained by Salop and Stiglitz (1977).

On the other hand, if there are too many captives in the economy, the competition might be discouraged. In this case, the economy has PSNE.

Proposition 2: If $u_1 \geq \frac{m}{c} - 1$, and $u_1 > u_2$, then the equilibrium is $p_1 = m$ and $p_2 = m - \epsilon$.

Proof: In this situation, too large share of population is captive and as a result firm 1 has no incentive to engage in competition. It is a dominant strategy for firm 1 to charge the monopolistic price m. Responding to this the competitor slightly undercuts the monopolistic price to capture shared consumers. None of the firms have any incentive to deviate from such an arrangement. ■

Before going into more interesting case of no return option for shoppers, notice that the model discussed in this section is a simply extension of the original Bertrand setup. The only feature added is the existence of captives. Yet, this simple modification (addition of even just one captive) has changed the outcomes of the model dramatically. So much so that in the only possible equilibrium case prices converge to the monopolistic price rather than to the marginal cost.

4. SEQUENTIAL TRIPS WITH NO RETURN OPTION

In the version of the model analyzed in previous section, shoppers choose the cheapest option available. In this section, we analyze a variant of the model where consumers do not have an option to go back to the shop that they have already visited. This modification induces shoppers to decide where to purchase in presence of incomplete information. Effectively they have to take the decision after seeing only one price. This substantially alters the results of the model.

Obtaining complete information about the prices in the economy might be costly. Consumers might have to engage in price search in order to identify the cheapest shop. In economics literature typically there are two types of search rules are considered: sequential search (Benhabib and Bull, 1983) and fixed sample size non-sequential search (Manning and Morgan, 1982). However, several papers have identified search strategies that dominate these two.[2]

The model herein can be interpreted as the one assuming imperfect rationality in consumer behavior. This is, in fact the consequence of not explicitly modelling the search costs. Perfectly rational consumers would have to weight potential gains from further information search again the costs associated with this process, which our consumers do not do. However, this imperfect rationality in consumer search is not novel. In fact marketing scientists have been studying this behavior on the example of the 'consideration set' effect (Mehta *et al*., 2003), that is equivalent of the fixed sample size search in economics. Besides, marketing research has also identified that informational motives of consumer search are regularly dominated by other (recreational) motives (Bloch *et al*., 1986). This would make actual consumer search seem less-than-perfectly rational for an economist's taste, much like consumers in our model.

Our concern is with the sequential search. The seminal contribution in this area is due Stigler (1961). He has demonstrated that it is optimal for consumers to decide whether to continue the search for the best offer every time they receive the quote. Building on Stigler's work, Kohn and Shavell (1974) have demonstrated that sequential search would result in a switch-point level of price. This means that consumer would terminate the search as soon as a price quote would fall below this threshold level. Kohn and Shavell (1974) have also demonstrated the uniqueness of this threshold.

In this paper, we adopt this sequential threshold search approach. We assume consumers cannot obtain quotes without going to the shop in person. In the original description, when consumers decide to terminate their search, they can choose to buy from the cheapest shop they have visited during the search process. However, in certain cases this is not very realistic.

Consider a long street which has two gas stations in either end of it. If a person wants to top up her tank, she has to decide while being close to the street end. Some consumers live on the street. Therefore, going to work every morning they pass only one of the gas stations. Consumers who have to drive east to work can be U_1, while the consumers who have to drive west can be U_2. These consumers get one quote each and they have to buy

[2] For the review of these works see Morgan and Manning (1985).

from the respective gas stations, because driving to the other end of the street is simply too costly (imagine its a busy long street).

However, there are consumers who do not live on this street, but they have to pass it while going to work. These are consumers belonging to set S. They get price quotes from both gas stations, but they can buy only from the one. And as driving from one gas station to the other is costly they are not free of choosing the cheapest shop even if they have complete information about the prices. Consider the decision this type of consumer has to make. She arrives on the street and passes the first gas station. She can stop and buy gas. However, if she decide not to do so, she has to buy gas from the station at the other end of the street. Even if gas there is more expensive she cannot turn and drive all the way back to the first gas station. This might be too costly.

This behavior is not unique to gas stations. Many of us do our weekly shopping in two supermarkets. We only buy part of our shopping list in the first super market leaving the other part for the next shop. However, it is perhaps a negligible share of us who would go back to the first shop to buy a single item that she found to be more expensive in the second shop.

This is the consumer behavior we model in this section. We consider consumers following simple rule of thumb in their purchases. Each of them has an idiosyncratic thresholds $\hat{p}_s$, such that as soon as the price she sees in the first shop she enters $p \leq \hat{p}_s$, she buys the product.[3] If the first shop charges higher price she takes a chance of buying the product in the second shop. The optimality of the threshold approach in our framework is demonstrated in the Appendix.

Notice that because consumers do not have a return option, they are essentially taking a gamble. If the price in the first shop is too high to their taste, they gamble on the price in the second shop. This gamble might pay off, if the second shop they visit charges lower price. But they might have to pay even higher price in the second shop. Therefore, $\hat{p}_s$ is a measure of the risk attitude of consumer s. If $\hat{p}_s$ is low, consumer takes riskier gambles.

Consider $\hat{p}_s$ is distributed over S with a certain probability density function $f(\cdot)$. Then, the corresponding cumulative distribution function at point p, $F(p)$, gives the share of shoppers for whom $\hat{p}_s < p$. Thus, we can conclude that $F(p)$ share of consumers that will enter the given shop first, will not buy the product from this shop and will gamble on the price charged

[3] Idiosyncrasy of thresholds can be due to multiple reasons. For example, consumers might be heterogeneous in their risk attitudes, they (depending on their occupation) might have different value for time, or, relating to the earlier gasoline shopping example, they might have different transportation costs (depending on the car they drive).

by the second shop. This, in return implies that $1-F(p)$ share these consumers will buy the product from this shop. In the case when profit functions for producers can be written as

$$\pi_i = \left[\frac{S}{2}\left(1-F(p_i)+F(p_j)\right)+U_i\right](p_i-c) \tag{1}$$

where $i = 1, 2$ and $j = 2, 1$.

In order to discuss the implications of this kind of behavior, we analyze two setups. One where consumers are homogenous with regard to $\hat{p}_s$, or their risk attitudes, the other where they are heterogenous. In fact the former setup is only a limiting case of the latter, but it helps to demonstrate few important implications of the model in a simpler arrangement.

4.1 Homogenous consumers

We start off with the simple case where consumers are homogenous with respect to their risk attitudes. This means that $c < \hat{p}_s = \mu < m$, $\forall s$. The value of μ measures the risk attitudes of the society as a whole—the higher the μ, the less risk-taking is the society.

In order to characterize the equilibria of this game consider the following.

Remark 2: No matter the price charged by the competitor and the market structure, it is never optimal to charge $p_i \in [c; \mu) \cup (\mu; m)$.

If $S = 0$, we know that the optimal policy is to charge $p = m$. However, if $S > 0$ we have to discuss two components of the interval separately. We know that firms want to extract maximum possible profit from captives. Therefore, for those consumers they would prefer charging as high prices as possible. For half of the shoppers the shop i will be the first shop they visit. They will buy from the shop i as long as $p_i \leq \mu$, they will not buy from the shop otherwise. On the other hand, shoppers for whom the shop i will be the second shop (and who have not bought the product in the shop they entered first) will always buy the product no matter the price. Therefore, no matter the price charged by the competitor, shop i will have two possible strategies: either maintain the shoppers entering the shop first or give them up. If a firm wants to maintain these shoppers it is clearly suboptimal to charge any price lower than μ. If the firm is ready to give them up, it is suboptimal to charge a price lower than m.

Therefore, in homogenous consumer case without return option we have only two possible candidates for the equilibrium price—μ and m. Hence, we can have only tree kinds of pure strategy Nash equilibria: (1) where both firms charge m, (2) where both firms charge μ and (3) where one of the firms charges μ while the other charges m.

Denote $m-c \equiv r$ and $\mu-c \equiv r'$. Note that r' is linearly related to μ, and therefore r' also measures the shoppers' risk attitudes.

Proposition 3: If $r' \leq \frac{\frac{1}{2}+u_i}{1+u_i} r$, $\forall i$, then $p_1=p_2=m$ is an equilibrium of the game.

The proof of the proposition is straight forward as the incentive we have to exclude is any of the firms wanting to jump down to μ.

Proposition 4: If $r' \geq \frac{u_i}{\frac{1}{2}+u_i} r$, $\forall i$, then $p_1=p_2=\mu$ is an equilibrium of the game.

The proof of this proposition is similarly simple. These results point to the fact that more risky consumers (lower r') imply higher equilibrium prices in the economy.

Proposition 5: If $\frac{\frac{1}{2}+u_2}{1+u_2} r \leq r' \leq \frac{u_1}{\frac{1}{2}+u_1} r$ and $u_1 > u_2$, then $p_1=m$ and $p_2=\mu$ is the equilibrium of the game.

The proof of this proposition is also straight forward as it is sufficient to ensure that firm 1 does not have an incentive to decrease the price to μ and that firm 2 does not to have an incentive to charge the monopolistic price. This is quite interesting as the model implies the price dispersion, a widely observed phenomenon on actual retail markets (Baye *et al.*, 2004; Barron *et al.*, 2004), in the equilibrium of a simple duopolistic setup.

Proposition 5 implies that the shop with larger number of captives charges the higher price. This is intuitive as the shop with more captives has to incur larger costs (in form of profits given up) for lowering the price. While for the firm with lower u, shoppers will be of greater importance, therefore, it will be willing to fiercely high for them by price cutting.

The summary of the results with homogenous consumers can be seen on figure 1. In this figure, $\frac{r'}{r}$ is measured on the ordinate and u_1 and u_2 are measured on abscissa. In this space, the market setup can be represented by two points on the same horizontal line, left one corresponding to firm 2 and the right one corresponding to firm 1. Note that the low values of $\frac{r'}{r}$ mean that the society is more risk-taking.

We have three regions on the figure. 1f the market setup is such that both firms are in the region below both curves—both firms are charging monopolistic price $p_1=p_2=m$, this is the unique equilibrium. If market

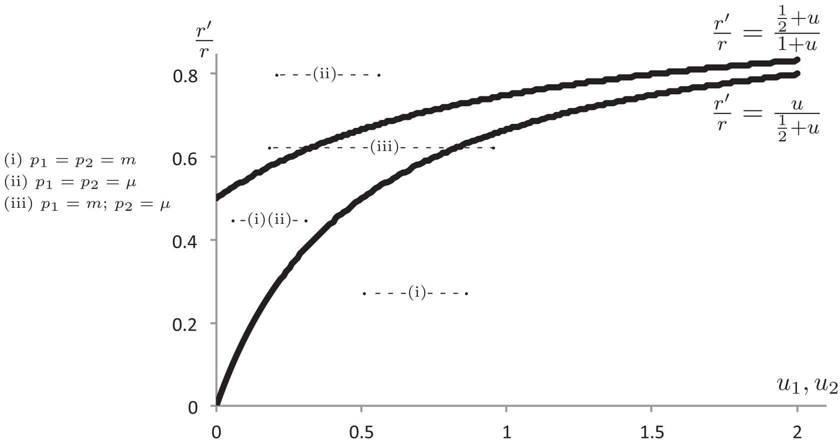

Figure 1. Summary of the results with homogenous consumers.

arrangement puts both firms above both curves—firms are charging $p_1 = p_2 = \mu$. If both firms are in the middle region both type (i) and type (ii) equilibria are possible. If market arrangement is such that only one of the firms is placed in the region between the two curves there exists no pure strategy Nash equilibrium. If one of the firm falls below both of the curves and the other above both of them then we have the unique equilibrium with a price dispersion: the firm above the two curves charges μ while the firm below the curves charges m. In order to understand why this is the case, note that being below both of the curves means that the firm has relatively high share of shoppers. In general, we can see that riskier consumers induce higher prices in the economy.

4.2 Heterogenous consumers

If there is some variance is shoppers' risk attitudes, the model requires more elaboration. Firms' profit functions are given by (1). We need to derive the best response functions for each of the firms. For this, we need to maximize profits by choosing p_i under the condition that $p_i \in [c; m]$. The first order condition for maximizing profits in the interior of the interval $[c; m]$ is

$$\frac{\partial \pi_i}{\partial p_i^{\diamond}} = \frac{S}{2}\left[1 - F(p_i^{\diamond}) + F(p_j)\right] + U_i - \frac{S}{2} f(p_i^{\diamond})[p_i^{\diamond} - c] = 0$$

where $p_i^{\diamond}$ represents the value of p_i maximizing π_i in the interior of interval $[c;m]$. This condition can be rewritten as

$$F(p_i^{\diamond})+f(p_i^{\diamond})(p_i^{\diamond}-c)=1+2u_i+F(p_j) \tag{2}$$

Solving equation (2) $p_i^{\diamond}$ will give the reaction function of firm i as a function of the price charged by firm j (p_j). The solution in implicit form is impossible, however, we can characterize the reaction of a firm to its competitor's actions.

Remark 3: In most of the cases, the higher price from the competitor implies the higher price for the firm (as long as the price has not hit the boundary at m). However, if $f'(p_i^{\diamond}) < -2\frac{f(p_i^{\diamond})}{p_i^{\diamond}-c}$, the higher price from the competitor calls for the decrease of the price of firm i.

For deriving the response of $p_i^{\diamond}$ to changes in p_j in the interior of the interval $[c;m]$, we can differentiate equation (2) with respect to p_j. This results into

$$\frac{\partial p_i^{\diamond}}{\partial p_j}=\frac{f(p_j)}{2f(p_i^{\diamond})+f'(p_i^{\diamond})(p_i^{\diamond}-c)} \tag{3}$$

Given that $f(\cdot)$ is a probability density function, it is never negative. The only way for the differential to be negative is for the denominator to be negative.

This is an intriguing result. It implies that in certain situations firm behavior will starkly contrast undercutting. Instead of increasing the price in response to price hike by the competitor, it is optimal for the firm to lower its price. In order to understand the intuition behind this result notice that there are two mechanisms at work for increasing firm's profits. Increasing the price helps profits as it extracts more revenue from captives as well as well as from shoppers who entered the competitor's shop first and decided not to buy there. Lowering the price, on the other hand, allows the firm to capture more shoppers who entered its shop first. If the second effect dominates the first, it is optimal for the firm to lower the prize retaining larger number of shoppers.

Remark 3 also gives us the hint of when such situation might occur. For this to happen $f'(p_i^{\diamond})$ should take a large negative value. This happens on a steeply downward sloping portions of the probability density function. This is intuitive, as in these areas a small price decrease results into a substantial increase in captured shoppers. For other situations, the reaction function implies more conventional response to competitor's price changes.

At this point, we have to point out that equation (1) and the analysis of this section so far is only a partial view. It takes into account only the possibility of continuous price adjustments within the boundaries of the interval $[c;m]$. However, as already seen in this paper, in this model the optimal response might be a discrete jump to the monopolistic price even in the interior of the interval. This would also imply a discrete jump from $\pi_i^\diamond$ to

$$\pi_i^m = \left(U_i + \frac{S}{2}F(p_j)\right)(m-c) \tag{4}$$

The jump in reaction function will occur when $\pi_i^m > \pi_i^\diamond$. Which implies the condition:

$$\left(1-F(p_i^\diamond)\right)(p_i^\diamond - c) < \left(2u_i + F(p_j)\right)(m - p_i^\diamond) \tag{5}$$

If the inequality (5) is satisfied, the firm i is better off charging m.

This possible discontinuity complicates more detailed analysis of the model. Model's behavior depends on the form of $f(\cdot)$, which makes it impossible to characterize equilibria of the game for a general functional form. However, significant insight can be drawn even from the specific distribution of $f(\cdot)$. In order to obtain analytic results, in what follows, we assume that thresholds are distributed uniformly along some interval $[a;b]$. This allows for the analytical solution for the reaction function and for the discussion of the economic effects of consumer heterogeneity with respect to risk attitudes. Even in such a restrictive case the model delivers important insight into the implication of the general setup.

We concentrate on the effects risk-taking behavior on equilibrium prices. In particular, we concentrate on the effects of the average threshold across consumers which represents a measure of the average risk-lovingness in the society. Recall that the lower $\hat{p}_s$ implies that consumer will turn down lower prices in the first shop taking on the gamble on prices she will see in the second shop. The mean of our distribution is a monotonically increasing function of $(a+b)$. We also examine how the heterogeneity in risk attitudes across population affects equilibrium prices. The heterogeneity can be measured by the variance of our distribution, which is a monotonically increasing function of $(b-a)$.

In case of the uniform distribution, $\hat{p}_s \sim U(a,b)$, the cumulative distribution function is $F(\hat{p}_s) = \frac{\hat{p}_s - a}{b-a}$. Notice that in this particular case, there is no circumstance when the condition in remark 3 is satisfied (as $f'(\cdot)=0$). As a consequence, we never see the perverse reaction to competitor's price changes in the interior of the interval.

Proposition 6: If $\hat{p}_s \sim U(a,b)$, $\{p_1^*, p_2^*\} \in ([a;b] \cap [c;m]) \cup \{m\}$

This effectively mean that prices are confined to the intersection between the $[a;b]$ and $[c;m]$ intervals, and can also take the value of m even if it is not included in the intersection. The proof of the proposition is as follows.

Proof: First, note that prices will never go neither above m (as it implies no sales and thus zero revenues) nor below c (as it implies negative profits). Therefore, if $a < c$ prices will never be charged in the interval $[a;c)$. At the other end of the interval, if $b > m$, prices still have to bounded from above by m. On the other hand, if $b < m$, it does not make any sense to charge prices in interval $(b;m)$. To see this consider producer's incentives for charging prices less then m. This is for making some of the shared first entrants buy in his shop. However, if the price he charges is above b his aim is not reached, as everybody's threshold is still lower than the price. Therefore, it is strictly better for the producer to charge m instead and extract higher revenues from shared second entrants and captives. ■

The proposition above has one degenerate case.

Corollary 1: If $\hat{p}_s \sim U(a,b)$ *and* $[a;b] \cap [c;m] \in \emptyset$, $p_1^* = p_2^* = m$

In such a degenerate case, it is obvious that changes neither in the average risk-taking behavior nor in heterogeneity will affect equilibrium prices in this situation.[4] This is, clearly, not a very interesting scenario to discuss.

A more interesting case is the instance where $[c;m] \cap [a;b]$ is not empty. In principle there are two different constellations for each end of the interval. At the lower end, we can have $a > c$ or $a \leq c$. At the upper end, we can have $b < m$ or $b \geq m$. The condition at the lower end of the intersection of the interval does not affect the subsequent analysis. The condition at the top end, does introduce slight differences in terms of reaction functions. However, all the qualitative results are similar between the $b < m$ and $b \geq m$ cases. For the slight advantage in tractability and considerable advantage in terms of economic meaningfulness, in what follows we concentrate on the case of $b < m$. The case $b \geq m$ implies that there are consumers who have so high thresholds that they would buy the product in the first shop they enter even if the price was high than their budget. Which, clearly does not make much economic sense.

[4] Here, an in subsequent discussions, we abstract from the possibility that altering mean and/or variance of $\hat{p}_s$ distribution will alter the conditions for the existence of a given equilibrium. This alteration might imply that the system will move from one equilibrium to another, which might also imply changes in prices.

Based on condition (2) we can calculate the reaction function of firm i. This will have two difference instances. When the opponent is charging the price in the interior of the intersection between $[c;m]$ and $[a;b]$. In this case

$$p_i^{\diamond}|_{p_j<m} = \frac{(1+2u_i)(b-a)+c+p_j}{2} \tag{6}$$

The other instance is then the opponent is charging m. In this case, the optimal price is

$$p_i^{\diamond}|_{p_j=m} = \frac{(2+2u_i)(b-a)+c+a}{2}.^{5} \tag{7}$$

Notice that in equilibrium we will never have an instance when firm 2 is charging monopolistic price, while firm 1 reacts with (7). This is similar to the situation with homogeneous consumers that we discussed in previous section, and is due to our assumption $U_1 > U_2$. Because of this assumption, firm 1 will always have more incentive to charge the monopolistic price. If in any situation firm 2 charges m, firm 1's only optimal response will always be to also charge m. However, we can have an equilibrium setup where firm 1 is charging m, while firm 2 is charging the price given by the equation (7).

Similar to the case with homogeneous consumers, we will have three different types of equilibria: (1) when both firms charge m; (2) when both firms charge prices in the interior of the interval; (3) when firm 1 charges m, while firm 2 charges the price in the interior of $(a;b) \cap (c;m)$.

Proposition 7: If the parameters of the model are such that $\left(\frac{1}{2}+u_2\right)(m-c) \geq \left(\frac{1}{2}+\frac{1}{2}u_2+\frac{1}{4}\frac{a-c}{b-a}\right)\left((1+u_2)(b-a)+\frac{a-c}{2}\right)$, then $p_1^*=p_2^*=m$ is an equilibrium of the game.

Proof: For $p_1^*=p_2^*=m$ to be an equilibrium it is sufficient to ensure that firm 2 not to have incentive to jump to charging the price in the interior. When both firm charge m, firm 2 profits are $\left(\frac{S}{2}+U_2\right)(m-c)$. On the other hand, profits in case of the jump can be calculated using the reaction function (7). In this case, firm 2's profits would be $\left(\frac{S}{2}\left(1-u_2+\frac{1}{2}\frac{a-c}{b-a}\right)+U_2\right)\left((1+u_2)(b-a)+\frac{a-c}{2}\right)$. Setting the latter to be no greater than the former results in the condition given in the proposition. ■

[5] Notice that in case if $b \geq m$, $p_i^{\diamond}|_{p_j=m} = \frac{(1+2u_i)(b-a)+c+m}{2}$.

This type of equilibrium is similar to the degenerate case we have discussed above. Due to the fact that both firms fix prices to the monopolistic level and therefore concentrating in extracting maximum revenues from captives and second entrant shared consumers, we see no price changes neither due to changing mean, not variance of the threshold distributions.

Proposition 8: *If* $\frac{1}{3}\frac{b-a}{m-c} \geq \frac{3+10u_1+8u_2+3c/(b-a)}{(3+4u_1+2u_2)^2}$, then $p_i^* = c + (b-a)\left(1+\frac{4u_i+2u_j}{3}\right)$ for $i,j = \{1,2\}$ and $i \neq j$ is the equilibrium.

Proof: For both firms to charge the price in the interior it is sufficient to ensure that the firm 1 does not have an incentive to jump to charging the monopolistic price m. In the proposed equilibrium setup firm 1's profits are given by $\frac{S(b-a)}{18}(3+4u_1+2u_2)^2$, while if it decided to charge m, it would earn $\frac{S(m-c)}{6}\left(3+10u_1+8u_2+\frac{3c}{b-a}\right)$. Setting the latter to be no greater than the former results in the condition given in the proposition. ■

Type (ii) equilibrium discussed in the proposition above is more interesting than type (i). Prices in this case do depend on the actual distribution of thresholds. To derive implications of the distributional changes note that changes in mean and variance are intricately related in case of uniform distribution. We have to keep this in mind as when we want to examine the effects of the change in the mean of the distribution, we have to make sure that the variance of the distribution stays unchanged. And on the contrary, when we want to examine the changes in brought by the changes in variance, we have to make sure that the mean of the distribution stays constant. Hence, the changes in the mean can be understood in changing parameter b (alternatively a), but these changes have to be counter-balanced by similar changes in parameter a (alternatively b) so that b—a stays constant. On the other hand, the change in the variance can be also understood by changing b (alternatively a), while imposing mirroring changes in a (alternatively b), in order to keep $b + a$ constant.

Remark 4: In case of type (ii) equilibrium, changing the average value of the price-acceptance threshold does not have an effect on prices. However, increasing the heterogeneity in threshold distribution drives the prices up.

This is easy to see if we simply look at the equilibrium prices. These prices depend on $b-a$. Therefore, increasing the mean with constant variance will not change this price. Which increasing variance would directly imply increasing $b-a$. It is clear that $\frac{\partial p_i^*}{\partial(b-a)} > 0$, therefore, higher variance implies, higher equilibrium prices. In order to understand the intuition behind these results recall the two mechanisms for profit generation in the no return option setup. One is to exploit captives and second entrants by higher price, the other is to increase the consumer base using the lower price. Thus, there are incentives to decrease, as well as incentives to increase the price in response to the price change by the competitor.

In light of this discussion, consider the type (ii) equilibrium. When prices are low, the mass of the $\hat{p}_s$ distribution to the left of p_2 is small. The reason for setting the price low is to retain as many first entrants as possible (this is measured by $F(p_1)$). The reason for setting the price high is to exploit the second entrants that have rejected the offer from firm 2 (measured by $1-F(p_2)$). Consider how the magnitude of these incentives changes with increasing $b-a$. Higher σ implies that more mass of the distribution is concentrated to the left of the p_2 (and potentially to the left of p_1).[6] This gives an extra incentive to increase the price and take advantage of the (marginally lower number of) first entrants and the large number of second entrants that have rejected the offer from firm 2. This is the reason why the larger consumer heterogeneity implies higher prices at low price equilibrium.

Proposition 9: *If* $\left((1+u_2)(b-a)+\frac{a-c}{2}\right)^2 \geq (1+u_2)(m-c)(b-a)$ and $\left(1+u_2+2u_1+\frac{a-c}{b-a}\right)(m-c) \geq \left(1+u_1+u_2+\frac{5}{2}\frac{c-a}{b-a}\right)\left(\left(1+u_1+\frac{1}{2}u_2\right)(b-a)-\frac{c-a}{4}\right)$, then $p_1^*=m$ and $p_2^*=(1+u_2)(b-a)+\frac{c+a}{2}$ is the equilibrium of the game.

Proof: For this setup to be an equilibrium we need firm 1 not to have an incentive to jump to the interior, and firm 2 not to have an incentive to jump to m. In the equilibrium setup the profits are

$$\pi_1=\left(\frac{S}{2}\left(1+u_2+\frac{1}{2}\frac{c-a}{b-a}\right)+U_1\right)(m-c)$$

and

$$\pi_2=\left(\frac{S}{2}\left(1+u_2-\frac{1}{2}\frac{c-a}{b-a}\right)+U_2\right)\left((1+u_2)(b-a)-\frac{c-a}{2}\right).$$

However, in case of jumps respective profits will be

$$\pi_1=\left(\frac{S}{2}\left(1-u_1+\frac{1}{2}u_2+\frac{5}{4}\frac{c-a}{b-a}\right)+U_1\right)\left(\left(1+u_1+\frac{1}{2}u_2\right)(b-a)-\frac{c-a}{4}\right)$$

and

$$\pi_2=\left(\frac{S}{2}+U_2\right)(m-c)$$

[6] Note that low price equilibrium will usually be lower than the mean of the distribution.

Setting the latter values to be no greater than corresponding former values results in conditions in the proposition. ■

In type (iii) equilibrium only firm 2's price can change as firm 1 charges the monopolistic price.

Remark 5: The equilibrium price of firm 2 increases with the increase of both, the average value of the price acceptance threshold and the degree of heterogeneity in the threshold distribution.

In order to demonstrate this remark let's take the mean of the distribution first. The distribution mean is linearly related to a that appears in the equilibrium price equation. However, we have to ensure that $b-a$ is constant. Therefore, the sign of the $\frac{\partial p_2^*}{\partial a}|_{(b-a)=\text{const}}$ will give us the indication to what happens to prices in equilibrium with the increase of the average threshold. $\frac{\partial p_2^*}{\partial a}|_{(b-a)=\text{const}}=\frac{1}{2}>0$, which means that prices in equilibrium increase with the mean threshold value.

In order to understand the intuition behind this result, notice that this happens as the first firm charges the monopolistic price. So, effectively only firm 2's prices increase in equilibrium. As consumers start accepting higher and higher prices at the first shop they enter, firm 2 has extra incentive to increase the price and extract higher returns for the consumer base.

Now turn to the effect of the heterogeneity in risk attitudes. In order to increase variance in threshold distribution while keeping the mean of the distribution constant we have to increase b with a small value (ϵ) and decrease the value of a with by the same value at the same time. Thus, we can compare the equilibrium price when when thresholds are distributed uniformly on the interval $[a;b]$, to an equilibrium price when thresholds are distributed on the interval $[a-\epsilon;b+\epsilon]$. Plugging the latter interval boundaries in the equilibrium price given by the proposition 9, and applying some algebra we obtain that the $p_2{}'{*}=p_2^*+\frac{3+4u_2}{2}\epsilon$. As $\frac{3+4u_2}{2}\epsilon>0$, we conclude that with increasing consumer heterogeneity the prices in equilibrium increase. This is a similar result as the one discussed in remark 4 and occurs essentially for the same reason we outlined above.

Even though the analysis with uniform distribution is restrictive, it has an important advantage (besides allowing the derivation of closed-form solutions to reaction functions). It gives us an opportunity to discuss the implications of the model with respect to the discontinuity in the best response function in a general setting. Consider the condition (5), together with the implicit condition that $p_j\in[c;m]$. In order to identify the point where reaction function has a discrete jump, we can calculate the difference

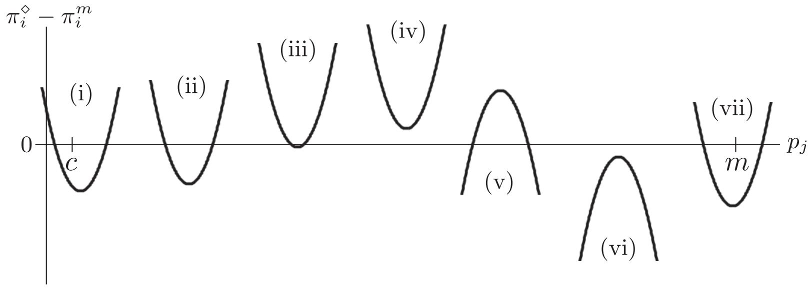

Figure 2. Few of the possible cases of for the $\pi_i^{\diamond}-\pi_i^m$ condition.

$\pi_i^{\diamond}-\pi_i^m$. Then this value is negative, the producer is better of charging the monopolistic price.

Using our specific case of the uniform distribution, we can calculate the value of $\pi_i^{\diamond}-\pi_i^m$. Using the reaction function (6),

$$\pi_i^{\diamond}-\pi_i^m = S\frac{\left((1+u_i)(b-a)+p_j-c\right)\left(p_j-c-u_i(b-a)\right)}{4(b-a)} - \left(\frac{S}{2}+U_i\right)(m-c) \quad (8)$$

Notice that this equation is quadratic in competitor's price. Therefore, the requirement for the optimal price jump represents the parabola in the case of the uniform distribution. Naturally, for any arbitrary shape of cutoff distribution, the requirement on p_j for the break in reaction function will be an inequality on a polynomial (possibly of higher order than 2) of p_j. There are several qualitatively distinct different setups in which the $\pi_i^{\diamond}-\pi_i^m < 0$ condition will be satisfied. This will cause a discontinuity in firm's reaction function.

It is interesting to discuss intuitions behind these discontinuities. We can do this by sticking with the shape of parabola in (not to complicate the interpretations even further) and considering the condition playing out in different ways. Figure 2 presents the illustration of seven (out of many) cases the condition could result in a general setup. We plot the value of $\pi_i^{\diamond}$ $-\pi_i^m$ as a function of the competitor's price (p_j). If this value is negative, it is optimal for the firm i to charge the monopolistic price.

For the start in the case (iii) parabola lies weekly above zero and thus the equilibria are perfectly described by π_i functions. The same is true in the case (iv). The case (vi) on the other hand implies that the firm i should charge m no matter the price of the competitor. In this case the firm does not engage in any kind of price undercutting behavior.

The cases (i) and (vii) are more interesting. In the former instance, the firm charges the monopolistic price if the competitor's price is too low. The latter, the firm charges the monopolistic price if the competitor's price is too high. The reason for why the firm might want to charge the monopolistic price is as follows. If the competitor charges a low price, it retains large number of shoppers that enter its shop first. In this situation, the continuous reaction function would normally dictate to also charge a low price in order to maximize the number of retained shoppers. However, if the number of captives is large enough, the firm might prefer charging m and extracting the monopolistic profit from captives (and however few shoppers who escaped the competitor). The reason why the firm might want to charge the monopolistic price in response to the high price charged by the competitor is more obvious. The higher the p_j, the more shoppers refuse to buy from the competitor, increasing the temptation of the firm to jump to m and charge those shoppers (as well as its captives) the highest price possible.

The cases (ii) and (v) demonstrate that intuitions put forward in the previous paragraph act on relative, rather than absolute scale. In case (v), the firm charges m in response to high and low values of the competitor's price, while charging $p_i^{\diamond} < m$ for the intermediate values of p_j. The case (ii) is the opposite of the case (v), here the monopolistic price is charged for in response to the intermediate values of the competitor's price. The difference between the two cases can be explained by the relative number of captives (u_i). Consider two firms. The first with the low share of captives, the second with the high share of captives. If the competitor charges too low price (thus retains many shoppers who enter his shop first), the first firm will have an incentive to charge m in order to extract profit from captives. For the second firm, however, this might not be optimal. It might be forced to charge the low price aiming at retaining more shoppers. As the competitor increases the price, the first firm might see the value from competing as now more and more shoppers pass by the competitor's shop. Thus, putting effort on retaining some of the shoppers that enter its shop first might pay off. In the similar situation, the second firm might have an incentive to monetize the shoppers that refused to buy from the competitor together with its captives. As competitor's price hikes even higher, reposes from the two firms could very well be contrasting again.

Now recall that in the general case the function $\pi_i^{\diamond} - \pi_i^m$ need not be a parabola. If it is a polynomial of higher order, it might cross from positive to negative area and back few times on interval $[c; m]$. This will depend on the distribution of $\hat{p}_s$, as well as parameter values, and notably the share of captives for the firm. Implied jumps to the monopolistic price and back can be explained by the intuitions outlined above.

5. CONCLUSION

In this paper, we have extended the classical duopolistic price competition setup to include captive consumers. This clearly adds to the realism of the model as not every consumer in the economy will have a chance to choose between competing suppliers. This starkly changes the equilibrium outcomes of the model. Addition of captives presents the firms with the opportunity to extract monopolistic profit. This dis-equilibrates the classical outcome of marginal cost pricing and makes the game discontinuous. In case of pricing at marginal cost firm extracts zero profits. As long as the firm has at least one captive, it is strictly better off to charging the monopolistic price and giving up competing for shoppers. As a result, the likely price dynamics in the economy is the one where starting from the monopolistic price the firms undercut each-other driving price gradually down, until one of them jumps back to the monopolistic price. The competitor follows and the loop starts over. However, we have also shown that if one of the firms has large share of captives he might not be motivated to engage in competition for shoppers at all. In this case equilibrium emerges. This equilibrium is very different from the equilibrium implied by the model without captives. In our case, the firm with the most captives charges the monopolistic price m, while the competitor charges just under m.

Next, we have altered the behavior of shoppers and modeled them as performing the price search in an environment where they cannot go back to the shop they have already visited to make the purchase. This changes many things in the model significantly. Effectively this forces shoppers to make the decision on where to buy the product under incomplete information. If the shopper decides not to buy in the first shop she enters, she is taking a gamble on the competitor's price. Consumer's risk-attitude comes into play. The distribution summarizing shopper's risk attitudes becomes the central technical hurdle. Due to the discontinuity of the game, the equilibrium analysis for an arbitrary distribution becomes impossible.

Instead we have proceeded with the analysis of the case when the distribution is uniform. This setup allows for the analytic solution and yields surprisingly deep insight into the general conclusions of the model. In this environment firms need not necessarily respond monotonically to the competitor's behavior. In this setup, firms have the incentive to increase the price in order to capture higher profits from captives, but now they also have an incentive to decrease the price in order to persuade higher number of shoppers to make the purchase from their shop. From the economic standpoint, it is the interplay between these two incentives makes the game discontinuous.

We have also looked at the implications of the heterogeneity in consumer risk attitudes on equilibrium prices in duopolistic setup. We have found, somewhat surprisingly, that the higher variance in consumer risk attitudes implies higher prices in equilibrium in most of the cases. This points to the fact that few of the risk-taking consumers might push the duopolistic price competition into the equilibrium where all consumers, captives and shoppers alike, are exploited by the sellers.

APPENDIX

OPTIMALITY OF THE THRESHOLD APPROACH

We demonstrate the optimality of the threshold approach by assuming the specific shape of the utility function which we have introduced in footnote 1. However, intuitions hold for general form of utility function that satisfies requirements outlined in the text. The only necessary condition is that the utility function has to be continuous in price of the product.

When there is no return option consumer solves his maximization problem in the first shop he enters. In this situation, he knows p_1, but does not know p_2. He has two options: either to buy from this shop and have utility $V_{s,1}=v+m-p_1$, or to pass on the opportunity and have utility $V_{s,2}=v+m-p_2$. In this situation, he is taking a gamble. Denote the conditional probability that the second shop charges higher price by $P=Prob(p_2 \leq p_1|p_1)$. This implies that $Prob(V_{s,2} \geq V_{s,1}|p_1)=1-P$. As competitors know level of m, prices for both of the firms are $p_i \in [c,m]$. Then as set of potential prices in the second shop is compact, it must be true that $\partial P/\partial p_1 > 0 \; \forall p_1$.

Now we introduce two more parameters. Denote the expectation of the price the consumer s would have if she were told that the second shop was charging the lower price by a_s, and the expectation of the price she would have if she was told the other shop was charging higher price by b_s. By assumption $a_s < p_1 < b_s$. When she does not have additional signal whether the other shop is charging higher or lower price, the utility she expects from passing on the option to buy the product in the first shop can be written as

$$E(V_{s,2}|p_1)=P(v+m-a_s)+(1-P)(v+m-b_s) \tag{9}$$

which can be rewritten as

$$E(V_{s,2}|p_1)=v+m-[Pa_s+(1-P)b_s] \tag{10}$$

Equation (10) and property $\partial P/\partial p_1 > 0$ together imply that $E(V_{s,2}|p_1)/\partial p_1 > 0$.

Then as p_1 goes up, the utility of immediate purchase goes down, while expected utility of waiting goes up. As out functions are continuous (in p_1), fixed point theorem implies that there must exist a threshold $\hat{p}_1 = [Pa_s + (1-P)b_s]$. When p_1 goes above this threshold it is optimal for the consumer to take a risk of buying the product in the second shop, while as long as $p_1 < \hat{p}_1$ it is optimal to jump at the opportunity in the first shop. Now, level of the threshold is consumer specific. It is determined by the probability function each consumer uses to derive the expectations about whether the price will be above or lower than the price in the first shop and by the conditional probabilities each consumer uses to derive expectations about the actual price she thinks she will see in the shop.

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